

## t59\_lattice2

(TMEjLWHzZZx9DxziPcHPNy1CxSfeKk2XEJg)

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Let  $v1\_xboole\_0 : \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k5\_finsub\_1 : \iota \Rightarrow \iota$  be given. Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v10\_lattices : \iota \Rightarrow o$  be given. Let  $v14\_lattices : \iota \Rightarrow o$  be given. Let  $l3\_lattices : \iota \Rightarrow o$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_funct\_2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $k1\_zfmisc\_1 : \iota \Rightarrow \iota$  be given. Let  $k2\_zfmisc\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k3\_funct\_2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k3\_lattice2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v13\_lattices : \iota \Rightarrow o$  be given. Let  $k2\_lattice2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_lattice2 : \iota \Rightarrow \iota$  be given. Let  $k1\_funct\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $g3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v3\_lattices : \iota \Rightarrow o$  be given. Let  $u1\_lattices : \iota \Rightarrow \iota$  be given. Let  $v3\_binop\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u2\_lattices : \iota \Rightarrow \iota$  be given. Let  $l2\_lattices : \iota \Rightarrow o$  be given. Let  $l1\_lattices : \iota \Rightarrow o$  be given. Let  $k7\_setwiseo : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned}
 & \forall X0. (\neg v1\_xboole\_0 X0) \Rightarrow (\forall X1. (m1\_subset\_1 X1 (k5\_finsub\_1 \\
 & X0)) \Rightarrow (\forall X2. ((\neg v2\_struct\_0 X2) \wedge ((v10\_lattices X2) \wedge ((v13\_lattices \\
 & X2) \wedge (l3\_lattices X2)))) \Rightarrow (\forall X3. ((v1\_funct\_1 X3) \wedge ((v1\_funct\_2 \\
 & X3 X0 (u1\_struct\_0 X2)) \wedge (m1\_subset\_1 X3 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\
 & X0 (u1\_struct\_0 X2)))))) \Rightarrow (\forall X4. (m1\_subset\_1 X4 (u1\_struct\_0 \\
 & X2)) \Rightarrow ((\forall X5. (m1\_subset\_1 X5 X0) \Rightarrow ((X5 \in X1) \Rightarrow (r3\_lattices \\
 & X2 (k3\_funct\_2 X0 (u1\_struct\_0 X2) X3 X5) X4)) \Rightarrow (r3\_lattices X2 \\
 & (k2\_lattice2 X0 X2 X1 X3) X4))))))
 \end{aligned} \tag{1}$$

Assume the following.

$$\forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices X0))) \Rightarrow ((v14\_lattices X0) \Leftrightarrow (v13\_lattices (k1\_lattice2 X0))) \tag{2}$$

Assume the following.

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\ & X0))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. \\ & (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (\forall X3.(m1\_subset\_1 X3 \\ & (u1\_struct\_0 (k1\_lattice2 X0)) \Rightarrow (\forall X4.(m1\_subset\_1 X4 \\ & (u1\_struct\_0 (k1\_lattice2 X0)) \Rightarrow (((r3\_lattices (k1\_lattice2 \\ & X0) X3 X4) \wedge ((X1 = X3) \wedge (X2 = X4)) \Rightarrow (r3\_lattices X0 X2 X1))))))) \end{aligned} \quad (3)$$

Assume the following.

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\ & X0))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. \\ & (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow ((r3\_lattices X0 X1 X2) \Rightarrow (\forall X3. \\ & (m1\_subset\_1 X3 (u1\_struct\_0 (k1\_lattice2 X0)) \Rightarrow (\forall X4. \\ & (m1\_subset\_1 X4 (u1\_struct\_0 (k1\_lattice2 X0)) \Rightarrow (((X1 = X3) \wedge \\ & X2 = X4) \Rightarrow (r3\_lattices (k1\_lattice2 X0) X4 X3))))))) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. \forall X3. ((\neg v1\_xboole\_0 X0) \wedge \\ & (((v1\_funct\_1 X2) \wedge ((v1\_funct\_2 X2 X0 X1) \wedge (m1\_subset\_1 X2 (k1\_zfmisc\_1 \\ & (k2\_zfmisc\_1 X0 X1)))))) \wedge (m1\_subset\_1 X3 X0))) \Rightarrow (k3\_funct\_2 X0 \\ & X1 X2 X3 = k1\_funct\_1 X2 X3) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1\_funct\_1 X1) \wedge ((v1\_funct\_2 \\ & X1 (k2\_zfmisc\_1 X0 X0) X0) \wedge (m1\_subset\_1 X1 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\ & (k2\_zfmisc\_1 X0 X0) X0)))))) \wedge ((v1\_funct\_1 X2) \wedge ((v1\_funct\_2 X2 \\ & (k2\_zfmisc\_1 X0 X0) X0) \wedge (m1\_subset\_1 X2 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\ & (k2\_zfmisc\_1 X0 X0) X0)))))) \Rightarrow (\forall X3. \forall X4. \forall X5. \\ & (g3\_lattices X0 X1 X2 = g3\_lattices X3 X4 X5) \Rightarrow ((X0 = X3) \wedge ((X1 = X4) \wedge \\ & (X2 = X5)))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\ & X0))) \Rightarrow ((v3\_lattices (k1\_lattice2 X0)) \wedge (v10\_lattices (k1\_lattice2 \\ & X0))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\ & X0))) \Rightarrow ((v1\_funct\_1 (u1\_lattices X0)) \wedge ((v1\_funct\_2 (u1\_lattices \\ & X0) (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)) (u1\_struct\_0 \\ & X0)) \wedge (v3\_binop\_1 (u1\_lattices X0) (u1\_struct\_0 X0)))) \end{aligned} \quad (8)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices X0))) \Rightarrow ((v1\_funct\_1 (u2\_lattices X0)) \wedge ((v1\_funct\_2 (u2\_lattices X0) (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)) (u1\_struct\_0 X0)) \wedge (v3\_binop\_1 (u2\_lattices X0) (u1\_struct\_0 X0)))) \quad (9)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0) \wedge (l3\_lattices X0)) \Rightarrow ((\neg v2\_struct\_0 (k1\_lattice2 X0)) \wedge (v3\_lattices (k1\_lattice2 X0))) \quad (10)$$

Assume the following.

$$\forall X0.(l2\_lattices X0) \Rightarrow ((v1\_funct\_1 (u2\_lattices X0)) \wedge ((v1\_funct\_2 (u2\_lattices X0) (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)) (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 (u2\_lattices X0) (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)) (u1\_struct\_0 X0)))))) \quad (11)$$

Assume the following.

$$\forall X0.(l1\_lattices X0) \Rightarrow ((v1\_funct\_1 (u1\_lattices X0)) \wedge ((v1\_funct\_2 (u1\_lattices X0) (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)) (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 (u1\_lattices X0) (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)) (u1\_struct\_0 X0)))))) \quad (12)$$

Assume the following.

$$\forall X0.(l3\_lattices X0) \Rightarrow ((l1\_lattices X0) \wedge (l2\_lattices X0)) \quad (13)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.\forall X3.((\neg v1\_xboole\_0 X0) \wedge (((\neg v2\_struct\_0 X1) \wedge ((v10\_lattices X1) \wedge (l3\_lattices X1))) \wedge ((m1\_subset\_1 X2 (k5\_finsub\_1 X0)) \wedge ((v1\_funct\_1 X3) \wedge ((v1\_funct\_2 X3 X0 (u1\_struct\_0 X1)) \wedge (m1\_subset\_1 X3 (k1\_zfmisc\_1 (k2\_zfmisc\_1 X0 (u1\_struct\_0 X1)))))))))) \Rightarrow (m1\_subset\_1 (k3\_lattice2 X0 X1 X2 X3) (u1\_struct\_0 X1)) \quad (14)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.\forall X3.((\neg v1\_xboole\_0 X0) \wedge (((v1\_funct\_1 X2) \wedge ((v1\_funct\_2 X2 X0 X1) \wedge (m1\_subset\_1 X2 (k1\_zfmisc\_1 (k2\_zfmisc\_1 X0 X1)))))) \wedge (m1\_subset\_1 X3 X0)) \Rightarrow (m1\_subset\_1 (k3\_funct\_2 X0 X1 X2 X3) X1) \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1\_funct\_1 X1) \wedge ((v1\_funct\_2 \\ & X1 (k2\_zfmisc\_1 X0 X0) X0) \wedge (m1\_subset\_1 X1 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\ & (k2\_zfmisc\_1 X0 X0) X0)))) \wedge ((v1\_funct\_1 X2) \wedge ((v1\_funct\_2 X2 \\ & (k2\_zfmisc\_1 X0 X0) X0) \wedge (m1\_subset\_1 X2 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\ & (k2\_zfmisc\_1 X0 X0) X0)))))) \Rightarrow ((v3\_lattices (g3\_lattices X0 X1 \\ & X2)) \wedge (l3\_lattices (g3\_lattices X0 X1 X2))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall X0. (\neg v1\_xboole\_0 X0) \Rightarrow (\forall X1. ((\neg v2\_struct\_0 X1) \wedge \\ & ((v10\_lattices X1) \wedge (l3\_lattices X1))) \Rightarrow (\forall X2. (m1\_subset\_1 \\ & X2 (k5\_finsub\_1 X0)) \Rightarrow (\forall X3. ((v1\_funct\_1 X3) \wedge ((v1\_funct\_2 \\ & X3 X0 (u1\_struct\_0 X1)) \wedge (m1\_subset\_1 X3 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\ & X0 (u1\_struct\_0 X1)))))) \Rightarrow (k3\_lattice2 X0 X1 X2 X3 = k7\_setwiseo \\ & X0 (u1\_struct\_0 X1) (u1\_lattices X1) X2 X3))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall X0. (\neg v1\_xboole\_0 X0) \Rightarrow (\forall X1. ((\neg v2\_struct\_0 X1) \wedge \\ & ((v10\_lattices X1) \wedge (l3\_lattices X1))) \Rightarrow (\forall X2. (m1\_subset\_1 \\ & X2 (k5\_finsub\_1 X0)) \Rightarrow (\forall X3. ((v1\_funct\_1 X3) \wedge ((v1\_funct\_2 \\ & X3 X0 (u1\_struct\_0 X1)) \wedge (m1\_subset\_1 X3 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\ & X0 (u1\_struct\_0 X1)))))) \Rightarrow (k2\_lattice2 X0 X1 X2 X3 = k7\_setwiseo \\ & X0 (u1\_struct\_0 X1) (u2\_lattices X1) X2 X3))) \end{aligned} \quad (18)$$

Assume the following.

$$\forall X0. (l3\_lattices X0) \Rightarrow (k1\_lattice2 X0 = g3\_lattices (u1\_struct\_0 X0) (u1\_lattices X0) (u2\_lattices X0)) \quad (19)$$

Assume the following.

$$\forall X0. (l3\_lattices X0) \Rightarrow ((v3\_lattices X0) \Rightarrow (X0 = g3\_lattices (u1\_struct\_0 X0) (u2\_lattices X0) (u1\_lattices X0))) \quad (20)$$

### Theorem 1

$$\begin{aligned} & \forall X0. (\neg v1\_xboole\_0 X0) \Rightarrow (\forall X1. (m1\_subset\_1 X1 (k5\_finsub\_1 \\ & X0)) \Rightarrow (\forall X2. ((\neg v2\_struct\_0 X2) \wedge ((v10\_lattices X2) \wedge ((v14\_lattices \\ & X2) \wedge (l3\_lattices X2)))) \Rightarrow (\forall X3. ((v1\_funct\_1 X3) \wedge ((v1\_funct\_2 \\ & X3 X0 (u1\_struct\_0 X2)) \wedge (m1\_subset\_1 X3 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\ & X0 (u1\_struct\_0 X2)))))) \Rightarrow (\forall X4. (m1\_subset\_1 X4 (u1\_struct\_0 \\ & X2)) \Rightarrow ((\forall X5. (m1\_subset\_1 X5 X0) \Rightarrow ((X5 \in X1) \Rightarrow (r3\_lattices \\ & X2 X4 (k3\_funct\_2 X0 (u1\_struct\_0 X2) X3 X5)))) \Rightarrow (r3\_lattices X2 \\ & X4 (k3\_lattice2 X0 X2 X1 X3)))))) \end{aligned}$$