

# t5\_normform (TMZUTP- kUy4yUJRnMyBbY71K6U9WLuNTk1R4)

October 27, 2020

Let  $v1\_xboole\_0 : \iota \Rightarrow o$  be given. Let  $v4\_finsub\_1 : \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k2\_zfmisc\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_normform : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k3\_xboole\_0 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_xboole\_0 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k3\_finsub\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k3\_domain\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_xtuple\_0 : \iota \Rightarrow \iota$  be given. Let  $k2\_domain\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_xtuple\_0 : \iota \Rightarrow \iota$  be given. Let  $k1\_finsub\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_domain\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k4\_tarski : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall X0.\forall X1.\forall X2.k3\_xboole\_0 X0 (k2\_xboole\_0 X1 X2) = k2\_xboole\_0 (k3\_xboole\_0 X0 X1) (k3\_xboole\_0 X0 X2) \quad (1)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v1\_xboole\_0 X0) \wedge (v4\_finsub\_1 X0)) \wedge ((m1\_subset\_1 X1 X0) \wedge (m1\_subset\_1 X2 X0))) \Rightarrow (k3\_finsub\_1 X0 X1 X2 = k3\_xboole\_0 X1 X2) \quad (2)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v1\_xboole\_0 X0) \wedge ((\neg v1\_xboole\_0 X1) \wedge (m1\_subset\_1 X2 (k2\_zfmisc\_1 X0 X1)))) \Rightarrow (k3\_domain\_1 X0 X1 X2 = k2\_xtuple\_0 X2)) \quad (3)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v1\_xboole\_0 X0) \wedge ((\neg v1\_xboole\_0 X1) \wedge (m1\_subset\_1 X2 (k2\_zfmisc\_1 X0 X1)))) \Rightarrow (k2\_domain\_1 X0 X1 X2 = k1\_xtuple\_0 X2)) \quad (4)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v1\_xboole\_0 X0) \wedge (v4\_finsub\_1 X0)) \wedge ((m1\_subset\_1 X1 X0) \wedge (m1\_subset\_1 X2 X0))) \Rightarrow (k1\_finsub\_1 X0 X1 X2 = k2\_xboole\_0 X1 X2) \quad (5)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.((\neg v1\_xboole\_0 X0)\wedge \\ & ((\neg v1\_xboole\_0 X1)\wedge((m1\_subset\_1 X2 X0)\wedge(m1\_subset\_1 X3 X1))))\Rightarrow \\ & (k1\_domain\_1 X0 X1 X2 X3 = k4\_tarski X2 X3) \end{aligned} \quad (6)$$

Assume the following.

$$\forall X0.\forall X1.k2\_xtuple\_0 (k4\_tarski X0 X1) = X1 \quad (7)$$

Assume the following.

$$\forall X0.\forall X1.k1\_xtuple\_0 (k4\_tarski X0 X1) = X0 \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((\neg v1\_xboole\_0 X0)\wedge(v4\_finsub\_1 \\ & X0))\wedge((m1\_subset\_1 X1 X0)\wedge(m1\_subset\_1 X2 X0)))\Rightarrow(m1\_subset\_1 \\ & (k3\_finsub\_1 X0 X1 X2) X0) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.((\neg v1\_xboole\_0 X0)\wedge((\neg v1\_xboole\_0 \\ & X1)\wedge(m1\_subset\_1 X2 (k2\_zfmisc\_1 X0 X1))))\Rightarrow(m1\_subset\_1 (k3\_domain\_1 \\ & X0 X1 X2) X1) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.(((\neg v1\_xboole\_0 \\ & X0)\wedge(v4\_finsub\_1 X0))\wedge(((\neg v1\_xboole\_0 X1)\wedge(v4\_finsub\_1 X1))\wedge \\ & ((m1\_subset\_1 X2 (k2\_zfmisc\_1 X0 X1))\wedge(m1\_subset\_1 X3 (k2\_zfmisc\_1 \\ & X0 X1))))))\Rightarrow(m1\_subset\_1 (k2\_normform X0 X1 X2 X3) (k2\_zfmisc\_1 \\ & X0 X1)) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.((\neg v1\_xboole\_0 X0)\wedge((\neg v1\_xboole\_0 \\ & X1)\wedge(m1\_subset\_1 X2 (k2\_zfmisc\_1 X0 X1))))\Rightarrow(m1\_subset\_1 (k2\_domain\_1 \\ & X0 X1 X2) X0) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.(((\neg v1\_xboole\_0 \\ & X0)\wedge(v4\_finsub\_1 X0))\wedge(((\neg v1\_xboole\_0 X1)\wedge(v4\_finsub\_1 X1))\wedge \\ & ((m1\_subset\_1 X2 (k2\_zfmisc\_1 X0 X1))\wedge(m1\_subset\_1 X3 (k2\_zfmisc\_1 \\ & X0 X1))))))\Rightarrow(m1\_subset\_1 (k1\_normform X0 X1 X2 X3) (k2\_zfmisc\_1 \\ & X0 X1)) \end{aligned} \quad (13)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v1\_xboole\_0 X0)\wedge(v4\_finsub\_1 X0))\wedge((m1\_subset\_1 X1 X0)\wedge(m1\_subset\_1 X2 X0)))\Rightarrow(m1\_subset\_1 (k1\_finsub\_1 X0 X1 X2) X0) \quad (14)$$

Assume the following.

$$\begin{aligned} &\forall X0.((\neg v1\_xboole\_0 X0)\wedge(v4\_finsub\_1 X0))\Rightarrow(\forall X1. \\ &((\neg v1\_xboole\_0 X1)\wedge(v4\_finsub\_1 X1))\Rightarrow(\forall X2.(m1\_subset\_1 \\ &X2 (k2\_zfmisc\_1 X0 X1))\Rightarrow(\forall X3.(m1\_subset\_1 X3 (k2\_zfmisc\_1 \\ &X0 X1))\Rightarrow(k2\_normform X0 X1 X2 X3 = k1\_domain\_1 X0 X1 (k3\_finsub\_1 \\ &X0 (k2\_domain\_1 X0 X1 X2) (k2\_domain\_1 X0 X1 X3)) (k3\_finsub\_1 X1 \\ &(k3\_domain\_1 X0 X1 X2) (k3\_domain\_1 X0 X1 X3)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} &\forall X0.((\neg v1\_xboole\_0 X0)\wedge(v4\_finsub\_1 X0))\Rightarrow(\forall X1. \\ &((\neg v1\_xboole\_0 X1)\wedge(v4\_finsub\_1 X1))\Rightarrow(\forall X2.(m1\_subset\_1 \\ &X2 (k2\_zfmisc\_1 X0 X1))\Rightarrow(\forall X3.(m1\_subset\_1 X3 (k2\_zfmisc\_1 \\ &X0 X1))\Rightarrow(k1\_normform X0 X1 X2 X3 = k1\_domain\_1 X0 X1 (k1\_finsub\_1 \\ &X0 (k2\_domain\_1 X0 X1 X2) (k2\_domain\_1 X0 X1 X3)) (k1\_finsub\_1 X1 \\ &(k3\_domain\_1 X0 X1 X2) (k3\_domain\_1 X0 X1 X3)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v1\_xboole\_0 X0)\wedge(v4\_finsub\_1 X0))\wedge((m1\_subset\_1 X1 X0)\wedge(m1\_subset\_1 X2 X0)))\Rightarrow(k3\_finsub\_1 X0 X1 X2 = k3\_finsub\_1 X0 X2 X1) \quad (17)$$

Assume the following.

$$\begin{aligned} &\forall X0.\forall X1.\forall X2.\forall X3.(((\neg v1\_xboole\_0 \\ &X0)\wedge(v4\_finsub\_1 X0))\wedge(((\neg v1\_xboole\_0 X1)\wedge(v4\_finsub\_1 X1))\wedge \\ &((m1\_subset\_1 X2 (k2\_zfmisc\_1 X0 X1))\wedge(m1\_subset\_1 X3 (k2\_zfmisc\_1 \\ &X0 X1))))))\Rightarrow(k2\_normform X0 X1 X2 X3 = k2\_normform X0 X1 X3 X2) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} &\forall X0.\forall X1.\forall X2.\forall X3.(((\neg v1\_xboole\_0 \\ &X0)\wedge(v4\_finsub\_1 X0))\wedge(((\neg v1\_xboole\_0 X1)\wedge(v4\_finsub\_1 X1))\wedge \\ &((m1\_subset\_1 X2 (k2\_zfmisc\_1 X0 X1))\wedge(m1\_subset\_1 X3 (k2\_zfmisc\_1 \\ &X0 X1))))))\Rightarrow(k1\_normform X0 X1 X2 X3 = k1\_normform X0 X1 X3 X2) \end{aligned} \quad (19)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v1\_xboole\_0 X0)\wedge(v4\_finsub\_1 X0))\wedge((m1\_subset\_1 X1 X0)\wedge(m1\_subset\_1 X2 X0)))\Rightarrow(k1\_finsub\_1 X0 X1 X2 = k1\_finsub\_1 X0 X2 X1) \quad (20)$$

**Theorem 1**

$$\begin{aligned} & \forall X0.((\neg v1\_xboole\_0 X0) \wedge (v4\_finsub\_1 X0)) \Rightarrow (\forall X1. \\ & ((\neg v1\_xboole\_0 X1) \wedge (v4\_finsub\_1 X1)) \Rightarrow (\forall X2.(m1\_subset\_1 \\ & X2 (k2\_zfmisc\_1 X0 X1)) \Rightarrow (\forall X3.(m1\_subset\_1 X3 (k2\_zfmisc\_1 \\ & X0 X1)) \Rightarrow (\forall X4.(m1\_subset\_1 X4 (k2\_zfmisc\_1 X0 X1)) \Rightarrow (k2\_normform \\ & X0 X1 X2 (k1\_normform X0 X1 X3 X4) = k1\_normform X0 X1 (k2\_normform \\ & X0 X1 X2 X3) (k2\_normform X0 X1 X2 X4)))))) \end{aligned}$$