

t60\_boolealg  
(TMX4bJKNsAw937RCqk63JRtg1oTqgJLaCkT)

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Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v10\_lattices : \iota \Rightarrow o$  be given. Let  $v17\_lattices : \iota \Rightarrow o$  be given. Let  $l3\_lattices : \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $r1\_boolealg : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k3\_boolealg : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k4\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_boolealg : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k7\_lattices : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k5\_lattices : \iota \Rightarrow \iota$  be given. Let  $k2\_boolealg : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v13\_lattices : \iota \Rightarrow o$  be given. Let  $k3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $l1\_lattices : \iota \Rightarrow o$  be given. Let  $l2\_lattices : \iota \Rightarrow o$  be given. Let  $v6\_lattices : \iota \Rightarrow o$  be given. Let  $v4\_lattices : \iota \Rightarrow o$  be given. Let  $v11\_lattices : \iota \Rightarrow o$  be given. Let  $v15\_lattices : \iota \Rightarrow o$  be given. Let  $v16\_lattices : \iota \Rightarrow o$  be given. Let  $v14\_lattices : \iota \Rightarrow o$  be given. Let  $v5\_lattices : \iota \Rightarrow o$  be given. Let  $v7\_lattices : \iota \Rightarrow o$  be given. Let  $v8\_lattices : \iota \Rightarrow o$  be given. Let  $v9\_lattices : \iota \Rightarrow o$  be given. Assume the following.

$$\begin{aligned} & \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\ & X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 \\ & X0)) \Rightarrow (\forall X2. (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow ((r3\_lattices \\ & X0 X1 X2) \Leftrightarrow (k4\_lattices X0 X1 (k7\_lattices X0 X2) = k5\_lattices X0)))) \end{aligned} \quad (1)$$

Assume the following.

$$\begin{aligned} & \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\ & X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 \\ & X0)) \Rightarrow (\forall X2. (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (r1\_boolealg \\ & X0 (k1\_boolealg X0 X1 (k4\_lattices X0 X1 X2)) (k1\_boolealg X0 X1 X2)))) \end{aligned} \quad (2)$$

Assume the following.

$$\begin{aligned} & \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\ & X0))) \Rightarrow (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. \\ & (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (\forall X3. (m1\_subset\_1 X3 \\ & (u1\_struct\_0 X0)) \Rightarrow ((r3\_lattices X0 X1 X2) \Rightarrow (r3\_lattices X0 (k4\_lattices \\ & X0 X1 X3) X2)))))) \end{aligned} \quad (3)$$

Assume the following.

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\ & X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 \\ & X0)) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow ((r1\_boolealg \\ & X0 (k1\_boolealg X0 X1 X2) (k5\_lattices X0)) \Leftrightarrow (r3\_lattices X0 X1 X2)))) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices \\ & X0) \wedge (l3\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge ( \\ & m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow (r1\_boolealg X0 X1 X1) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices \\ & X0) \wedge (l3\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge ( \\ & m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow ((r1\_boolealg X0 X1 X2) \Leftrightarrow (X1 = \\ & X2)) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices \\ & X0) \wedge (l3\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge ( \\ & m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow (k3\_boolealg X0 X1 X2 = k2\_boolealg \\ & X0 X1 X2) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.(((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge \\ & ((v13\_lattices X0) \wedge (l3\_lattices X0)))) \wedge (m1\_subset\_1 X1 (u1\_struct\_0 \\ & X0))) \Rightarrow (k3\_lattices X0 (k5\_lattices X0) X1 = X1) \end{aligned} \quad (8)$$

Assume the following.

$$\forall X0.(l3\_lattices X0) \Rightarrow ((l1\_lattices X0) \wedge (l2\_lattices X0)) \quad (9)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0) \wedge (l1\_lattices X0)) \Rightarrow (m1\_subset\_1 (k5\_lattices X0) (u1\_struct\_0 X0)) \quad (10)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0) \wedge ((v6\_lattices \\ & X0) \wedge (l1\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge ( \\ & m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow (m1\_subset\_1 (k4\_lattices \\ & X0 X1 X2) (u1\_struct\_0 X0)) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((\neg v2\_struct\_0 X0) \wedge (v10\_lattices \\ & X0) \wedge (l3\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge \\ & m1\_subset\_1 X2 (u1\_struct\_0 X0))) \Rightarrow (m1\_subset\_1 (k1\_boolealg \\ & X0 X1 X2) (u1\_struct\_0 X0)) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & \forall X0. ((\neg v2\_struct\_0 X0) \wedge (v10\_lattices X0) \wedge (l3\_lattices \\ & X0)) \Rightarrow (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. \\ & (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow ((r1\_boolealg X0 X1 X2) \Leftrightarrow ((r3\_lattices \\ & X0 X1 X2) \wedge (r3\_lattices X0 X2 X1)))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & \forall X0. ((\neg v2\_struct\_0 X0) \wedge (v10\_lattices X0) \wedge (l3\_lattices \\ & X0)) \Rightarrow (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. \\ & (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (k2\_boolealg X0 X1 X2 = k3\_lattices \\ & X0 (k1\_boolealg X0 X1 X2) (k1\_boolealg X0 X2 X1)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & \forall X0. ((\neg v2\_struct\_0 X0) \wedge (v10\_lattices X0) \wedge (l3\_lattices \\ & X0)) \Rightarrow (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. \\ & (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (k1\_boolealg X0 X1 X2 = k4\_lattices \\ & X0 X1 (k7\_lattices X0 X2)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((\neg v2\_struct\_0 X0) \wedge (v4\_lattices \\ & X0) \wedge (l2\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge \\ & m1\_subset\_1 X2 (u1\_struct\_0 X0))) \Rightarrow (k3\_lattices X0 X1 X2 = k3\_lattices \\ & X0 X2 X1) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall X0. (l3\_lattices X0) \Rightarrow (((\neg v2\_struct\_0 X0) \wedge (v17\_lattices \\ & X0)) \Rightarrow ((\neg v2\_struct\_0 X0) \wedge ((v11\_lattices X0) \wedge ((v15\_lattices \\ & X0) \wedge (v16\_lattices X0)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall X0. (l3\_lattices X0) \Rightarrow (((\neg v2\_struct\_0 X0) \wedge (v15\_lattices \\ & X0)) \Rightarrow ((\neg v2\_struct\_0 X0) \wedge ((v13\_lattices X0) \wedge (v14\_lattices X0)))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & \forall X0. (l3\_lattices X0) \Rightarrow (((\neg v2\_struct\_0 X0) \wedge (v10\_lattices \\ & X0)) \Rightarrow ((\neg v2\_struct\_0 X0) \wedge ((v4\_lattices X0) \wedge ((v5\_lattices X0) \wedge \\ & ((v6\_lattices X0) \wedge ((v7\_lattices X0) \wedge ((v8\_lattices X0) \wedge (v9\_lattices \\ & X0)))))))))) \end{aligned} \quad (19)$$

**Theorem 1**

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0) \wedge (v10\_lattices X0) \wedge ((v17\_lattices \\ & X0) \wedge (l3\_lattices X0))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 \\ & X0)) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (r1\_boolealg \\ & X0 (k3\_boolealg X0 X1 (k4\_lattices X0 X1 X2)) (k1\_boolealg X0 X1 X2)))) \end{aligned}$$