

# t61\_boolealg (TMUcHhvjqwR- wCs8hUofD3jGHaXYYpFNHxXh)

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Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v10\_lattices : \iota \Rightarrow o$  be given. Let  $v17\_lattices : \iota \Rightarrow o$  be given. Let  $l3\_lattices : \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $r1\_boolealg : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k3\_boolealg : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k4\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_boolealg : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v6\_lattices : \iota \Rightarrow o$  be given. Let  $l1\_lattices : \iota \Rightarrow o$  be given. Let  $k2\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v4\_lattices : \iota \Rightarrow o$  be given. Let  $l2\_lattices : \iota \Rightarrow o$  be given. Let  $k1\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_boolealg : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k7\_lattices : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v8\_lattices : \iota \Rightarrow o$  be given. Let  $v9\_lattices : \iota \Rightarrow o$  be given. Let  $v5\_lattices : \iota \Rightarrow o$  be given. Let  $v7\_lattices : \iota \Rightarrow o$  be given. Assume the following.

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\ & X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 \\ & X0)) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (r1\_boolealg \\ & X0 (k3\_lattices X0 (k4\_lattices X0 X1 X2) (k1\_boolealg X0 X1 X2)) \\ & X1))) \end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\ & X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 \\ & X0)) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (r1\_boolealg \\ & X0 (k3\_lattices X0 X1 (k1\_boolealg X0 X2 X1)) (k3\_lattices X0 X1 X2)))) \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices \\ & X0) \wedge (l3\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge \\ & (m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow ((r1\_boolealg X0 X1 X2) \Leftrightarrow (X1 = \\ & X2)) \end{aligned} \tag{3}$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge(v6\_lattices X0)\wedge(l1\_lattices X0)))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0)))\Rightarrow(k4\_lattices X0 X1 X2 = k2\_lattices X0 X1 X2) \quad (4)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge(v4\_lattices X0)\wedge(l2\_lattices X0)))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0)))\Rightarrow(k3\_lattices X0 X1 X2 = k1\_lattices X0 X1 X2) \quad (5)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge(v10\_lattices X0)\wedge(l3\_lattices X0)))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0)))\Rightarrow(k3\_boolealg X0 X1 X2 = k2\_boolealg X0 X1 X2) \quad (6)$$

Assume the following.

$$\forall X0.\forall X1.(((\neg v2\_struct\_0 X0)\wedge(v10\_lattices X0)\wedge((v17\_lattices X0)\wedge(l3\_lattices X0))))\wedge(m1\_subset\_1 X1 (u1\_struct\_0 X0))\Rightarrow(k7\_lattices X0 (k7\_lattices X0 X1) = X1) \quad (7)$$

Assume the following.

$$\forall X0.\forall X1.(((\neg v2\_struct\_0 X0)\wedge(v6\_lattices X0)\wedge((v8\_lattices X0)\wedge(v9\_lattices X0)\wedge(l3\_lattices X0))))\wedge(m1\_subset\_1 X1 (u1\_struct\_0 X0))\Rightarrow(k1\_lattices X0 X1 X1 = X1) \quad (8)$$

Assume the following.

$$\forall X0.(l3\_lattices X0)\Rightarrow((l1\_lattices X0)\wedge(l2\_lattices X0)) \quad (9)$$

Assume the following.

$$\forall X0.\forall X1.(((\neg v2\_struct\_0 X0)\wedge(l3\_lattices X0))\wedge(m1\_subset\_1 X1 (u1\_struct\_0 X0)))\Rightarrow(m1\_subset\_1 (k7\_lattices X0 X1) (u1\_struct\_0 X0)) \quad (10)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge(v6\_lattices X0)\wedge(l1\_lattices X0)))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0)))\Rightarrow(m1\_subset\_1 (k4\_lattices X0 X1 X2) (u1\_struct\_0 X0)) \quad (11)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge(v4\_lattices X0)\wedge(l2\_lattices X0))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow(m1\_subset\_1 (k3\_lattices X0 X1 X2) (u1\_struct\_0 X0)) \quad (12)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge(v10\_lattices X0)\wedge(l3\_lattices X0))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow(m1\_subset\_1 (k3\_boolealg X0 X1 X2) (u1\_struct\_0 X0)) \quad (13)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge(l2\_lattices X0))\Rightarrow((v5\_lattices X0)\Leftrightarrow(\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0))\Rightarrow(\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0))\Rightarrow(\forall X3.(m1\_subset\_1 X3 (u1\_struct\_0 X0))\Rightarrow(k1\_lattices X0 X1 (k1\_lattices X0 X2 X3) = k1\_lattices X0 (k1\_lattices X0 X1 X2) X3)))))) \quad (14)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge((v10\_lattices X0)\wedge(l3\_lattices X0)))\Rightarrow(\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0))\Rightarrow(\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0))\Rightarrow(k2\_boolealg X0 X1 X2 = k3\_lattices X0 (k1\_boolealg X0 X1 X2) (k1\_boolealg X0 X2 X1)))) \quad (15)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge((v10\_lattices X0)\wedge(l3\_lattices X0)))\Rightarrow(\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0))\Rightarrow(\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0))\Rightarrow(k1\_boolealg X0 X1 X2 = k4\_lattices X0 X1 (k7\_lattices X0 X2)))) \quad (16)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge(v6\_lattices X0)\wedge(l1\_lattices X0))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow(k4\_lattices X0 X1 X2 = k4\_lattices X0 X2 X1) \quad (17)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge(v4\_lattices X0)\wedge(l2\_lattices X0))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow(k3\_lattices X0 X1 X2 = k3\_lattices X0 X2 X1) \quad (18)$$

Assume the following.

$$\begin{aligned} \forall X0. (&l3\_lattices\ X0) \Rightarrow (((\neg v2\_struct\_0\ X0) \wedge (v10\_lattices \\ &X0)) \Rightarrow ((\neg v2\_struct\_0\ X0) \wedge ((v4\_lattices\ X0) \wedge ((v5\_lattices\ X0) \wedge \\ &((v6\_lattices\ X0) \wedge ((v7\_lattices\ X0) \wedge ((v8\_lattices\ X0) \wedge (v9\_lattices \\ &X0)))))))) \end{aligned} \quad (19)$$

**Theorem 1**

$$\begin{aligned} \forall X0. ((\neg v2\_struct\_0\ X0) \wedge ((v10\_lattices\ X0) \wedge ((v17\_lattices \\ &X0) \wedge (l3\_lattices\ X0)))) \Rightarrow (\forall X1. (m1\_subset\_1\ X1\ (u1\_struct\_0 \\ &X0)) \Rightarrow (\forall X2. (m1\_subset\_1\ X2\ (u1\_struct\_0\ X0)) \Rightarrow (r1\_boolealg \\ &X0\ (k3\_lattices\ X0\ X1\ X2)\ (k3\_lattices\ X0\ (k3\_boolealg\ X0\ X1\ X2)\ ( \\ &k4\_lattices\ X0\ X1\ X2)))))) \end{aligned}$$