

## t62\_boolealg

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Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v10\_lattices : \iota \Rightarrow o$  be given. Let  $v17\_lattices : \iota \Rightarrow o$  be given. Let  $l3\_lattices : \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $r1\_boolealg : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k3\_boolealg : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k4\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_boolealg : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k7\_lattices : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k6\_lattices : \iota \Rightarrow \iota$  be given. Let  $k5\_lattices : \iota \Rightarrow \iota$  be given. Let  $v11\_lattices : \iota \Rightarrow o$  be given. Let  $v6\_lattices : \iota \Rightarrow o$  be given. Let  $l1\_lattices : \iota \Rightarrow o$  be given. Let  $k2\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v4\_lattices : \iota \Rightarrow o$  be given. Let  $l2\_lattices : \iota \Rightarrow o$  be given. Let  $k1\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_boolealg : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v14\_lattices : \iota \Rightarrow o$  be given. Let  $v13\_lattices : \iota \Rightarrow o$  be given. Let  $v8\_lattices : \iota \Rightarrow o$  be given. Let  $v9\_lattices : \iota \Rightarrow o$  be given. Let  $v7\_lattices : \iota \Rightarrow o$  be given. Let  $v5\_lattices : \iota \Rightarrow o$  be given. Let  $v15\_lattices : \iota \Rightarrow o$  be given. Let  $v16\_lattices : \iota \Rightarrow o$  be given. Assume the following.

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\ & X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 \\ & X0)) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (r1\_boolealg \\ & X0 (k3\_lattices X0 X1 (k1\_boolealg X0 X2 X1)) (k3\_lattices X0 X1 X2)))) \end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\ & X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 \\ & X0)) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (k7\_lattices \\ & X0 (k3\_lattices X0 X1 X2) = k4\_lattices X0 (k7\_lattices X0 X1) (k7\_lattices \\ & X0 X2)))) \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\ X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 \\ X0)) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (k7\_lattices \\ X0 (k4\_lattices X0 X1 X2) = k3\_lattices X0 (k7\_lattices X0 X1) (k7\_lattices \\ X0 X2)))) \end{aligned} \quad (3)$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\ X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 \\ X0)) \Rightarrow (k3\_lattices X0 (k7\_lattices X0 X1) X1 = k6\_lattices X0)) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\ X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 \\ X0)) \Rightarrow (k4\_lattices X0 (k7\_lattices X0 X1) X1 = k5\_lattices X0)) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v11\_lattices \\ X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 \\ X0)) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (\forall X3. \\ (m1\_subset\_1 X3 (u1\_struct\_0 X0)) \Rightarrow (k3\_lattices X0 X1 (k4\_lattices \\ X0 X2 X3) = k4\_lattices X0 (k3\_lattices X0 X1 X2) (k3\_lattices X0 X1 \\ X3)))))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0) \wedge ((v6\_lattices \\ X0) \wedge (l1\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge ( \\ m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow (k4\_lattices X0 X1 X2 = k2\_lattices \\ X0 X1 X2) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0) \wedge ((v4\_lattices \\ X0) \wedge (l2\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge ( \\ m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow (k3\_lattices X0 X1 X2 = k1\_lattices \\ X0 X1 X2) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices \\ X0) \wedge (l3\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge ( \\ m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow (k3\_boolealg X0 X1 X2 = k2\_boolealg \\ X0 X1 X2) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. (((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge \\ & ((v17\_lattices X0) \wedge (l3\_lattices X0)))) \wedge (m1\_subset\_1 X1 (u1\_struct\_0 \\ & X0))) \Rightarrow (k7\_lattices X0 (k7\_lattices X0 X1) = X1) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. (((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge \\ & ((v14\_lattices X0) \wedge (l3\_lattices X0)))) \wedge (m1\_subset\_1 X1 (u1\_struct\_0 \\ & X0))) \Rightarrow (k3\_lattices X0 (k6\_lattices X0) X1 = k6\_lattices X0) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. (((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge \\ & ((v14\_lattices X0) \wedge (l3\_lattices X0)))) \wedge (m1\_subset\_1 X1 (u1\_struct\_0 \\ & X0))) \Rightarrow (k4\_lattices X0 (k6\_lattices X0) X1 = X1) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. (((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge \\ & ((v13\_lattices X0) \wedge (l3\_lattices X0)))) \wedge (m1\_subset\_1 X1 (u1\_struct\_0 \\ & X0))) \Rightarrow (k3\_lattices X0 (k5\_lattices X0) X1 = X1) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. (((\neg v2\_struct\_0 X0) \wedge ((v6\_lattices X0) \wedge \\ & ((v8\_lattices X0) \wedge ((v9\_lattices X0) \wedge (l3\_lattices X0)))) \wedge ( \\ & m1\_subset\_1 X1 (u1\_struct\_0 X0))) \Rightarrow (k4\_lattices X0 X1 X1 = X1) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. (((\neg v2\_struct\_0 X0) \wedge ((v6\_lattices X0) \wedge \\ & ((v8\_lattices X0) \wedge ((v9\_lattices X0) \wedge (l3\_lattices X0)))) \wedge ( \\ & m1\_subset\_1 X1 (u1\_struct\_0 X0))) \Rightarrow (k1\_lattices X0 X1 X1 = X1) \end{aligned} \quad (15)$$

Assume the following.

$$\forall X0. (l3\_lattices X0) \Rightarrow ((l1\_lattices X0) \wedge (l2\_lattices X0)) \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. (((\neg v2\_struct\_0 X0) \wedge (l3\_lattices X0)) \wedge \\ & (m1\_subset\_1 X1 (u1\_struct\_0 X0))) \Rightarrow (m1\_subset\_1 (k7\_lattices \\ & X0 X1) (u1\_struct\_0 X0)) \end{aligned} \quad (17)$$

Assume the following.

$$\forall X0. ((\neg v2\_struct\_0 X0) \wedge (l2\_lattices X0)) \Rightarrow (m1\_subset\_1 (k6\_lattices X0) (u1\_struct\_0 X0)) \quad (18)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0) \wedge (l1\_lattices X0)) \Rightarrow (m1\_subset\_1 (k5\_lattices X0) (u1\_struct\_0 X0)) \quad (19)$$

Assume the following.

$$\forall X0. \forall X1. \forall X2. (((\neg v2\_struct\_0 X0) \wedge ((v6\_lattices X0) \wedge (l1\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow (m1\_subset\_1 (k4\_lattices X0 X1 X2) (u1\_struct\_0 X0)) \quad (20)$$

Assume the following.

$$\forall X0. \forall X1. \forall X2. (((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow (m1\_subset\_1 (k3\_boolealg X0 X1 X2) (u1\_struct\_0 X0)) \quad (21)$$

Assume the following.

$$\forall X0. \forall X1. \forall X2. (((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow (m1\_subset\_1 (k1\_boolealg X0 X1 X2) (u1\_struct\_0 X0)) \quad (22)$$

Assume the following.

$$\forall X0. ((\neg v2\_struct\_0 X0) \wedge (l3\_lattices X0)) \Rightarrow ((v9\_lattices X0) \Leftrightarrow (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (k2\_lattices X0 X1 (k1\_lattices X0 X1 X2) = X1)))) \quad (23)$$

Assume the following.

$$\forall X0. ((\neg v2\_struct\_0 X0) \wedge (l3\_lattices X0)) \Rightarrow ((v8\_lattices X0) \Leftrightarrow (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (k1\_lattices X0 (k2\_lattices X0 X1 X2) X2 = X2)))) \quad (24)$$

Assume the following.

$$\forall X0. ((\neg v2\_struct\_0 X0) \wedge (l1\_lattices X0)) \Rightarrow ((v7\_lattices X0) \Leftrightarrow (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (\forall X3. (m1\_subset\_1 X3 (u1\_struct\_0 X0)) \Rightarrow (k2\_lattices X0 X1 (k2\_lattices X0 X2 X3) = k2\_lattices X0 (k2\_lattices X0 X1 X2) X3)))))) \quad (25)$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge (l2\_lattices X0)) \Rightarrow ((v5\_lattices \\ X0) \Leftrightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. \\ (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (\forall X3.(m1\_subset\_1 X3 \\ (u1\_struct\_0 X0)) \Rightarrow (k1\_lattices X0 X1 (k1\_lattices X0 X2 X3) = k1\_lattices \\ X0 (k1\_lattices X0 X1 X2) X3)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\ X0))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. \\ (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (k2\_boolealg X0 X1 X2 = k3\_lattices \\ X0 (k1\_boolealg X0 X1 X2) (k1\_boolealg X0 X2 X1)))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\ X0))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. \\ (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (k1\_boolealg X0 X1 X2 = k4\_lattices \\ X0 X1 (k7\_lattices X0 X2)))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge (l3\_lattices X0)) \Rightarrow ((v11\_lattices \\ X0) \Leftrightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. \\ (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (\forall X3.(m1\_subset\_1 X3 \\ (u1\_struct\_0 X0)) \Rightarrow (k2\_lattices X0 X1 (k1\_lattices X0 X2 X3) = k1\_lattices \\ X0 (k2\_lattices X0 X1 X2) (k2\_lattices X0 X1 X3)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge (l3\_lattices X0)) \Rightarrow ((v10\_lattices \\ X0) \Leftrightarrow ((v4\_lattices X0) \wedge ((v5\_lattices X0) \wedge ((v8\_lattices X0) \wedge \\ ((v6\_lattices X0) \wedge ((v7\_lattices X0) \wedge (v9\_lattices X0))))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} \forall X0. \forall X1. \forall X2. (((\neg v2\_struct\_0 X0) \wedge ((v6\_lattices \\ X0) \wedge (l1\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge ( \\ m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow (k4\_lattices X0 X1 X2 = k4\_lattices \\ X0 X2 X1) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} \forall X0. \forall X1. \forall X2. (((\neg v2\_struct\_0 X0) \wedge ((v4\_lattices \\ X0) \wedge (l2\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge ( \\ m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow (k3\_lattices X0 X1 X2 = k3\_lattices \\ X0 X2 X1) \end{aligned} \quad (32)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge(v10\_lattices X0)\wedge(l3\_lattices X0))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow(k3\_boolealg X0 X1 X2 = k3\_boolealg X0 X2 X1) \quad (33)$$

Assume the following.

$$\forall X0.(l3\_lattices X0)\Rightarrow(((\neg v2\_struct\_0 X0)\wedge(v17\_lattices X0))\Rightarrow((\neg v2\_struct\_0 X0)\wedge((v11\_lattices X0)\wedge((v15\_lattices X0)\wedge(v16\_lattices X0)))) \quad (34)$$

Assume the following.

$$\forall X0.(l3\_lattices X0)\Rightarrow(((\neg v2\_struct\_0 X0)\wedge(v15\_lattices X0))\Rightarrow((\neg v2\_struct\_0 X0)\wedge((v13\_lattices X0)\wedge(v14\_lattices X0)))) \quad (35)$$

**Theorem 1**

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge((v10\_lattices X0)\wedge((v17\_lattices X0)\wedge(l3\_lattices X0))))\Rightarrow(\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0))\Rightarrow(\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0))\Rightarrow(r1\_boolealg X0 (k3\_boolealg X0 (k3\_boolealg X0 X1 X2) (k4\_lattices X0 X1 X2)) (k3\_lattices X0 X1 X2))))$$