

t62_pboole
(TMSbM8NYWURRp4qbX9DkBkv3WSjV7YW98xA)

October 27, 2020

Let $v1_relat_1 : \iota \Rightarrow o$ be given. Let $v4_relat_1 : \iota \Rightarrow \iota \Rightarrow o$ be given. Let $v1_funct_1 : \iota \Rightarrow o$ be given. Let $v1_partfun1 : \iota \Rightarrow \iota \Rightarrow o$ be given. Let $r6_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$ be given. Let $k3_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $k4_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $k3_xboole_0 : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $k4_xboole_0 : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $k6_subset_1 : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $k1_funct_1 : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall X0. \forall X1. \forall X2. k3_xboole_0 X0 (k4_xboole_0 X1 X2) = k4_xboole_0 (k3_xboole_0 X0 X1) X2 \quad (1)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1_relat_1 X1) \wedge ((v4_relat_1 \\ & X1 X0) \wedge ((v1_funct_1 X1) \wedge (v1_partfun1 X1 X0)))) \wedge ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))))) \Rightarrow \\ & ((r6_pboole X0 X1 X2) \Rightarrow (r6_pboole X0 X2 X1)) \end{aligned} \quad (2)$$

Assume the following.

$$\forall X0. \forall X1. k6_subset_1 X0 X1 = k4_xboole_0 X0 X1 \quad (3)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1_relat_1 X1) \wedge ((v4_relat_1 \\ & X1 X0) \wedge ((v1_funct_1 X1) \wedge (v1_partfun1 X1 X0)))) \wedge ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))))) \Rightarrow \\ & (k3_pboole X0 X1 X1 = X1) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1_relat_1 X1) \wedge ((v4_relat_1 \\ & X1 X0) \wedge ((v1_funct_1 X1) \wedge (v1_partfun1 X1 X0)))) \wedge ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))))) \Rightarrow \\ & ((v1_relat_1 (k4_pboole X0 X1 X2)) \wedge ((v4_relat_1 (k4_pboole X0 \\ & X1 X2) X0) \wedge ((v1_funct_1 (k4_pboole X0 X1 X2)) \wedge (v1_partfun1 (k4_pboole \\ & X0 X1 X2) X0)))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1_relat_1 X1) \wedge ((v4_relat_1 \\ & X1 X0) \wedge ((v1_funct_1 X1) \wedge (v1_partfun1 X1 X0)))) \wedge ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\ & ((v1_relat_1 (k3_pboole X0 X1 X2)) \wedge ((v4_relat_1 (k3_pboole X0 \\ & X1 X2) X0) \wedge ((v1_funct_1 (k3_pboole X0 X1 X2)) \wedge (v1_partfun1 (k3_pboole \\ & X0 X1 X2) X0)))) \end{aligned} \tag{6}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge (\\ & (v1_funct_1 X1) \wedge (v1_partfun1 X1 X0)))) \Rightarrow (\forall X2. ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\ & (\forall X3. ((v1_relat_1 X3) \wedge ((v4_relat_1 X3 X0) \wedge ((v1_funct_1 \\ & X3) \wedge (v1_partfun1 X3 X0)))) \Rightarrow ((X3 = k4_pboole X0 X1 X2) \Leftrightarrow (\forall X4. \\ & (X4 \in X0) \Rightarrow (k1_funct_1 X3 X4 = k6_subset_1 (k1_funct_1 X1 X4) (k1_funct_1 \\ & X2 X4)))))) \end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge (\\ & (v1_funct_1 X1) \wedge (v1_partfun1 X1 X0)))) \Rightarrow (\forall X2. ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\ & (\forall X3. ((v1_relat_1 X3) \wedge ((v4_relat_1 X3 X0) \wedge ((v1_funct_1 \\ & X3) \wedge (v1_partfun1 X3 X0)))) \Rightarrow ((X3 = k3_pboole X0 X1 X2) \Leftrightarrow (\forall X4. \\ & (X4 \in X0) \Rightarrow (k1_funct_1 X3 X4 = k3_xboole_0 (k1_funct_1 X1 X4) (k1_funct_1 \\ & X2 X4)))))) \end{aligned} \tag{8}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge (\\ & (v1_funct_1 X1) \wedge (v1_partfun1 X1 X0)))) \Rightarrow (\forall X2. ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\ & ((r6_pboole X0 X1 X2) \Leftrightarrow (\forall X3. (X3 \in X0) \Rightarrow (k1_funct_1 X1 X3 = k1_funct_1 \\ & X2 X3)))) \end{aligned} \tag{9}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1_relat_1 X1) \wedge ((v4_relat_1 \\ & X1 X0) \wedge ((v1_funct_1 X1) \wedge (v1_partfun1 X1 X0)))) \wedge ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\ & (k3_pboole X0 X1 X2 = k3_pboole X0 X2 X1) \end{aligned} \tag{10}$$

Theorem 1

$$\begin{aligned} & \forall X0. \forall X1. ((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge \\ & (v1_funct_1 X1) \wedge (v1_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\ & (\forall X3. ((v1_relat_1 X3) \wedge ((v4_relat_1 X3 X0) \wedge ((v1_funct_1 \\ & X3) \wedge (v1_partfun1 X3 X0)))) \Rightarrow (r6_pboole X0 (k3_pboole X0 X1 (k4_pboole \\ & X0 X2 X3)) (k4_pboole X0 (k3_pboole X0 X1 X2) X3)))) \end{aligned}$$