

t63_pboole (TMHzE- oFNLA vFZ4i5xbwK1WsgLTTPjqvjRHj)

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Let $v1_relat_1 : \iota \Rightarrow o$ be given. Let $v4_relat_1 : \iota \Rightarrow \iota \Rightarrow o$ be given. Let $v1_funct_1 : \iota \Rightarrow o$ be given. Let $v1_partfun1 : \iota \Rightarrow \iota \Rightarrow o$ be given. Let $r6_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$ be given. Let $k3_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $k4_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $k1_pboole : \iota \Rightarrow \iota$ be given. Let $r2_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$ be given. Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge \\ & (v1_funct_1 X1) \wedge (v1_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\ & ((r2_pboole X0 X1 (k4_pboole X0 X2 X1)) \Rightarrow (r6_pboole X0 X1 (k1_pboole \\ & X0)))) \end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge \\ & (v1_funct_1 X1) \wedge (v1_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\ & (\forall X3. ((v1_relat_1 X3) \wedge ((v4_relat_1 X3 X0) \wedge ((v1_funct_1 \\ & X3) \wedge (v1_partfun1 X3 X0)))) \Rightarrow ((r2_pboole X0 X1 X2) \Rightarrow (r2_pboole X0 \\ & (k4_pboole X0 X3 X2) (k4_pboole X0 X3 X1)))) \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge \\ & (v1_funct_1 X1) \wedge (v1_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\ & (r2_pboole X0 (k3_pboole X0 X1 X2) X1)) \end{aligned} \tag{3}$$

Assume the following.

$$\begin{aligned}
& \forall X0. \forall X1. ((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge \\
& (v1_funct_1 X1) \wedge (v1_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1_relat_1 \\
& X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\
& (\forall X3. ((v1_relat_1 X3) \wedge ((v4_relat_1 X3 X0) \wedge ((v1_funct_1 \\
& X3) \wedge (v1_partfun1 X3 X0)))) \Rightarrow ((r2_pboole X0 X1 X2) \wedge (r2_pboole \\
& X0 X2 X3)) \Rightarrow (r2_pboole X0 X1 X3)))
\end{aligned} \tag{4}$$

Assume the following.

$$\begin{aligned}
& \forall X0. \forall X1. \forall X2. (((v1_relat_1 X1) \wedge ((v4_relat_1 \\
& X1 X0) \wedge ((v1_funct_1 X1) \wedge (v1_partfun1 X1 X0)))) \wedge ((v1_relat_1 \\
& X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\
& ((v1_relat_1 (k4_pboole X0 X1 X2)) \wedge ((v4_relat_1 (k4_pboole X0 \\
& X1 X2) X0) \wedge ((v1_funct_1 (k4_pboole X0 X1 X2)) \wedge (v1_partfun1 (k4_pboole \\
& X0 X1 X2) X0))))
\end{aligned} \tag{5}$$

Assume the following.

$$\begin{aligned}
& \forall X0. \forall X1. \forall X2. (((v1_relat_1 X1) \wedge ((v4_relat_1 \\
& X1 X0) \wedge ((v1_funct_1 X1) \wedge (v1_partfun1 X1 X0)))) \wedge ((v1_relat_1 \\
& X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\
& ((v1_relat_1 (k3_pboole X0 X1 X2)) \wedge ((v4_relat_1 (k3_pboole X0 \\
& X1 X2) X0) \wedge ((v1_funct_1 (k3_pboole X0 X1 X2)) \wedge (v1_partfun1 (k3_pboole \\
& X0 X1 X2) X0))))
\end{aligned} \tag{6}$$

Assume the following.

$$\begin{aligned}
& \forall X0. \forall X1. \forall X2. (((v1_relat_1 X1) \wedge ((v4_relat_1 \\
& X1 X0) \wedge ((v1_funct_1 X1) \wedge (v1_partfun1 X1 X0)))) \wedge ((v1_relat_1 \\
& X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\
& (k3_pboole X0 X1 X2 = k3_pboole X0 X2 X1)
\end{aligned} \tag{7}$$

Theorem 1

$$\begin{aligned}
& \forall X0. \forall X1. ((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge \\
& (v1_funct_1 X1) \wedge (v1_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1_relat_1 \\
& X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\
& (r6_pboole X0 (k3_pboole X0 (k4_pboole X0 X1 X2) X2) (k1_pboole X0)))
\end{aligned}$$