

t68\_boolealg  
(TMTR9prHhtRcmBgt7MYarNY7NRiLYo25JwD)

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Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v10\_lattices : \iota \Rightarrow o$  be given. Let  $v17\_lattices : \iota \Rightarrow o$  be given. Let  $l3\_lattices : \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $r1\_boolealg : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k7\_lattices : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k3\_boolealg : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k4\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k4\_filter\_0 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k7\_filter\_0 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k5\_lattices : \iota \Rightarrow \iota$  be given. Let  $k6\_lattices : \iota \Rightarrow \iota$  be given. Let  $v6\_lattices : \iota \Rightarrow o$  be given. Let  $v8\_lattices : \iota \Rightarrow o$  be given. Let  $v9\_lattices : \iota \Rightarrow o$  be given. Let  $k2\_boolealg : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v14\_lattices : \iota \Rightarrow o$  be given. Let  $v13\_lattices : \iota \Rightarrow o$  be given. Let  $l1\_lattices : \iota \Rightarrow o$  be given. Let  $l2\_lattices : \iota \Rightarrow o$  be given. Let  $k1\_boolealg : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v4\_lattices : \iota \Rightarrow o$  be given. Let  $v5\_lattices : \iota \Rightarrow o$  be given. Let  $v7\_lattices : \iota \Rightarrow o$  be given. Let  $v11\_lattices : \iota \Rightarrow o$  be given. Let  $v15\_lattices : \iota \Rightarrow o$  be given. Let  $v16\_lattices : \iota \Rightarrow o$  be given. Assume the following.

$$\begin{aligned}
& \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\
& X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 \\
& X0)) \Rightarrow (\forall X2. (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (\forall X3. \\
& (m1\_subset\_1 X3 (u1\_struct\_0 X0)) \Rightarrow (r1\_boolealg X0 (k3\_boolealg \\
& X0 (k3\_boolealg X0 X1 X2) X3) (k3\_boolealg X0 X1 (k3\_boolealg X0 X2 \\
& X3))))))
\end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\
& X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 \\
& X0)) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow ((k7\_lattices \\
& X0 (k4\_filter\_0 X0 X1 X2) = k4\_lattices X0 X1 (k7\_lattices X0 X2)) \wedge \\
& ((k7\_lattices X0 (k7\_filter\_0 X0 X1 X2) = k3\_lattices X0 (k4\_lattices \\
& X0 X1 (k7\_lattices X0 X2)) (k4\_lattices X0 (k7\_lattices X0 X1) X2)) \wedge \\
& ((k7\_lattices X0 (k7\_filter\_0 X0 X1 X2) = k7\_filter\_0 X0 X1 (k7\_lattices \\
& X0 X2)) \wedge (k7\_lattices X0 (k7\_filter\_0 X0 X1 X2) = k7\_filter\_0 X0 ( \\
& k7\_lattices X0 X1) X2))))))
\end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\
& X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 \\
& X0)) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (k7\_filter\_0 \\
& X0 X1 X2 = k3\_lattices X0 (k4\_lattices X0 X1 X2) (k4\_lattices X0 (k7\_lattices \\
& X0 X1) (k7\_lattices X0 X2))))))
\end{aligned} \tag{3}$$

Assume the following.

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\
& X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 \\
& X0)) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow ((r3\_lattices \\
& X0 X1 X2) \Leftrightarrow (k4\_lattices X0 X1 (k7\_lattices X0 X2) = k5\_lattices X0))))
\end{aligned} \tag{4}$$

Assume the following.

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\
& X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 \\
& X0)) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (k4\_filter\_0 \\
& X0 X1 X2 = k3\_lattices X0 (k7\_lattices X0 X1) X2)))
\end{aligned} \tag{5}$$

Assume the following.

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\
& X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 \\
& X0)) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (k7\_lattices \\
& X0 (k3\_lattices X0 X1 X2) = k4\_lattices X0 (k7\_lattices X0 X1) (k7\_lattices \\
& X0 X2))))
\end{aligned} \tag{6}$$

Assume the following.

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\
& X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 \\
& X0)) \Rightarrow (k3\_lattices X0 (k7\_lattices X0 X1) X1 = k6\_lattices X0))
\end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((\neg v2\_struct\_0 X0) \wedge (v6\_lattices \\ & X0) \wedge (v8\_lattices X0) \wedge (v9\_lattices X0) \wedge (l3\_lattices X0))) \wedge \\ & ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 X2 (u1\_struct\_0 \\ & X0))) \Rightarrow (r3\_lattices X0 X1 X1) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((\neg v2\_struct\_0 X0) \wedge (v10\_lattices \\ & X0) \wedge (l3\_lattices X0)) \wedge (m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge \\ & m1\_subset\_1 X2 (u1\_struct\_0 X0))) \Rightarrow (k3\_boolealg X0 X1 X2 = k2\_boolealg \\ & X0 X1 X2) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. (((\neg v2\_struct\_0 X0) \wedge (v10\_lattices X0) \wedge \\ & (v17\_lattices X0) \wedge (l3\_lattices X0))) \wedge (m1\_subset\_1 X1 (u1\_struct\_0 \\ & X0)) \Rightarrow (k7\_lattices X0 (k7\_lattices X0 X1) = X1) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. (((\neg v2\_struct\_0 X0) \wedge (v10\_lattices X0) \wedge \\ & (v14\_lattices X0) \wedge (l3\_lattices X0))) \wedge (m1\_subset\_1 X1 (u1\_struct\_0 \\ & X0)) \Rightarrow (k3\_lattices X0 (k6\_lattices X0) X1 = k6\_lattices X0) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. (((\neg v2\_struct\_0 X0) \wedge (v10\_lattices X0) \wedge \\ & (v14\_lattices X0) \wedge (l3\_lattices X0))) \wedge (m1\_subset\_1 X1 (u1\_struct\_0 \\ & X0)) \Rightarrow (k4\_lattices X0 (k6\_lattices X0) X1 = X1) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. (((\neg v2\_struct\_0 X0) \wedge (v10\_lattices X0) \wedge \\ & (v13\_lattices X0) \wedge (l3\_lattices X0))) \wedge (m1\_subset\_1 X1 (u1\_struct\_0 \\ & X0)) \Rightarrow (k4\_lattices X0 (k5\_lattices X0) X1 = k5\_lattices X0) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. (((\neg v2\_struct\_0 X0) \wedge (v10\_lattices X0) \wedge \\ & (v13\_lattices X0) \wedge (l3\_lattices X0))) \wedge (m1\_subset\_1 X1 (u1\_struct\_0 \\ & X0)) \Rightarrow (k3\_lattices X0 (k5\_lattices X0) X1 = X1) \end{aligned} \quad (14)$$

Assume the following.

$$\forall X0. (l3\_lattices X0) \Rightarrow ((l1\_lattices X0) \wedge (l2\_lattices X0)) \quad (15)$$

Assume the following.

$$\forall X0.\forall X1.(((\neg v2\_struct\_0 X0)\wedge(l3\_lattices X0))\wedge(m1\_subset\_1 X1 (u1\_struct\_0 X0)))\Rightarrow(m1\_subset\_1 (k7\_lattices X0 X1) (u1\_struct\_0 X0)) \quad (16)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge(l2\_lattices X0))\Rightarrow(m1\_subset\_1 (k6\_lattices X0) (u1\_struct\_0 X0)) \quad (17)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge(l1\_lattices X0))\Rightarrow(m1\_subset\_1 (k5\_lattices X0) (u1\_struct\_0 X0)) \quad (18)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge((v10\_lattices X0)\wedge(l3\_lattices X0)))\Rightarrow(\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0))\Rightarrow(\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0))\Rightarrow(k2\_boolealg X0 X1 X2 = k3\_lattices X0 (k1\_boolealg X0 X1 X2) (k1\_boolealg X0 X2 X1)))) \quad (19)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge((v10\_lattices X0)\wedge(l3\_lattices X0)))\Rightarrow(\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0))\Rightarrow(\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0))\Rightarrow(k1\_boolealg X0 X1 X2 = k4\_lattices X0 X1 (k7\_lattices X0 X2)))) \quad (20)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge(l3\_lattices X0))\Rightarrow(((v10\_lattices X0)\Leftrightarrow(((v4\_lattices X0)\wedge((v5\_lattices X0)\wedge((v8\_lattices X0)\wedge((v6\_lattices X0)\wedge((v7\_lattices X0)\wedge(v9\_lattices X0)))))))) \quad (21)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge((v6\_lattices X0)\wedge(l1\_lattices X0)))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow(k4\_lattices X0 X1 X2 = k4\_lattices X0 X2 X1) \quad (22)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge((v4\_lattices X0)\wedge(l2\_lattices X0)))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow(k3\_lattices X0 X1 X2 = k3\_lattices X0 X2 X1) \quad (23)$$

Assume the following.

$$\forall X0.(l3\_lattices\ X0)\Rightarrow(((\neg v2\_struct\_0\ X0)\wedge(v17\_lattices\ X0))\Rightarrow((\neg v2\_struct\_0\ X0)\wedge((v11\_lattices\ X0)\wedge((v15\_lattices\ X0)\wedge(v16\_lattices\ X0)))))) \quad (24)$$

Assume the following.

$$\forall X0.(l3\_lattices\ X0)\Rightarrow(((\neg v2\_struct\_0\ X0)\wedge(v15\_lattices\ X0))\Rightarrow((\neg v2\_struct\_0\ X0)\wedge((v13\_lattices\ X0)\wedge(v14\_lattices\ X0)))) \quad (25)$$

**Theorem 1**

$$\forall X0.((\neg v2\_struct\_0\ X0)\wedge((v10\_lattices\ X0)\wedge((v17\_lattices\ X0)\wedge(l3\_lattices\ X0))))\Rightarrow(\forall X1.(m1\_subset\_1\ X1\ (u1\_struct\_0\ X0))\Rightarrow(\forall X2.(m1\_subset\_1\ X2\ (u1\_struct\_0\ X0))\Rightarrow(r1\_boolealg\ X0\ (k7\_lattices\ X0\ (k3\_boolealg\ X0\ X1\ X2))\ (k3\_lattices\ X0\ (k4\_lattices\ X0\ X1\ X2))\ (k4\_lattices\ X0\ (k7\_lattices\ X0\ X1))\ (k7\_lattices\ X0\ X2))))))$$