# t6_member_1 <br> (TMPiPUMzJboStgZgwkJas4xSLVN3vrtztmf) 

October 27, 2020

Let $v 2 \_$membered : $\iota \Rightarrow 0$ be given. Let $k 4 \_$member_1 : $\iota \Rightarrow \iota$ be given. Let $k 3 \_x b o o l e \_0: \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $v 1 \_x x r e a l \_0: \iota \Rightarrow o$ be given. Let $k 2 \_x x r e a l \_3$ : $\iota \Rightarrow \iota$ be given. Let m1_subset_1: $\iota \Rightarrow \iota \Rightarrow o$ be given. Let $k 7$ _numbers $: \iota$ be given. Let $k 1 \_$member_1 : $\iota \Rightarrow \iota$ be given. Assume the following.

$$
\begin{gather*}
\forall X 0 .\left(v 2 \_ \text {membered } X 0\right) \Rightarrow\left(\forall X 1 .\left(v 1 \_x x r e a l \_0 X 1\right) \Rightarrow(( \right. \\
\left.\left.\left.k 2 \_x x r e a l \_3 X 1 \in X 0\right) \Leftrightarrow\left(X 1 \in k 4 \_m e m b e r \_1 X 0\right)\right)\right) \tag{1}
\end{gather*}
$$

Assume the following.

$$
\begin{equation*}
\forall X 0 .\left(v 2 \_m e m b e r e d X 0\right) \Rightarrow\left(k 4 \_m e m b e r \_1\left(k 4 \_m e m b e r \_1 X 0\right)=X 0\right) \tag{2}
\end{equation*}
$$

Assume the following.

$$
\begin{equation*}
\forall X 0 .\left(v 1 \_x x r e a l \_0 X 0\right) \Rightarrow\left(k 2 \_x x r e a l \_3\left(k 2 \_x x r e a l \_3 X 0\right)=X 0\right) \tag{3}
\end{equation*}
$$

Assume the following.

$$
\begin{gather*}
\forall X 0 . \forall X 1 \text {. }\left(v 2 \_m e m b e r e d ~\right. \\
X 0) \Rightarrow\left(v 2 \_ m e m b e r e d ~ \left(k 3 \_x b o o l e \_0\right.\right.  \tag{4}\\
X 1 X 0))
\end{gather*}
$$

Assume the following.

$$
\begin{gather*}
\forall X 0 . \forall X 1 .\left(v 2 \_m e m b e r e d X 0\right) \Rightarrow\left(v 2 \_ m e m b e r e d ~ \left(k 3 \_x b o o l e \_0\right.\right.  \tag{5}\\
X 0 X 1))
\end{gather*}
$$

Assume the following.

$$
\begin{equation*}
\forall X 0 .\left(v 2 \_m e m b e r e d \quad X 0\right) \Rightarrow\left(v 2 \_m e m b e r e d\left(k 4 \_m e m b e r \_1 X 0\right)\right) \tag{6}
\end{equation*}
$$

Assume the following.

$$
\begin{equation*}
\forall X 0 .\left(v 1 \_x x r e a l \_0 X 0\right) \Rightarrow\left(v 1 \_x x r e a l \_0\left(k 2 \_x x r e a l \_3 X 0\right)\right) \tag{7}
\end{equation*}
$$

Assume the following.

$$
\begin{gather*}
\forall X 0 . \forall X 1 . \forall X 2 .\left(X 2=k 3 \_x b o o l e \_0 X 0 X 1\right) \Leftrightarrow(\forall X 3 . \\
(X 3 \in X 2) \Leftrightarrow((X 3 \in X 0) \wedge(X 3 \in X 1))) \tag{8}
\end{gather*}
$$

Assume the following.

$$
\begin{aligned}
& \forall X 0 .(v 2 \text { _membered } X 0) \Rightarrow\left(k 4 \_ \text {member_ } 1 X 0=\text { ReplSep }(\text { toset }\right. \\
& \left.\left.\lambda X 1: \iota . m 1 \_ \text {subset_1 } X 1 \text { k7_numbers }\right)\right)(\lambda X 1: \iota . X 1 \in X 0)( \\
& \left.\left.\lambda X 1: \iota . k 1 \_m e m b e r \_1 X 1\right)\right)
\end{aligned}
$$

Assume the following.

$$
\begin{gather*}
\forall X 0 .\left(v 2 \_ \text {membered } X 0\right) \Rightarrow(\forall X 1 .(\text { v2_membered } X 1) \Rightarrow(( \\
\left.\left.X 0=X 1) \Leftrightarrow\left(\forall X 2 .\left(v 1 \_x x r e a l \_0 X 2\right) \Rightarrow((X 2 \in X 0) \Leftrightarrow(X 2 \in X 1))\right)\right)\right) \tag{10}
\end{gather*}
$$

Assume the following.

$$
\begin{equation*}
\forall X 0 . \forall X 1 . k 3 \_x b o o l e \_0 X 0 X 1=k 3 \_x b o o l e \_0 X 1 X 0 \tag{11}
\end{equation*}
$$

## Theorem 1

$\forall X 0 .\left(v 2 \_\right.$membered $\left.X 0\right) \Rightarrow\left(\forall X 1 .\left(v 2 \_m e m b e r e d ~ X 1\right) \Rightarrow\left(k 4 \_m e m b e r \_1\right.\right.$ ( $k 3 \_x b o o l e \_0 X 0 X 1$ ) $=k 3 \_x b o o l e \_0\left(k 4 \_m e m b e r \_1 ~ X 0\right) ~\left(k 4 \_m e m b e r \_1\right.$ X1)))

