

t6\_real\_lat (TMd-  
cDW167NG4bvbnBvdQN36K7AVtiJKpXTN)

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Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $k3\_real\_lat : \iota$  be given. Let  $k1\_binop\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_real\_lat : \iota$  be given. Let  $k2\_real\_lat : \iota$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_funct\_2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k2\_zfmisc\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_zfmisc\_1 : \iota \Rightarrow \iota$  be given. Let  $k5\_binop\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $g3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v3\_lattices : \iota \Rightarrow o$  be given. Let  $l2\_lattices : \iota \Rightarrow o$  be given. Let  $u2\_lattices : \iota \Rightarrow \iota$  be given. Let  $l1\_lattices : \iota \Rightarrow o$  be given. Let  $u1\_lattices : \iota \Rightarrow \iota$  be given. Let  $l3\_lattices : \iota \Rightarrow o$  be given. Let  $k1\_numbers : \iota$  be given. Assume the following.

$$\begin{aligned} & \forall X0.(m1\_subset\_1 X0 (u1\_struct\_0 k3\_real\_lat)) \Rightarrow (\forall X1. \\ & (m1\_subset\_1 X1 (u1\_struct\_0 k3\_real\_lat)) \Rightarrow (k1\_binop\_1 k1\_real\_lat \\ & X0 X1 = k1\_binop\_1 k1\_real\_lat X1 X0)) \end{aligned} \quad (1)$$

Assume the following.

$$\begin{aligned} & \forall X0.(m1\_subset\_1 X0 (u1\_struct\_0 k3\_real\_lat)) \Rightarrow (\forall X1. \\ & (m1\_subset\_1 X1 (u1\_struct\_0 k3\_real\_lat)) \Rightarrow (k1\_binop\_1 k2\_real\_lat \\ & X0 X1 = k1\_binop\_1 k2\_real\_lat X1 X0)) \end{aligned} \quad (2)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.(((v1\_funct\_1 X1) \wedge \\ & ((v1\_funct\_2 X1 (k2\_zfmisc\_1 X0 X0) X0) \wedge (m1\_subset\_1 X1 (k1\_zfmisc\_1 \\ & (k2\_zfmisc\_1 (k2\_zfmisc\_1 X0 X0) X0)))))) \wedge ((m1\_subset\_1 X2 X0) \wedge \\ & (m1\_subset\_1 X3 X0))) \Rightarrow (k5\_binop\_1 X0 X1 X2 X3 = k1\_binop\_1 X1 X2 X3) \end{aligned} \quad (3)$$

Assume the following.

$$\begin{aligned} & \forall X0.(m1\_subset\_1 X0 (u1\_struct\_0 k3\_real\_lat)) \Rightarrow (\forall X1. \\ & (m1\_subset\_1 X1 (u1\_struct\_0 k3\_real\_lat)) \Rightarrow (k2\_lattices k3\_real\_lat \\ & X0 (k1\_lattices k3\_real\_lat X0 X1) = X0)) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall X0.(m1\_subset\_1 X0 (u1\_struct\_0 k3\_real\_lat)) \Rightarrow (\forall X1. \\ & (m1\_subset\_1 X1 (u1\_struct\_0 k3\_real\_lat)) \Rightarrow (k1\_lattices k3\_real\_lat \\ & X0 X1 = k1\_lattices k3\_real\_lat X1 X0)) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((v1\_funct\_1 X1) \wedge ((v1\_funct\_2 \\ & X1 (k2\_zfmisc\_1 X0 X0) X0) \wedge (m1\_subset\_1 X1 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\ & (k2\_zfmisc\_1 X0 X0) X0)))))) \wedge ((v1\_funct\_1 X2) \wedge ((v1\_funct\_2 X2 \\ & (k2\_zfmisc\_1 X0 X0) X0) \wedge (m1\_subset\_1 X2 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\ & (k2\_zfmisc\_1 X0 X0) X0)))))) \Rightarrow (\forall X3.\forall X4.\forall X5. \\ & (g3\_lattices X0 X1 X2 = g3\_lattices X3 X4 X5) \Rightarrow ((X0 = X3) \wedge ((X1 = X4) \wedge \\ & (X2 = X5)))) \end{aligned} \quad (6)$$

Assume the following.

$$(\neg v2\_struct\_0 k3\_real\_lat) \wedge (v3\_lattices k3\_real\_lat) \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall X0.(l2\_lattices X0) \Rightarrow ((v1\_funct\_1 (u2\_lattices X0)) \wedge \\ & ((v1\_funct\_2 (u2\_lattices X0) (k2\_zfmisc\_1 (u1\_struct\_0 X0) ( \\ & u1\_struct\_0 X0)) (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 (u2\_lattices \\ & X0) (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 (u1\_struct\_0 X0) ( \\ & u1\_struct\_0 X0)) (u1\_struct\_0 X0)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall X0.(l1\_lattices X0) \Rightarrow ((v1\_funct\_1 (u1\_lattices X0)) \wedge \\ & ((v1\_funct\_2 (u1\_lattices X0) (k2\_zfmisc\_1 (u1\_struct\_0 X0) ( \\ & u1\_struct\_0 X0)) (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 (u1\_lattices \\ & X0) (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 (u1\_struct\_0 X0) ( \\ & u1\_struct\_0 X0)) (u1\_struct\_0 X0)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\forall X0.(l3\_lattices X0) \Rightarrow ((l1\_lattices X0) \wedge (l2\_lattices X0)) \quad (10)$$

Assume the following.

$$(v3\_lattices k3\_real\_lat) \wedge (l3\_lattices k3\_real\_lat) \quad (11)$$

Assume the following.

$$\begin{aligned} & (v1\_funct\_1 k2\_real\_lat) \wedge ((v1\_funct\_2 k2\_real\_lat (k2\_zfmisc\_1 \\ & k1\_numbers k1\_numbers) k1\_numbers) \wedge (m1\_subset\_1 k2\_real\_lat \\ & (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 k1\_numbers k1\_numbers) \\ & k1\_numbers)))) \end{aligned} \quad (12)$$

Assume the following.

$$(v1\_funct\_1\ k1\_real\_lat) \wedge ((v1\_funct\_2\ k1\_real\_lat\ (k2\_zfmisc\_1\ k1\_numbers\ k1\_numbers)\ k1\_numbers) \wedge (m1\_subset\_1\ k1\_real\_lat\ (k1\_zfmisc\_1\ (k2\_zfmisc\_1\ (k2\_zfmisc\_1\ k1\_numbers\ k1\_numbers)\ k1\_numbers)))) \quad (13)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0\ X0) \wedge (l2\_lattices\ X0)) \wedge ((m1\_subset\_1\ X1\ (u1\_struct\_0\ X0)) \wedge (m1\_subset\_1\ X2\ (u1\_struct\_0\ X0)))) \Rightarrow (m1\_subset\_1\ (k1\_lattices\ X0\ X1\ X2)\ (u1\_struct\_0\ X0)) \quad (14)$$

Assume the following.

$$k3\_real\_lat = g3\_lattices\ k1\_numbers\ k2\_real\_lat\ k1\_real\_lat \quad (15)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0\ X0) \wedge (l1\_lattices\ X0)) \Rightarrow (\forall X1.\ (m1\_subset\_1\ X1\ (u1\_struct\_0\ X0)) \Rightarrow (\forall X2.(m1\_subset\_1\ X2\ (u1\_struct\_0\ X0)) \Rightarrow (k2\_lattices\ X0\ X1\ X2 = k5\_binop\_1\ (u1\_struct\_0\ X0)\ (u1\_lattices\ X0)\ X1\ X2)))) \quad (16)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0\ X0) \wedge (l2\_lattices\ X0)) \Rightarrow (\forall X1.\ (m1\_subset\_1\ X1\ (u1\_struct\_0\ X0)) \Rightarrow (\forall X2.(m1\_subset\_1\ X2\ (u1\_struct\_0\ X0)) \Rightarrow (k1\_lattices\ X0\ X1\ X2 = k5\_binop\_1\ (u1\_struct\_0\ X0)\ (u2\_lattices\ X0)\ X1\ X2)))) \quad (17)$$

Assume the following.

$$\forall X0.(l3\_lattices\ X0) \Rightarrow ((v3\_lattices\ X0) \Rightarrow (X0 = g3\_lattices\ (u1\_struct\_0\ X0)\ (u2\_lattices\ X0)\ (u1\_lattices\ X0))) \quad (18)$$

**Theorem 1**

$$\forall X0.(m1\_subset\_1\ X0\ (u1\_struct\_0\ k3\_real\_lat)) \Rightarrow (\forall X1.\ (m1\_subset\_1\ X1\ (u1\_struct\_0\ k3\_real\_lat)) \Rightarrow ((k1\_binop\_1\ k1\_real\_lat\ X0\ (k1\_binop\_1\ k2\_real\_lat\ X0\ X1) = X0) \wedge ((k1\_binop\_1\ k1\_real\_lat\ (k1\_binop\_1\ k2\_real\_lat\ X1\ X0)\ X0 = X0) \wedge ((k1\_binop\_1\ k1\_real\_lat\ X0\ (k1\_binop\_1\ k2\_real\_lat\ X1\ X0) = X0) \wedge (k1\_binop\_1\ k1\_real\_lat\ (k1\_binop\_1\ k2\_real\_lat\ X0\ X1)\ X0 = X0))))))$$