

t72\_pboole  
(TMPdYxuViZ8ZiRGhgVLb8foiLsgQd6L3ZQG)

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Let  $v1\_relat\_1 : \iota \Rightarrow o$  be given. Let  $v4\_relat\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_partfun1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $r6\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k4\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k4\_xboole\_0 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_xboole\_0 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k6\_subset\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_funct\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall X0. \forall X1. \forall X2. k4\_xboole\_0 (k2\_xboole\_0 X0 X1) X2 = k2\_xboole\_0 (k4\_xboole\_0 X0 X2) (k4\_xboole\_0 X1 X2) \quad (1)$$

Assume the following.

$$\forall X0. \forall X1. k6\_subset\_1 X0 X1 = k4\_xboole\_0 X0 X1 \quad (2)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 \\ & X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \wedge ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))))) \Rightarrow \\ & (k2\_pboole X0 X1 X1 = X1) \end{aligned} \quad (3)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 \\ & X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \wedge ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))))) \Rightarrow \\ & ((v1\_relat\_1 (k4\_pboole X0 X1 X2)) \wedge ((v4\_relat\_1 (k4\_pboole X0 \\ & X1 X2) X0) \wedge ((v1\_funct\_1 (k4\_pboole X0 X1 X2)) \wedge (v1\_partfun1 (k4\_pboole \\ & X0 X1 X2) X0)))) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 \\ & X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \wedge ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & ((v1\_relat\_1 (k2\_pboole X0 X1 X2)) \wedge ((v4\_relat\_1 (k2\_pboole X0 \\ & X1 X2) X0) \wedge ((v1\_funct\_1 (k2\_pboole X0 X1 X2)) \wedge (v1\_partfun1 (k2\_pboole \\ & X0 X1 X2) X0)))) \end{aligned} \tag{5}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge ( \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & (\forall X3. ((v1\_relat\_1 X3) \wedge ((v4\_relat\_1 X3 X0) \wedge ((v1\_funct\_1 \\ & X3) \wedge (v1\_partfun1 X3 X0)))) \Rightarrow ((X3 = k4\_pboole X0 X1 X2) \Leftrightarrow (\forall X4. \\ & (X4 \in X0) \Rightarrow (k1\_funct\_1 X3 X4 = k6\_subset\_1 (k1\_funct\_1 X1 X4) (k1\_funct\_1 \\ & X2 X4)))))) \end{aligned} \tag{6}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge ( \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & (\forall X3. ((v1\_relat\_1 X3) \wedge ((v4\_relat\_1 X3 X0) \wedge ((v1\_funct\_1 \\ & X3) \wedge (v1\_partfun1 X3 X0)))) \Rightarrow ((X3 = k2\_pboole X0 X1 X2) \Leftrightarrow (\forall X4. \\ & (X4 \in X0) \Rightarrow (k1\_funct\_1 X3 X4 = k2\_xboole\_0 (k1\_funct\_1 X1 X4) (k1\_funct\_1 \\ & X2 X4)))))) \end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge ( \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & ((r6\_pboole X0 X1 X2) \Leftrightarrow (\forall X3. (X3 \in X0) \Rightarrow (k1\_funct\_1 X1 X3 = k1\_funct\_1 \\ & X2 X3)))) \end{aligned} \tag{8}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 \\ & X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \wedge ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & (k2\_pboole X0 X1 X2 = k2\_pboole X0 X2 X1) \end{aligned} \tag{9}$$

**Theorem 1**

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & (\forall X3. ((v1\_relat\_1 X3) \wedge ((v4\_relat\_1 X3 X0) \wedge ((v1\_funct\_1 \\ & X3) \wedge (v1\_partfun1 X3 X0)))) \Rightarrow (r6\_pboole X0 (k4\_pboole X0 (k2\_pboole \\ & X0 X1 X2) X3) (k2\_pboole X0 (k4\_pboole X0 X1 X3) (k4\_pboole X0 X2 X3)))))) \end{aligned}$$