# t73_member_1 <br> (TMFujQ23Cr8N2fkLWz19HR3tjcKfWuG6cuH) 

October 27, 2020

Let $v 1 \_$membered $: \iota \Rightarrow 0$ be given. Let $k 11 \_$member_1 $: \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $k 9 \_m e m b e r \_1: \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $k 5$ _member_1 : $\iota \Rightarrow \iota$ be given. Assume the following.
$\forall X 0 .\left(v 1 \_m e m b e r e d ~ X 0\right) \Rightarrow\left(\forall X 1 .\left(v 1 \_m e m b e r e d ~ X 1\right) \Rightarrow(\forall X 2\right.$.
(v1_membered $X 2) \Rightarrow\left(k 9 \_m e m b e r \_1 ~ X 0\left(k 11 \_m e m b e r \_1 ~ X 1 ~ X 2\right) ~=~ k 11 \_m e m b e r \_1 ~\right.$ (k9_member_1 X0 X1) X2)))

Assume the following.
$\forall X 0$. $\left(v 1 \_\right.$membered $\left.X 0\right) \Rightarrow\left(\forall X 1\right.$. $\left(v 1 \_m e m b e r e d ~ X 1\right) \Rightarrow\left(k 5 \_m e m b e r \_1\right.$
(k9_member_1 X0 X1) = k9_member_1 (k5_member_1 X0) (k5_member_1
X1)))
Assume the following.

$$
\begin{gather*}
\forall X 0 . \forall X 1 .\left(\left(v 1 \_m e m b e r e d X 0\right) \wedge\left(v 1 \_m e m b e r e d ~ X 1\right)\right) \Rightarrow( \\
\left.v 1 \_m e m b e r e d\left(k 9 \_m e m b e r \_1 X 0 X 1\right)\right) \tag{3}
\end{gather*}
$$

Assume the following.

$$
\begin{equation*}
\forall X 0 .\left(v 1 \_m e m b e r e d ~ X 0\right) \Rightarrow\left(v 1 \_m e m b e r e d\left(k 5 \_m e m b e r \_1 X 0\right)\right) \tag{4}
\end{equation*}
$$

Assume the following.

$$
\begin{gather*}
\forall X 0 .\left(v 1 \_m e m b e r e d \quad X 0\right) \Rightarrow\left(\forall X 1 .\left(v 1 \_m e m b e r e d ~ X 1\right) \Rightarrow\left(k 11 \_m e m b e r \_1\right.\right. \\
\left.\left.X 0 X 1=k 9 \_m e m b e r \_1 X 0\left(k 5 \_m e m b e r \_1 X 1\right)\right)\right) \tag{5}
\end{gather*}
$$

## Theorem 1

$\forall X 0 .\left(v 1 \_m e m b e r e d \quad X 0\right) \Rightarrow\left(\forall X 1 .\left(v 1 \_m e m b e r e d ~ X 1\right) \Rightarrow(\forall X 2\right.$.
(v1_membered $X 2) \Rightarrow\left(k 11 \_m e m b e r \_1 ~ X 0\left(k 9 \_m e m b e r \_1 ~ X 1 ~ X 2\right) ~=~ k 11 \_m e m b e r \_1 ~\right.$ (k11_member_1 X0 X1) X2)))

