

t76\_filter\_2 (TMWX-  
uWRhhZrTNx7rrDaEL8E59vHpEWAbEFT)

October 27, 2020

Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v10\_lattices : \iota \Rightarrow o$  be given. Let  $l3\_lattices : \iota \Rightarrow o$  be given. Let  $v1\_xboole\_0 : \iota \Rightarrow o$  be given. Let  $v20\_lattices : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v21\_lattices : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_zfmisc\_1 : \iota \Rightarrow \iota$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $v12\_lattices : \iota \Rightarrow o$  be given. Let  $k10\_filter\_2 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k4\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $m2\_nat\_lat : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v6\_lattices : \iota \Rightarrow o$  be given. Let  $v8\_lattices : \iota \Rightarrow o$  be given. Let  $v9\_lattices : \iota \Rightarrow o$  be given. Let  $r1\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $l1\_lattices : \iota \Rightarrow o$  be given. Let  $k2\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v4\_lattices : \iota \Rightarrow o$  be given. Let  $l2\_lattices : \iota \Rightarrow o$  be given. Let  $k1\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v3\_lattices : \iota \Rightarrow o$  be given. Let  $k5\_binop\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $u2\_lattices : \iota \Rightarrow \iota$  be given. Let  $v5\_lattices : \iota \Rightarrow o$  be given. Let  $v7\_lattices : \iota \Rightarrow o$  be given. Let  $g3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $u1\_lattices : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned}
 & \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\
 & X0))) \Rightarrow (\forall X1. ((\neg v1\_xboole\_0 X1) \wedge ((v20\_lattices X1 X0) \wedge \\
 & ((v21\_lattices X1 X0) \wedge (m1\_subset\_1 X1 (k1\_zfmisc\_1 (u1\_struct\_0 \\
 & X0)))))) \Rightarrow (\forall X2. (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (\forall X3. \\
 & (m1\_subset\_1 X3 (u1\_struct\_0 X0)) \Rightarrow (\forall X4. (m1\_subset\_1 X4 \\
 & (u1\_struct\_0 (k10\_filter\_2 X0 X1))) \Rightarrow (\forall X5. (m1\_subset\_1 \\
 & X5 (u1\_struct\_0 (k10\_filter\_2 X0 X1))) \Rightarrow (((X2 = X4) \wedge (X3 = X5)) \Rightarrow ( \\
 & (r3\_lattices X0 X2 X3) \Leftrightarrow (r3\_lattices (k10\_filter\_2 X0 X1) X4 X5)))))))))
 \end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\
& X0))) \Rightarrow (\forall X1.((\neg v1\_xboole\_0 X1) \wedge ((v20\_lattices X1 X0) \wedge \\
& ((v21\_lattices X1 X0) \wedge (m1\_subset\_1 X1 (k1\_zfmisc\_1 (u1\_struct\_0 \\
& X0)))))) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (\forall X3. \\
& (m1\_subset\_1 X3 (u1\_struct\_0 X0)) \Rightarrow (\forall X4.(m1\_subset\_1 X4 \\
& (u1\_struct\_0 (k10\_filter\_2 X0 X1))) \Rightarrow (\forall X5.(m1\_subset\_1 \\
& X5 (u1\_struct\_0 (k10\_filter\_2 X0 X1))) \Rightarrow (((X2 = X4) \wedge (X3 = X5)) \Rightarrow ( \\
& (k3\_lattices X0 X2 X3 = k3\_lattices (k10\_filter\_2 X0 X1) X4 X5) \wedge ( \\
& k4\_lattices X0 X2 X3 = k4\_lattices (k10\_filter\_2 X0 X1) X4 X5))))))))) \tag{2}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\
& X0))) \Rightarrow (\forall X1.(m2\_nat\_lat X1 X0) \Rightarrow (\forall X2.(m1\_subset\_1 \\
& X2 (u1\_struct\_0 X1)) \Rightarrow (m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \tag{3}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0) \wedge ((v6\_lattices \\
& X0) \wedge ((v8\_lattices X0) \wedge ((v9\_lattices X0) \wedge (l3\_lattices X0)))))) \wedge \\
& ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 X2 (u1\_struct\_0 \\
& X0))) \Rightarrow ((r3\_lattices X0 X1 X2) \Leftrightarrow (r1\_lattices X0 X1 X2)) \tag{4}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0) \wedge ((v6\_lattices \\
& X0) \wedge (l1\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge ( \\
& m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow (k4\_lattices X0 X1 X2 = k2\_lattices \\
& X0 X1 X2) \tag{5}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0) \wedge ((v4\_lattices \\
& X0) \wedge (l2\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge ( \\
& m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow (k3\_lattices X0 X1 X2 = k1\_lattices \\
& X0 X1 X2) \tag{6}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall X0.\forall X1.(((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge \\
& (l3\_lattices X0))) \wedge ((\neg v1\_xboole\_0 X1) \wedge ((v20\_lattices X1 X0) \wedge \\
& ((v21\_lattices X1 X0) \wedge (m1\_subset\_1 X1 (k1\_zfmisc\_1 (u1\_struct\_0 \\
& X0)))))) \Rightarrow (v3\_lattices (k10\_filter\_2 X0 X1)) \tag{7}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\
& X0))) \Rightarrow (\forall X1.(m2\_nat\_lat X1 X0) \Rightarrow ((\neg v2\_struct\_0 X1) \wedge ((v10\_lattices \\
& X1) \wedge (l3\_lattices X1)))) \tag{8}
\end{aligned}$$

Assume the following.

$$\forall X0.(l3\_lattices\ X0)\Rightarrow((l1\_lattices\ X0)\wedge(l2\_lattices\ X0)) \quad (9)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0\ X0)\wedge((v6\_lattices \\ X0)\wedge(l1\_lattices\ X0)))\wedge((m1\_subset\_1\ X1\ (u1\_struct\_0\ X0))\wedge \\ m1\_subset\_1\ X2\ (u1\_struct\_0\ X0)))\Rightarrow(m1\_subset\_1\ (k4\_lattices \\ X0\ X1\ X2)\ (u1\_struct\_0\ X0)) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0\ X0)\wedge(l2\_lattices \\ X0))\wedge((m1\_subset\_1\ X1\ (u1\_struct\_0\ X0))\wedge(m1\_subset\_1\ X2\ (u1\_struct\_0 \\ X0))))\Rightarrow(m1\_subset\_1\ (k1\_lattices\ X0\ X1\ X2)\ (u1\_struct\_0\ X0)) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.(((\neg v2\_struct\_0\ X0)\wedge((v10\_lattices\ X0)\wedge \\ (l3\_lattices\ X0)))\wedge((\neg v1\_xboole\_0\ X1)\wedge((v20\_lattices\ X1\ X0)\wedge \\ ((v21\_lattices\ X1\ X0)\wedge(m1\_subset\_1\ X1\ (k1\_zfmisc\_1\ (u1\_struct\_0 \\ X0))))))\Rightarrow(m2\_nat\_lat\ (k10\_filter\_2\ X0\ X1)\ X0) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0\ X0)\wedge(l2\_lattices\ X0))\Rightarrow(\forall X1. \\ (m1\_subset\_1\ X1\ (u1\_struct\_0\ X0))\Rightarrow(\forall X2.(m1\_subset\_1\ X2 \\ (u1\_struct\_0\ X0))\Rightarrow(k1\_lattices\ X0\ X1\ X2 = k5\_binop\_1\ (u1\_struct\_0 \\ X0)\ (u2\_lattices\ X0)\ X1\ X2))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0\ X0)\wedge(l3\_lattices\ X0))\Rightarrow((v12\_lattices \\ X0)\Leftrightarrow(\forall X1.(m1\_subset\_1\ X1\ (u1\_struct\_0\ X0))\Rightarrow(\forall X2. \\ (m1\_subset\_1\ X2\ (u1\_struct\_0\ X0))\Rightarrow(\forall X3.(m1\_subset\_1\ X3 \\ (u1\_struct\_0\ X0))\Rightarrow((r1\_lattices\ X0\ X1\ X3)\Rightarrow(k1\_lattices\ X0\ X1\ ( \\ k2\_lattices\ X0\ X2\ X3) = k2\_lattices\ X0\ (k1\_lattices\ X0\ X1\ X2)\ X3)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0\ X0)\wedge((v6\_lattices \\ X0)\wedge(l1\_lattices\ X0)))\wedge((m1\_subset\_1\ X1\ (u1\_struct\_0\ X0))\wedge \\ m1\_subset\_1\ X2\ (u1\_struct\_0\ X0)))\Rightarrow(k4\_lattices\ X0\ X1\ X2 = k4\_lattices \\ X0\ X2\ X1) \end{aligned} \quad (15)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge(v4\_lattices X0)\wedge(l2\_lattices X0))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow(k3\_lattices X0 X1 X2 = k3\_lattices X0 X2 X1) \quad (16)$$

Assume the following.

$$\forall X0.(l3\_lattices X0)\Rightarrow(((\neg v2\_struct\_0 X0)\wedge(v10\_lattices X0))\Rightarrow((\neg v2\_struct\_0 X0)\wedge(v4\_lattices X0)\wedge(v5\_lattices X0)\wedge(v6\_lattices X0)\wedge(v7\_lattices X0)\wedge(v8\_lattices X0)\wedge(v9\_lattices X0)))) \quad (17)$$

Assume the following.

$$\forall X0.(l3\_lattices X0)\Rightarrow((v3\_lattices X0)\Rightarrow(X0 = g3\_lattices (u1\_struct\_0 X0) (u2\_lattices X0) (u1\_lattices X0))) \quad (18)$$

**Theorem 1**

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge((v10\_lattices X0)\wedge(l3\_lattices X0)))\Rightarrow(\forall X1.((\neg v1\_xboole\_0 X1)\wedge((v20\_lattices X1 X0)\wedge((v21\_lattices X1 X0)\wedge(m1\_subset\_1 X1 (k1\_zfmisc\_1 (u1\_struct\_0 X0))))))\Rightarrow((v12\_lattices X0)\Rightarrow(v12\_lattices (k10\_filter\_2 X0 X1))))$$