

t76\_pboole (TMFMHhP-  
KDqLXG1QTz97wxDCSgdcb7RjKcVE)

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Let  $v1\_relat\_1 : \iota \Rightarrow o$  be given. Let  $v4\_relat\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_partfun1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $r2\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k2\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k4\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r6\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & (r6\_pboole X0 (k4\_pboole X0 (k2\_pboole X0 X1 X2) X2) (k4\_pboole X0 \\ & X1 X2))) \end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & (r2\_pboole X0 (k4\_pboole X0 X1 X2) X1)) \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & (\forall X3. ((v1\_relat\_1 X3) \wedge ((v4\_relat\_1 X3 X0) \wedge ((v1\_funct\_1 \\ & X3) \wedge (v1\_partfun1 X3 X0)))) \Rightarrow ((r2\_pboole X0 X1 X2) \Rightarrow (r2\_pboole X0 \\ & (k4\_pboole X0 X1 X3) (k4\_pboole X0 X2 X3)))) \end{aligned} \tag{3}$$

Assume the following.

$$\begin{aligned}
& \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\
& (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\
& X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\
& (\forall X3. ((v1\_relat\_1 X3) \wedge ((v4\_relat\_1 X3 X0) \wedge ((v1\_funct\_1 \\
& X3) \wedge (v1\_partfun1 X3 X0)))) \Rightarrow (((r2\_pboole X0 X1 X2) \wedge (r2\_pboole \\
& X0 X2 X3)) \Rightarrow (r2\_pboole X0 X1 X3)))
\end{aligned} \tag{4}$$

Assume the following.

$$\begin{aligned}
& \forall X0. \forall X1. \forall X2. (((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 \\
& X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \wedge ((v1\_relat\_1 \\
& X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\
& ((r6\_pboole X0 X1 X2) \Leftrightarrow (X1 = X2))
\end{aligned} \tag{5}$$

Assume the following.

$$\begin{aligned}
& \forall X0. \forall X1. \forall X2. (((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 \\
& X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \wedge ((v1\_relat\_1 \\
& X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\
& ((v1\_relat\_1 (k4\_pboole X0 X1 X2)) \wedge ((v4\_relat\_1 (k4\_pboole X0 \\
& X1 X2) X0) \wedge ((v1\_funct\_1 (k4\_pboole X0 X1 X2)) \wedge (v1\_partfun1 (k4\_pboole \\
& X0 X1 X2) X0))))
\end{aligned} \tag{6}$$

Assume the following.

$$\begin{aligned}
& \forall X0. \forall X1. \forall X2. (((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 \\
& X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \wedge ((v1\_relat\_1 \\
& X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\
& ((v1\_relat\_1 (k2\_pboole X0 X1 X2)) \wedge ((v4\_relat\_1 (k2\_pboole X0 \\
& X1 X2) X0) \wedge ((v1\_funct\_1 (k2\_pboole X0 X1 X2)) \wedge (v1\_partfun1 (k2\_pboole \\
& X0 X1 X2) X0))))
\end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned}
& \forall X0. \forall X1. \forall X2. (((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 \\
& X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \wedge ((v1\_relat\_1 \\
& X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\
& (k2\_pboole X0 X1 X2 = k2\_pboole X0 X2 X1)
\end{aligned} \tag{8}$$

**Theorem 1**

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & (\forall X3. ((v1\_relat\_1 X3) \wedge ((v4\_relat\_1 X3 X0) \wedge ((v1\_funct\_1 \\ & X3) \wedge (v1\_partfun1 X3 X0)))) \Rightarrow ((r2\_pboole X0 X1 (k2\_pboole X0 X2 X3)) \Rightarrow \\ & ((r2\_pboole X0 (k4\_pboole X0 X1 X2) X3) \wedge (r2\_pboole X0 (k4\_pboole \\ & X0 X1 X3) X2)))))) \end{aligned}$$