

t7_pboole (TMPTtF-
bqQW5ThABhrmuFsUEMZy9RydMLEnd)

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Let $v1_relat_1 : \iota \Rightarrow o$ be given. Let $v4_relat_1 : \iota \Rightarrow \iota \Rightarrow o$ be given. Let $v1_funct_1 : \iota \Rightarrow o$ be given. Let $v1_partfun1 : \iota \Rightarrow \iota \Rightarrow o$ be given. Let $r1_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$ be given. Let $k2_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $k1_funct_1 : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $k2_xboole_0 : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1_relat_1 X1) \wedge ((v4_relat_1 \\ & X1 X0) \wedge ((v1_funct_1 X1) \wedge (v1_partfun1 X1 X0)))) \wedge ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))))) \Rightarrow \\ & ((v1_relat_1 (k2_pboole X0 X1 X2)) \wedge ((v4_relat_1 (k2_pboole X0 \\ & X1 X2) X0) \wedge ((v1_funct_1 (k2_pboole X0 X1 X2)) \wedge (v1_partfun1 (k2_pboole \\ & X0 X1 X2) X0)))) \end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge (\\ & (v1_funct_1 X1) \wedge (v1_partfun1 X1 X0)))) \Rightarrow (\forall X2. ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\ & (\forall X3. ((v1_relat_1 X3) \wedge ((v4_relat_1 X3 X0) \wedge ((v1_funct_1 \\ & X3) \wedge (v1_partfun1 X3 X0)))) \Rightarrow ((X3 = k2_pboole X0 X1 X2) \Leftrightarrow (\forall X4. \\ & (X4 \in X0) \Rightarrow (k1_funct_1 X3 X4 = k2_xboole_0 (k1_funct_1 X1 X4) (k1_funct_1 \\ & X2 X4)))))) \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (X2 = k2_xboole_0 X0 X1) \Leftrightarrow (\forall X3. \\ & (X3 \in X2) \Leftrightarrow ((X3 \in X0) \vee (X3 \in X1))) \end{aligned} \tag{3}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge (\\ & (v1_funct_1 X1) \wedge (v1_partfun1 X1 X0)))) \Rightarrow (\forall X2. ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\ & ((r1_pboole X0 X1 X2) \Leftrightarrow (\forall X3. (X3 \in X0) \Rightarrow (k1_funct_1 X1 X3 \in k1_funct_1 \\ & X2 X3)))) \end{aligned} \tag{4}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1_relat_1 X1) \wedge ((v4_relat_1 \\ & X1 X0) \wedge ((v1_funct_1 X1) \wedge (v1_partfun1 X1 X0)))) \wedge ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))))) \Rightarrow \\ & (k2_pboole X0 X1 X2 = k2_pboole X0 X2 X1) \end{aligned} \tag{5}$$

Theorem 1

$$\begin{aligned} & \forall X0. \forall X1. ((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge \\ & (v1_funct_1 X1) \wedge (v1_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\ & (\forall X3. ((v1_relat_1 X3) \wedge ((v4_relat_1 X3 X0) \wedge ((v1_funct_1 \\ & X3) \wedge (v1_partfun1 X3 X0)))) \Rightarrow (((r1_pboole X0 X1 X2) \vee (r1_pboole \\ & X0 X1 X3)) \Rightarrow (r1_pboole X0 X1 (k2_pboole X0 X2 X3)))))) \end{aligned}$$