

t87_pboole (TMTYwC- cWcm6fuGNLcTBd82EPiAyC2uKxZ7U)

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Let $v1_relat_1 : \iota \Rightarrow o$ be given. Let $v4_relat_1 : \iota \Rightarrow \iota \Rightarrow o$ be given. Let $v1_funct_1 : \iota \Rightarrow o$ be given. Let $v1_partfun1 : \iota \Rightarrow \iota \Rightarrow o$ be given. Let $r6_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$ be given. Let $k5_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $k4_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $k2_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $k3_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge \\ & (v1_funct_1 X1) \wedge (v1_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\ & (r6_pboole X0 (k4_pboole X0 (k2_pboole X0 X1 X2) (k3_pboole X0 X1 \\ & X2)) (k2_pboole X0 (k4_pboole X0 X1 X2) (k4_pboole X0 X2 X1)))) \end{aligned} \quad (1)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1_relat_1 X1) \wedge ((v4_relat_1 \\ & X1 X0) \wedge ((v1_funct_1 X1) \wedge (v1_partfun1 X1 X0)))) \wedge ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\ & ((r6_pboole X0 X1 X2) \Rightarrow (r6_pboole X0 X2 X1)) \end{aligned} \quad (2)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1_relat_1 X1) \wedge ((v4_relat_1 \\ & X1 X0) \wedge ((v1_funct_1 X1) \wedge (v1_partfun1 X1 X0)))) \wedge ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\ & ((v1_relat_1 (k4_pboole X0 X1 X2)) \wedge ((v4_relat_1 (k4_pboole X0 \\ & X1 X2) X0) \wedge ((v1_funct_1 (k4_pboole X0 X1 X2)) \wedge (v1_partfun1 (k4_pboole \\ & X0 X1 X2) X0)))) \end{aligned} \quad (3)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((v1_relat_1 X1)\wedge((v4_relat_1 \\ & X1 X0)\wedge((v1_funct_1 X1)\wedge(v1_partfun1 X1 X0))))\wedge((v1_relat_1 \\ & X2)\wedge((v4_relat_1 X2 X0)\wedge((v1_funct_1 X2)\wedge(v1_partfun1 X2 X0))))\Rightarrow \\ & ((v1_relat_1 (k3_pboole X0 X1 X2))\wedge((v4_relat_1 (k3_pboole X0 \\ & X1 X2) X0)\wedge((v1_funct_1 (k3_pboole X0 X1 X2))\wedge(v1_partfun1 (k3_pboole \\ & X0 X1 X2) X0)))) \end{aligned} \tag{4}$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((v1_relat_1 X1)\wedge((v4_relat_1 \\ & X1 X0)\wedge((v1_funct_1 X1)\wedge(v1_partfun1 X1 X0))))\wedge((v1_relat_1 \\ & X2)\wedge((v4_relat_1 X2 X0)\wedge((v1_funct_1 X2)\wedge(v1_partfun1 X2 X0))))\Rightarrow \\ & ((v1_relat_1 (k2_pboole X0 X1 X2))\wedge((v4_relat_1 (k2_pboole X0 \\ & X1 X2) X0)\wedge((v1_funct_1 (k2_pboole X0 X1 X2))\wedge(v1_partfun1 (k2_pboole \\ & X0 X1 X2) X0)))) \end{aligned} \tag{5}$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.((v1_relat_1 X1)\wedge((v4_relat_1 X1 X0)\wedge(\\ & (v1_funct_1 X1)\wedge(v1_partfun1 X1 X0))))\Rightarrow(\forall X2.((v1_relat_1 \\ & X2)\wedge((v4_relat_1 X2 X0)\wedge((v1_funct_1 X2)\wedge(v1_partfun1 X2 X0))))\Rightarrow \\ & (k5_pboole X0 X1 X2 = k2_pboole X0 (k4_pboole X0 X1 X2) (k4_pboole \\ & X0 X2 X1))) \end{aligned} \tag{6}$$

Theorem 1

$$\begin{aligned} & \forall X0.\forall X1.((v1_relat_1 X1)\wedge((v4_relat_1 X1 X0)\wedge(\\ & (v1_funct_1 X1)\wedge(v1_partfun1 X1 X0))))\Rightarrow(\forall X2.((v1_relat_1 \\ & X2)\wedge((v4_relat_1 X2 X0)\wedge((v1_funct_1 X2)\wedge(v1_partfun1 X2 X0))))\Rightarrow \\ & (r6_pboole X0 (k5_pboole X0 X1 X2) (k4_pboole X0 (k2_pboole X0 X1 \\ & X2) (k3_pboole X0 X1 X2)))) \end{aligned}$$