

# t89\_sublemma (TMHFnytMCCX- cNT4v44jQK1zaBexYMjfdvyN)

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Let  $m1\_qc\_lang1 : \iota \Rightarrow o$  be given. Let  $v1\_xboole\_0 : \iota \Rightarrow o$  be given. Let  $m1\_valuat\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $m2\_subset\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k16\_subst1 : \iota \Rightarrow \iota$  be given. Let  $k38\_subst1 : \iota \Rightarrow \iota$  be given. Let  $m2\_funct\_2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k3\_qc\_lang1 : \iota \Rightarrow \iota$  be given. Let  $k2\_valuat\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r1\_valuat\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k39\_subst1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r1\_sublemma : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_sublemma : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k3\_sublemma : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_qc\_lang1 : \iota \Rightarrow \iota$  be given. Let  $m1\_subst1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k7\_sublemma : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v3\_subst1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k9\_sublemma : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v2\_subst1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k19\_subst1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k6\_sublemma : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k5\_sublemma : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k5\_numbers : \iota$  be given. Let  $k6\_qc\_lang1 : \iota \Rightarrow \iota$  be given. Let  $k8\_qc\_lang1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v5\_relat\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v3\_card\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $m2\_finseq\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_subst1 : \iota \Rightarrow \iota$  be given. Let  $k4\_sublemma : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned}
& \forall X0.(m1\_qc\_lang1 X0) \Rightarrow (\forall X1.(m2\_subset\_1 X1 (k2\_qc\_lang1 \\
& \quad X0) (k3\_qc\_lang1 X0)) \Rightarrow (\forall X2.(\neg v1\_xboole\_0 X2) \Rightarrow (\forall X3. \\
& \quad (m1\_valuat\_1 X3 X0 X2) \Rightarrow (\forall X4.(m2\_subset\_1 X4 (k16\_subst1 \\
& \quad X0) (k38\_subst1 X0)) \Rightarrow (\forall X5.(m1\_subst1 X5 X0 (k7\_sublemma \\
& \quad X0 X4 X1)) \Rightarrow (((v3\_subst1 (k7\_sublemma X0 X4 X1) X0) \wedge (\forall X6. \\
& \quad (m2\_funct\_2 X6 (k3\_qc\_lang1 X0) X2 (k2\_valuat\_1 X0 X2)) \Rightarrow ((r1\_valuat\_1 \\
& \quad X0 X2 (k39\_subst1 X0 X4) X3 X6) \Leftrightarrow (r1\_sublemma X0 X4 X2 (k1\_sublemma \\
& \quad X0 X2 X6 (k3\_sublemma X0 X4 X2 X6)) X3)))) \Rightarrow (\forall X6.(m2\_funct\_2 \\
& \quad X6 (k3\_qc\_lang1 X0) X2 (k2\_valuat\_1 X0 X2)) \Rightarrow ((r1\_valuat\_1 X0 X2 \\
& \quad (k39\_subst1 X0 (k9\_sublemma X0 (k7\_sublemma X0 X4 X1) X5)) X3 X6) \Leftrightarrow \\
& \quad (r1\_sublemma X0 (k9\_sublemma X0 (k7\_sublemma X0 X4 X1) X5) X2 (k1\_sublemma \\
& \quad X0 X2 X6 (k3\_sublemma X0 (k9\_sublemma X0 (k7\_sublemma X0 X4 X1) X5) \\
& \quad \quad X2 X6)) X3)))))))))
\end{aligned}$$

(1)

Assume the following.

$$\begin{aligned}
& \forall X0.(m1\_qc\_lang1\ X0) \Rightarrow (\forall X1.(\neg v1\_xboole\_0\ X1) \Rightarrow ( \\
& \quad \forall X2.(m1\_valuat\_1\ X2\ X0\ X1) \Rightarrow (\forall X3.(m2\_subset\_1\ X3 \\
& \quad (k16\_subst1\ X0)\ (k38\_subst1\ X0)) \Rightarrow ((v2\_subst1\ X3\ X0) \Rightarrow (\forall X4. \\
& \quad (m2\_funct\_2\ X4\ (k3\_qc\_lang1\ X0)\ X1\ (k2\_valuat\_1\ X0\ X1)) \Rightarrow ((r1\_valuat\_1 \\
& \quad X0\ X1\ (k39\_subst1\ X0\ X3)\ X2\ X4) \Leftrightarrow (r1\_sublemma\ X0\ X3\ X1\ (k1\_sublemma \\
& \quad X0\ X1\ X4\ (k3\_sublemma\ X0\ X3\ X1\ X4))\ X2))))))
\end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned}
& \forall X0.(m1\_qc\_lang1\ X0) \Rightarrow (\forall X1.(\neg v1\_xboole\_0\ X1) \Rightarrow ( \\
& \quad \forall X2.(m1\_valuat\_1\ X2\ X0\ X1) \Rightarrow (\forall X3.(m2\_subset\_1\ X3 \\
& \quad (k16\_subst1\ X0)\ (k38\_subst1\ X0)) \Rightarrow (\forall X4.(m2\_subset\_1 \\
& \quad X4\ (k16\_subst1\ X0)\ (k38\_subst1\ X0)) \Rightarrow (((k19\_subst1\ X0\ X3 = \\
& \quad k19\_subst1\ X0\ X4) \wedge ((\forall X5.(m2\_funct\_2\ X5\ (k3\_qc\_lang1 \\
& \quad X0)\ X1\ (k2\_valuat\_1\ X0\ X1)) \Rightarrow ((r1\_valuat\_1\ X0\ X1\ (k39\_subst1\ X0 \\
& \quad X3)\ X2\ X5) \Leftrightarrow (r1\_sublemma\ X0\ X3\ X1\ (k1\_sublemma\ X0\ X1\ X5\ (k3\_sublemma \\
& \quad X0\ X3\ X1\ X5))\ X2)))) \wedge (\forall X5.(m2\_funct\_2\ X5\ (k3\_qc\_lang1\ X0) \\
& \quad X1\ (k2\_valuat\_1\ X0\ X1)) \Rightarrow ((r1\_valuat\_1\ X0\ X1\ (k39\_subst1\ X0\ X4) \\
& \quad X2\ X5) \Leftrightarrow (r1\_sublemma\ X0\ X4\ X1\ (k1\_sublemma\ X0\ X1\ X5\ (k3\_sublemma\ X0 \\
& \quad X4\ X1\ X5))\ X2)))))) \Rightarrow (\forall X5.(m2\_funct\_2\ X5\ (k3\_qc\_lang1\ X0) \\
& \quad X1\ (k2\_valuat\_1\ X0\ X1)) \Rightarrow ((r1\_valuat\_1\ X0\ X1\ (k39\_subst1\ X0\ (k6\_sublemma \\
& \quad X0\ X3\ X4))\ X2\ X5) \Leftrightarrow (r1\_sublemma\ X0\ (k6\_sublemma\ X0\ X3\ X4)\ X1\ (k1\_sublemma \\
& \quad X0\ X1\ X5\ (k3\_sublemma\ X0\ (k6\_sublemma\ X0\ X3\ X4)\ X1\ X5))\ X2))))))
\end{aligned} \tag{3}$$

Assume the following.

$$\begin{aligned}
& \forall X0.(m1\_qc\_lang1\ X0) \Rightarrow (\forall X1.(\neg v1\_xboole\_0\ X1) \Rightarrow ( \\
& \quad \forall X2.(m1\_valuat\_1\ X2\ X0\ X1) \Rightarrow (\forall X3.(m2\_subset\_1\ X3 \\
& \quad (k16\_subst1\ X0)\ (k38\_subst1\ X0)) \Rightarrow ((\forall X4.(m2\_funct\_2 \\
& \quad X4\ (k3\_qc\_lang1\ X0)\ X1\ (k2\_valuat\_1\ X0\ X1)) \Rightarrow ((r1\_valuat\_1\ X0\ X1 \\
& \quad (k39\_subst1\ X0\ X3)\ X2\ X4) \Leftrightarrow (r1\_sublemma\ X0\ X3\ X1\ (k1\_sublemma\ X0 \\
& \quad X1\ X4\ (k3\_sublemma\ X0\ X3\ X1\ X4))\ X2)))) \Rightarrow (\forall X4.(m2\_funct\_2\ X4 \\
& \quad (k3\_qc\_lang1\ X0)\ X1\ (k2\_valuat\_1\ X0\ X1)) \Rightarrow ((r1\_valuat\_1\ X0\ X1\ (k39\_subst1 \\
& \quad X0\ (k5\_sublemma\ X0\ X3))\ X2\ X4) \Leftrightarrow (r1\_sublemma\ X0\ (k5\_sublemma\ X0\ X3) \\
& \quad X1\ (k1\_sublemma\ X0\ X1\ X4\ (k3\_sublemma\ X0\ (k5\_sublemma\ X0\ X3)\ X1\ X4)) \\
& \quad X2))))))
\end{aligned} \tag{4}$$

Assume the following.

$$\begin{aligned}
& \forall X0.(m1\_qc\_lang1\ X0) \Rightarrow (\forall X1.(m1\_subset\_1\ X1\ k5\_numbers) \Rightarrow \\
& \quad (\forall X2.(\neg v1\_xboole\_0\ X2) \Rightarrow (\forall X3.(m1\_valuat\_1\ X3\ X0 \\
& \quad X2) \Rightarrow (\forall X4.(m2\_subset\_1\ X4\ (k6\_qc\_lang1\ X0)\ (k8\_qc\_lang1 \\
& \quad X0\ X1)) \Rightarrow (\forall X5.((v5\_relat\_1\ X5\ (k3\_qc\_lang1\ X0)) \wedge ((v3\_card\_1 \\
& \quad X5\ X1) \wedge (m2\_finseq\_1\ X5\ (k2\_qc\_lang1\ X0)))) \Rightarrow (\forall X6.(m1\_subset\_1 \\
& \quad X6\ (k1\_subst1\ X0)) \Rightarrow (\forall X7.(m2\_funct\_2\ X7\ (k3\_qc\_lang1 \\
& \quad X0)\ X2\ (k2\_valuat\_1\ X0\ X2)) \Rightarrow ((r1\_valuat\_1\ X0\ X2\ (k39\_subst1\ X0 \\
& \quad (k4\_sublemma\ X1\ X0\ X4\ X5\ X6))\ X3\ X7) \Leftrightarrow (r1\_sublemma\ X0\ (k4\_sublemma \\
& \quad X1\ X0\ X4\ X5\ X6)\ X2\ (k1\_sublemma\ X0\ X2\ X7\ (k3\_sublemma\ X0\ (k4\_sublemma \\
& \quad X1\ X0\ X4\ X5\ X6)\ X2\ X7))\ X3))))))))) \\
& \hspace{15em} (5)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall X0 : \iota \Rightarrow o. \forall X1.(m1\_qc\_lang1\ X1) \Rightarrow ((\forall X2. \\
& \quad (m2\_subset\_1\ X2\ (k16\_subst1\ X1)\ (k38\_subst1\ X1)) \Rightarrow (\forall X3. \\
& \quad (m2\_subset\_1\ X3\ (k16\_subst1\ X1)\ (k38\_subst1\ X1)) \Rightarrow (\forall X4. \\
& \quad (m2\_subset\_1\ X4\ (k2\_qc\_lang1\ X1)\ (k3\_qc\_lang1\ X1)) \Rightarrow (\forall X5. \\
& \quad (m1\_subst1\ X5\ X1\ (k7\_sublemma\ X1\ X2\ X4)) \Rightarrow (\forall X6.(m1\_subset\_1 \\
& \quad X6\ k5\_numbers) \Rightarrow (\forall X7.((v5\_relat\_1\ X7\ (k3\_qc\_lang1\ X1)) \wedge \\
& \quad ((v3\_card\_1\ X7\ X6) \wedge (m2\_finseq\_1\ X7\ (k2\_qc\_lang1\ X1)))) \Rightarrow (\forall X8. \\
& \quad (m2\_subset\_1\ X8\ (k6\_qc\_lang1\ X1)\ (k8\_qc\_lang1\ X1\ X6)) \Rightarrow (\forall X9. \\
& \quad (m1\_subset\_1\ X9\ (k1\_subst1\ X1)) \Rightarrow ((X0\ (k4\_sublemma\ X6\ X1\ X8\ X7 \\
& \quad X9)) \wedge (((v2\_subst1\ X2\ X1) \Rightarrow (X0\ X2)) \wedge (((X0\ X2) \Rightarrow (X0\ (k5\_sublemma \\
& \quad X1\ X2))) \wedge (((k19\_subst1\ X1\ X2 = k19\_subst1\ X1\ X3) \wedge ((X0\ X2) \wedge \\
& \quad (X0\ X3))) \Rightarrow (X0\ (k6\_sublemma\ X1\ X2\ X3))) \wedge (((v3\_subst1\ (k7\_sublemma \\
& \quad X1\ X2\ X4)\ X1) \wedge (X0\ X2)) \Rightarrow (X0\ (k9\_sublemma\ X1\ (k7\_sublemma\ X1\ X2\ X4) \\
& \quad X5))))))))) \Rightarrow (\forall X2.(m2\_subset\_1\ X2\ (k16\_subst1 \\
& \quad X1)\ (k38\_subst1\ X1)) \Rightarrow (X0\ X2)) \\
& \hspace{15em} (6)
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& \forall X0.(m1\_qc\_lang1\ X0) \Rightarrow (\forall X1.(\neg v1\_xboole\_0\ X1) \Rightarrow ( \\
& \quad \forall X2.(m1\_valuat\_1\ X2\ X0\ X1) \Rightarrow (\forall X3.(m2\_subset\_1\ X3 \\
& \quad (k16\_subst1\ X0)\ (k38\_subst1\ X0)) \Rightarrow (\forall X4.(m2\_funct\_2 \\
& \quad X4\ (k3\_qc\_lang1\ X0)\ X1\ (k2\_valuat\_1\ X0\ X1)) \Rightarrow ((r1\_valuat\_1\ X0\ X1 \\
& \quad (k39\_subst1\ X0\ X3)\ X2\ X4) \Leftrightarrow (r1\_sublemma\ X0\ X3\ X1\ (k1\_sublemma\ X0 \\
& \quad X1\ X4\ (k3\_sublemma\ X0\ X3\ X1\ X4))\ X2))))))
\end{aligned}$$