

t9\_heyting3  
(TMGZ8PbgKeXdbMaPNUSbjFM4F9DMLv7NheJ)

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Let  $r1\_tarski : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_substlat : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k4\_partfun1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_zfmisc\_1 : \iota \Rightarrow \iota$  be given. Let  $v1\_xboole\_0 : \iota \Rightarrow o$  be given. Let  $k5\_finsub\_1 : \iota \Rightarrow \iota$  be given. Let  $v4\_finsub\_1 : \iota \Rightarrow o$  be given. Let  $v1\_finset\_1 : \iota \Rightarrow o$  be given. Assume the following.

$$\forall X0.\forall X1.\forall X2.\forall X3.((r1\_tarski X0 X1) \wedge (r1\_tarski X2 X3)) \Rightarrow (r1\_tarski (k4\_partfun1 X0 X2) (k4\_partfun1 X1 X3)) \quad (1)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.((X0 \in X1) \wedge (m1\_subset\_1 X1 (k1\_zfmisc\_1 X2))) \Rightarrow (m1\_subset\_1 X0 X2) \quad (2)$$

Assume the following.

$$\forall X0.\forall X1.(m1\_subset\_1 X0 (k1\_zfmisc\_1 X1)) \Leftrightarrow (r1\_tarski X0 X1) \quad (3)$$

Assume the following.

$$\forall X0.\forall X1.(m1\_subset\_1 X0 X1) \Rightarrow ((v1\_xboole\_0 X1) \vee (X0 \in X1)) \quad (4)$$

Assume the following.

$$\forall X0.\forall X1.(X0 \in X1) \Rightarrow (m1\_subset\_1 X0 X1) \quad (5)$$

Assume the following.

$$\forall X0.\forall X1.(r1\_tarski X0 X1) \Rightarrow (r1\_tarski (k5\_finsub\_1 X0) (k5\_finsub\_1 X1)) \quad (6)$$

Assume the following.

$$\forall X0.(\neg v1\_xboole\_0 (k5\_finsub\_1 X0)) \wedge (v4\_finsub\_1 (k5\_finsub\_1 X0)) \quad (7)$$

Assume the following.

$$\forall X0.v4\_finsub\_1 (k5\_finsub\_1 X0) \quad (8)$$

Assume the following.

$$\forall X0.\forall X1.(v4\_finsub\_1 X1) \Rightarrow ((X1 = k5\_finsub\_1 X0) \Leftrightarrow (\forall X2.(X2 \in X1) \Leftrightarrow ((r1\_tarski X2 X0) \wedge (v1\_finset\_1 X2)))) \quad (9)$$

Assume the following.

$$\forall X0.\forall X1.(r1\_tarski X0 X1) \Leftrightarrow (\forall X2.(X2 \in X0) \Rightarrow (X2 \in X1)) \quad (10)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.k1\_substlat X0 X1 = ReplSep (toset (\lambda X2 : \\ \iota.m1\_subset\_1 X2 (k5\_finsub\_1 (k4\_partfun1 X0 X1)))) (\lambda X2 : \\ \iota.(\forall X3.(X3 \in X2) \Rightarrow (v1\_finset\_1 X3)) \wedge (\forall X3.(m1\_subset\_1 \\ X3 (k4\_partfun1 X0 X1)) \Rightarrow (\forall X4.(m1\_subset\_1 X4 (k4\_partfun1 \\ X0 X1)) \Rightarrow (((X3 \in X2) \wedge ((X4 \in X2) \wedge (r1\_tarski X3 X4))) \Rightarrow (X3 = X4)))))) \\ (\lambda X2 : \iota.X2) \end{aligned} \quad (11)$$

**Theorem 1**

$$\forall X0.\forall X1.\forall X2.\forall X3.((r1\_tarski X0 X1) \wedge (r1\_tarski X2 X3)) \Rightarrow (r1\_tarski (k1\_substlat X0 X2) (k1\_substlat X1 X3))$$