

## t9\_lattice5

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Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v10\_lattices : \iota \Rightarrow o$  be given. Let  $l3\_lattices : \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $k3\_lattice3 : \iota \Rightarrow \iota$  be given. Let  $k13\_lattice3 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k5\_lattice3 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $r3\_orders\_2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k4\_lattice3 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v4\_lattices : \iota \Rightarrow o$  be given. Let  $v5\_lattices : \iota \Rightarrow o$  be given. Let  $v6\_lattices : \iota \Rightarrow o$  be given. Let  $v8\_lattices : \iota \Rightarrow o$  be given. Let  $v9\_lattices : \iota \Rightarrow o$  be given. Let  $r1\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v5\_orders\_2 : \iota \Rightarrow o$  be given. Let  $v1\_lattice3 : \iota \Rightarrow o$  be given. Let  $l1\_orders\_2 : \iota \Rightarrow o$  be given. Let  $r1\_orders\_2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v3\_orders\_2 : \iota \Rightarrow o$  be given. Let  $l2\_lattices : \iota \Rightarrow o$  be given. Let  $k2\_lattice3 : \iota \Rightarrow \iota$  be given. Let  $k8\_filter\_1 : \iota \Rightarrow \iota$  be given. Let  $k1\_zfmisc\_1 : \iota \Rightarrow \iota$  be given. Let  $k2\_zfmisc\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $g1\_orders\_2 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v1\_orders\_2 : \iota \Rightarrow o$  be given. Let  $v4\_orders\_2 : \iota \Rightarrow o$  be given. Let  $v2\_lattice3 : \iota \Rightarrow o$  be given. Let  $l1\_lattices : \iota \Rightarrow o$  be given. Let  $v1\_partfun1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v1\_relat\_2 : \iota \Rightarrow o$  be given. Let  $v4\_relat\_2 : \iota \Rightarrow o$  be given. Let  $v8\_relat\_2 : \iota \Rightarrow o$  be given. Let  $v7\_lattices : \iota \Rightarrow o$  be given. Let  $u1\_orders\_2 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices X0))) \Rightarrow (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. \\ & (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow ((r3\_lattices X0 X1 X2) \Leftrightarrow (r3\_orders\_2 \\ & (k3\_lattice3 X0) (k4\_lattice3 X0 X1) (k4\_lattice3 X0 X2)))))) \end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned} & \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v4\_lattices X0) \wedge ((v5\_lattices X0) \wedge ((v6\_lattices X0) \wedge ((v8\_lattices X0) \wedge ((v9\_lattices X0) \wedge \\ & (l3\_lattices X0)))))))) \Rightarrow (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (\forall X3. \\ & (m1\_subset\_1 X3 (u1\_struct\_0 X0)) \Rightarrow (((r3\_lattices X0 X1 X3) \wedge (r3\_lattices X0 X2 X3)) \Rightarrow (r3\_lattices X0 (k3\_lattices X0 X1 X2) X3)))))) \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v5\_lattices X0) \wedge ((v6\_lattices \\ & X0) \wedge ((v8\_lattices X0) \wedge ((v9\_lattices X0) \wedge (l3\_lattices X0)))))) \Rightarrow \\ & (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2.(m1\_subset\_1 \\ & X2 (u1\_struct\_0 X0)) \Rightarrow (r1\_lattices X0 X1 (k1\_lattices X0 X1 X2)))) \end{aligned} \quad (3)$$

Assume the following.

$$\begin{aligned} & \forall X0.((v5\_orders\_2 X0) \wedge ((v1\_lattice3 X0) \wedge (l1\_orders\_2 \\ & X0))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. \\ & (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (\forall X3.(m1\_subset\_1 X3 \\ & (u1\_struct\_0 X0)) \Rightarrow ((X3 = k13\_lattice3 X0 X1 X2) \Leftrightarrow ((r1\_orders\_2 \\ & X0 X1 X3) \wedge ((r1\_orders\_2 X0 X2 X3) \wedge (\forall X4.(m1\_subset\_1 X4 ( \\ & u1\_struct\_0 X0)) \Rightarrow ((r1\_orders\_2 X0 X1 X4) \wedge (r1\_orders\_2 X0 X2 X4)) \Rightarrow \\ & (r1\_orders\_2 X0 X3 X4)))))))) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((\neg v2\_struct\_0 X0) \wedge ((v3\_orders\_2 \\ & X0) \wedge (l1\_orders\_2 X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge ( \\ & m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow ((r3\_orders\_2 X0 X1 X2) \Leftrightarrow (r1\_orders\_2 \\ & X0 X1 X2)) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((\neg v2\_struct\_0 X0) \wedge ((v6\_lattices \\ & X0) \wedge ((v8\_lattices X0) \wedge ((v9\_lattices X0) \wedge (l3\_lattices X0)))) \wedge \\ & ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 X2 (u1\_struct\_0 \\ & X0)))) \Rightarrow ((r3\_lattices X0 X1 X2) \Leftrightarrow (r1\_lattices X0 X1 X2)) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((\neg v2\_struct\_0 X0) \wedge ((v4\_lattices \\ & X0) \wedge (l2\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge ( \\ & m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow (k3\_lattices X0 X1 X2 = k1\_lattices \\ & X0 X1 X2) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\ & X0))) \Rightarrow (k2\_lattice3 X0 = k8\_filter\_1 X0) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. (m1\_subset\_1 X1 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\ & X0 X0))) \Rightarrow (\forall X2. \forall X3. (g1\_orders\_2 X0 X1 = g1\_orders\_2 \\ & X2 X3) \Rightarrow ((X0 = X2) \wedge (X1 = X3))) \end{aligned} \quad (9)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices X0))) \Rightarrow ((\neg v2\_struct\_0 (k3\_lattice3 X0)) \wedge ((v1\_orders\_2 (k3\_lattice3 X0)) \wedge ((v3\_orders\_2 (k3\_lattice3 X0)) \wedge ((v4\_orders\_2 (k3\_lattice3 X0)) \wedge (v5\_orders\_2 (k3\_lattice3 X0)))))) \quad (10)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices X0))) \Rightarrow ((v1\_orders\_2 (k3\_lattice3 X0)) \wedge ((v3\_orders\_2 (k3\_lattice3 X0)) \wedge ((v4\_orders\_2 (k3\_lattice3 X0)) \wedge ((v5\_orders\_2 (k3\_lattice3 X0)) \wedge ((v1\_lattice3 (k3\_lattice3 X0)) \wedge (v2\_lattice3 (k3\_lattice3 X0))))))) \quad (11)$$

Assume the following.

$$\forall X0.(l3\_lattices X0) \Rightarrow ((l1\_lattices X0) \wedge (l2\_lattices X0)) \quad (12)$$

Assume the following.

$$\forall X0.\forall X1.(((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices X0))) \wedge (m1\_subset\_1 X1 (u1\_struct\_0 (k3\_lattice3 X0)))) \Rightarrow (m1\_subset\_1 (k5\_lattice3 X0 X1) (u1\_struct\_0 X0)) \quad (13)$$

Assume the following.

$$\forall X0.\forall X1.(((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices X0))) \wedge (m1\_subset\_1 X1 (u1\_struct\_0 X0))) \Rightarrow (m1\_subset\_1 (k4\_lattice3 X0 X1) (u1\_struct\_0 (k3\_lattice3 X0))) \quad (14)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0) \wedge ((v4\_lattices X0) \wedge (l2\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow (m1\_subset\_1 (k3\_lattices X0 X1 X2) (u1\_struct\_0 X0)) \quad (15)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices X0))) \Rightarrow ((v1\_orders\_2 (k3\_lattice3 X0)) \wedge ((v3\_orders\_2 (k3\_lattice3 X0)) \wedge ((v4\_orders\_2 (k3\_lattice3 X0)) \wedge ((v5\_orders\_2 (k3\_lattice3 X0)) \wedge (l1\_orders\_2 (k3\_lattice3 X0)))))) \quad (16)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices X0))) \Rightarrow ((v1\_partfun1 (k2\_lattice3 X0) (u1\_struct\_0 X0)) \wedge ((v1\_relat\_2 (k2\_lattice3 X0)) \wedge ((v4\_relat\_2 (k2\_lattice3 X0)) \wedge ((v8\_relat\_2 (k2\_lattice3 X0)) \wedge (m1\_subset\_1 (k2\_lattice3 X0) (k1\_zfmisc\_1 (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)))))))) \quad (17)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices X0))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 (k3\_lattice3 X0))) \Rightarrow (k5\_lattice3 X0 X1 = X1)) \quad (18)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices X0))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (k4\_lattice3 X0 X1 = X1)) \quad (19)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices X0))) \Rightarrow (k3\_lattice3 X0 = g1\_orders\_2 (u1\_struct\_0 X0) (k2\_lattice3 X0)) \quad (20)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0) \wedge ((v4\_lattices X0) \wedge (l2\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow (k3\_lattices X0 X1 X2 = k3\_lattices X0 X2 X1) \quad (21)$$

Assume the following.

$$\forall X0.(l3\_lattices X0) \Rightarrow (((\neg v2\_struct\_0 X0) \wedge (v10\_lattices X0)) \Rightarrow ((\neg v2\_struct\_0 X0) \wedge ((v4\_lattices X0) \wedge ((v5\_lattices X0) \wedge ((v6\_lattices X0) \wedge ((v7\_lattices X0) \wedge ((v8\_lattices X0) \wedge (v9\_lattices X0)))))))) \quad (22)$$

Assume the following.

$$\forall X0.(l1\_orders\_2 X0) \Rightarrow ((v1\_orders\_2 X0) \Rightarrow (X0 = g1\_orders\_2 (u1\_struct\_0 X0) (u1\_orders\_2 X0))) \quad (23)$$

**Theorem 1**

$$\forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices X0))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 (k3\_lattice3 X0))) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 (k3\_lattice3 X0))) \Rightarrow (k13\_lattice3 (k3\_lattice3 X0) X1 X2 = k3\_lattices X0 (k5\_lattice3 X0 X1) (k5\_lattice3 X0 X2))))$$