FORMALIZED MATHEMATICS
Vol. 1, No. 1, Pages 1–4,
DOI: 10.1515/forma-2015-0010

DE GRUYTER OPEN
degruyter.com/view/j/forma

MSC: 03B35

Keywords:

MML identifier: AAA, version:

1.

From now on x, y, z, E, E_1, E_2, E_3 denote sets, s_9 denotes a family of subsets of E, f denotes a function from E into E, and k, l, m, n denote elements of $\mathbb{N}$.

LLCC38:10 LLCC39:19 Let i be an integer. EELLCC39:19 LLCC40:6 We say that i is even if and only if

(Def. 1) $2 \mid i$.

EELLCC40:6 EELLCC38:10 LLCC46:8 LLCC48:11 We introduce the notation i is odd as an antonym for i is even. EELLCC48:11 EELLCC46:8 LLCC51:10 LLCC52:15 Let n be a natural number. EELLCC52:15 LLCC53:15 Let us observe that n is even if and only if the condition (Def. 2) is satisfied.

(Def. 2) there exists k such that $n = 2 \cdot k$.

EELLCC53:15 EELLCC51:10 LLCC58:12 LLCC59:9 Note that there exists an element of $\mathbb{N}$ which is even EELLCC59:9 LLCC60:9 One can verify that there exists an element of $\mathbb{N}$ which is odd EELLCC60:9 LLCC61:9 and there exists an integer which is even.

EELLCC61:9 LLCC62:9 and there exists an integer which is odd.

EELLCC62:9 EELLCC58:12 LLCC65:7 Now we state the proposition:

(1) Let us consider an integer i. Then i is odd if and only if there exists an integer j such that $i = 2 \cdot j + 1$.

EELLCC65:7 LLCC68:12 LLCC69:19 Let i be an integer. EELLCC69:19 LLCC70:9 Observe that $2 \cdot i$ is even.

EELLCC70:9 EELLCC68:12 LLCC73:12 LLCC74:24 Let i be an even integer. EELLCC74:24 LLCC75:9 One can verify that $i + 1$ is odd.

EELLCC75:9 EELLCC73:12 LLCC78:12 LLCC79:23 Let i be an odd integer. EELLCC79:23 LLCC80:9 Note that $i + 1$ is even.

EELLCC80:9 EELLCC78:12 LLCC83:12 LLCC84:24 Let i be an even integer. EELLCC84:24 LLCC85:9 Observe that $i - 1$ is odd.

EELLCC85:9 EELLCC83:12 LLCC88:12 LLCC89:23 Let i be an odd integer. EELLCC89:23 LLCC90:9 One can verify that $i - 1$ is even.

ISSN 1426–2630(Print), 1898-9934(Online)

EELLCC90:9 EELLCC88:12 LLCC93:12 LLCC94:38 Let i be an even integer and EELLCC94:38 LLCC94:38 j be an integer. EELLCC94:38 LLCC95:9 Let us note that $i \cdot j$ is even.

EELLCC95:9 LLCC96:9 One can verify that $j \cdot i$ is even.

EELLCC96:9 EELLCC93:12 LLCC99:12 LLCC100:26 Let i, j be odd integers. EELLCC100:26 LLCC101:9 One can check that $i \cdot j$ is odd.

EELLCC101:9 EELLCC99:12 LLCC104:12 LLCC105:27 Let i, j be even integers. EELLCC105:27 LLCC106:9 Note that $i + j$ is even.

EELLCC106:9 EELLCC104:12 LLCC109:12 LLCC110:42 Let i be an even integer and EELLCC110:42 LLCC110:42 j be an odd integer. EELLCC110:42 LLCC111:9 Let us observe that $i + j$ is odd.

EELLCC111:9 LLCC112:9 Note that $j + i$ is odd.

EELLCC112:9 EELLCC109:12 LLCC115:12 LLCC116:26 Let i, j be odd integers. EELLCC116:26 LLCC117:9 Let us note that $i + j$ is even.

EELLCC117:9 EELLCC115:12 LLCC120:12 LLCC121:42 Let i be an even integer and EELLCC121:42 LLCC121:42 j be an odd integer. EELLCC121:42 LLCC122:9 One can verify that $i - j$ is odd.

EELLCC122:9 LLCC123:9 Let us note that $j - i$ is odd.

EELLCC123:9 EELLCC120:12 LLCC126:12 LLCC127:26 Let i, j be odd integers. EELLCC127:26 LLCC128:9 Observe that $i - j$ is even.

EELLCC128:9 EELLCC126:12 LLCC131:12 LLCC132:24 Let m be an even integer. EELLCC132:24 LLCC133:9 One can verify that $m + 2$ is even.

EELLCC133:9 EELLCC131:12 LLCC136:12 LLCC137:23 Let m be an odd integer. EELLCC137:23 LLCC138:9 Note that $m + 2$ is odd.

EELLCC138:9 EELLCC136:12 LLCC141:10 LLCC142:11 Let us consider E and EELLCC142:11 LLCC142:11 f. EELLCC142:11 LLCC142:25 Let n be a natural number. EELLCC142:25 LLCC143:15 Observe that the functor f^n yields a function from E into E. EELLCC143:15 EELLCC141:10 LLCC146:7 Now we state the propositions:

(2) Let us consider a non empty subset S of $\mathbb{N}$. If $0 \in S$, then $\min S = 0$. EELLCC146:7 LLCC149:7

(3) Let us consider a non empty set E, a function f from E into E, and an element x of E. Then $f^0(x) = x$.

EELLCC149:7 LLCC155:10 LLCC156:34 Let x be an object and EELLCC156:34 LLCC156:34 f be a function. EELLCC156:34 LLCC157:6 We say that x is a fixpoint of f if and only if

(Def. 3) $x \in \operatorname{dom} f$ and $x = f(x)$.

EELLCC157:6 EELLCC155:10 LLCC163:10 LLCC164:67 Let A be a non empty set, EELLCC164:67 LLCC164:67 a be an element of A, and EELLCC164:67 LLCC164:67 f be a function from A into A. EELLCC164:67 LLCC165:15 One can verify that a is a fixpoint of f if and only if the condition (Def. 4) is satisfied.

(Def. 4) $a = f(a)$.

EELLCC165:15 EELLCC163:10 LLCC171:10 LLCC172:20 Let f be a function. EELLCC172:20 LLCC173:6 We say that f has fixpoints if and only if

(Def. 5) there exists an object x such that x is a fixpoint of f.

EELLCC173:6 EELLCC171:10 LLCC179:8 LLCC181:11 We introduce the notation f is without fixpoints as an antonym for f has fixpoints. EELLCC181:11 EELLCC179:8 LLCC184:10

LLCC185:34 Let X be a set and EELLCC185:34 LLCC185:34 x be an element of X. EELLCC185:34 LLCC186:6 We say that x is covering if and only if

(Def. 6) $\bigcup x = \bigcup \bigcup X$.

EELLCC186:6 EELLCC184:10 LLCC192:7 Now we state the proposition:

(4) s_9 is covering if and only if $\bigcup s_9 = E$.

EELLCC192:7 LLCC195:12 LLCC196:8 Let us consider E. EELLCC196:8 LLCC197:9 Observe that there exists a family of subsets of E which is non empty, finite, and covering.

EELLCC197:9 EELLCC195:12 LLCC200:7 Now we state the proposition:

(5) Let us consider a set E, a function f from E into E, and a non empty, covering family s_9 of subsets of E. Suppose for every element X of s_9, X misses $f^\circ X$. Then f is without fixpoints.

EELLCC200:7 LLCC205:10 LLCC206:11 Let us consider E and EELLCC206:11 LLCC206:11 f. EELLCC206:11 LLCC207:6 The functor $f_{\equiv}$ yielding an equivalence relation of E is defined by

(Def. 7) for every x and y such that $x, y \in E$ holds $\langle x, y \rangle \in it$ iff there exists k and there exists l such that $f^k(x) = f^l(y)$.

EELLCC207:6 EELLCC205:10 LLCC214:7 Now we state the propositions:

(6) Let us consider a non empty set E, a function f from E into E, an element c of Classes $f_{\equiv}$, and an element e of c. Then $f(e) \in c$. EELLCC214:7 LLCC218:7

(7) Let us consider a non empty set E, a function f from E into E, an element c of Classes $f_{\equiv}$, an element e of c, and n. Then $f^n(e) \in c$.

EELLCC218:7 LLCC222:12 LLCC223:9 One can check that every set which is empty-membered is also trivial.

EELLCC223:9 EELLCC222:12 LLCC226:12 LLCC227:48 Let A be a set and EELLCC227:48 LLCC227:48 B be a set with a non-empty element. EELLCC227:48 LLCC228:9 One can verify that there exists a function from A into B which is non-empty.

EELLCC228:9 EELLCC226:12 LLCC231:12 LLCC233:38 Let A be a non empty set, EELLCC233:38 LLCC233:38 f be a non-empty function from A into B, and EELLCC233:38 LLCC233:38 a be an element of A. EELLCC233:38 LLCC234:9 One can check that $f(a)$ is non empty.

EELLCC234:9 EELLCC231:12 LLCC237:12 LLCC238:25 Let X be a non empty set. EELLCC238:25 LLCC239:9 One can check that 2^X has a non-empty element.

EELLCC239:9 EELLCC237:12 LLCC242:7 Now we state the proposition:

(8) Let us consider a non empty set E, and a function f from E into E. Suppose f is without fixpoints. Then there exists E_1 and there exists E_2 and there exists E_3 such that $(E_1 \cup E_2) \cup E_3 = E$ and $f^\circ E_1$ misses E_1 and $f^\circ E_2$ misses E_2 and $f^\circ E_3$ misses E_3.

EELLCC242:7

2.

LLCC250:7 Now we state the propositions:

(9) Let us consider a natural number n. Then n is odd if and only if there exists an element k of $\mathbb{N}$ such that $n = 2 \cdot k + 1$. EELLCC250:7 LLCC256:7

(10) Let us consider a non empty set A, a function f from A into A, and an element x of A. Then $f^{n+1}(x) = f(f^n(x))$. EELLCC256:7 LLCC260:7

(11) Let us consider an integer i. Then i is even if and only if there exists an integer j such that $i = 2 \cdot j$.

EELLCC260:7 LLCC265:12 LLCC266:9 Let us note that there exists a natural number which is odd.

EELLCC266:9 LLCC267:9 One can check that there exists a natural number which is even.

EELLCC267:9 EELLCC265:12 LLCC270:7 Now we state the proposition:

(12) Let us consider an odd natural number n. Then $1 \leqslant n$.

EELLCC270:7 LLCC273:12 LLCC274:8 One can verify that every integer which is odd is also non zero.

EELLCC274:8 EELLCC273:12

Received April 13, 2018

The English version of this volume of Formalized Mathematics was financed under agreement 548/P-DUN/2016 with the funds from the Polish Minister of Science and Higher Education for the dissemination of science.