Informal2Formal: Automating Formalization by Statistical and Semantic Parsing of Mathematics

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Bonn, February 2nd, 2020

European Research Council
Established by the European Commission

Outline

Autoformalization

Demos

PCFG-based Parsing

Neural Parsing

Chad's Remarks

My Remarks and Reactions

Autoformalization

- Goal: Learn understanding of informal math formulas and reasoning
- Experiments with the CYK chart parser linked to semantic methods
- Experiments with neural methods
- Combined with semantic methods: Type checking, theorem proving
- Feedback loops between the learning and the semantic methods
- Math is a much nicer area than unrestricted NLP:
- We (believe we) can express informal math formally, prove things, etc.
- If we achieve grounding math, we might ground scientific texts, law, etc.
- Corpora: Flyspeck, Mizar, Proofwiki, Stacks, Arxiv, etc.
- Isabelle/AFP?, Coq/Feit-Thompson?, Lean/Mathlib?, Naproche/SAD?
- Some aligned corpora - Flyspeck, Feit-Thompson, Compendium of Cont. Lattices, Rewriting and All That; but most not aligned (requires unsupervised MT methods)

Demos

- Inf2formal over HOL Light: `http://grid01.ciirc.cvut.cz/~mptp/demo.ogv`
- Inf2formal over Mizar: `http://grid01.ciirc.cvut.cz/~mptp/t2m/`
- Nearest neighbor search for similar sentences in Arxiv: `http://grid01.ciirc.cvut.cz/~mptp/arxsim.html`
- ForSet – Chad's set theory backend: `https://github.com/JUrban/ForSet/`

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Statistical/Semantic Parsing of Informalized HOL

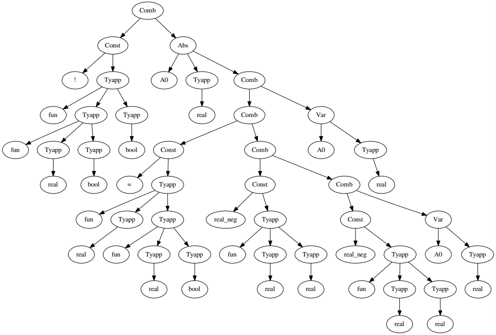
- Training and testing examples exported form Flyspeck formulas
  - Along with their informalized versions
- Grammar parse trees
  - Annotate each (nonterminal) symbol with its HOL type
  - Also “semantic (formal)” nonterminals annotate overloaded terminals
  - guiding analogy: word-sense disambiguation using CYK is common
- Terminals exactly compose the textual form, for example:
- REAL_NEGNEG: $\forall x. - -x = x$

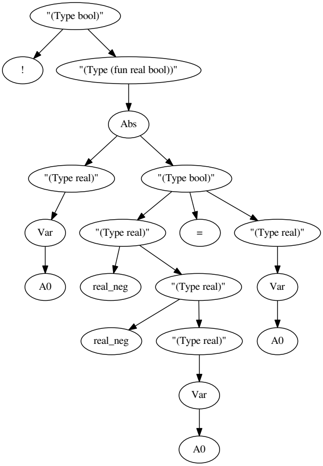
```
(Comb (Const "!" (Tyapp "fun" (Tyapp "fun" (Tyapp "real") (Tyapp "bool"))
(Tyapp "bool"))) (Abs "A0" (Tyapp "real") (Comb (Comb (Const "=" (Tyapp "fun"
(Tyapp "real") (Tyapp "fun" (Tyapp "real") (Tyapp "bool")))) (Comb (Const
"real_neg" (Tyapp "fun" (Tyapp "real") (Tyapp "real"))) (Comb (Const
"real_neg" (Tyapp "fun" (Tyapp "real") (Tyapp "real"))) (Var "A0" (Tyapp
"real"))))) (Var "A0" (Tyapp "real")))))
```

- becomes

```
("(Type bool)" ! ("(Type (fun real bool))" (Abs ("(Type real)"
(Var A0)) ("(Type bool)" ("(Type real)" real_neg ("(Type real)"
real_neg ("(Type real)" (Var A0)))) = ("(Type real)" (Var A0))))))
```

Example grammars





CYK Learning and Parsing (KUV, ITP 17)

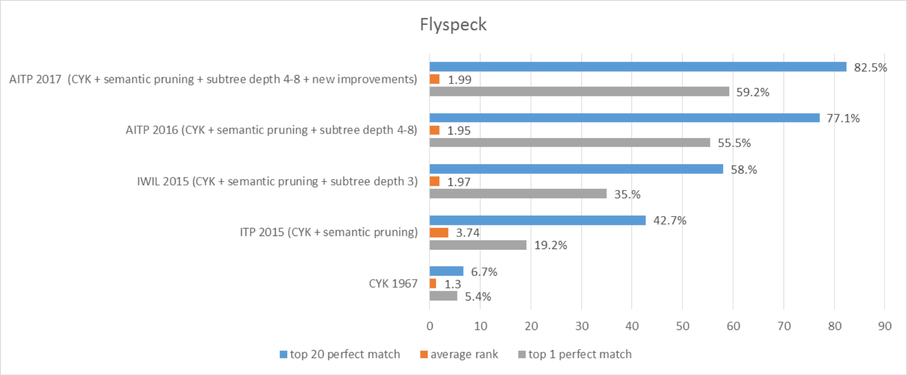
- Induce PCFG (probabilistic context-free grammar) from the trees
  - Grammar rules obtained from the inner nodes of each grammar tree
  - Probabilities are computed from the frequencies
- The PCFG grammar is binarized for efficiency
  - New nonterminals as shortcuts for multiple nonterminals
- CYK: dynamic-programming algorithm for parsing ambiguous sentences
  - input: sentence – a sequence of words and a binarized PCFG
  - output: N most probable parse trees
- Additional semantic pruning
  - Compatible types for free variables in subtrees
- Allow small probability for each symbol to be a variable
- Top parse trees are de-binarized to the original CFG
  - Transformed to HOL parse trees (preterms, Hindley-Milner)
  - typed checked in HOL and then given to an ATP (hammer)

Autoformalization based on PCFG and semantics

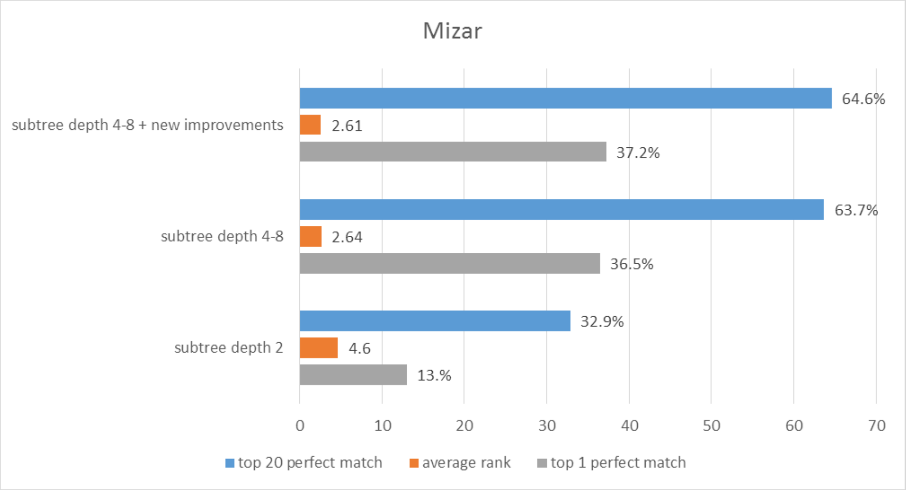
- "`sin ( 0 * x ) = cos pi / 2`"
- produces 16 parses
- of which 11 get type-checked by HOL Light as follows
- with all but three being proved by HOL(y)Hammer
- demo: `http://grid01.ciirc.cvut.cz/~mptp/demo.ogv`

```
sin (&0 * A0) = cos (pi / &2) where A0:real
sin (&0 * A0) = cos pi / &2 where A0:real
sin (&0 * &A0) = cos (pi / &2) where A0:num
sin (&0 * &A0) = cos pi / &2 where A0:num
sin (&(0 * A0)) = cos (pi / &2) where A0:num
sin (&(0 * A0)) = cos pi / &2 where A0:num
csin (Cx (&0 * A0)) = ccos (Cx (pi / &2)) where A0:real
csin (Cx (&0) * A0) = ccos (Cx (pi / &2)) where A0:real^2
Cx (sin (&0 * A0)) = ccos (Cx (pi / &2)) where A0:real
csin (Cx (&0 * A0)) = Cx (cos (pi / &2)) where A0:real
csin (Cx (&0) * A0) = Cx (cos (pi / &2)) where A0:real^2
```

Flyspeck Progress



First Mizar Results (100-fold Cross-validation)



Outline

Neural Autoformalization (Wang et al., 2018,2020)

- generate about 1M Latex - Mizar pairs based on Bancerek's work
- train neural seq-to-seq translation models (Luong – NMT)
- evaluate on about 100k examples
- many architectures tested, some work much better than others
- very important latest invention: *attention* in the seq-to-seq models
- more data very important for neural training – our biggest bottleneck (you can help!)
- Recent addition: unsupervised methods (Lample et all 2018) – no need for aligned data!

Neural Autoformalization data

Rendered LaTeX	If $X \subseteq Y \subseteq Z$, then $X \subseteq Z$.
Mizar	`X c= Y & Y c= Z implies X c= Z;`
Tokenized Mizar	`X c= Y & Y c= Z implies X c= Z ;`
LaTeX	`If $X \subseteq Y \subseteq Z$, then $X \subseteq Z$.`
Tokenized LaTeX	`If $ X \subseteq Y \subseteq Z $ , then $ X \subseteq Z $ .`

Neural Autoformalization results

Parameter	Final Test Perplexity	Final Test BLEU	Identical Statements (%)	Identical No-overlap (%)
128 Units	3.06	41.1	40121 (38.12%)	6458 (13.43%)
256 Units	1.59	64.2	63433 (60.27%)	19685 (40.92%)
512 Units	1.6	**67.9**	66361 (63.05%)	21506 (44.71%)
1024 Units	**1.51**	61.6	**69179 (65.73%)**	**22978 (47.77%)**
2048 Units	2.02	60	59637 (56.66%)	16284 (33.85%)

Neural Fun – Performance after Some Training

Rendered LaTeX	Suppose s_8 is convergent and s_7 is convergent . Then $\lim(s_8+s_7) = \lim s_8 + \lim s_7$
Input LaTeX	`Suppose $ { s _ { 8 } } $ is convergent and $ { s _ { 7 } } $ is convergent . Then $ \mathop { \rm lim } ( { s _ { 8 } } { + } { s _ { 7 } } } ) \mathrel { = } \mathop { \rm lim } { s _ { 8 } } { + } \mathop { \rm lim } { s _ { 7 } } $ .`
Correct	`seq1 is convergent & seq2 is convergent implies lim ( seq1 + seq2 ) = ( lim seq1 ) + ( lim seq2 ) ;`
Snapshot-1000	`x in dom f implies ( x * y ) * ( f | ( x | ( y | ( y | y ) ) ) ) = ( x | ( y | ( y | ( y | y ) ) ) ) ) ;`
Snapshot-2000	`seq is summable implies seq is summable ;`
Snapshot-3000	`seq is convergent & lim seq = 0c implies seq = seq ;`
Snapshot-4000	`seq is convergent & lim seq = lim seq implies seq1 + seq2 is convergent ;`
Snapshot-5000	`seq1 is convergent & lim seq2 = lim seq2 implies lim_inf seq1 = lim_inf seq2 ;`
Snapshot-6000	`seq is convergent & lim seq = lim seq implies seq1 + seq2 is convergent ;`
Snapshot-7000	`seq is convergent & seq9 is convergent implies lim ( seq + seq9 ) = ( lim seq ) + ( lim seq9 ) ;`

Unsupervised NMT Fun on Short Formulas

```
len <* a *> = 1 ;                    len <* a *> = 1 ;
assume i < len q ;                   i < len q ;
len <* q *> = 1 ;                    len <* q *> = 1 ;
s = apply ( v2 , v1 ast t ) ;        s = apply ( v2 , v1 ) . t ;
s . ( i + 1 ) = tt . ( i + 1 )       s . ( i + 1 ) = tau1 . ( i + 1 )
1 + j <= len v2 ;                    1 + j <= len v2 ;
1 + j + 0 <= len v2 + 1 ;            1 + j + 0 <= len v2 + 1 ;
let i be Nat ;                       i is_at_least_length_of p ;
assume v is_applicable_to t ;        not v is applicable ;
let t be type of T ;                 t is_orientedpath_of v1 , v2 , T ;
a ast t in downarrow t ;             a *' in downarrow t ;
t9 in types a ;                      t `2 in types a ;
a ast t <= t ;                       a *' <= t ;
A is_applicable_to t ;               A is applicable ;
Carrier ( f ) c= B                   support ppf n c= B
u in B or u in { v } ;               u in B or u in { v } ;
F . w in w & F . w in I ;            F . w in F & F . w in I ;
GG . y in rng HH ;                   G0 . y in rng ( H1 ./. y ) ;
a * L = Z_ZeroLC ( V ) ;             a * L = ZeroLC ( V ) ;
not u in { v } ;                     u >> v ;
u <> v ;                             u <> v ;
v - w = v1 - w1 ;                    vw = v1 - w1 ;
v + w = v1 + w1 ;                    v + w = v1 + w1 ;
x in A & y in A ;                    assume [ x , y ] in A ;
```

Outline

Chad's remarks

- Naproche/Forthel encoding of notions
- Set vs Type Theory encodings of groups and structures
- `http://grid01.ciirc.cvut.cz/~chad/setsslides.pdf`

Outline

Mizar and Andrzej Trybulec – Unsung Hero of Mathematician-Friendly Formalization

- His motivation: proof checker for refactoring his topology PhD thesis
- Started (1970's) by analyzing a topology paper by H. Patkowska.
- Mizar's proof style and language: Jaskowski (1934) style natural deduction proofs – *On the rules of suppositions*.
- The Discourse Representation Theory motivating Naproche might be its derivative via Montague and Tarski. Jeff Pelletier (Montague's PhD student, funny stories about Montague) wrote a comparison of Gentzen and Jaskowski style ND (good to learn about eigenvariables).
- Obvious inferences: work on the right human-friendly granularity of the proof steps (started by M. Davis, continued by Rudnicki and Trybulec).
- Fast internal proof checker *critical* for library refactoring (not hammers).

Mizar and Andrzej Trybulec

- ENOD: Experience Not Only Doctrine! – build a large math library (in the 90s!) Move away from just the formula/proof language tweaking.
- Math is full of soft types and overloading (200 meanings of + in MML). Mizar's advanced soft types (adjectives and registration) precede Haskell type classes and derived typeclass mechanisms in other ITPs.
- Freek Wiedijk (repeating Andrzej): mathematicians don't use type theory. They use set theory and soft types.
- Jeremy Avigad showing me Gonthier's groups encoding in 2009 - my lack of faith in type theory.
- Freek again: COBOL's too much natural language was a failure – beware.
- Peter Koepke – translating Mizar to German in 2000s (connected to Solovay's Mizar visit in 2002?)

Grzegorz Bancerek, CCL, $\TeX$, ProofWiki

- Grzegorz Bancerek (and team): Compendium of Continuous Lattices (CCL) - Mizar aligned with the book
- I got the tex sources of CCL in 2004 planning to use them for statistical translation to Mizar
- But it took 10 years to declare and start the autoformalization project (CICM 2014)
- Grzegorz's other achievements: translation of Mizar to $\TeX$ and PDF
- If our goal is good controlled language for READING, Mizar + this translation might be a good approach – see the latest Mizar PDFs.
- Also translation of Mizar to ProofWiki (ca 500 ProofWiki pages)
- (Co-)author of 124 Mizar articles (10% of the library)

Some Replies on Automation and Machine Learning

- 2014 AI/TP challenges: `http://ai4reason.org/aichallenges.html`. Unlike the recent PR efforts by the poor Google/Facebook/Microsoft companies and AI teams, I have put my money where my mouth is - you can still bet me.
- Hammers: yes, use strong AI/TP systems to find the proofs, but then refactor the proofs into readable Mizar-style proofs.
- And YES, combinations of ML and AR/TP are very useful and a very cool AI topic.
- And NO, deep learning (even if interesting) has NOT been the critical missing piece for developing AI/TP so far. The largest 40-70% ATP improvements in real time (rlCoP, ENIGMA) are so far done with gradient boosted trees.
- Example Mizar proof (Knaster-Tarski) found by ENIGMA: `http://grid01.ciirc.cvut.cz/~mptp/7.13.01_4.181.1147/html/knaster#T21`
- Its E-ENIGMA proof: `http://grid01.ciirc.cvut.cz/~mptp/t21_knaster`.

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- C. Kaliszyk: `http://cl-informatik.uibk.ac.at/teaching/ss18/mltp/content.php`

Thanks and Advertisement

- Thanks for your attention!
- AITP – Artificial Intelligence and Theorem Proving
- March 22–27, 2020, Aussois, France, `aitp-conference.org`
- ATP/ITP/Math vs AI/Machine-Learning people, Computational linguists
- Discussion-oriented and experimental - submit a talk abstract!
- Grown to 80 people in 2019