

# MACHINE LEARNING AND AUTOMATED REASONING

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# Learning vs Reasoning – Alan Turing 1950 – AI



- 1950: *Computing machinery and intelligence* – AI, Turing test
- “We may hope that machines will eventually compete with men in *all purely intellectual fields*.” (regardless of his 1936 undecidability result!)
- last section on **Learning Machines**:
- “But which are the best ones [fields] to start [learning on] with?”
- “... Even this is a difficult decision. Many people think that a very abstract activity, like the *playing of chess*, would be best.”
- Why not try with **math**? It is much more (universally?) expressive ...

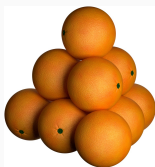
# Learning vs Reasoning – Alan Turing 1950 – AI



- Successes: Chess - DeepBlue vs Kasparov in the 1990s
- AlphaGo/Zero - 2016/17 - self-improvement by combining search and learning
- Why not try with **math**? It is much more (universally?) expressive ...

# Big Math Game: The Flyspeck project

- Kepler conjecture (1611): The most compact way of stacking balls of the same size in space is a pyramid.



$$V = \frac{\pi}{\sqrt{18}} \approx 74\%$$

- Formal proof finished in 2014
- 20000 lemmas in geometry, analysis, graph theory
- All of it at <https://code.google.com/p/flyspeck/>
- All of it **computer-understandable and verified** in HOL Light:
  - `polyhedron s /\ c face_of s ==> polyhedron c`
- However, this took **20 – 30 person-years!**
- Our AlphaGo/Zero-style systems for Math: 40-60% automatically

# AlphaGo/Zero for Automated Reasoning

- set of first-order clauses, *extension* and *reduction* steps
- proof finished when all branches are closed
- a lot of nondeterminism, requires backtracking
- can be used as a setting for AlphaG/Zero-style search/learning self-improvement

Clauses:

$$c_1 : P(x)$$

$$c_2 : R(x, y) \vee \neg P(x) \vee Q(y)$$

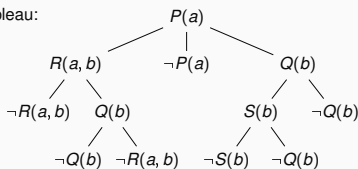
$$c_3 : S(x) \vee \neg Q(b)$$

$$c_4 : \neg S(x) \vee \neg Q(x)$$

$$c_5 : \neg Q(x) \vee \neg R(a, x)$$

$$c_6 : \neg R(a, x) \vee Q(x)$$

Closed Connection Tableau:



Imagine doing this over all of mathematical knowledge, like Flyspeck!