Formal Objects and Propositions in Megalodon as of February 2021

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Contents

Preface

This document lists most of the objects defined and propositions proven in the Egal theory of Proofgold (using Megalodon) as of February 2021. When relevant, corresponding 256-bit identifiers and 162-bit addresses are given for objects and propositions so that users can find them in the Proofgold blockchain. The objects and propositions here are those distributed with Megalodon in the file `PfgEFeb2021Preamble.mgs`.

The document is a draft and much of it was automatically generated (using the -latex option in Megalodon), so please overlook any formatting problems.

Chapter 1

Basic Logic

Primitive. The name Eps_i is a primitive term of type $(\iota \to o) \to \iota$. Specifically it is "Primitive 0" in the Proofgold Egal theory.

Axiom 1.1 $\forall P : \iota \to o. \forall x : \iota. P\ x \to P\ (\mathsf{Eps\_i}\ P)$. *The proposition is identified by the following information:*

Pure Prop Id:	*c3f0de4cb966012957ca752938aa96a32c594389e7aea45227d571c0506618ba*
Pure Prop Address:	*TMaqNdrw1xUUqfmGqr4T5HNsS6X1kThiGJp*
Theory Prop Id:	*756d6e520a540dee983a2bc983f030fca1e15bc5c01cd10daa59ba9846775a89*
Theory Prop Address:	*TMZeDQEPZjtPy34irdW6qoDmhfGuT9rLoo3*

Definition 1.1 *We define* True *to be* $\forall p : o.p \to p$ *of type* o *identified by the following information:*

Pure Object Id:	*f81b3934a73154a8595135f10d1564b0719278d3976cc83da7fda60151d770da*
Pure Object Address:	*TMXh1PeTNoRdeQMkipsnTZ1NBoLMCSQGszg*
Theory Object Id:	*94e6a74d7fb4010e37ca58528b5eefb6d84e85f0fcd9598afcfdb3000334a38a*
Theory Object Address:	*TMdZywkjk5BP16zSdFbfK4CcoVzmBS54DHA*

Definition 1.2 *We define* False *to be* $\forall p : o.p$ *of type* o *identified by the following information:*

Pure Object Id:	*1db7057b60c1ceb81172de1c3ba2207a1a8928e814b31ea13b9525be180f05af*
Pure Object Address:	*TMVcBGjSsw48XJ4L45VWUk51f7pywF6kvVX*
Theory Object Id:	*266ad502104b0eadbdf70b5ce22ec14cce255c0b312cfc3719813f538ed6de1b*
Theory Object Address:	*TMZx4Q2psRTfsQjwWdVEVQnyn3ZRxXosKPq*

Definition 1.3 *We define* not *to be* $\lambda A : o.A \to \mathsf{False}$ *of type* $o \to o$ *identified by the following information:*

Pure Object Id:	*f30435b6080d786f27e1adaca219d7927ddce994708aacaf4856c5b701fe9fa1*
Pure Object Address:	*TMWAvR96ZU4RCj94CSoZhrtMR5rdaTaLB5R*
Theory Object Id:	*0dc3c779c7cc720c63503b5ae618908388fe9f8ef7b20ce33a5baa0de6389c1b*
Theory Object Address:	*TMJJZawymNdUcMFLBwRqSW8DSQsHg76ans7*

Notation. We use $\neg-$ as a prefix operator corresponding to applying term not.

Definition 1.4 *We define* and *to be* $\lambda AB : o.\forall p : o.(A \to B \to p) \to p$ *of type* $o \to o \to o$ *identified by the following information:*

Pure Object Id:	*2ba7d093d496c0f2909a6e2ea3b2e4a943a97740d27d15003a815d25340b379a*
Pure Object Address:	*TMHVPoyLuccMxozW2JkzTcKaFNorhQvcqVz*
Theory Object Id:	*37da83639bcf9d8c8a37734df867cfc15d7d22573b6c8ccfc52e5c2e7a5abc81*
Theory Object Address:	*TMMEUAXx5wQ61b6khohvEMcR5eUbjjJSYAw*

Notation. We use $\wedge$ as a left associative infix operator corresponding to applying term and.

Definition 1.5 *We define* or *to be* $\lambda AB : o.\forall p : o.(A \to p) \to (B \to p) \to p$ *of type* $o \to o \to o$ *identified by the following information:*

Pure Object Id:	*95774681991614 70abc0815b6a25e03706a4e61d5e628c203bf1f793606b1153*
Pure Object Address:	*TMRrSAiwhsGG8gNN8EukN1zLVibyVKQXaFq*
Theory Object Id:	*86ae6491fbd8f20f1f7e4986f09463cdcc489bdedfda845c840df66cb8f76edb*
Theory Object Address:	*TMbacaYWYcRwAFxG4ALNMm3EyDKWEtRrh42*

Notation. We use $\vee$ as a left associative infix operator corresponding to applying term or.

Definition 1.6 *We define* iff *to be* $\lambda AB : o.$and $(A \to B)$ $(B \to A)$ *of type* $o \to o \to o$ *identified by the following information:*

Pure Object Id:	*98aaaf225067eca7b3f9af03e4905bbbf48fc0ccbe2b4777422caed3e8d4dfb9*
Pure Object Address:	*TMNPkfzmoJmg4Qsbe28fcbyaXL1XTynPhik*
Theory Object Id:	*6588e5a76078e7919b50307e23503d854af1b869a2eedd6b091f6610609777ba*
Theory Object Address:	*TMc6fJsUiuiG3KCWP4EAREq7NWM56fQqJHi*

Notation. We use $\Leftrightarrow$ as an infix operator corresponding to applying term iff.

Chapter 2

Equality

2.1 Eq

Let α be a type.

Definition 2.1 *We define* eq *to be* $\lambda xy : \alpha.\forall Q : \alpha \to \alpha \to o.Q\ x\ y \to Q\ y\ x$ *of type* $\alpha \to \alpha \to o$.

Since Proofgold has no polymorphism, there is no single object corresponding to eq. Instead there is a constructor Eq that takes a type argument and is always expanded away using the definition above as part of normalization.

Definition 2.2 *We define* neq *to be* $\lambda xy : \alpha.\neg$eq $x\ y$ *of type* $\alpha \to \alpha \to o$.

Again, there is no single object in Proofgold corresponding to neq. Instead it is represented by its definition which is then expanded away as part of normalization.

2.2 Notation for Equality

Notation. We use $=$ as an infix operator corresponding to applying term eq. **Notation.** We use $\neq$ as an infix operator corresponding to applying term neq.

2.3 Functional Extensionality

Let $\alpha\beta$ be types.

Axiom 2.1 $\forall fg : \alpha \to \beta . (\forall x : \alpha . f \ x = g \ x) \to f = g$.

Functional extensionality is built in as part of the proof term data structure in Proofgold, so this proposition has no counterpart as a proposition published in the chain.

Chapter 3

Existential Quantifiers

3.1 Ex

Let α be a type.

Definition 3.1 *We define* ex *to be* $\lambda Q : \alpha \to o.\forall P : o.(\forall x : \alpha.Q\ x \to P) \to P$ *of type* $(\alpha \to o) \to o$.

Since Proofgold has no polymorphism there is no single object corresponding to existential quantification. Instead there is a constructor Ex that takes a type argument and is always expanded away using the definition above as part of normalization.

3.2 Notation for Existential Quantifiers

Notation. We use $\exists x, \ldots, y.$ as a binder notation corresponding to a term constructed using ex.

Chapter 4

Further Primitives and Axioms

Axiom 4.1 $\forall pq : o.\mathsf{iff}\ p\ q \to p = q$. *The proposition is identified by the following information:*

Pure Prop Id:	*00ac81fe17c156a4ac6f8a4b0f730686a7acf0e26a713a5d0e7f9aab342ef273*
Pure Prop Address:	*TMT4ViLziW9cx8676T7siQNUsc4pLGajCAh*
Theory Prop Id:	*1d1e565be0b2225dbdd84a1b9f843374545e89ad6f25ebf6856e801170f8d60b*
Theory Prop Address:	*TMK6YdqYUKB5vJfwfTMJNgJJGj1TmDiA7rj*

Primitive. The name In is a primitive term of type $\iota \to \iota \to o$. In particular it is "Primitive 1" in the Proofgold Egal theory.

Definition 4.1 **Notation.** *We use* $\in$ *in infix notation for* In.

We define Subq *to be* $\lambda AB.\forall x \in A.x \in B$ *of type* $\iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*81c0efe6636cef7bc8041820a3ebc8dc5fa3bc816854d8b7f507e30425fc1cef*
Pure Object Address:	*TMbQvibiSdPYTcyHirAiT7dojaZd9QVHnnx*
Theory Object Id:	*317007aa687cfc3fa6d33a4d45b72e2b84878595f194108dee11852a04950639*
Theory Object Address:	*TMJciJ4WVfdZbE7XT2SUcAkkoSYiF2gabRX*

Notation. *We use* $\subseteq$ *in infix notation for* Subq.

Axiom 4.2 $\forall XY : \iota.X \subseteq Y \to Y \subseteq X \to X = Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*b3b6981b6de9a71189167e6208f1fa2be38ade4bd2a77084c6878323e6d97ab9*
Pure Prop Address:	*TMP6dV9kMfpLcfVgkH1rTbsurTnQKfFkFUf*
Theory Prop Id:	*21238ac2a61503233d86f0ab5bbd1cfebd49fecaa7aa3a131715c5e08003430e*
Theory Prop Address:	*TMQ3CJdXpjT9Vyjdd8x58bzAfVa2uHKMmnZ*

Axiom 4.3 $\forall P : \iota \to o.(\forall X : \iota.(\forall x \in X.P\ x) \to P\ X) \to \forall X : \iota.P\ X$. *The proposition is identified by the following information:*

Pure Prop Id:	*04521121bc85ca3c89ee96767237629584fc96b34de1cf1a3f790ea4af4b0047*
Pure Prop Address:	*TMZDd5XWnW8qoDTqVGuvgh8AQGEhcN5haVH*
Theory Prop Id:	*20faa51007fd5aff9dc54e343b9c7b57fab2eb829c71a0daa2e90c858707d2b7*
Theory Prop Address:	*TMMzAhfU4QKJhywTdGXVuR4P8u6KoKixxYw*

Notation. We use $\exists x, \ldots, y.$ as a binder notation corresponding to a term constructed using ex and handling $\in$ or $\subseteq$ ascriptions using and. **Primitive.** The name Empty is a primitive term of type ι. In particular it is "Primitive 2" in the Proofgold Egal theory.

Axiom 4.4 $\neg\exists x : \iota. x \in \mathsf{Empty}$. *The proposition is identified by the following information:*

Pure Prop Id:	*4da93150e41630a98ea3fc30e9bc894dee0da4364b020ac8ee0a79d04508434f*
Pure Prop Address:	*TMQMLLe8Cz5vwfXB4uUFfsKFVfvwBMf4VPJ*
Theory Prop Id:	*c7b0d7f466fd9528f6264e7a3a9eaeff8c18159e8679b3d6b840d2689bda8486*
Theory Prop Address:	*TMaPKWsC7HDHhrC2zYBmYcX19RqLy2VefbW*

Primitive. The name Union is a primitive term of type $\iota \to \iota$. In particular it is "Primitive 3" in the Proofgold Egal theory.

Axiom 4.5 $\forall X x. x \in \mathsf{Union}\ X \Leftrightarrow \exists Y. x \in Y \land Y \in X$. *The proposition is identified by the following information:*

Pure Prop Id:	*bf4c263ac4776c12ed6de40b226c687445f1e634eddf121ad52b2d095224ec74*
Pure Prop Address:	*TMHRWvQi72bggXNdrukFL23tqhkHk6iri2y*
Theory Prop Id:	*40660fae1988a12c0dd433fed65b5971371ce274cfa7acb0152f6c8b11856323*
Theory Prop Address:	*TMdiuFh9LBKb2nXjbQi7GzQCn57wX2izbEV*

Primitive. The name Power is a primitive term of type $\iota \to \iota$. Specifically it is "Primitive 4" in the Proofgold Egal theory.

Axiom 4.6 $\forall X Y : \iota. Y \in \mathsf{Power}\ X \Leftrightarrow Y \subseteq X$. *The proposition is identified by the following information:*

Pure Prop Id:	*746e5a9564518580f673371a7891c1db2e38c91762155f302694c6b4cbf7c649*
Pure Prop Address:	*TMTzabXXxRmuTxrdG42VtpMDWCUmgK2UMgY*
Theory Prop Id:	*d67d718c245e91b738c984751e77f84e6db2b59131a6dd3dbf033e5c5b151dc0*
Theory Prop Address:	*TMbRJTDNmcR6KZC6t4TGXakKw4XSAYS3VN8*

Primitive. The name Repl is a primitive term of type $\iota \to (\iota \to \iota) \to \iota$. Specifically it is "Primitive 5" in the Proofgold Egal theory.

Notation. We write $\mathsf{Repl}\ A\ (\lambda x.s)$ as $\{s | x \in A\}$.

Axiom 4.7 $\forall A : \iota. \forall F : \iota \to \iota. \forall y : \iota. y \in \{F\ x | x \in A\} \Leftrightarrow \exists x \in A. y = F\ x$. *The proposition is identified by the following information:*

Pure Prop Id:	*4f2357276dc7f58ae12c278fb7f133f1de0a4e899d0a1b4df22db544ef8fbfea*
Pure Prop Address:	*TMHvGDzbjKwgKUfmhz4YstLTy6FK6VEf8VX*
Theory Prop Id:	*471122ee9aecdb9185d0ed855c4fd074596525d6ade7150ea945d05f732c1d91*
Theory Prop Address:	*TMQwYTqizk3ouisSfLQUbv5JP258W4VhxxV*

Definition 4.2 *We define* TransSet *to be* $\lambda U : \iota.\forall x \in U.x \subseteq U$ *of type* $\iota \to o$ *identified by the following information:*

Pure Object Id:	*538bb76a522dc0736106c2b620bfc2d628d5ec8a27fe62fc505e3a0cdb60a5a2*
Pure Object Address:	*TMFFkFhggUWb8iaKbBVFF2NXQtBDp9ocvey*
Theory Object Id:	*e69f92130adf04e6fb850fe93c130a82c6a84226e2914139b5b46a4c8b46dd59*
Theory Object Address:	*TMaE1Wfu3Ekit1RP2q2rs1eExhVT4eYQUWZ*

Definition 4.3 *We define* Union_closed *to be* $\lambda U : \iota.\forall X : \iota.X \in U \to \mathsf{Union}\ X \in U$ *of type* $\iota \to o$ *identified by the following information:*

Pure Object Id:	*57561da78767379e0c78b7935a6858f63c7c4be20fe81fe487471b6f2b30c085*
Pure Object Address:	*TMGQhGvaoAR8Ac54JYsvH3PiWjPoz8NS8J6*
Theory Object Id:	*e45ed946b4201c2ac3bce08c594bcc5b37e16935b9de8d06f9515256bfcb6c5d*
Theory Object Address:	*TML288688fkgDpJdibigbTFnrScrqv2iKt8*

Definition 4.4 *We define* Power_closed *to be* $\lambda U : \iota.\forall X : \iota.X \in U \to \mathsf{Power}\ X \in U$ *of type* $\iota \to o$ *identified by the following information:*

Pure Object Id:	*8b9cc0777a4e6a47a5191b7e6b041943fe822daffc2eb14f3a7baec83fa16020*
Pure Object Address:	*TMGEQQpyASUMyKRHqWbnFXopjtYsATE5AVk*
Theory Object Id:	*d2954ea0f8a59a40ddc3fc5be9d473a03e8b92e4a279ea7e2bf10debed186ffd*
Theory Object Address:	*TMNbYYR8P3gh4AeJAfTdNEMMkiD5HvA97jx*

Definition 4.5 *We define* Repl_closed *to be*

$$\lambda U : \iota.\forall X : \iota.X \in U \to \forall F : \iota \to \iota.(\forall x : \iota.x \in X \to F\ x \in U) \to \{F\ x | x \in X\} \in U$$

of type $\iota \to o$ *identified by the following information:*

Pure Object Id:	*5574bbcac2e27d8e3345e1dc66374aa209740ff86c1db14104bc6c6129aee558*
Pure Object Address:	*TMHsmhDUjBaGfXeB9XmeViaPhpFM3GPGFEK*
Theory Object Id:	*5135346e3c2b9f78fd2ca0f8dc77f2075a100d43dfdecf64bc2792a54493f446*
Theory Object Address:	*TMFer2uiF5iECeDKX8Yjf4jvbfj2RQYen9J*

Definition 4.6 *We define* ZF_closed *to be* $\lambda U : \iota.\mathsf{Union\_closed}\ U \land \mathsf{Power\_closed}\ U \land \mathsf{Repl\_closed}\ U$ *of type* $\iota \to o$ *identified by the following information:*

Pure Object Id:	*1bd4aa0ec0b5e627dce9a8a1ddae929e58109ccbb6f4bb3d08614abf740280c0*
Pure Object Address:	*TMK8id4rjoGzRQvGv2RVDkaCpE2gvidq2jS*
Theory Object Id:	*e1eccf55c627a83d82a0c5fbb2d8effe908afbba6259c3a3a9f7af4839904407*
Theory Object Address:	*TMKM8HgiAgaT2ShpWewk4PuTK8LpKuESdvs*

Primitive. The name UnivOf is a primitive term of type $\iota \to \iota$. In particular it is "Primitive 6" in the Proofgold Egal theory.

Axiom 4.8 $\forall N : \iota . N \in \mathsf{UnivOf}\ N$. *The proposition is identified by the following information:*

Pure Prop Id:	*efcc1652fa565b11ad9b7e6b61992946e170a56766124011245e25a95ef533d8*
Pure Prop Address:	*TMWDMXfHXNT3hXXgPN8FdFXa6TtH5VEC8L2*
Theory Prop Id:	*e805d44e6e3b129108fe3a5d7b86456ab1ded7fbabdb1e013db339c19e74fc3c*
Theory Prop Address:	*TMXQAwXZaXUD3yTSAm1fLH7g2HqyDMYutzR*

Axiom 4.9 $\forall N : \iota . \mathsf{TransSet}\ (\mathsf{UnivOf}\ N)$. *The proposition is identified by the following information:*

Pure Prop Id:	*05357566d69c1e942fba918ba630fe1befcd3b76747edaa23bf9e22c07e0bf80*
Pure Prop Address:	*TMPFnHy6wH7xHdqPH1LVnFGKXNXBVDCGhcc*
Theory Prop Id:	*cf4f5cdd03fb3fcd9b880d27f1bb10dfa697dda82fb891d4f01e8901f99e0ddd*
Theory Prop Address:	*TMQDPgN5NEyK5uGnLoLPsCtuYVhgM8BFwd8*

Axiom 4.10 $\forall N : \iota . \mathsf{ZF\_closed}\ (\mathsf{UnivOf}\ N)$. *The proposition is identified by the following information:*

Pure Prop Id:	*770e356024df94789dc767ad29154fb817b6339b6526f6b4826cc678ae6937ab*
Pure Prop Address:	*TMUaRKj3xqMnJ3tpKZ46beaVE82Vps7qYVG*
Theory Prop Id:	*0e736334ea6ea7d6454eede353e88ba0fa7a5e4c3a17f6c90921a2414dab6d08*
Theory Prop Address:	*TMK8JSDxgUygbLA49dTtQCjnRY2ijj3pG6u*

Axiom 4.11 $\forall N U : \iota . N \in U \to \mathsf{TransSet}\ U \to \mathsf{ZF\_closed}\ U \to \mathsf{UnivOf}\ N \subseteq U$. *The proposition is identified by the following information:*

Pure Prop Id:	*042dda2b34a2ebfa193a104324faf5219666788a71551ab9dd079136a7c6281d*
Pure Prop Address:	*TMSbghKuyRptdLDe7mXjA1e8YRJUc3PRgkx*
Theory Prop Id:	*1f793348811ae94e9d72c764a3e780c39a9a2f45f55507133ecc732e15133ce6*
Theory Prop Address:	*TMGeTmqMVx11mKzRV46CRSj6xUcaf13udQk*

Chapter 5

Some Basic Results

`FalseE`

Theorem 5.1 $\mathsf{False} \to \forall p : o.p$. *The proposition is identified by the following information:*

Pure Prop Id:	*0dcfe33e27797b7b436d2f4d048a94573a53d6753db4a3cc25933a3dd6f89015*
Pure Prop Address:	*TMKsmG8CTue9Uq3xFgtUwj8xZhW6Vsv7LaE*
Theory Prop Id:	*f9a230def2f4412d999116d577818e7c13149b3ffd6ec8c0f216fb2570056eab*
Theory Prop Address:	*TMJrbzrCp7MPJvNNanPW79aqzPDUV4gcM4o*

`TrueI`

Theorem 5.2 True. *The proposition is identified by the following information:*

Pure Prop Id:	*f81b3934a73154a8595135f10d1564b0719278d3976cc83da7fda60151d770da*
Pure Prop Address:	*TMXh1PeTNoRdeQMkipsnTZ1NBoLMCSQGszg*
Theory Prop Id:	*eb5cbf9e00c1dc6fbac87f0e7dd91901a69a5afc430de9f265def7009d06523d*
Theory Prop Address:	*TMdaMQ9TGYHTUYWtAiezq6j2m9Lh8nTGaJX*

`notI`

Theorem 5.3 $\forall A : o.(A \to \mathsf{False}) \to \neg A$. *The proposition is identified by the following information:*

Pure Prop Id:	*67ebfbb7241f6a92dbb75a0571d5396f165877e75a006c85956aeff1c756032a*
Pure Prop Address:	*TMMfBXPbPGW7MmkNMsTQVtkyifxwbPw9VeV*
Theory Prop Id:	*12f948374053b3cf6da14578610534d466feec847f6e53f73e2bb362ee644abb*
Theory Prop Address:	*TMUAmMSt4Wx5Red5pwuUbb3SEBepWAW1UwN*

`notE`

Theorem 5.4 $\forall A : o. \neg A \to A \to \mathsf{False}$. *The proposition is identified by the following information:*

Pure Prop Id:	*23bb78c7c174d96fa3a932c82b14d7e88bfb71751aaaf84541df8ff1a4ceaf2e*
Pure Prop Address:	*TMVPdAHpYAjBMsM3in5F2sssApoxWNtPMfR*
Theory Prop Id:	*856e325af2a33a3529471e559a4573ad6338bfc260d1c5073519e9b223161c81*
Theory Prop Address:	*TMe11m2tYgGjWUJtGEX5X1tvrSDZWn8w2j1*

`andI`

Theorem 5.5 $\forall AB : o. A \to B \to A \wedge B$. *The proposition is identified by the following information:*

Pure Prop Id:	*3c4f59ac38ec13161d275eff419ecea194e7a962d51e721d2c93249a9130e97c*
Pure Prop Address:	*TMdtgzJ3v8tqbJfC6ZLrrzp79xerKpJKvkB*
Theory Prop Id:	*3bf1985f2198e37de5686038da57a16773e504ab6257e9686d49bfa2edd1762f*
Theory Prop Address:	*TMFJPpp7jnkJTJUfrH6ULPhVR9zhYrkB16u*

`andEL`

Theorem 5.6 $\forall AB : o. A \wedge B \to A$. *The proposition is identified by the following information:*

Pure Prop Id:	*c2f006baa588b23089bf32c5f1ecbbf59d67cec883a690fc7a3dfd4f92b52e28*
Pure Prop Address:	*TMNrPqMjXhvQDz1y7fMcUrWvqz8EvpMAuGq*
Theory Prop Id:	*718be46f06f0025b5dbef8d8ac0138d307c9b79d1e220278818af7e7137afaf1*
Theory Prop Address:	*TMP2MCwM6msNpa6CvfXzjby7fxBAXQCafdT*

`andER`

Theorem 5.7 $\forall AB : o. A \wedge B \to B$. *The proposition is identified by the following information:*

Pure Prop Id:	*f66b82a6432b84ea5d3544eec5c9459bd603e2c74b7dc20fdb37c6c0278a30d2*
Pure Prop Address:	*TML2fqTH52b5jHHUqj4UWsE6pCJqVKiMDrP*
Theory Prop Id:	*5caea9b570346a0a4827977bc317350a63bfb51530b8489373b29970868e822a*
Theory Prop Address:	*TMRHXWMGqfM8cUBGJnz16hksWXqTpSxL6AN*

`orIL`

Theorem 5.8 $\forall AB : o. A \to A \vee B$. *The proposition is identified by the following information:*

Pure Prop Id:	*55be8c084d3d69aaadcdfedd53b12314b3f00b7aa0fe62bba95d8e20a6913eef*
Pure Prop Address:	*TMcR7JViW1Pt5HqJG1ZrAu2LnZ6BoYWBfiw*
Theory Prop Id:	*38267393635d01946334424cfe8e37c8739b8cefeb185f7310c8efc17536d3b6*
Theory Prop Address:	*TMWwPCgtJ9GA5byYB2oN3wyknGgpGw6sABs*

`orIR`

Theorem 5.9 $\forall AB : o.B \to A \lor B$. *The proposition is identified by the following information:*

Pure Prop Id:	*3c63fc5739c09fc9b12fb6d378e9de249f610fbbf2df1cce28f85000280f7bfb*
Pure Prop Address:	*TMTfYZuPbL4xJdZTnLMEYGVXQTCZrDeAcef*
Theory Prop Id:	*8d8e400885f8d52e5c13e42b0127916d2d04fa1a6277be7ce7b01afd1b7e90bc*
Theory Prop Address:	*TMShT5hWbYuTfR7XBLwEnzy4XoJLhwhTWfs*

orE

Theorem 5.10 $\forall ABC : o.(A \to C) \to (B \to C) \to A \lor B \to C$. *The proposition is identified by the following information:*

Pure Prop Id:	*5af08e7b4b57e0dbdc28c2fff18dfe38e04e61787986420a7b92d8802bd7ebef*
Pure Prop Address:	*TMRxb9x7hsQcQdArwJPp9wuFPEjCqtSpjvD*
Theory Prop Id:	*27f42ce3892a9e7634f07197c79efa6d1892500cb92de0ba160f103333bae4e5*
Theory Prop Address:	*TMXA3QBiXM4MyCoYLxr1N5ZWgF5hLhEUP3r*

5.1 PropN

Let $P1P2P3 : o$ be given.

and3I

Theorem 5.11 $P1 \to P2 \to P3 \to P1 \land P2 \land P3$. *The proposition is identified by the following information:*

Pure Prop Id:	*23ac9ca708caa1dd38b9d722781b028bb2d1c60d27249f613f0f58a46f811df6*
Pure Prop Address:	*TMTCLZz943DGXmGagi5cxC1VhkU7Xh7gmxj*
Theory Prop Id:	*cb999bf5d18c324be0801b18f227a943efbdaa196adbe869b96f43be18f1790d*
Theory Prop Address:	*TMHU8VN1xhmJmu7CW62fpKC6YQYCHXWhdNt*

and3E

Theorem 5.12 $P1 \land P2 \land P3 \to (\forall p : o.(P1 \to P2 \to P3 \to p) \to p)$. *The proposition is identified by the following information:*

Pure Prop Id:	*445f7614b5a460ac2ee2c65f60676f43814b16323277b97c9e03b7e2d5ef1bca*
Pure Prop Address:	*TMHw71AzVXN296b5iiE9yrMayr1gPK6Dmgn*
Theory Prop Id:	*50fabe4d504ce92226b4ca92d7a1ab747f919e6c3d6c87a33765f260310d73ef*
Theory Prop Address:	*TMRpTuB7NQrKHNzC82mBhAzGZdBLPpwn8YZ*

or3I1

Theorem 5.13 $P1 \to P1 \lor P2 \lor P3$. *The proposition is identified by the following information:*

Pure Prop Id:	*e44f34f452d8efd0440da0051ba3966fc604cdf0f1a7d5a43a24ebca3f6477e6*
Pure Prop Address:	*TMY8tfGHE5SpXjBtYW9GnGTRRRYZjTkVC1h*
Theory Prop Id:	*ef9071fe3a6dd6111bf8b4b88913d688c289a81f6af948df2b99c0f80d2ac6c1*
Theory Prop Address:	*TMKCha5JJm2rxV2CyYeXrokWFDR2ryrF4rE*

`or3I2`

Theorem 5.14 $P2 \to P1 \lor P2 \lor P3$. *The proposition is identified by the following information:*

Pure Prop Id:	*8df6e153401808c8d1877008f68f33f35f4bb6901c96d857f1e0f13e1519cce6*
Pure Prop Address:	*TMcANDxLDCL4xKQuqhsiQV2MPpGnTrcX2tb*
Theory Prop Id:	*827a91555bc9a5f9fbc87e42c706559e78927705587b35109c356ece2a13307d*
Theory Prop Address:	*TMJjiRdEq1FHszQpepkDPdNLaNYqcVQvb3g*

`or3I3`

Theorem 5.15 $P3 \to P1 \lor P2 \lor P3$. *The proposition is identified by the following information:*

Pure Prop Id:	*017a00202c1447a85eee3f717fabbb57b1194539474715a7958654c3eeff6dba*
Pure Prop Address:	*TMMKVc34VAUfiURqFmHXpwwfDmcdTEMi8p9*
Theory Prop Id:	*3b7f5abd745db6a80ff6dd817b2bc916034448d1d8cca1a8e2958f0b3257c71c*
Theory Prop Address:	*TMc5vyzo3ArE8GAUf3UUvpSQdgk1mgJAYae*

`or3E`

Theorem 5.16 $P1 \lor P2 \lor P3 \to (\forall p : o.(P1 \to p) \to (P2 \to p) \to (P3 \to p) \to p)$. *The proposition is identified by the following information:*

Pure Prop Id:	*5d069d8e90aea128ccbc62404954d9fa5e38854f6dea8d51e13e68ff413e0ab1*
Pure Prop Address:	*TML79YQmK3xkoSvhsjHcVf5xiAdsKJpEJBe*
Theory Prop Id:	*2120c9770c0268721584c931577292b619113ab7eeb88e02ffbf738291ce46f1*
Theory Prop Address:	*TMTxTUaNh3Y288kongaMx7qkfusZvuByRau*

Let $P4 : o$ be given.

`and4I`

Theorem 5.17 $P1 \to P2 \to P3 \to P4 \to P1 \land P2 \land P3 \land P4$. *The proposition is identified by the following information:*

Pure Prop Id:	*ed32cbf9688b8877bdd20c59f473bb9c69432ffd24de548cfc4c59616dd11c5a*
Pure Prop Address:	*TMdAoQLQ3RM117hTZoE6J4XtvGzkLFHKGQm*
Theory Prop Id:	*906aaf2cf039cec16718707b69fda8d19357ca96611472679c8c38ac5552d505*
Theory Prop Address:	*TMXRatgrgSzbMm1bvkukK5JVrsGh2z8rQ6u*

`and4E`

Theorem 5.18 $P1 \wedge P2 \wedge P3 \wedge P4 \rightarrow (\forall p : o.(P1 \rightarrow P2 \rightarrow P3 \rightarrow P4 \rightarrow p) \rightarrow p)$. *The proposition is identified by the following information:*

Pure Prop Id:	*7f6e6c52947184716f0a59a54ee2c3fd4562ee529d8e109ad2527c908611d3be*
Pure Prop Address:	*TMTwtsu5L9Ezi887h92X1qCCtTSgg6Z5Fxr*
Theory Prop Id:	*b41cb764baa71ac5c454b1c9bff373c0bc8ceac5c10a56f18ba2e4a30db98bd1*
Theory Prop Address:	*TMT4yM4NV4sxuvWZUkXew6X7hJ9zigrwUfZ*

`or4I1`

Theorem 5.19 $P1 \rightarrow P1 \vee P2 \vee P3 \vee P4$. *The proposition is identified by the following information:*

Pure Prop Id:	*d61309b1a6dfcdb5b57f074782c88d8e6004fb5bd8c217f4ef2e88a3376de181*
Pure Prop Address:	*TMNvs8HY31Qn4Ju5WNUWKVjbaoTkg7MDXDG*
Theory Prop Id:	*d10b2eb23a5fa505396dff509fa1c442d6870e72d887a41bf455239abe6dff54*
Theory Prop Address:	*TMaTMcRZ9E1Rb4Nuq4C1Ho7bwKAyCZhJbn1*

`or4I2`

Theorem 5.20 $P2 \rightarrow P1 \vee P2 \vee P3 \vee P4$. *The proposition is identified by the following information:*

Pure Prop Id:	*17c763a32537244fd99d77c2973672e33ccdcb0195284690a5b20a50d917f1bc*
Pure Prop Address:	*TMMFmRtJXqQaL2pJyCq2NXmUoQo6iEcBUeF*
Theory Prop Id:	*f9e4cf5b7be5972585b09b77e8fe2de549ffcd217050934f83ca2a0269d7c22e*
Theory Prop Address:	*TMXYRWPJ6eBJ2veji2CB4sPXSweJfbMW9NC*

`or4I3`

Theorem 5.21 $P3 \rightarrow P1 \vee P2 \vee P3 \vee P4$. *The proposition is identified by the following information:*

Pure Prop Id:	*7500b33566aaf93de84ea48148bfb3bb21b30ec225170d993339f8bb0cbbaaf1*
Pure Prop Address:	*TMZiwfSWdAkAZ7WSEnmx8EfCq26LV9zrvDT*
Theory Prop Id:	*1a5881c849fb86fd77a8a4f11ef6783644b06e8c3f74fb02af0c13b7a816c3a7*
Theory Prop Address:	*TMHnh1oKCafRDqpe4JxXfg2qfSkei64vT35*

`or4I4`

Theorem 5.22 $P4 \rightarrow P1 \vee P2 \vee P3 \vee P4$. *The proposition is identified by the following information:*

Pure Prop Id:	*7a20061f67b705b52da52d1619188bf47b727e3bf41cab7aab5e3b9c6c9e1e9f*
Pure Prop Address:	*TMVBHMZoCoxkQBLGMKjpPfCyWzaL2nkaZx4*
Theory Prop Id:	*1ecd5bc77d1ec7549783816ee07b8441f43a51ff5ab63ec30ca559c442813060*
Theory Prop Address:	*TMFhHjtCoxKWhG8QRGUQ92dvizsEVBwWvYQ*

`or4E`

Theorem 5.23

$$P1 \vee P2 \vee P3 \vee P4 \to (\forall p : o.(P1 \to p) \to (P2 \to p) \to (P3 \to p) \to (P4 \to p) \to p).$$

The proposition is identified by the following information:

Pure Prop Id:	*d05b31b0be8cd483fa07a3f0f19264b60e47f6c21659902935462906bdf6b76d*
Pure Prop Address:	*TMQwRKYH5ei57PtG8MGuyMNQBKe8cuszmF4*
Theory Prop Id:	*1fdc2a30b796702db155c2eb5a9c6e71c48a99beca4fc05f20102b22e9cdce47*
Theory Prop Address:	*TMWhPKKnyM7dRKMXmnAxawenezrpwrV9cgz*

Let $P5 : o$ be given.

`and5I`

Theorem 5.24 $P1 \to P2 \to P3 \to P4 \to P5 \to P1 \wedge P2 \wedge P3 \wedge P4 \wedge P5$. *The proposition is identified by the following information:*

Pure Prop Id:	*041ced6dc277faece2d470de6befa0e97006998af0c9cb1989fc9901b3ec866f*
Pure Prop Address:	*TMVd1Ghg1sxR7zcgk4GPNy72pYJ9AQdBzd6*
Theory Prop Id:	*11fc478665305a3b2e27aa913e8dcb933d84e00ee923c3e503f0ed51d4ed7d8f*
Theory Prop Address:	*TMFEndoHPA1ez9Y8dc2y2YAf6QxYxCFg4BB*

`and5E`

Theorem 5.25

$$P1 \wedge P2 \wedge P3 \wedge P4 \wedge P5 \to (\forall p : o.(P1 \to P2 \to P3 \to P4 \to P5 \to p) \to p).$$

The proposition is identified by the following information:

Pure Prop Id:	*b9e36eb0772059378de6e2bae57aec129036e5378cd37e203e1791e7b44f92cf*
Pure Prop Address:	*TMJBWvgmbPvfKX7ymNvDkg1M46iQe7Ny1z7*
Theory Prop Id:	*a02e0e4e7793f2f93af907335f12098d37abc74e46d2eeb4465324ba9d8046db*
Theory Prop Address:	*TMMmsE2QkKEBNxNaoo9zCfmr5o7QBZgTPGQ*

`or5I1`

Theorem 5.26 $P1 \to P1 \vee P2 \vee P3 \vee P4 \vee P5$. *The proposition is identified by the following information:*

Pure Prop Id:	*39ea457dead1717f0a108c7304916bfd9cab247cf4cf2d3b67f5065814a8e0b4*
Pure Prop Address:	*TMUs12JtV2vEaiiahyRUXpuovbU11fq7zLB*
Theory Prop Id:	*4e8f4064dae12df31e7df3da1e5c8535b89046a7b5319d8f2305e5de1b24f829*
Theory Prop Address:	*TMTPcpiEwfkrghjEEposf5yS1v869ke7Rrj*

`or5I2`

Theorem 5.27 *$P2 \to P1 \vee P2 \vee P3 \vee P4 \vee P5$. The proposition is identified by the following information:*

Pure Prop Id:	*8494de963a2912613d2bfb1d747fc055d6eb362d422be1896eee6d866c358af9*
Pure Prop Address:	*TMFZtXVUhAJgMrzbc8SGQr8PvkqfircW7cL*
Theory Prop Id:	*32035af2da626c1acd3d71cbf3197f52b67159384f068ebb3b5f4c50a08ce3db*
Theory Prop Address:	*TMR8LzrcpDPkw2uf7T8WHukPntFRQWVakmR*

`or5I3`

Theorem 5.28 *$P3 \to P1 \vee P2 \vee P3 \vee P4 \vee P5$. The proposition is identified by the following information:*

Pure Prop Id:	*09182b1c056c3e6ec773e5fb3c476f7e026643a15998bb79deb35e131fc516a4*
Pure Prop Address:	*TMV1nEkVMZR4hjji67oiYpLCFwcPavF6bSp*
Theory Prop Id:	*4aca859beb67b87c07448642e2f80b02e346c431898060774f6806896b8bc060*
Theory Prop Address:	*TMVdimQJwC5PcNHpaQeD8vVev4mkSnqKUDh*

`or5I4`

Theorem 5.29 *$P4 \to P1 \vee P2 \vee P3 \vee P4 \vee P5$. The proposition is identified by the following information:*

Pure Prop Id:	*51cd1ca65c14f555453c13f820eac600735e6825f73e4e668138659cb752ce82*
Pure Prop Address:	*TMRxkMs8kLmmDi8zdv7a2mgmiiwm8gtbeQC*
Theory Prop Id:	*8e109e9f1c205c91c5b79a556c11e2b2fb94d2d77c537f904faaf5e7cdf2fce4*
Theory Prop Address:	*TMTZe6z6z3GDdCznmKAm5d5ZAas3rSPduhh*

`or5I5`

Theorem 5.30 *$P5 \to P1 \vee P2 \vee P3 \vee P4 \vee P5$. The proposition is identified by the following information:*

Pure Prop Id:	*1d6362c9b2c69a1b4b1360eef0b6f9f9763f1089e1bbde2904b3db2c6544b5b9*
Pure Prop Address:	*TMHM6WDtzUk5yH5hFkLWwTSXzie4fHKJkeR*
Theory Prop Id:	*5947729b14c31f7f5dfb495f3dc278156afc20b954ecaa24e6839650aef497bf*
Theory Prop Address:	*TMWqVbPVCB7MunfAfT8ReFmNyoB4QbkEzA2*

`or5E`

Theorem 5.31

$$P1 \vee P2 \vee P3 \vee P4 \vee P5 \to (\forall p : o.(P1 \to p) \to (P2 \to p) \to (P3 \to p) \to (P4 \to p) \to (P5 \to p) \to p$$

The proposition is identified by the following information:

Pure Prop Id:	*b956eb2b7204f5a6e780e90bc1bd534e682ab77b2ad4cb7c5d03d8acacfc9a2e*
Pure Prop Address:	*TMURL8fhzF9KpcCx1BootRK4CFuzooNhaUw*
Theory Prop Id:	*780a9f1fdc60b8ddc55d630d2b78a745575857 51e69471682b41605c6a27eef9*
Theory Prop Address:	*TMVMNDhbZznQnqiXkLo51F83SmRwT6czcWC*

Let $P6 : o$ be given.

`and6I`

Theorem 5.32 $P1 \to P2 \to P3 \to P4 \to P5 \to P6 \to P1 \wedge P2 \wedge P3 \wedge P4 \wedge P5 \wedge P6.$
The proposition is identified by the following information:

Pure Prop Id:	*9817c7a535d81bed18efaa95b110653ebba78a7ad991eaca8054c41020b664da*
Pure Prop Address:	*TMM8hfvyGGBPHT3XKYSQ5WjPZBdQu8CHFmh*
Theory Prop Id:	*a5dc2935b019c23338edba6ccdea3a654b6a51efe6170f8887e55dc4e5d6e4ff*
Theory Prop Address:	*TMNPT8qknaGSp8UzvMyiNy5UD7HRGHNMvZ9*

`and6E`

Theorem 5.33

$$P1 \wedge P2 \wedge P3 \wedge P4 \wedge P5 \wedge P6 \to (\forall p : o.(P1 \to P2 \to P3 \to P4 \to P5 \to P6 \to p) \to p).$$

The proposition is identified by the following information:

Pure Prop Id:	*3668da80c73d5b9a85e7b685ab4ed93a92bdbb2a7cacf0c764a2353bffd98bd5*
Pure Prop Address:	*TMQ5JF8b5hxhEB7utWT6iBHdgtNte14WFKZ*
Theory Prop Id:	*1c4fc650221a6c544c529f279d39c9effe360f4bbb342bf30ac2ef1bb0cae7bd*
Theory Prop Address:	*TMXE3S7PKbVYmocgEe4jSS2d628TPGWZpTj*

Let $P7 : o$ be given.

`and7I`

Theorem 5.34 $P1 \to P2 \to P3 \to P4 \to P5 \to P6 \to P7 \to P1 \wedge P2 \wedge P3 \wedge P4 \wedge P5 \wedge P6$
The proposition is identified by the following information:

Pure Prop Id:	*6a4d8ce6f0dd012e01148d6cfcd0336a7bdfd1f6e0a108700387aa6534f21e3a*
Pure Prop Address:	*TMcFEeC2wp56VXpGCwrrXAy9KX7iPCggM9d*
Theory Prop Id:	*c89388f0b732abd74a2fb4d6fef23202a20e588058361e4617d162de86cd6b7d*
Theory Prop Address:	*TMFwsYdcsUc2UYbNFsd4RzP6m2K8FrCgFrq*

`and7E`

Theorem 5.35

$$P1 \wedge P2 \wedge P3 \wedge P4 \wedge P5 \wedge P6 \wedge P7 \to (\forall p : o.(P1 \to P2 \to P3 \to P4 \to P5 \to P6 \to P7 \to$$

The proposition is identified by the following information:

Pure Prop Id:	*2ea6ee206fc0e0bac89e94ecdeb5cc8218a37b0e2ad93881ee4b947fe261aca5*
Pure Prop Address:	*TMarbzMgS52Zmt6DZPQwD7NrgfFnGmniZBg*
Theory Prop Id:	*02714a2b656b0d600f52d924edb42df9d13fd49d79bacc042df6ff567767a5a5*
Theory Prop Address:	*TMJ3TDaD9hfZwexV6a4jq3FXE9KhjambBpi*

5.2 Further Results

`iffI`

Theorem 5.36 $\forall AB : o.(A \to B) \to (B \to A) \to (A \Leftrightarrow B)$. *The proposition is identified by the following information:*

Pure Prop Id:	*7516b9382e323d500a2b396c81bf98c3859ca15e7e7b359310c7cea2aeed5c8a*
Pure Prop Address:	*TMMjuJFnsPng1UMvQzKS18LFFsgWsi9d8xU*
Theory Prop Id:	*4d966a5f5da0679b05fd8ce1619ac4165c9654be7a87ee2cceaaec3ceac10633*
Theory Prop Address:	*TMddTKwGeWwMHruZq1JPdHyJmpeTrFg89Xi*

`iffEL`

Theorem 5.37 $\forall AB : o.(A \Leftrightarrow B) \to A \to B$. *The proposition is identified by the following information:*

Pure Prop Id:	*0e94c69dbdcf6a1baca24d804ad190121b2de563e5642fa8c452be1c6595979b*
Pure Prop Address:	*TMPAKwBu2btn7LfWCbudNDBAqu86wzbzmEY*
Theory Prop Id:	*9a07f60730cc4e5044b6b2af10fd8467b0bedf6f66364ca33c0818495467c2ed*
Theory Prop Address:	*TMTL1Jyg3GdY9swgWWQPXN6ceqVnBuFt1Dn*

`iffER`

Theorem 5.38 $\forall AB : o.(A \Leftrightarrow B) \to B \to A$. *The proposition is identified by the following information:*

Pure Prop Id:	*54702aec0e7fabe49df0fa4b3a00910bf49c4c03c646ddc286485e7af2899fe8*
Pure Prop Address:	*TMLxQTofs8NSTdPqLWVf9T5mVck9C5B8m8s*
Theory Prop Id:	*f5dfc0631a7727945b67f112d323c8a1558a724a0d6b5c261d4a11649d925f64*
Theory Prop Address:	*TMGZN34A8TcN1Z21rxqNAMYJRjG67dAotvb*

`iff_ref`

Theorem 5.39 $\forall A : o.A \Leftrightarrow A$. *The proposition is identified by the following information:*

Pure Prop Id:	*01338dd4902fcd80dd89af75713f451fe1b1e4804229f05955b962fe6def6250*
Pure Prop Address:	*TMSKz5gXPFakcgFRBJ5RJYfDQC2q8ocrFi1*
Theory Prop Id:	*7b71b52f0f8ece18ad34af695ce60729d9853085b81694137894 7b88e44cbf0a*
Theory Prop Address:	*TML3Gcc3WbCE559uqmKxvpDahXs8iDJjURL*

`neq_i_sym`

Theorem 5.40 $\forall xy.x \neq y \to y \neq x$. *The proposition is identified by the following information:*

Pure Prop Id:	*5c0ec9ea191d0f6e263cc182b469ec839768d633284966a6a6a2058c1778ed0f*
Pure Prop Address:	*TMXXdxoTHLJ7BfG4hU2CwgemEXjQ8FmtQ1C*
Theory Prop Id:	*1a98981725a2236f37b1c465a0e41336ebef451702e4d9da44b1fab316a2e844*
Theory Prop Address:	*TMdSA41VnJVw63YmxPtgsvson8sPkVFMoKc*

Definition 5.1 *We define* nIn *to be* $\lambda xX.\neg \mathsf{In}\ x\ X$ *of type* $\iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*36808cca33f2b3819497d3a1f10fcc77486a0cddbcb304e44cbf2d818e181c3e*
Pure Object Address:	*TMSqgddy21cUnLoRkMPcBAxXhAmCfaQeLQj*
Theory Object Id:	*253451c995b7e8af5baae1333a990b55d0dea9af34d44b09b418d3f09fe72de4*
Theory Object Address:	*TMReNhKc6t82pWW6stKo9YNy1BuLWXDCyeC*

Notation. We use $\notin$ as an infix operator corresponding to applying term nIn.

`Eps_i_ex`

Theorem 5.41 $\forall P : \iota \to o.(\exists x.P\ x) \to P\ (\mathsf{Eps\_i}\ P)$. *The proposition is identified by the following information:*

Pure Prop Id:	*be36f0573e3745e47e74354bb117860bd387b3f6c26ba3a752a7388e6b66d647*
Pure Prop Address:	*TMFoJytvVebF2Sg18ua9AU1WgCbeqEijvbZ*
Theory Prop Id:	*6a28328c813ae82b72e327564626a89c19306d9b1f89653ada2b22e954e90881*
Theory Prop Address:	*TMXYa9wuYkRrAmfCgqhH6hY5uK5GFw8gzHB*

`pred_ext`

Theorem 5.42 $\forall PQ : \iota \to o.(\forall x.P\ x \Leftrightarrow Q\ x) \to P = Q$. *The proposition is identified by the following information:*

Pure Prop Id:	*e9e64b253f257e12e8cc34a6936324678cd05c9763263d2d12e48da4bd7198e7*
Pure Prop Address:	*TMYnqc7V192uGaHR6Z4JQYWcpukZNrMq6cy*
Theory Prop Id:	*d4b2d3f3eabb3ff09d4d1a4e7e1663d78c31373aac18adcfbef2e2c081bb44fa*
Theory Prop Address:	*TMFdQW2ooCy3wDncDrzFGtMSsTwAzHRg5kG*

`prop_ext_2`

Theorem 5.43 $\forall pq : o.(p \to q) \to (q \to p) \to p = q$. *The proposition is identified by the following information:*

Pure Prop Id:	*a9ede2c9e3cf1d70dfbf9f57d775f61afa29105059e99ada89c0de099e2c1a26*
Pure Prop Address:	*TMNH7NvYuoPP9nYGA3ZMWdycLUsdiXP5REb*
Theory Prop Id:	*a56fc7bc69f2eed7db4340adfe025ace2f0f83cdfc470f09b018498a097775ab*
Theory Prop Address:	*TMYdTtZfAJ1wEsDSWCBMpXk181wbLRKQ2tT*

`pred_ext_2`

Theorem 5.44 $\forall PQ : \iota \to o.P \subseteq Q \to Q \subseteq P \to P = Q$. *The proposition is identified by the following information:*

Pure Prop Id:	*c558ffab38d9b7b5cc1866022b11c5f544a9b798f1745926e15f3391ff6cc87d*
Pure Prop Address:	*TML1d3NrCUcMsEZrsdaVLuyf7VGJq9zKYD8*
Theory Prop Id:	*c5df84914 88c0d5344f2a38c75caa07e6cfe3d5e89feb23ebc08798a9fdccfd4*
Theory Prop Address:	*TMW8biyFW9jcdzzGjDz5sXDkojWntfdbQ1t*

`Subq_ref`

Theorem 5.45 $\forall X : \iota.X \subseteq X$. *The proposition is identified by the following information:*

Pure Prop Id:	*4c53952cc6ecb22968534a4d43adab78af1f0b047ed070da10ea521d270d1f2a*
Pure Prop Address:	*TMWfjRz3PusvtrSBv2umMamuPFQ3vpfqq5R*
Theory Prop Id:	*2efbacece8d60c274641e6775400fab460246bacd8ac2d18f883d059a7ac8358*
Theory Prop Address:	*TMLkYwQzhxE5mhkLWGyedCPNtZ5dCRp8qmB*

`Subq_tra`

Theorem 5.46 $\forall XYZ : \iota.X \subseteq Y \rightarrow Y \subseteq Z \rightarrow X \subseteq Z$. *The proposition is identified by the following information:*

Pure Prop Id:	*79accb4060735cc396e0e3ea36014c3e94c87d8e52a22ed4c18d55d6af348d9e*
Pure Prop Address:	*TMXhwEo6mEGCoaDosK1LZDEK8xXZr8kji3H*
Theory Prop Id:	*fed662887c1b855fd3ff74e7894edbaebabacb12dcae138964b49804bddac9a6*
Theory Prop Address:	*TMaozUoYZxHxHHTvoggGVukYwGVgw9VLyP8*

`Subq_contra`

Theorem 5.47 $\forall XYz : \iota.X \subseteq Y \rightarrow z \notin Y \rightarrow z \notin X$. *The proposition is identified by the following information:*

Pure Prop Id:	*d15e87bdbc1fd0a307b0f0b273414e5fca8c31ec8256241a668bcab8d25dc157*
Pure Prop Address:	*TMZtpMMhjtkFakD25QWYgBYDJcz4naaYj5w*
Theory Prop Id:	*bd5a2f337d80e9eca8abab55ca08c824896fb8274ddacef26a0d1e1bf8b8f0a3*
Theory Prop Address:	*TMFK5BFNrPEtMNeyxT8Acey4o4GybQWYQ8i*

`EmptyE`

Theorem 5.48 $\forall x : \iota.x \notin \mathsf{Empty}$. *The proposition is identified by the following information:*

Pure Prop Id:	*93a77a0c78f3c49660c44dd07c723ad23e5e55e64c0dcd527a22c0278b829b8e*
Pure Prop Address:	*TMaSWKbJHDuPvqXN5ddrdRfghAejYyXef7K*
Theory Prop Id:	*8349abf2aeba6cf6d62d407fc6062a629c409a81df869ba5e46a2adde87b67a5*
Theory Prop Address:	*TMUoYxyibdoLdU1WGAmWKr2UQqgAiyrf8sb*

`Subq_Empty`

Theorem 5.49 $\forall X : \iota.\mathsf{Empty} \subseteq X$. *The proposition is identified by the following information:*

Pure Prop Id:	*a517cdaf3478a733b128a31cce59618dd840c79c17df99f29410ff3e9ba455e4*
Pure Prop Address:	*TMWWMiBhRYozNqky8uYhTyxKxDXXVBjHmqH*
Theory Prop Id:	*2faec377f13468a82bbfcb310b43ad7a193b5d26e72c513a4b6fbb139dd1f852*
Theory Prop Address:	*TMJxeX1k56XRyrVDH9k6F9DZp2n6xFpPAck*

`Empty_Subq_eq`

Theorem 5.50 $\forall X : \iota . X \subseteq \mathsf{Empty} \to X = \mathsf{Empty}$. *The proposition is identified by the following information:*

Pure Prop Id:	*2dd95966ce09dd2345607d594b51d1640857501d96dea096bacb21e292697f31*
Pure Prop Address:	*TMbzyCseCE8NavdH4tvfvqjNSXHHLkNaTk4*
Theory Prop Id:	*988691971197671404c5f64923b3e6f170a71f5f8f48e44c06089873913e08c9*
Theory Prop Address:	*TMQENPUZ2wP5s7vyaLEtBCdaJErFFSUpJSV*

`Empty_eq`

Theorem 5.51 $\forall X : \iota . (\forall x . x \notin X) \to X = \mathsf{Empty}$. *The proposition is identified by the following information:*

Pure Prop Id:	*3de0fdba2afc8aa47ba046f6d57a7edaabc38eebe8925bdc1b3803ca0a447373*
Pure Prop Address:	*TMM1agcdRVrGxbz8kV5AmjKvcKDfDHWAxHb*
Theory Prop Id:	*8b915bb396e264c2d82d33dc8dfd93f4374585f0ab37c01fc964b73f4e28b7a3*
Theory Prop Address:	*TMcosVQ8txADpELPSRpWSvATFdw3vWrW8H2*

`UnionI`

Theorem 5.52 $\forall X x Y : \iota . x \in Y \to Y \in X \to x \in \mathsf{Union}\ X$. *The proposition is identified by the following information:*

Pure Prop Id:	*51b5499aea657de4516b761629ee7a8bf5292810ac70c13d0745541a55023b02*
Pure Prop Address:	*TMQCYZzADyCKcXXBekJX5crLwK9MZzh7paV*
Theory Prop Id:	*bade054056415b264e8e0ad162a917d91a5399de193fc12ede490c8bf786acf3*
Theory Prop Address:	*TMHDQ27UKqnJLKFeQDx27tH4XqzefSAZp1w*

`UnionE`

Theorem 5.53 $\forall X x : \iota . x \in \mathsf{Union}\ X \to \exists Y : \iota . x \in Y \wedge Y \in X$. *The proposition is identified by the following information:*

Pure Prop Id:	*db4bc1188573af6ee726c91f30668d0d047de9916b17056407fb50b728e30efe*
Pure Prop Address:	*TMF2PbBRuwVtGUAm2dBcakxEFts1b3gfeHi*
Theory Prop Id:	*ab781b05690ff7302616c39024e38d17990dfb88f8dc15950b3d5f47ab0a4324*
Theory Prop Address:	*TMPGvCnxEDNcLzGEpegCEggB87LUw7GvgQ9*

`UnionE_impred`

Theorem 5.54 $\forall X x : \iota . x \in \mathsf{Union}\ X \to \forall p : o . (\forall Y : \iota . x \in Y \to Y \in X \to p) \to p$. *The proposition is identified by the following information:*

Pure Prop Id:	*8efd578687dc029da349c22ca087953113491e4794018f04c73b2c7ae5084bf0*
Pure Prop Address:	*TMH5ng38evRBjZnMvcn9FJLMLXbDC1Bz9aY*
Theory Prop Id:	*d621b2d16dc72ef52c1fe29162e3508201e053e42f163701b3713472780e9a5e*
Theory Prop Address:	*TMHeeXhX7g1GDga7C7LQdPcR4t9R2vstffp*

`Union_Empty`

Theorem 5.55 Union Empty = Empty. *The proposition is identified by the following information:*

Pure Prop Id:	*2c511ebf784d1ac4263638dde61f03ebe06b666994224125d685445a9af596e3*
Pure Prop Address:	*TMLm3kKCuXxhm2w4H3avGFTRVMtJZ4TZQPW*
Theory Prop Id:	*72b1c4f60fbbe561816842cf8d137a012a23b49ea41913fed5ee67e33bbf6c52*
Theory Prop Address:	*TMMwTUayoXbEd6iFoK2PNm3DAtTEe5cdhtf*

`PowerI`

Theorem 5.56 $\forall XY : \iota . Y \subseteq X \to Y \in \mathsf{Power}\ X$. *The proposition is identified by the following information:*

Pure Prop Id:	*7143d71ecf102401ab2636034e61d377bec399d276d42b9e418431501853f4c6*
Pure Prop Address:	*TMYd9dR9ghFpJALxMSh1bifCcoHmd6XFuvA*
Theory Prop Id:	*3c48d3c62b94e7a41c69deefa90b2e8c771cbc0aa279fcba058fefb98579331d*
Theory Prop Address:	*TMXdAMd1HZUyw6mE1ZFaV71nZ63jBYMLiUj*

`PowerE`

Theorem 5.57 $\forall XY : \iota . Y \in \mathsf{Power}\ X \to Y \subseteq X$. *The proposition is identified by the following information:*

Pure Prop Id:	*149030a919d6402241e63172077fe438016089bfde0e7da4cbc536dc04901bb1*
Pure Prop Address:	*TMWSJ1X7N37P2Pemk8UyDMfYjkY3zie7nYZ*
Theory Prop Id:	*6470703fa38f09b9dbdffe7bcefe88a4d4c94d929d7e93c64556a9f00ad14694*
Theory Prop Address:	*TMTYtQq2xbReLwQoZqj15EvuksZjaG3A9fA*

`Power_Subq`

Theorem 5.58 $\forall XY : \iota . X \subseteq Y \to \mathsf{Power}\ X \subseteq \mathsf{Power}\ Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*6d8f173a14a5bbfef48f6ce86bee8512556a0deed405c0bd072d5141d2299af9*
Pure Prop Address:	*TMUvnczMnpH9X3CybY6TzCw5SCHqkt7ir7u*
Theory Prop Id:	*9dd5f3b3f3808a5118155ae75d2e1c08c28ea7ba30690f84aae27623efd601dd*
Theory Prop Address:	*TMLB1eXPkLwvDDtkFvqtauiz4FckV2UwQPk*

`Empty_In_Power`

Theorem 5.59 $\forall X : \iota . \mathsf{Empty} \in \mathsf{Power}\ X$. *The proposition is identified by the following information:*

Pure Prop Id:	*d2656b52d4bf2bab93fe5255f13b797a5512a056ca12c4fb73ccdc10829f6f17*
Pure Prop Address:	*TMaK4ZHB63mPURsGQ3iXFc6dERVy8jXzgG5*
Theory Prop Id:	*487855dcfff4149b1efb5bd9ea0572b18fe9005d660d86fcfc5ff90110ba6df9*
Theory Prop Address:	*TMa8XHamyspUjhkeiNgAYuj8aecCrFsHqjg*

`Self_In_Power`

Theorem 5.60 $\forall X : \iota . X \in \mathsf{Power}\ X$. *The proposition is identified by the following information:*

Pure Prop Id:	*f81ef2a25f07c69ddd40acce90e3caf25609ce25f8183e2509b613cb0092508e*
Pure Prop Address:	*TMQycQfNrWABmzjhJNUDx6UCqmAXeAy1qsv*
Theory Prop Id:	*8336ff563585b5335f5d748d44c68d6e04e2e671537cfd5537a4b37acde6a25a*
Theory Prop Address:	*TMKY4E4GGAWwCfDftsZmANC17noKbnuCrbo*

`Union_Power_Subq`

Theorem 5.61 $\forall X : \iota . \mathsf{Union}\ (\mathsf{Power}\ X) \subseteq X$. *The proposition is identified by the following information:*

Pure Prop Id:	*7f4d2bc5abb24988ada92aecad1eeb9b7e0902f2bf91f6062f31024af17a8bed*
Pure Prop Address:	*TMTgvEtxJEiGcbTTuKLumZaRiNe3WkRbVBG*
Theory Prop Id:	*1b4311aee3fe38a1eb6d381638c72194c02e1025b68cb8fd7f24d84b4aa8ff10*
Theory Prop Address:	*TMQdwUwhhyCEESAhoqgTeHBAY85kLo5KAYP*

`xm`

Theorem 5.62 $\forall P : o . P \vee \neg P$. *The proposition is identified by the following information:*

Pure Prop Id:	*62e4c3fec424181797ab4ef1d385902e9cdc35c7558a8f861b90f48a494c0e0e*
Pure Prop Address:	*TMKtBvuSXX3aDsty6aeAaXbywwMJa4KhJpC*
Theory Prop Id:	*7b6737622a32781568aa9679a7eb1928791917e8dbf1a5fea21e2f0cbccee791*
Theory Prop Address:	*TMMPzhDBTSXgihVRhAwaiREjdQ5Jeotv34y*

`dneg`

Theorem 5.63 $\forall P : o . \neg\neg P \rightarrow P$. *The proposition is identified by the following information:*

Pure Prop Id:	*852f346d61a871c2db340ac4fd7de308d10ae313849575cd39cfac1469b8995e*
Pure Prop Address:	*TMHNYQKxvrS4ZDs2FctrNBCPfYUgZB6RFrx*
Theory Prop Id:	*b7cc631d047502f5a0c47618dc8c76a4947e2224be400faf040b7ef95f7994da*
Theory Prop Address:	*TMRYv1F8fFeWtceTAuu9zbNET7tH5xtFHQn*

`imp_not_or`

Theorem 5.64 $\forall pq : o . (p \rightarrow q) \rightarrow \neg p \vee q$. *The proposition is identified by the following information:*

Pure Prop Id:	*a8655639be6ebff69772dc3e37a7cf1414005132611dfae586f7e43fa02f82f9*
Pure Prop Address:	*TMcvyPAhaTi7kXdf8YYSBsEH4XcVThUkX9L*
Theory Prop Id:	*30c98c972cfd8712dbdce74f465991aad8aa72159a4511fc01e975a23d063a5c*
Theory Prop Address:	*TMH4ymv2c9JczjQGEQqzDcP9xj2yX1JHHfx*

not_and_or_demorgan

Theorem 5.65 $\forall pq : o.\neg(p \wedge q) \to \neg p \vee \neg q$. *The proposition is identified by the following information:*

Pure Prop Id:	*29877a368f94842edeff462e056244efa22376525930141a2d2926e5f3d0ef3b*
Pure Prop Address:	*TMQfs5TGwUrjea616pdTxbLXGNvhBxY4UzL*
Theory Prop Id:	*63ddfe069e38d4763ee4f112c78b39e12a120ccaa496fd47e6a4646fc9391093*
Theory Prop Address:	*TMaCC7RwYa5bssu9qewCUzTz5xikj6JohLz*

5.3 Exactly 1 of 2

Definition 5.2 exactly1of2 *is the opaque object of type* $o \to o \to o$ *identified by the following information:*

Pure Object Id:	*163602f90de012a7426ee39176523ca58bc964ccde619b652cb448bd678f7e21*
Pure Object Address:	*TMS9Bz8WFNT8hbFokWDmzwFmNr2ZD7CfP2e*
Theory Object Id:	*a0b94402d8edc93e77f0dd488bd6709a5ab850b3f07e42bedc0d1e40163aec06*
Theory Object Address:	*TMEqbBVsvC394VsHqRdcJB6GggoMgLnuS6A*

exactly1of2_I1

Theorem 5.66 $\forall AB : o.A \to \neg B \to \mathsf{exactly1of2}\ A\ B$. *The proposition is identified by the following information:*

Pure Prop Id:	*8e1e4515e8d2e1265d837e207a0eaa000d82c55e2fa708c4ebd9ad229b5a5d38*
Pure Prop Address:	*TMQnjH6acMFmZdz797stCauUdrSebKX7Mev*
Theory Prop Id:	*4f94fafdd5f7f8d4dc0f3e1b7277b17a0c7c19add3a6af231254dcedab30fe39*
Theory Prop Address:	*TMJL9YXWiTdUUzhvi5WWHy9wvvoebH1tQzv*

exactly1of2_I2

Theorem 5.67 $\forall AB : o.\neg A \to B \to \mathsf{exactly1of2}\ A\ B$. *The proposition is identified by the following information:*

Pure Prop Id:	*c9fd90a0917801b2b7fae79f83bdc259f931342830e8158acece4f3c1a283659*
Pure Prop Address:	*TMMAKooscST9mjX4iuDPzg3FY58bpufyfVU*
Theory Prop Id:	*6c1ad919f0b4255c719d573f707feb239d05426a99ece78b2f738ef43e3e1a51*
Theory Prop Address:	*TML5oMK64kBLn2JLS7ATshVeqCgaCsuNK6e*

exactly1of2_impI1

Theorem 5.68 $\forall AB : o.(A \to \neg B) \to (\neg A \to B) \to \mathsf{exactly1of2}\ A\ B$. *The proposition is identified by the following information:*

Pure Prop Id: *51838fa63f26d38944c5e35f2413ef312130a04668affc103545649e6845b841*
Pure Prop Address: *TMRhwQ9VVpjkPMMkpzHArgyLqFJ2sLxakNb*
Theory Prop Id: *517be2b6e298510b19ae2a4576b522c9cc8fe9bf89dfa1630bc8b79219852955*
Theory Prop Address: *TMcoNtefvy7eJpxYPtwBmyQuGKfMJ9PMpkn*

`exactly1of2_impI2`

Theorem 5.69 $\forall AB : o.(B \to \neg A) \to (\neg B \to A) \to \mathsf{exactly1of2}\ A\ B$. *The proposition is identified by the following information:*

Pure Prop Id: *f32aab556c283c02cca82ae1401040f6003bf6a1ac60788c12ad6329d832a293*
Pure Prop Address: *TMSY4dBSVErbAWxEf4c743PqBZ9EABV3BJY*
Theory Prop Id: *811f013a6ac0abc6e0c8f752b4c6fbd661919f540a84f0f8b6fc99dd0d22aa07*
Theory Prop Address: *TMGJHhrbg2Ea8nrQBjPv3SdM2aoAj4dxq8y*

`exactly1of2_E`

Theorem 5.70 $\forall AB : o.\mathsf{exactly1of2}\ A\ B \to \forall p : o.(A \to \neg B \to p) \to (\neg A \to B \to p) \to p$. *The proposition is identified by the following information:*

Pure Prop Id: *29b901ba4db12851de74e31368539f6d1cc25bbc1c301edaa62335d7e6c17545*
Pure Prop Address: *TMRCeD4ey2b4rXZn7HkMnX7YburjvMK1Eyg*
Theory Prop Id: *c674348923fa7045ff80c55a4bb504730f6f8ee0dcdf90e8c71d35291ea85112*
Theory Prop Address: *TMJHeaLcDFsg71BZKr8vRobyqEDxdcim8fW*

`exactly1of2_or`

Theorem 5.71 $\forall AB : o.\mathsf{exactly1of2}\ A\ B \to A \vee B$. *The proposition is identified by the following information:*

Pure Prop Id: *18af7395e3694a283ae114ccec2d06247033786f17700e124d0fd73c83572675*
Pure Prop Address: *TMLi8xLyPBxTfc12noiHmSmqavSb4tpNQeQ*
Theory Prop Id: *e092f672ff0d584cbaf46120347c86b03fbcec240a9d5100380e8240373fd2be*
Theory Prop Address: *TMXLeWxQ5A2VGtDdKgtMuBn5HCa3VzxF1pb*

`exactly1of2_impn12`

Theorem 5.72 $\forall AB : o.\mathsf{exactly1of2}\ A\ B \to A \to \neg B$. *The proposition is identified by the following information:*

Pure Prop Id: *d3009840b3d85b74b9e0a9620f4308a8e1436d22cdd2a69e83dd3c46c1487323*
Pure Prop Address: *TMWzgfM5wbPGyJhNsxsijwiYzmX4TthmdKH*
Theory Prop Id: *ffdec3681a2e06d83ddcbdc151520cfe5b3b623d2db529d7a849209f1d6fe7cc*
Theory Prop Address: *TMG66wx9tzCkMhQc9NRiUxq74peErmdEviC*

`exactly1of2_impn21`

Theorem 5.73 $\forall AB : o.\mathsf{exactly1of2}\ A\ B \to B \to \neg A$. *The proposition is identified by the following information:*

Pure Prop Id:	*29929fd4030494b67c9eeeaa7dc35578573efb328bcafaf20f5efb6ad854e87b*
Pure Prop Address:	*TMJz2j2BW2BvWFzAm664pFqbVtvTqWXzGq7*
Theory Prop Id:	*7499d10110c69f4c2d84763d224e28a23a5525fa48cfc823789b09051b07db66*
Theory Prop Address:	*TMPjFxrRCRMeS8z17aDuupoUhLQX4Kodx6A*

`exactly1of2_nimp12`

Theorem 5.74 $\forall AB : o.\mathsf{exactly1of2}\ A\ B \to \neg A \to B$. *The proposition is identified by the following information:*

Pure Prop Id:	*0fb71ab4a166db8b8a5493187a1623cb2573bdafdb80c215e1df61d68b8fa34b*
Pure Prop Address:	*TMYo2ThWofRLhEMmrLih7wWsVmnmor928bi*
Theory Prop Id:	*d000b6d72c072278302ade061ba0e7a3735507b341fa9f3cb4b8ea13a683c623*
Theory Prop Address:	*TMdXoSSvPrdQxAK2M2kCdSMvu74LozCHPWp*

`exactly1of2_nimp21`

Theorem 5.75 $\forall AB : o.\mathsf{exactly1of2}\ A\ B \to \neg B \to A$. *The proposition is identified by the following information:*

Pure Prop Id:	*0ef7cfe1ed1435414acfd763fb0f561715bba0ebfee6abe5163770fb6289d861*
Pure Prop Address:	*TMdxUZ8ab8VgywY6cY6h1Q7GH65rTvgRbZM*
Theory Prop Id:	*bced87455f1dbd71dca44499cef836fc13af9c4cd9d900b0e299d0289a36ec08d*
Theory Prop Address:	*TMXMKQXQCPoboD5XPkTZaaJc8c6rPbjnF7n*

5.4 Exactly 1 of 3

Definition 5.3 $\mathsf{exactly1of3}$ *is the opaque object of type* $o \to o \to o \to o$ *identified by the following information:*

Pure Object Id:	*aa4bcd059b9a4c99635877362627f7d5998ee755c58679934cc62913f8ef06e0*
Pure Object Address:	*TMKyEfBd2FYAx6zajz2sxvYYYXLUo86oiwP*
Theory Object Id:	*c2eb06c534871c653d9f25994d6e8a39d134bed3c7ab7fe19a4f2ecc1eaa1c9c*
Theory Object Address:	*TMVfRiAA5KRi6JvZP4W3VJUw9mtsQVKMdhf*

`exactly1of3_I1`

Theorem 5.76 $\forall ABC : o.A \to \neg B \to \neg C \to \mathsf{exactly1of3}\ A\ B\ C$. *The proposition is identified by the following information:*

Pure Prop Id:	*1310b63ae562af7333aee9625e0aa6979394ff104503c204c828d7b627b814d9*
Pure Prop Address:	*TMcMEtNxcD358TggS5kHahcWnA74uxTqbqr*
Theory Prop Id:	*b7468ae93994040f6662cbfbfdfebbcd1b25bb59711010960cc711c9538dfac3*
Theory Prop Address:	*TMYCtwze3JZKavf143XCtrbzsmDzM8qK318*

`exactly1of3_I2`

Theorem 5.77 $\forall ABC : o.\neg A \to B \to \neg C \to \mathsf{exactly1of3}\ A\ B\ C$. *The proposition is identified by the following information:*

Pure Prop Id:	*00b5b6f76b51f729be0782aed25cf933b24c8784fde8d05bdc69f9b47b28a170*
Pure Prop Address:	*TMbGNDSr1GGAFnxH4SmSg37nzwWa3ctnxUP*
Theory Prop Id:	*9660f910a33194ba73b4fbc257bf1c9c423ef83597391b5889dfe8f6a38be5cb*
Theory Prop Address:	*TMXqa8cMXntrgJc53wodmauXTvXn3M1bkAY*

`exactly1of3_I3`

Theorem 5.78 $\forall ABC : o.\neg A \to \neg B \to C \to \mathsf{exactly1of3}\ A\ B\ C$. *The proposition is identified by the following information:*

Pure Prop Id:	*fb6319ee090e002b279304bb4d2ec36b0740f58ea070dd36a44eed28f3b20035*
Pure Prop Address:	*TMUH4ZTnmgc4sLH4v3sQNX3n4SQUyCZmnRE*
Theory Prop Id:	*98b137b027f483863b18f8083c21acbebd5fb61bcf6f96cf6dbd18c4e987f7fe*
Theory Prop Address:	*TMR4EdHrWcDi4aPaboYPdGxLPTj5h4zc5Jb*

`exactly1of3_impI1`

Theorem 5.79

$$\forall ABC : o.(A \to \neg B) \to (A \to \neg C) \to (B \to \neg C) \to (\neg A \to B \vee C) \to \mathsf{exactly1of3}\ A\ B\ C.$$

The proposition is identified by the following information:

Pure Prop Id:	*ad228087f6c62536f8e84aa14c71244af2a1324cab874e74e539fce731299438*
Pure Prop Address:	*TMTheuMKMYKhVdkDXgyaffeAMHxELJkUktD*
Theory Prop Id:	*83079842b76616f048df954d49c762e5ac75af2a19adeb1da721b43ed57eb0bf*
Theory Prop Address:	*TMREFPJ1oU8sqtHxNQZq9p3RdA4ch42V8qu*

`exactly1of3_impI2`

Theorem 5.80

$$\forall ABC : o.(B \to \neg A) \to (B \to \neg C) \to (A \to \neg C) \to (\neg B \to A \vee C) \to \mathsf{exactly1of3}\ A\ B\ C.$$

The proposition is identified by the following information:

Pure Prop Id:	*92d78d63c2f16429dfa172220b50c32bf70c813aefa4c1fbfc82982a1db5b261*
Pure Prop Address:	*TMPq35SCVMNKL4cFiGZvRwpmRBR7XyHk3FM*
Theory Prop Id:	*e9f574b7e9b23c6e5e976545161fe61fed9af69c6425e8992913d0188d809d95*
Theory Prop Address:	*TMFXVhkyu6LNHsoHcGv18AE41FHiuVDFocM*

`exactly1of3_impI3`

Theorem 5.81

$$\forall ABC : o.(C \to \neg A) \to (C \to \neg B) \to (A \to \neg B) \to (\neg A \to B) \to \mathsf{exactly1of3}\ A\ B\ C.$$

The proposition is identified by the following information:

Pure Prop Id:	*80acac2dac94c4ea11d970424ef6748ca187d4a2d65bb70dd89cbb140688d23c*
Pure Prop Address:	*TMXWj2i7Ck9NwC1rdBpeVZCnufshBHrocD1*
Theory Prop Id:	*78f24ba81b345b2c6ab858eba1eb4f9a5b836a0669931ce7c85b99e2a517410e*
Theory Prop Address:	*TMPZhiykV6QMjsj1rdDoZ3FEXBoTtzSGsW1*

`exactly1of3_E`

Theorem 5.82

$$\forall ABC : o.\mathsf{exactly1of3}\ A\ B\ C \to$$
$$\forall p : o.(A \to \neg B \to \neg C \to p) \to (\neg A \to B \to \neg C \to p) \to (\neg A \to \neg B \to C \to p) \to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*9abcef9e10449a86443719e4eccde7cc0122aea0cdd87559e72070b5056c68a3*
Pure Prop Address:	*TMSs9kYqUVNU4be4UnVpebwjefQ88pwnfPh*
Theory Prop Id:	*f4999a084ffdbda508b13e281a016002d1099fdaf2727b624c9a352ff505061e*
Theory Prop Address:	*TMRGu2Bkor4inX6t54rtKgCqaA85yMnm4Fn*

`exactly1of3_or`

Theorem 5.83 $\forall ABC : o.\mathsf{exactly1of3}\ A\ B\ C \to A \lor B \lor C$. *The proposition is identified by the following information:*

Pure Prop Id:	*b9e1330f726a56d262ecb418f9a9f2892f85275d460094e7c8f3b8a38518f227*
Pure Prop Address:	*TMLoZZfbmUJwXoBoU6BXeji1WYei2vbwgjy*
Theory Prop Id:	*46638186a29363d865020e0cf0a5e9f76ca956ed7c669bee7f98f4d7fd242530*
Theory Prop Address:	*TMLUgggjEPAVsx7wBkaGiLQy8AdUeXJZWjM*

`exactly1of3_impn12`

Theorem 5.84 $\forall ABC : o.\mathsf{exactly1of3}\ A\ B\ C \to A \to \neg B$. *The proposition is identified by the following information:*

Pure Prop Id:	*aa5009ff9a11492e840f32443e73a771ca267d22293893156f1d4deb4aaab2be*
Pure Prop Address:	*TMYv5ButtzZjAKGh8ZS12xVuGTD6YJQcH2b*
Theory Prop Id:	*ddde3fa5accd00c57075aa8e16e5ddd6096787e97651a7d00fe4c06313bd25e5*
Theory Prop Address:	*TMEsRotLbeZwM4Vb8CEFrHUrjwZobtVbQmC*

`exactly1of3_impn13`

Theorem 5.85 $\forall ABC : o.\mathsf{exactly1of3}\ A\ B\ C \to A \to \neg C$. *The proposition is identified by the following information:*

Pure Prop Id:	*610826600590b2f8df19bcab4fe7ab0aaf66cf995d6f68f1d9ef9fd3139ef3b3*
Pure Prop Address:	*TMadDers8VBnavfoVVcRpPTxDYWgEMaqYaA*
Theory Prop Id:	*829d4d5e39a13c5555db82570159451aaf7dedfe2b643ebc2c21720e8128ade4*
Theory Prop Address:	*TMKWE5kXYcypJ3VwhHAfwnUZ524Pbhidn2h*

`exactly1of3_impn21`

Theorem 5.86 $\forall ABC : o.\mathsf{exactly1of3}\ A\ B\ C \to B \to \neg A$. *The proposition is identified by the following information:*

Pure Prop Id:	*1d346d501d1ba647921c3025a1335977b63c27a8a61bd46533fcfc6ba14b2d81*
Pure Prop Address:	*TMMzqWU52BqgHAJK8WaH7Z9CCq7JwxqJcP4*
Theory Prop Id:	*397117cbef66df36202b151aefd2ddef44318f936aabf85d3a81c5b6185a89a7*
Theory Prop Address:	*TMRitWJaC1PAk8gRhTEJE7Eoz2VanS86r9f*

`exactly1of3_impn23`

Theorem 5.87 $\forall ABC : o.\mathsf{exactly1of3}\ A\ B\ C \to B \to \neg C$. *The proposition is identified by the following information:*

Pure Prop Id:	*e61d04bf5bbf41805809956d45bdde949bb6e8b00f4defdce071649f26b9bd42*
Pure Prop Address:	*TMXB6Z3uL2sQmzDCqC3abT4uqUmke5iLtk6*
Theory Prop Id:	*a5f7c26af5306214a6ff6a38a92231f9ad04f3095b211b4147ed6e6fa1272d23*
Theory Prop Address:	*TMSaFCqhniFsaMDK85YeQmyVxVZbpNzbFHt*

`exactly1of3_impn31`

Theorem 5.88 $\forall ABC : o.\mathsf{exactly1of3}\ A\ B\ C \to C \to \neg A$. *The proposition is identified by the following information:*

Pure Prop Id:	*fca5376e31e8f8fc2e5c6f166569ff186415ff18941d23f3f6f029f569148dd7*
Pure Prop Address:	*TMFM62rQpPoQyXj5ji7YpQmUU3nG3SpUV6h*
Theory Prop Id:	*35250014509cb9f52a6cd6ac464fff7cd4df30035a9ea5b3c48aaedd2b42946b*
Theory Prop Address:	*TMTbHqaT1XHTxp9kCkBgrphFzfDCg9TtuTi*

`exactly1of3_impn32`

Theorem 5.89 $\forall ABC : o.\mathsf{exactly1of3}\ A\ B\ C \to C \to \neg B$. *The proposition is identified by the following information:*

Pure Prop Id:	*b27fbaab4bdf4912ad94528a90059e4fe9b1aa06f9c87ceed32c02a2c49b5dc5*
Pure Prop Address:	*TMN18VWDLkauUPx1tHnX2hvXJFtMx1JohBA*
Theory Prop Id:	*fd95f6b42e5f234c90b8218b2e2ebe7858c531af928f420deb984c1f39e731d6*
Theory Prop Address:	*TMY9goAmFbnGXqWRMRHhrFiULnAUs9A1FYY*

`exactly1of3_nimp1`

Theorem 5.90 $\forall ABC : o.\mathsf{exactly1of3}\ A\ B\ C \to \neg A \to B \vee C$. *The proposition is identified by the following information:*

Pure Prop Id:	*6f09b4a396feb709ceb93d0a17d24519b1d958895abedbaba2715b65b7e7584e*
Pure Prop Address:	*TMJM8ao3vrQPrWM5rgtqDZ9GARGusrQ5wm3*
Theory Prop Id:	*21dd5ba0fc3af8a67d61c902bc798489c825a4490a81e54581755b00b24c8b73*
Theory Prop Address:	*TMQ1J7pdfn7NmxfAzvuJMKcSZ7sdKVx5yff*

`exactly1of3_nimp2`

Theorem 5.91 $\forall ABC : o.\mathsf{exactly1of3}\ A\ B\ C \to \neg B \to A \vee C$. *The proposition is identified by the following information:*

Pure Prop Id:	*f93f16a2acab0e2c70aaa6ba0f16c5f8fae767769ebf1191ccb689aae347f908*
Pure Prop Address:	*TMan3tfnxxCb6QyS1xebfjHrUsCXBfkbrta*
Theory Prop Id:	*0a20f6a1f5ec9ea9dc84d10e81affaaeca954a6fb32ae14805896ac1d06bfb30*
Theory Prop Address:	*TMTyG7NfHu1i8TzsieyTpERQxhdGXmw7o4P*

`exactly1of3_nimp3`

Theorem 5.92 $\forall ABC : o.\mathsf{exactly1of3}\ A\ B\ C \to \neg C \to A \vee B$. *The proposition is identified by the following information:*

Pure Prop Id:	*6e599c436e1c33f48b986f575992a80e57b99534e289fc843916f91e1d1a347d*
Pure Prop Address:	*TMbF9MFUNWCLgbMpnwTvAAwrKRm9QfXyEba*
Theory Prop Id:	*d19918a97b442e1a1f214592d4b8db4cdda02c3fe87acf67f5a1951ba2cab40a*
Theory Prop Address:	*TMaKsz23V9zHBq8Nc44i1QWCrQiM7XnB9N3*

5.5 More Basic Results

`ReplI`

Theorem 5.93 $\forall A : \iota.\forall F : \iota \to \iota.\forall x : \iota.x \in A \to F\ x \in \{F\ x|x \in A\}$. *The proposition is identified by the following information:*

Pure Prop Id:	*b146c5f4439d8676c0dccf44c83e811ebb81f9038983f510c8607ddc1eb34420*
Pure Prop Address:	*TMP2e3huiS7VJFLqC2ppXhrjDJ2bUGLJV5e*
Theory Prop Id:	*f84982ee3b076ecba0e4ff60abdc2f37c622ad1afac9429103f323e4cdfe3e19*
Theory Prop Address:	*TMHGmtrvwBUQsPubrpbb6tLvjthQVG8iwCD*

`ReplE`

Theorem 5.94 $\forall A : \iota.\forall F : \iota \to \iota.\forall y : \iota.y \in \{F\ x|x \in A\} \to \exists x \in A.y = F\ x$. *The proposition is identified by the following information:*

Pure Prop Id:	*311cdfe6686cf4d3ef41da3e91d25bb9180eb27a0f3a38739e52ece413817da1*
Pure Prop Address:	*TMU6c8sDinZc5CK9bdAsnrw7LPnQPhim3R3*
Theory Prop Id:	*d07e78df51e6c40a1fb15182455062bac21a7cd73536d47d7e4aeba8426d8f70*
Theory Prop Address:	*TMdxK4ZQijyUW6813F1x5brcPHDKGCWdQpm*

`ReplE_impred`

Theorem 5.95 $\forall A : \iota.\forall F : \iota \to \iota.\forall y : \iota.y \in \{F\ x|x \in A\} \to \forall p : o.(\forall x : \iota.x \in A \to y = F\ x \to p) \to p$. *The proposition is identified by the following information:*

Pure Prop Id:	*31e90a1cfe1339b3fdd723ef9c177dda85e4f3f565d6cd3cfb4f371ac4c304f8*
Pure Prop Address:	*TMFAa4xtcBS2jBu91NQ7stWLB3d3RE3qzYR*
Theory Prop Id:	*34fefdfa28cb805cd6e029487d3c32cc301dc5cfe9635fa1b9dded4506aca32e*
Theory Prop Address:	*TMJVpXF9QNoP2mDr6CyNe7oNaDmpm2R1zMb*

`Repl_Empty`

Theorem 5.96 $\forall F : \iota \to \iota.\{F\ x | x \in \mathsf{Empty}\} = \mathsf{Empty}$. *The proposition is identified by the following information:*

Pure Prop Id:	*2af2ac513652a1800583c65d812613479314c35762cfa051dd456a4370470c27*
Pure Prop Address:	*TMNTKJzUTMuevhemTED7aYZL43yxKSfLyaw*
Theory Prop Id:	*0835bc06e82fa4c2e4d9eb7e519d643457109db965d61841af4879944ee45344*
Theory Prop Address:	*TMdRuwDbtmFquZoGEj4pWkc2saYwQmcLziT*

`ReplEq_ext_sub`

Theorem 5.97 $\forall X.\forall FG : \iota \to \iota.(\forall x \in X.F\ x = G\ x) \to \{F\ x | x \in X\} \subseteq \{G\ x | x \in X\}$. *The proposition is identified by the following information:*

Pure Prop Id:	*eb4c1d386f80c2ee581f2c1069f67f723e405c7993ee30a79f8bbd10d84d9ca3*
Pure Prop Address:	*TMcq6sFfV1su5tjWvGTgkj2EP4oNFAVRcup*
Theory Prop Id:	*56aad0710a7b30d465d2dbe0bd8d3c290b9f836ccb5a6fa4519911a80b05626a*
Theory Prop Address:	*TMZZYGkpW7fMLbAob3MNpFsZFKRvPdWXvXH*

`ReplEq_ext`

Theorem 5.98 $\forall X.\forall FG : \iota \to \iota.(\forall x \in X.F\ x = G\ x) \to \{F\ x | x \in X\} = \{G\ x | x \in X\}$. *The proposition is identified by the following information:*

Pure Prop Id:	*81eb906670f9d334d181dc7e4b9de214b26ca6dcc378c218bba5f27088c69e41*
Pure Prop Address:	*TMNrQK5e5dPgQCprQNuNJ3RRcsb98VaqbXM*
Theory Prop Id:	*7818462d4aac5c1290215c65d85d4a4cae2160757b04afdf041748738daa94de*
Theory Prop Address:	*TMHUULG8EX1BGFc9jYacJPvM5JWuzmqSJoh*

5.6 If-then-else on Sets

Definition 5.4 $\mathsf{If\_i}$ *is the opaque object of type* $o \to \iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*b8ff52f838d0ff97beb955ee0b26fad79602e1529f8a2854bda0ecd4193a8a3c*
Pure Object Address:	*TMFXposu1uQxEse8ZSjwvaRFgnCw5z9YTK6*
Theory Object Id:	*76c4c0ead9836400beb62a9590f920012eeb5b529d51bc098f0a4540dd3492bf*
Theory Object Address:	*TMTZcwmiXZdp3zV8ehxEX8eEg2jwH6B4gXM*

Notation. We write $\mathsf{if}\ p\ \mathsf{then}\ x\ \mathsf{else}\ y$ for $\mathsf{If\_i}\ p\ x\ y$.

`If_i_correct`

Theorem 5.99 $\forall p : o.\forall xy : \iota.p \wedge (\mathsf{if}\ p\ \mathsf{then}\ x\ \mathsf{else}\ y) = x \vee \neg p \wedge (\mathsf{if}\ p\ \mathsf{then}\ x\ \mathsf{else}\ y) = y$. *The proposition is identified by the following information:*

Pure Prop Id:	*2a17ae8615c55763933cbf62cfdea60aab440ec5370b1a166e5a90b2dbbdaa33*
Pure Prop Address:	*TMYAPcTTzBuQuezPwJWXPnQTtHjtuAmrh3a*
Theory Prop Id:	*b5c26bc5c00a1fae5b9eb6e6e2dddacab303e5c17f31736c985646e26d160188*
Theory Prop Address:	*TMSSvVBAsGTpqvzGHpFj5YS4oCq5oKaLmoJ*

`If_i_0`

Theorem 5.100 $\forall p : o.\forall xy : \iota.\neg p \to (\textsf{if } p \textsf{ then } x \textsf{ else } y) = y.$ *The proposition is identified by the following information:*

Pure Prop Id:	*fde1a8047aea787315fd3c48c3bf4b8dcf677b9f13975abdc0cf25a46bd38dbe*
Pure Prop Address:	*TMS3vZha9dDF2eKGvMvTFmh9tT8CjUJYmHN*
Theory Prop Id:	*a9abd320bb96a93c0bfd6f8cd02d5a4b36f1c06bc80d8a0d61d8f7f21e978b60*
Theory Prop Address:	*TMXimAMUQhCm2DKHwaiTu61r3YFoxo2t7aF*

`If_i_1`

Theorem 5.101 $\forall p : o.\forall xy : \iota.p \to (\textsf{if } p \textsf{ then } x \textsf{ else } y) = x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*b1aa3ff40ccd3b5e52ea574b197c5921dcbf1d479f4fc972636f9a536dc4c6c7*
Pure Prop Address:	*TMG6G29dgsYC3Bvq1bUdKCsiS18QXwMmERv*
Theory Prop Id:	*a93c2c61afa1f7d589c4078389126876aa054a9fb356be52dbf9a1ee994afe29*
Theory Prop Address:	*TMLX1L6KQv9yGQm2cqdp146LHmN8xmhBiSp*

`If_i_or`

Theorem 5.102 $\forall p : o.\forall xy : \iota.(\textsf{if } p \textsf{ then } x \textsf{ else } y) = x \vee (\textsf{if } p \textsf{ then } x \textsf{ else } y) = y.$ *The proposition is identified by the following information:*

Pure Prop Id:	*3e14398be827f1c6e32094ea4dc2734502882cbca27eddafe5dce31ef4356336*
Pure Prop Address:	*TMVLUMbzPPFhysrpDbLFxMCLeFJ1VVL1jZC*
Theory Prop Id:	*4de742c8e4700f7927be7ee9ab705ba6fd76b46b20d1ba83576b15b076f51a7e*
Theory Prop Address:	*TMRFxe5VNckUFijgfvt4yKBVqJJFGDGD9Un*

`If_i_eta`

Theorem 5.103 $\forall p : o.\forall x : \iota.(\textsf{if } p \textsf{ then } x \textsf{ else } x) = x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*c790ef8c8cbc1d8b8baccef44b4431f462ec3f875068c16063b3f8e4f9606c96*
Pure Prop Address:	*TMEsYnnoa7TBfEDReC6dhETQtLYNdEGfVEm*
Theory Prop Id:	*e413bd92fbd8dffea8c40f40fa4c82bd8732a0d4a82572f9264e5767a06a8ade*
Theory Prop Address:	*TMTiR3dSMwWpEh2ReAKGGJZjp5K2AjJ4xrH*

Chapter 6

Basic Set Theory

Definition 6.1 UPair *is the opaque object of type $\iota \to \iota \to \iota$ identified by the following information:*

Pure Object Id:	*74243828e4e6c9c0b467551f19c2ddaebf843f72e2437cc2dea41d079a31107f*
Pure Object Address:	*TMVRUmJTotqxWfVB8wE3prBP8W149nntsis*
Theory Object Id:	*dc313483331e27588bf1a1b69cd5f5c7e098db82519bb4f97f7410379392841d*
Theory Object Address:	*TMX4EvisNRDE5LYNME7fAsXyPiC8WnAhWDf*

Notation. We write $\{x, y\}$ for UPair x y.

`UPairE`

Theorem 6.1 $\forall xyz : \iota . x \in \{y, z\} \to x = y \lor x = z$. *The proposition is identified by the following information:*

Pure Prop Id:	*5009cc2fb54e181e885e6ba3ebb81d1e7e8a680ac7694595d03d2bae261e7b96*
Pure Prop Address:	*TMGwnYYsQikhZ63aRncU1cXAkHjKyj9bmC5*
Theory Prop Id:	*e67fc59933588258cbf12faa14d08a2171e75c4be3c6375f1c244269e8f0a620*
Theory Prop Address:	*TMZokDUxWLSRz89jtEJyPSvuwJgZmLVRkEK*

`UPairI1`

Theorem 6.2 $\forall yz : \iota . y \in \{y, z\}$. *The proposition is identified by the following information:*

Pure Prop Id:	*b1eea2f239b14cf1d5593f79c76232b611aa52ae4809066c3fff5897fe4a9d56*
Pure Prop Address:	*TMPfeeVFA54LBD1V3Ejp5GiM71yeFdjdpEG*
Theory Prop Id:	*bf36495fbbdc72eb575c69fdfc681c506ae9eebfb94538f10549bc8f191ce07b*
Theory Prop Address:	*TMd3cJnuukmmggqVuKQroUQUyw9xFNmdytb*

`UPairI2`

Theorem 6.3 $\forall yz : \iota . z \in \{y, z\}$. *The proposition is identified by the following information:*

Pure Prop Id:	*27455b29a050cd7d072a6fecd331e4463360653ad07926e47c46ee56e5b66f75*
Pure Prop Address:	*TMdg8pGYbTqgx8xPL9PwPu2pJJbSoxCnRHy*
Theory Prop Id:	*a0a5b852859049d097662ba32a64381a0c4251956d0027ceea65617e12ec7e68*
Theory Prop Address:	*TMTrsMdWZgjAwNgkT5uqy2DqzBnBXnxWupZ*

`UPair_com`

Theorem 6.4 $\forall xy : \iota . \{x, y\} = \{y, x\}$. *The proposition is identified by the following information:*

Pure Prop Id:	*b0847098a703dd9518520ce0538ef2bb61b423ef3bfce3e056d6ddebc96a5b10*
Pure Prop Address:	*TMTDNchhQ5726g1za89cNkp72gXjpPYs7Nm*
Theory Prop Id:	*792d7d18b9ffdad302a08ed667c940da3d4f02f0bfbaf07fef81aa2961d74531*
Theory Prop Address:	*TMVjMcbkMeWtsqfqTEQnrRhtUFG4TMoXw5s*

Definition 6.2 Sing *is the opaque object of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*bd01a809e97149be7e091bf7cbb44e0c2084c018911c24e159f585455d8e6bd0*
Pure Object Address:	*TMRoFJh3XPuDqcMDh3fauvfQL55yG4ZW8xi*
Theory Object Id:	*b7dfc1c317fc5a717f457d8782ced05f886230b550688a8ad8b1a60364455741*
Theory Object Address:	*TMYwdTEJpbYhdjjsshw7LP6uC2s7dSrhjMJ*

Notation. We write $\{x\}$ for Sing x.

`SingI`

Theorem 6.5 $\forall x : \iota . x \in \{x\}$. *The proposition is identified by the following information:*

Pure Prop Id:	*6591bd63306a8b20a34c3c62753b21a3972cf8174ef23e9454980a5b6b82618d*
Pure Prop Address:	*TMWZhsiyoAnszzrXM7jCkxtmgh4ETFtiEfk*
Theory Prop Id:	*2f93be9a22d244304c3bc2f3253e0b4a512c5f66bb0f005cb881e61f6eb26071*
Theory Prop Address:	*TMbi5x8pysc7XDzhg16jDTJ4KrkuJzZS7BK*

`SingE`

Theorem 6.6 $\forall xy : \iota . y \in \{x\} \to y = x$. *The proposition is identified by the following information:*

Pure Prop Id:	*9341fe885f477660bd3dfc0b46ef487b4623351d45911bddb267d44cdf6d0821*
Pure Prop Address:	*TMNoMEGQFgAVj7maHozhYTCSXBfUaNXtuAN*
Theory Prop Id:	*3b0dd829f31946c18010e2f35a58c321cf20e3464f3c10d63efcab64a4f018fb*
Theory Prop Address:	*TMGwutpkdmmtAbHgjLsWPgK3gW6vqLx2s43*

Definition 6.3 binunion *is the opaque object of type* $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*5e1ac4ac93257583d0e9e17d6d048ff7c0d6ccc1a69875b2a505a2d4da305784*
Pure Object Address:	*TMKadjsG1yH8Ra5dXxSA9jstbEvqFo88bxo*
Theory Object Id:	*e14989b9e54d0036d328f81b7c2ca9bf3b1c6d7d6dfd5cbc1d8421fd6cc0159e*
Theory Object Address:	*TMWh5oU44xPmpsrB4CwWV2e49m2HBPkrW2D*

Notation. We use $\cup$ as a left associative infix operator corresponding to applying term binunion.

`binunionI1`

Theorem 6.7 $\forall X Y z : \iota . z \in X \to z \in X \cup Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*753b6ef046ed0483fcc02f203ffb7e80213207fe9ca6b95995de9c4eaea00411*
Pure Prop Address:	*TMRRF1KJnf4XF9rqDvQv3UaJJUWLeXK8ZP9*
Theory Prop Id:	*e43c8db7c2b8d05d548b5a0c56a8d23f40736402d22a35cb8a212ee0ab4be72c*
Theory Prop Address:	*TMMufFv1jkCn3ZdkSG7Y7viPMpw7eZgnmeh*

`binunionI2`

Theorem 6.8 $\forall X Y z : \iota . z \in Y \to z \in X \cup Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*b8821c633c5f298b2822c44b4eecc8fdef6356f33c6b8bdd93a53e96d7729690*
Pure Prop Address:	*TMY2zmhR5aQU4DsXTEmDmSgQYhYPh1Py1VW*
Theory Prop Id:	*959535d02d0caf6be5dc6d781bb10a45795b705bd32a8de6d41ef5ccdca130ce*
Theory Prop Address:	*TMLpWxJ4RXSb2TWfpnMMgvDQbHM1A4XSH9X*

`binunionE`

Theorem 6.9 $\forall X Y z : \iota . z \in X \cup Y \to z \in X \lor z \in Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*5a319544d131da120ce102ea37cee203a594453cf4350bac27592a4628092716*
Pure Prop Address:	*TMHtDhNz67RzwWxi6YMkqKVbqx8c9XaiDyX*
Theory Prop Id:	*1ceeb045243b9d5012dc124e31a841805b4e6059c2e99973ef3d2663b5039471*
Theory Prop Address:	*TMMe9w3Qz1uA5B6MSbrBAwnM2Ydf6fFAksa*

Definition 6.4 *We define* SetAdjoin *to be* $\lambda X y . X \cup \{y\}$ *of type* $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*1f3a09356e470bff37ef2408148f440357b92f92f9a27c828b37d777eb41ddc6*
Pure Object Address:	*TMXya8p7m5AB5g1dVJ1MSDKpnXtcu2N87Mz*
Theory Object Id:	*5bdda84d481c907ad5ac9225d0210c23eca52ec933bd26ac51fc7debfa5cd394*
Theory Object Address:	*TMLYeA5y1CUhy319tM7uM8NUTufJJyM6eya*

Notation. We now write $\{x_1, \ldots, x_n\}$ in general with the following conventions. If $n = 0$, the notation means Empty. If $n = 1$, the notation means $\mathsf{Sing}\ x_1$. If $n = 2$, the notation means $\mathsf{UPair}\ x_1\ x_2$. If $n > 2$, then $\mathsf{SetAdjoin}$ is used to reduce to the case with one fewer elements.

`Power_0_Sing_0`

Theorem 6.10 $\mathsf{Power\ Empty} = \{\mathsf{Empty}\}$. *The proposition is identified by the following information:*

Pure Prop Id:	*6526a1550f9c7084aa6631fc5bd8b4d8d339fd0ce60e7d548edc293a22206f43*
Pure Prop Address:	*TMFtrGrnuqpdT6zKi1x2QYqakvTT82UnEGx*
Theory Prop Id:	*a39fd0f7312cd15f6f2e488850e84694dd268ddebb9a0a84a4dec2e6cce4b2e2*
Theory Prop Address:	*TMZJjQSh37Giw23bdQW2e8z3PhLqdGbYhbC*

`Repl_UPair`

Theorem 6.11

$$\forall F : \iota \to \iota . \forall x y : \iota . \{F\ z | z \in \{x, y\}\} = \{F\ x, F\ y\}.$$

The proposition is identified by the following information:

Pure Prop Id:	*cacbdd41cb5a9699e9d7e46d5d2d536a174a15a2dad0e76a3885d739f3bf9618*
Pure Prop Address:	*TMavxam5T7BVKzszyZpmyDGWmzwq22rrVr1*
Theory Prop Id:	*412745a32a74658e5f4747ff82265690389455bc948a7dc4ae5ff1cc468be3bd*
Theory Prop Address:	*TMSeQz641QN3jhU5ykuM6QagGpUJisdc52E*

`Repl_Sing`

Theorem 6.12 $\forall F : \iota \to \iota . \forall x : \iota . \{F\ z | z \in \{x\}\} = \{F\ x\}$. *The proposition is identified by the following information:*

Pure Prop Id:	*1b6f04154ad08fd4e5ac8a4a7e0666ebb6990dd65162ddc49c4c2aec615a9338*
Pure Prop Address:	*TMXmXBaEMHiXuMsDum7NZFR84rqzoKxHCpU*
Theory Prop Id:	*bd9e64fedf4189ba490aa0c11b9fbd79890c422133c09d3193c333247af88b1b*
Theory Prop Address:	*TMV9HYczMJjbDeoJuBm27QfyM9yXxF6FN3Y*

`Repl_restr`

Theorem 6.13 $\forall X : \iota . \forall F G : \iota \to \iota . (\forall x : \iota . x \in X \to F\ x = G\ x) \to \{F\ x | x \in X\} = \{G\ x | x \in$
The proposition is identified by the following information:

Pure Prop Id:	*81eb906670f9d334d181dc7e4b9de214b26ca6dcc378c218bba5f27088c69e41*
Pure Prop Address:	*TMNrQK5e5dPgQCprQNuNJ3RRcsb98VaqbXM*
Theory Prop Id:	*7818462d4aac5c1290215c65d85d4a4cae2160757b04afdf041748738daa94de*
Theory Prop Address:	*TMHUULG8EX1BGFc9jYacJPvM5JWuzmqSJoh*

Definition 6.5 famunion *is the opaque object of type* $\iota \to (\iota \to \iota) \to \iota$ *identified by the following information:*

Pure Object Id:	*b3e3bf86a58af5d468d398d3acad61ccc50261f43c856a68f8594967a06ec07a*
Pure Object Address:	*TMWQ76Uj1RnGeogmDxhYaxbaaPGB1LBoCiP*
Theory Object Id:	*241349d21f55f878b9cd5590cf66f9dc362c98552772a88eccdedc7910500fd0*
Theory Object Address:	*TMaTpVSSy7hrg8WXTVmsiohBmjmfrDfMMJG*

Notation. We use $\bigcup_{x \in -} .-$ as a binder notation corresponding to a term constructed using famunion.

`famunionI`

Theorem 6.14 $\forall X : \iota.\forall F : (\iota \to \iota).\forall xy : \iota.x \in X \to y \in F\ x \to y \in \bigcup_{x \in X} F\ x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*0c23d0ec3aeca92359022d4236f139bbce8322597bc7aa615dfd5957df202628*
Pure Prop Address:	*TMc9t7j1HpKhoSwH9rdLbnytztKsuCGA1n3*
Theory Prop Id:	*e4f490a807c7166931ad2baca93cfaa01c42669f58e0e3fa9d5a4e910da403f1*
Theory Prop Address:	*TMSPh7dPrn6WaLbJUMvCgyCRAXKpqw7tSxW*

`famunionE`

Theorem 6.15 $\forall X : \iota.\forall F : (\iota \to \iota).\forall y : \iota.y \in (\bigcup_{x \in X} F\ x) \to \exists x \in X.y \in F\ x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*48683a19e0b9cac431e35763caf26ab25d3b68d0eec6fb8193bb111ac8d2a57a*
Pure Prop Address:	*TMS1BrnhPG5Rb3BqURjxG5UF4FXEg1CWJ6h*
Theory Prop Id:	*ede6e5d506bad2c8dd1acc9526f74416f2e8ed084472c376ae92dd12e47b1b36*
Theory Prop Address:	*TMN9kEqZptjbB8TVx3pwjNpZJsSZxFeje71*

`famunionE_impred`

Theorem 6.16

$$\forall X : \iota.\forall F : (\iota \to \iota).\forall y : \iota.y \in (\bigcup_{x \in X} F\ x) \to \forall p : o.(\forall x.x \in X \to y \in F\ x \to p) \to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*ab22661d1f7c130e5684c80d6642dea2ccbe8caea270129cb32e218a75dc3b14*
Pure Prop Address:	*TMM2k1dfrqcpKhy5sotaNPvjvVMC2zbAF94*
Theory Prop Id:	*562023a05cb52f7958f9f74add2503404a86b5d121085bea792c6ae40d578143*
Theory Prop Address:	*TMbvpfPvzznj6iG7UVhksbQDUQpsJeGR69p*

`UnionEq_famunionId`

Theorem 6.17 $\forall X : \iota.\mathsf{Union}\ X = \bigcup_{x \in X} x.$ *The proposition is identified by the following information:*

Pure Prop Id: *c0407ac29149de6b96007edfd11708f9cad613bf44851fb9a688ec0b72823abc*
Pure Prop Address: *TMJq16zmpETvRMnCrNiWd4hPxiCF9wLQGvM*
Theory Prop Id: *5b5383939b1feed9224b810975652ed8da9eef8545e5df1b9311c75e151ca976*
Theory Prop Address: *TMazYyBoEesPvNn5L5cyG4oKnoYJWf1MTXS*

`ReplEq_famunion_Sing`

Theorem 6.18 $\forall X : \iota.\forall F : (\iota \to \iota).\{F\ x | x \in X\} = \bigcup_{x \in X} \{F\ x\}$. *The proposition is identified by the following information:*

Pure Prop Id: *dfa22e97a0f977492075ad87b7ec5f1e874e8cc726cad73016122eff2e495382*
Pure Prop Address: *TMcpQ6xMXyb8neoFWcHWRhPuHkeDjU7r5Sc*
Theory Prop Id: *9829d1ccabddee314bee7238491144e99da007ceb90bcc3b1aade28e6bc30824*
Theory Prop Address: *TMbPFYZ4Eaf5LC8DmM7MPALZ4kWFxfV1npd*

`Power_Sing`

Theorem 6.19 $\forall x : \iota.\mathsf{Power}\ \{x\} = \{\mathsf{Empty}, \{x\}\}$. *The proposition is identified by the following information:*

Pure Prop Id: *f735ed6a9bad818e002e12cc5ef0819b362cbc929180991433e3e3237282e883*
Pure Prop Address: *TMYYws5iekM1u7qxpPo65kGRFcdPoaPVCa9*
Theory Prop Id: *e5fb8fe5f5f2baf36fccd9d1f35398ed4a86da3345f90b8e9d2336d609a38a7b*
Theory Prop Address: *TMYxTSeb1gFn8ZUnNyFfjYHJv1cLqspoYGo*

`Power_Sing_0`

Theorem 6.20 $\mathsf{Power}\ \{\mathsf{Empty}\} = \{\mathsf{Empty}, \{\mathsf{Empty}\}\}$. *The proposition is identified by the following information:*

Pure Prop Id: *24373cc848ed23a872ca0879dfdbcf7a451233726845c72b4cfccef56ea08e80*
Pure Prop Address: *TMVoMhEC9xC2sTc7tiuSozHc8YaTWHcTdNa*
Theory Prop Id: *173fc92aca3f0c56bd3327162bd62d8aa42795423e3bd3a7692afa1bcbead4cc*
Theory Prop Address: *TMTeCtmtdQ9KwHpHJU1W4m7ytbUgCGJRUV5*

Definition 6.6 Sep *is the opaque object of type* $\iota \to (\iota \to o) \to \iota$ *identified by the following information:*

Pure Object Id: *f336a4ec8d55185095e45a638507748bac5384e04e0c48d008e4f6a9653e9c44*
Pure Object Address: *TMUtfXxDDLj9Tk47vMVXxn7X4YsuGxPH9iG*
Theory Object Id: *7008ec0ee19d11cc67c5f11635c82852ccb27360b8bd8431676b0c7e03219428*
Theory Object Address: *TMU5xEoVJ4JK7LsxiB8DMm5ouqBu3Rgy3CL*

Notation. We write $\{x \in A | varphi\}$ for $\mathsf{Sep}\ A\ (\lambda x.\varphi)$.

`SepI`

Theorem 6.21 $\forall X : \iota.\forall P : (\iota \to o).\forall x : \iota.x \in X \to P\ x \to x \in \{x \in X | P\ x\}$. *The proposition is identified by the following information:*

Pure Prop Id: *ea56bb93dea4f3624b90c68591a17a4b0e370a444cd3e405fd7fcdb52632635b*
Pure Prop Address: *TMR97AbUhdgTucfxJ7ujRoLGzK6qJ2E1jyn*
Theory Prop Id: *147a6ae90a60bb6d7a3135bb216628e19a94a3f9a5fb6e915e96bbbf26d60ba0*
Theory Prop Address: *TMUrJfQgr3jYT2UBYmVX9Mv2z2Jgr6upg8L*

`SepE`

Theorem 6.22 $\forall X : \iota.\forall P : (\iota \to o).\forall x : \iota.x \in \{x \in X | P\ x\} \to x \in X \wedge P\ x.$ *The proposition is identified by the following information:*

Pure Prop Id: *102830994311842627e3ed866fe29eb1f07e8f9431d5cb028658a623df943412*
Pure Prop Address: *TMGr4JhwYm65vLL1JdgtV7Z4udWfVUCyR7C*
Theory Prop Id: *00899a751488a4862ed01d1ae8f2ff0dbfbfe3fe46a1349c813bec079a6f9ede*
Theory Prop Address: *TMSaSd5PfDFDpCcrVNF6RUq6GkDHu9ks3pQ*

`SepE1`

Theorem 6.23 $\forall X : \iota.\forall P : (\iota \to o).\forall x : \iota.x \in \{x \in X | P\ x\} \to x \in X.$ *The proposition is identified by the following information:*

Pure Prop Id: *d83445946f710ccd1847efc45d29fcc99bebd491806d37ec544f5350bcfa0f86*
Pure Prop Address: *TMWSj1Q1gssEN1FRYzrkAPVjkpNyJhfDv8F*
Theory Prop Id: *6399b0a98035d8a70e8306896b0f872730b3288dcdbb66b851b1f84cc75089c8*
Theory Prop Address: *TMSjwkTcc2asZAUTrPPnJyXprfdL48BnKdS*

`SepE2`

Theorem 6.24 $\forall X : \iota.\forall P : (\iota \to o).\forall x : \iota.x \in \{x \in X | P\ x\} \to P\ x.$ *The proposition is identified by the following information:*

Pure Prop Id: *aa61ca17580417f6d32cdf9fba918e207307848ff899382c5acbd1e2027eebd5*
Pure Prop Address: *TMV6AB23w8THH3JMfPMPg1fJahEHtfBWk9A*
Theory Prop Id: *104227af0e258de974a06dacd30d9996af2cc290588f50a9c669aaf9868024f6*
Theory Prop Address: *TMUZTejdmRUG8mdTCdynTZZpHdZx8LNmpbi*

`Sep_Subq`

Theorem 6.25 $\forall X : \iota.\forall P : \iota \to o.\{x \in X | P\ x\} \subseteq X.$ *The proposition is identified by the following information:*

Pure Prop Id: *97d0809464d57d8f031c3968a64d1d7a35cc9bf2076517a32bfada2a891489f0*
Pure Prop Address: *TMKXby5zNJ4KVAvdU1gcKmDEYMkBvsYGdtv*
Theory Prop Id: *c17d45b17052435d24c1e2d23e7e0c0589d735640358f24c5893103ac9cd487f*
Theory Prop Address: *TMYQD6DWoPBDiMim118Sqg7B5nz4JVyqYui*

`Sep_In_Power`

Theorem 6.26 $\forall X : \iota.\forall P : \iota \to o.\{x \in X | P\ x\} \in \mathsf{Power}\ X.$ *The proposition is identified by the following information:*

Pure Prop Id:	*612c94a6047cff99b0373d060e43552169d05dddccd4ea06be3c8d6a86650764*
Pure Prop Address:	*TMHfdLZNWAmDjm1ZXXcZtP8QxYVC7ugYRTA*
Theory Prop Id:	*fba53ed8ea5e05c876d2ead08cbb41c2104737958be70e5c48d509f88ed420a6*
Theory Prop Address:	*TMd59svET4t7PBsSUNEFGENePU74pbhVRyk*

Definition 6.7 ReplSep *is the opaque object of type* $\iota \to (\iota \to o) \to (\iota \to \iota) \to \iota$ *identified by the following information:*

Pure Object Id:	*ec807b205da3293041239ff9552e2912636525180ddecb3a2b285b91b53f70d8*
Pure Object Address:	*TMLdtkt2SAyu6uRx3VwN4EmguAaBBbX15ky*
Theory Object Id:	*9bdb562173d8d14c707097187f2b9ec994d4d550da69fd27247ea7a1734dc0ea*
Theory Object Address:	*TMRDkBndFptKLHv3ZdFcEM1zsnPucRbrZsD*

Notation. We write $\{s|x \in A, \varphi\}$ for ReplSep A $(\lambda x.\varphi)$ $(\lambda x.s)$.

`ReplSepI`

Theorem 6.27 $\forall X : \iota.\forall P : \iota \to o.\forall F : \iota \to \iota.\forall x : \iota.x \in X \to P\ x \to F\ x \in \{F\ x|x \in X, P\ x\}$
The proposition is identified by the following information:

Pure Prop Id:	*35b8ecafee8ec7ce07622bb50b3833b0aab40abc8e7a7253fef934747ce888e9*
Pure Prop Address:	*TMWGYkRQTsDoJuJbmJZ1z6DVf1M5n7LNCgC*
Theory Prop Id:	*aa7b2b992fe9f4cd5864693019fc63fcdd7e92856cb452371ce4409e421bf1e7*
Theory Prop Address:	*TMGFrvHwbGaUn5voQsk3ha4eAC8RLvBtTSB*

`ReplSepE`

Theorem 6.28

$$\forall X : \iota.\forall P : \iota \to o.\forall F : \iota \to \iota.\forall y : \iota.y \in \{F\ x|x \in X, P\ x\} \to \exists x : \iota.x \in X \wedge P\ x \wedge y = F\ x.$$

The proposition is identified by the following information:

Pure Prop Id:	*415ff1723537afd685cb0fed04bae8d9d3e3c9fc8dd81f94e3e5ef00af2fcd62*
Pure Prop Address:	*TMErPUtuvpjwR2ReNB9nGcmnK8AYV664v6C*
Theory Prop Id:	*b1bc76c44d5f7b7b23a5be6966a678cb5d36886ab4685b00d3d5157e8daf69e3*
Theory Prop Address:	*TMQMpL9gjiXL2MUteH6hiemmLJG4AhbLTXr*

`ReplSepE_impred`

Theorem 6.29

$$\forall X : \iota.\forall P : \iota \to o.\forall F : \iota \to \iota.\forall y : \iota.y \in \{F\ x|x \in X, P\ x\} \to \forall p : o.(\forall x \in X.P\ x \to y = F\ x \to$$

The proposition is identified by the following information:

Pure Prop Id:	*5d26e1469193d5bd2136b195439a688359b4bf58fb7a17905abb3c0cf02e93af*
Pure Prop Address:	*TMMk3dLDDxdwMUP7GQkv6ep261PQbXFJKi5*
Theory Prop Id:	*063cead7e54785ba03db6bfc07a0999226785a5ad4252309bcbbf4d94eb92422*
Theory Prop Address:	*TMM2umBqoRLF5bftij5PkXw6MCfbhi2G26n*

Definition 6.8 ReplSep2 *is the opaque object of type* $\iota \to (\iota \to \iota) \to (\iota \to \iota \to o) \to (\iota \to \iota \to \iota) \to \iota$ *identified by the following information:*

Pure Object Id:	*da098a2dd3a59275101fdd49b6d2258642997171eac15c6b60570c638743e785*
Pure Object Address:	*TMNVwxznYhNZZoYoQL3ESJeSeewae4Z5ubB*
Theory Object Id:	*bd46a4454a608867c1da526c8a8d052f33ebc39c49a21d7e84f4d329218203b0*
Theory Object Address:	*TMdeKFiZBi4BgUrRypBtmVFrLRWnZ8EbVUP*

`ReplSep2I`

Theorem 6.30

$$\forall A.\forall B:\iota\to\iota.\forall P:\iota\to\iota\to o.\forall F:\iota\to\iota\to\iota.\forall x\in A.\forall y\in B\ x.P\ x\ y\to F\ x\ y\in \mathsf{ReplSep2}\ A\ B\ P\ F.$$

The proposition is identified by the following information:

Pure Prop Id:	*c012d962b28e32216b7c116bb9b93b568f22cdbd01c2ad49eb0973e193d89ca5*
Pure Prop Address:	*TMbDYFXkGgf7ZjkUpxAt2c8KfCPQnzBVHXo*
Theory Prop Id:	*d75c053450bb002a32e5862bcdf41f63858ceb9bca07b0b78edc448c3cd382e5*
Theory Prop Address:	*TMTZAhgLueBybAHKpFbownqVFw34ydwJv3A*

`ReplSep2E_impred`

Theorem 6.31

$$\forall A.\forall B:\iota\to\iota.\forall P:\iota\to\iota\to o.\forall F:\iota\to\iota\to\iota.$$
$$\forall r\in \mathsf{ReplSep2}\ A\ B\ P\ F.\forall p:o.(\forall x\in A.\forall y\in B\ x.P\ x\ y\to r=F\ x\ y\to p)\to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*9cf007f5a509413458fb2202520eeadc0e1282d8f5e08ea4e998973e0c49f6eb*
Pure Prop Address:	*TMH6JGjNiuhPqpFuX9oizzHFk5Le2zk5WAH*
Theory Prop Id:	*fc01e896457b69f97929e28b9ca1348b7bec5c0f73942fd10cd6424efd4ff8da*
Theory Prop Address:	*TMP5H56SXZ4FHy2J2NYCGC9QGApNByq6Fuv*

`ReplSep2E`

Theorem 6.32

$$\forall A.\forall B:\iota\to\iota.\forall P:\iota\to\iota\to o.\forall F:\iota\to\iota\to\iota.\forall r\in \mathsf{ReplSep2}\ A\ B\ P\ F.$$
$$\exists x\in A.\exists y\in B\ x.P\ x\ y\wedge r=F\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*7c43a59f9be76561c287a01d1503cef13020f9d5467c7fe07a42e5ca37944da3*
Pure Prop Address:	*TMWPJMkxvgzqUKqtFU66YkSgt15bbJMX2GN*
Theory Prop Id:	*fb571ab298cdb2355f09c7b4be2d8e161ca5df97f2f94f5907375ecfd6c02700*
Theory Prop Address:	*TMR6JYS9aq1RbPLrzpqs7eQpgMEUJFbekS2*

`binunion_asso`

Theorem 6.33 $\forall XYZ : \iota . X \cup (Y \cup Z) = (X \cup Y) \cup Z$. *The proposition is identified by the following information:*

Pure Prop Id:	*e0f701721ae7f584ad335901c46f861f37994ae2c421659e2fc7d24d52c4ad58*
Pure Prop Address:	*TMSkYkrEs3aZ5sGQH7NK9w6LMvc7ihPrtiY*
Theory Prop Id:	*61dfbad9a60ca916e2a74941dde462165a35addadb67dd1aa0e14bf13e972036*
Theory Prop Address:	*TMay4ygEWLRWTtSwLgj2avwuNTtFqy4VPaD*

`binunion_com`

Theorem 6.34 $\forall XY : \iota . X \cup Y = Y \cup X$. *The proposition is identified by the following information:*

Pure Prop Id:	*e20f4f80912d4279be9396097afd9e80c497a2f339edd6413343e5d23c789f0e*
Pure Prop Address:	*TMaLPUJawPt39v249vAR1DqP1n84oA3fd8g*
Theory Prop Id:	*bed33ed76997bd9546985ac652f8933bf140216a164aa11462375a79b95e7a08*
Theory Prop Address:	*TMY4TyspYtVRBA6PfYNL89qxpZvncatAvsk*

`binunion_idl`

Theorem 6.35 $\forall X : \iota . \mathsf{Empty} \cup X = X$. *The proposition is identified by the following information:*

Pure Prop Id:	*8dc61661d4d5e1f88a3162d142c0a3e25f64eacc329dfd0068a1dc72be818a05*
Pure Prop Address:	*TMHvo75eYy3DzHQyXDhy7nSXmaRpD1QL8Xv*
Theory Prop Id:	*52cd24837ea762ce63b82815496f8b7cf911a666c277332e8ecf1679702408c2*
Theory Prop Address:	*TMM6roc6rAT3Nk8tdS3jEzf3rjCYtEtKppo*

`binunion_idr`

Theorem 6.36 $\forall X : \iota . X \cup \mathsf{Empty} = X$. *The proposition is identified by the following information:*

Pure Prop Id:	*67e82fb8d1d026e5488f27f866055b8ad7b6ae3107546e436d50455676cb7db6*
Pure Prop Address:	*TMYUCbeWsYbEVYGjC69S1voGpXmnhiyPS1j*
Theory Prop Id:	*d5a0059cf51bb85c8614882cfb5aa46e5986047c17f59943f59e9afdca587577*
Theory Prop Address:	*TMK8JAPKwtcTnWyvdtnBF1r93mW75pcUzMM*

`binunion_idem`

Theorem 6.37 $\forall X : \iota . X \cup X = X$. *The proposition is identified by the following information:*

Pure Prop Id:	*810a82013ba606cd5918b3f6b58976e1a987615eaad568a3e32930a3239a3e37*
Pure Prop Address:	*TMbQmiBEMSBKRMaesDwRUx5iHKc86Jpvahe*
Theory Prop Id:	*10e6968d996355460bf1486862c7a957c7b64f3eaa21e7c5530de09d0acc76bd*
Theory Prop Address:	*TMRyZSEzQ6SeCXSBXN2HWBcfU7GgBsUCzp7*

binunion_Subq_1

Theorem 6.38 $\forall XY : \iota.X \subseteq X \cup Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*b2c65821a8c09a0b77af768d4af071f6f52e54e3ab9f874132e22d9b780fec03*
Pure Prop Address:	*TMNMiyzoBmmTgby8dHSUT9GNJXBRATKB1Je*
Theory Prop Id:	*a9b3a8f498f868b3e5f0ad0bf8f135aeabf5748b92ad79003393964533a788f6*
Theory Prop Address:	*TMKuARFTAuYGC1dr9qRt4msXcsg294CE5fx*

binunion_Subq_2

Theorem 6.39 $\forall XY : \iota.Y \subseteq X \cup Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*3cf2292964400c071542529f75f809fb1cccc1b48658e084626f9f60981211a2*
Pure Prop Address:	*TMWp6qtFQ6sNjLamnX9ZFCwWRY29tAeEj4n*
Theory Prop Id:	*1507ce49772284307aa14b5f6312cbe2050013469059484c4e6af749d27296c7*
Theory Prop Address:	*TMHECM3VikFfrZ52FBVEeJcMZtdHizHKgfk*

binunion_Subq_min

Theorem 6.40 $\forall XYZ : \iota.X \subseteq Z \to Y \subseteq Z \to X \cup Y \subseteq Z$. *The proposition is identified by the following information:*

Pure Prop Id:	*cc32a5be9e847d0d6b3321ae657ec25e33e4efa0fa1a8b187388477416c1b8a3*
Pure Prop Address:	*TMZ3UZw7snyBWjGXoD2PLSgQ7Z2tViLeZHE*
Theory Prop Id:	*d64da44c02293ec947fdce4b6a1334307011456947961dce8950499be075b837*
Theory Prop Address:	*TMEmr475JNDN72vef9HbrMFEnw3HeFp8ckR*

Subq_binunion_eq

Theorem 6.41 $\forall XY.(X \subseteq Y) = (X \cup Y = Y)$. *The proposition is identified by the following information:*

Pure Prop Id:	*f8ee53d9e4447bc52c14da3fcc90d5e8615f20617571fe5255ea2f6eb381e06d*
Pure Prop Address:	*TMbuiip99HVRqBDM9hnt3UzPZe6NvLXEw43*
Theory Prop Id:	*c2d39365f127c2c7b045442887fe24ea64a512918a28eaa5c725c8592f45e775*
Theory Prop Address:	*TMZAQ7e8J964d8E63qrdsixRgixW7twAP2T*

binunion_nIn_I

Theorem 6.42 $\forall XYz : \iota.z \notin X \to z \notin Y \to z \notin X \cup Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*a815887d2483bf0106a08715fcac061cae3e6bcbb8e465a668937ec74c34a252*
Pure Prop Address:	*TMHmep2oeAYUaJwZC3hiCXb2vG3pYtH1QiF*
Theory Prop Id:	*b26c78a45180d7dd20f8b0147d272da614cde185efd40b05c72c389284a5bdfd*
Theory Prop Address:	*TMNdwhoFsncxh4zt41JCJjMiky1UTdLveND*

`binunion_nIn_E`

Theorem 6.43 $\forall XYz : \iota . z \notin X \cup Y \to z \notin X \land z \notin Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*eb90dbeba507f6184d9a4519639032fd25e13e8a5665b55e204384457add4651*
Pure Prop Address:	*TMLKugoiqVxQHU6VmQg13s94YEdBHkpk3c6*
Theory Prop Id:	*64c8be6bcd58b6ff3005c6e68cea0923081c0c08b375ed3440f58e2ae4590507*
Theory Prop Address:	*TMMzSapwNEj6zYnr4sqrPXz964RVJfZhXvJ*

Definition 6.9 binintersect *is the opaque object of type* $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*b2abd2e5215c0170efe42d2fa0fb8a62cdafe2c8fbd0d37ca14e3497e54ba729*
Pure Object Address:	*TMLEtM84AAELgYDKCd7u4BvMP8XhNj6rBEU*
Theory Object Id:	*2c68d89742466253865fe4ac454f660dc5ef3aabe295adca6101b1cfe29a9b4b*
Theory Object Address:	*TMTxtnFNepi91FFerdTCcnaeQ6ZJWf8pc76*

Notation. We use $\cap$ as a left associative infix operator corresponding to applying term binintersect.

`binintersectI`

Theorem 6.44 $\forall XYz . z \in X \to z \in Y \to z \in X \cap Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*fd656d06038bf1e8213bf796c99d2730d0934ac5207ea1a324a38e8ffb7f1d72*
Pure Prop Address:	*TMavDaUgQY4Yc7jvGaUAZLBxvza2KYE7CTa*
Theory Prop Id:	*9081a09947c1d3d1991f93fd8066a329cc8dd137d5232ac2ec7d204ac6f93e3c*
Theory Prop Address:	*TMW6j5eRPzucAp43ZDsETbD83EUAMzqu3gS*

`binintersectE`

Theorem 6.45 $\forall XYz . z \in X \cap Y \to z \in X \land z \in Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*463b5a7d659e015ae77f37224a2da56f16082db6bae0d607624285937805d0ac*
Pure Prop Address:	*TMGgPwxMGdV2DoStuTEK7hiAAwBA496E6VG*
Theory Prop Id:	*ca06365460a4a4310d69039b45771ed26e9f64b8553c1685d7d26b38bc88fce3*
Theory Prop Address:	*TMT3UicyLqut6GdZ4E3a5HpLsx7pQRGYv4o*

`binintersectE1`

Theorem 6.46 $\forall XYz . z \in X \cap Y \to z \in X$. *The proposition is identified by the following information:*

Pure Prop Id:	*b12123fa4efe378e34ca200da469f86ccfc917210d536de59d52c5e9de77116c*
Pure Prop Address:	*TMVXNYZCzRuA7LZtTrfmGXLqpJKtWXchcg3*
Theory Prop Id:	*51005ef0a8ef018c9807d92cb93099aee35ab19d9b57fee1f06b6d001123d66e*
Theory Prop Address:	*TMGso6ks75EXfsrAgvoTysUxjJQVJBRAjXQ*

binintersectE2

Theorem 6.47 $\forall X Y z. z \in X \cap Y \to z \in Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*39656fd10aea14619f5a39d872360e27a8a2adbf407ac9f383cc565db9601f30*
Pure Prop Address:	*TMFtaVMiexDH341MBZktuvoMPVCTRcMZ5E6*
Theory Prop Id:	*599de44995ff57c083eb2f45a7abd687d47df353ae136fb3a5621a3444c2edaf*
Theory Prop Address:	*TMckjJj9KRQHyQMWWeL4Y7CKxNta6jpAGYB*

binintersect_Subq_1

Theorem 6.48 $\forall X Y : \iota. X \cap Y \subseteq X$. *The proposition is identified by the following information:*

Pure Prop Id:	*f296ad2186681350539ee33fda247d0b2db3407ef4c9b9878863d410f9f6c174*
Pure Prop Address:	*TMHYoEPaHzR6jsg5oRyMdnfEQqNRzrVv5fj*
Theory Prop Id:	*303833566de1cc91ab34cfc55ff0aa1141fcb51e6ffe3026695258fa36b408ef*
Theory Prop Address:	*TMbU54waGS53UwVgzDZ1rm2yMPzD36vBhkj*

binintersect_Subq_2

Theorem 6.49 $\forall X Y : \iota. X \cap Y \subseteq Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*fcd5f54826312c964c01eee203b1512d720124e0ee6cda87c6500675af973a10*
Pure Prop Address:	*TMdHZ7X2QicJLMked82RYwwjaLTcFLaALcQ*
Theory Prop Id:	*8b9fd275589ddd59b577f6592484143e175ec81d47f79fecbc691c373af53bf6*
Theory Prop Address:	*TMdhk1ZqZYfFHLHXb5kzMk3FTGGmYXyd6pv*

binintersect_Subq_eq_1

Theorem 6.50 $\forall X Y. X \subseteq Y \to X \cap Y = X$. *The proposition is identified by the following information:*

Pure Prop Id:	*277eecb6dd017782dd48738677bd8a5982157e4c5cd5cc05918e2124c974cd07*
Pure Prop Address:	*TMGURzfSeKMD3UYDfmxJ2DFVhggDoZwZnUy*
Theory Prop Id:	*ec1cc3fd8120069fd440f892bb823f090399876a574880710582ba7186a73de5*
Theory Prop Address:	*TMcUHjKDeFZpkMnMczJ2SUAQuA1B6GVgFMp*

binintersect_Subq_max

Theorem 6.51 $\forall X Y Z : \iota. Z \subseteq X \to Z \subseteq Y \to Z \subseteq X \cap Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*95624ddfbe331265c4ca2f86b896a19a583eb8540109d5bf381eca2fd9c5a2ce*
Pure Prop Address:	*TMZEyDaHk9qJtwS7tYcTVD3TPZbywzpMvLN*
Theory Prop Id:	*64636bafad5e1f11c7c5b8d776a60352971b7c848370b9d8c312eba0ba7c8a7d*
Theory Prop Address:	*TMYQdf7jp47hrnGmxiraKcX3aZTPo8rVu7B*

`binintersect_asso`

Theorem 6.52 $\forall XYZ : \iota.X \cap (Y \cap Z) = (X \cap Y) \cap Z$. *The proposition is identified by the following information:*

Pure Prop Id:	*888f8c948c07fff1208257cfdcb30e80aba97a0ea19895a0ddf3f2a7ccae2cdb*
Pure Prop Address:	*TMSNBUzV3aQxYYeZPY2TT8DCJH4y5uZtPS8*
Theory Prop Id:	*e32fe808b50d4a7d26feb7c47fba0677ea7248260462759537117a8ded2fb5dc*
Theory Prop Address:	*TMYMBJhEnU2xzWURsT3VUeTKQ6932cZ99sf*

`binintersect_com`

Theorem 6.53 $\forall XY : \iota.X \cap Y = Y \cap X$. *The proposition is identified by the following information:*

Pure Prop Id:	*1a2c6f83dfc7eef1f0f5d8a6ef71433168e521d79874af4f4612a7cae5c5b035*
Pure Prop Address:	*TMb2URdQekPfF7sFmwNvzkDv5BMnydvoeSe*
Theory Prop Id:	*ea8a1cf7428c5c48bad97c047bde68748dd123f1e3e2886bb9723b9685f19f52*
Theory Prop Address:	*TMPHGC6FLeDyJUBBy5raiX5Q2fa4jN5brmt*

`binintersect_annil`

Theorem 6.54 $\forall X : \iota.\mathsf{Empty} \cap X = \mathsf{Empty}$. *The proposition is identified by the following information:*

Pure Prop Id:	*19837ac3a4cc72e88c8423496b51c2617beb1ae51cfbefea7f1962c0b5f50a16*
Pure Prop Address:	*TMHFgk1LgJ2LLMW8CoCeuxb6mvWZeaznuN8*
Theory Prop Id:	*66dfcafe7e8daa11a68daa9116d60646fe5aa8d58830a016d31ad2426013468c*
Theory Prop Address:	*TMUGqogYhT1MpqtsHd9QtihJT2pr5trMXwD*

`binintersect_annir`

Theorem 6.55 $\forall X : \iota.X \cap \mathsf{Empty} = \mathsf{Empty}$. *The proposition is identified by the following information:*

Pure Prop Id:	*0e7e2f4e6df513f1e6b5ccaea6a1621ebbca518c2b68b695a894a4ce698c4e0c*
Pure Prop Address:	*TMS8kC9HsHPYsso9Ze468n6eVHK6b88et1P*
Theory Prop Id:	*347af54bba6e5ad1b72402cecf209ec4580f847c599c14715314311d43e1dcef*
Theory Prop Address:	*TMKWuV4D5gCFenDioXWGtrYFYLc52e3UzGG*

`binintersect_idem`

Theorem 6.56 $\forall X : \iota.X \cap X = X$. *The proposition is identified by the following information:*

Pure Prop Id:	*dbddbcb4a8c0a37d70f0630df520746c472b6a1a846e6011ecc825783815db34*
Pure Prop Address:	*TMHBW8eKU6ryBGgk4HWYuA3dnXogpEKcixW*
Theory Prop Id:	*d2a963ba345439222b37255ea718d0d0044869fef2d1dbf88d33f1c74f8181af*
Theory Prop Address:	*TMVpfMftXnY2WNrX82PsUg6Ffz8dhHZCCbY*

`binintersect_binunion_distr`

Theorem 6.57 $\forall XYZ:\iota.X\cap(Y\cup Z)=X\cap Y\cup X\cap Z$. *The proposition is identified by the following information:*

Pure Prop Id:	*36e5834dc2dad9ecc45b407b7436248dd124ef4599aa9404e9c6e2d285fdf8ce*
Pure Prop Address:	*TMJTm4UUo6gbzs6LVeQouXfnfj1XXyMHF1A*
Theory Prop Id:	*8127d38d4909f520e4a84c6845262a06189700ca1e234d7cbdaf3d2ce716f3ca*
Theory Prop Address:	*TMbjiZjgzVpkQ3KA7NniPjhJJdwASKPh6M6*

`binunion_binintersect_distr`

Theorem 6.58 $\forall XYZ:\iota.X\cup Y\cap Z=(X\cup Y)\cap(X\cup Z)$. *The proposition is identified by the following information:*

Pure Prop Id:	*a9a58a72e353790019b05940c85c859102eea0d847382f289d58e8bd468aacc5*
Pure Prop Address:	*TMUsf49mgzPX91G3rqfz1ovatYjDfYdvK98*
Theory Prop Id:	*4a9b047f4f17cdf61b7b7dd40930b1c044fa82525a4c6116a7e11a025b94040f*
Theory Prop Address:	*TMSNTa34aJse1X5BoYYckuaeBfJRNGqKuDG*

`Subq_binintersection_eq`

Theorem 6.59 $\forall XY:\iota.(X\subseteq Y)=(X\cap Y=X)$. *The proposition is identified by the following information:*

Pure Prop Id:	*86d529c3fd349b6b3715efdec80a7ca160419af4d312f017c9b6429831a8b6fa*
Pure Prop Address:	*TMS6VwxbSL2tMcsJ33Gzb3sYguz8uL14aJW*
Theory Prop Id:	*33cf2fc02a709c5c13da3cccb6bb79ee7947a9d4791092613388daca4cfe3096*
Theory Prop Address:	*TMM8Sp6PDTpeVhKhLLCUpaWCnnBiqynotsR*

`binintersect_nIn_I1`

Theorem 6.60 $\forall XYz:\iota.z\notin X\to z\notin X\cap Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*073b4038a060c4b1bde2ede64117c357d3bdce362ac825ec6a720138f563c8c5*
Pure Prop Address:	*TMVoJYkpJgivDmuJexxTuap5J3uDDg1wMek*
Theory Prop Id:	*9c3d131fc1d769080781e93eb8ec589b7a2c2d6c60bcbc11c1c14a15a604d93a*
Theory Prop Address:	*TMQojPTSE5pppDmrTox3ZJojc7GqQ5pR8nk*

`binintersect_nIn_I2`

Theorem 6.61 $\forall XYz:\iota.z\notin Y\to z\notin X\cap Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*4cd97a3920660cc1baab84a02cde2f0752cc800bdbf6948734149b840f916d31*
Pure Prop Address:	*TMHeuLBfC3AwL6jnzuUSDhrdaQmCoHoj5D3*
Theory Prop Id:	*095f40265118c2d28d3547f12e68d5c28ea77112d6ccc6b086c324fc60b2794e*
Theory Prop Address:	*TMMM6TarDYx9DpZCUpsD2z8vW1PAAJQ5vPa*

`binintersect_nIn_E`

Theorem 6.62 $\forall X Y z : \iota . z \notin X \cap Y \to z \notin X \vee z \notin Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*f6260f7735d4122a10a80856c4f9049d1519687f57b7b19edb745ee2cf5cfd3e*
Pure Prop Address:	*TMdweHzbtRnrtxQVKunco97o2a12LP8roAQ*
Theory Prop Id:	*84390c4eb6fdb43452a191bfde74ee0a790cd0285b2a874167fa19e4f893bf40*
Theory Prop Address:	*TML22Gpy4Bf79Bv1gy1Ndk2iWQUPF7ZTQMZ*

Definition 6.10 setminus *is the opaque object of type* $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*c68e5a1f5f57bc5b6e12b423f8c24b51b48bcc32149a86fc2c30a969a15d8881*
Pure Object Address:	*TMHxJ13rHG66sG7Z5d8qoQ2Wf71i3cMXRPW*
Theory Object Id:	*913da1489106a5031ed6923d7aac74c19bb471c7cb6d0b317001452fc6f29b30*
Theory Object Address:	*TMNfftowRQHpY83NVqg5iVfi5uncZiEp6pF*

Notation. We use $\setminus$ as an infix operator corresponding to applying term setminus.

`setminusI`

Theorem 6.63 $\forall X Y z . (z \in X) \to (z \notin Y) \to z \in X \setminus Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*0feeb8d8a9b7ef65218a930ce4e11ecfc7dbb1545c5ef07add1f4b81afb9c3e5*
Pure Prop Address:	*TMQPezxvx5iDwGozY2n44p1QSmzLW9c33gh*
Theory Prop Id:	*80db6afb2b95204c3816c5c17e3193ee6ef8bd580a5c82b1d40de15b0dd67e5c*
Theory Prop Address:	*TMdgvunw2TsmUp4GCmh9xtJANKkfVaELTtY*

`setminusE`

Theorem 6.64 $\forall X Y z . (z \in X \setminus Y) \to z \in X \wedge z \notin Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*6774f4f653e1e92356f11506e8157d8abe7dbb9be9e6bcb14db1f0bceab51ef3*
Pure Prop Address:	*TMXy1QEyBGVDaiEg8aD2Cam9qThxoomLi6f*
Theory Prop Id:	*71763af047667a6ca6327c62400dccb91551bc8f9f136c9e479e28698216380b*
Theory Prop Address:	*TMdogwAC88ANifD7adTtE8r86D6W3bUPM9S*

`setminusE1`

Theorem 6.65 $\forall X Y z . (z \in X \setminus Y) \to z \in X$. *The proposition is identified by the following information:*

Pure Prop Id:	*064bd5d663dc259ca997e99bb2bd4b8152814550c0aa05c43ce0df3746079700*
Pure Prop Address:	*TMTjEjKNmtdB3PuULs3bKw7WREdMMYnLNNi*
Theory Prop Id:	*d74bafc908140cfff43333693f1863756a00f38b654f98d03ff010b973d027f6*
Theory Prop Address:	*TMMW2tdJZPbYz3RFp8VEL3uz9UEjkov951Y*

`setminusE2`

Theorem 6.66 $\forall XYz.(z \in X \setminus Y) \to z \notin Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*3e696906861b398caf75e058badb44e8df22002a1dbcebbd1e432ae75ebf73e8*
Pure Prop Address:	*TMbw77MdhQRu71U2YgwKpdioxpTzvqPeTph*
Theory Prop Id:	*36b2737dca3d3ac20725828013e922045b7f000d58c5bd39fbdbde5ba29e8f17*
Theory Prop Address:	*TMK2S85jwBL3f1YkbqgioAc3U9L13yLZ4dq*

`setminus_Subq`

Theorem 6.67 $\forall XY : \iota.X \setminus Y \subseteq X$. *The proposition is identified by the following information:*

Pure Prop Id:	*5a497a676d0c43a84c28067a8d230220dfff42d4ecda3069868bd45d6864a3a1*
Pure Prop Address:	*TMZpkAydiSX6PF3eKuUhdGAjSwYNgM8nANN*
Theory Prop Id:	*66f982019f8b0f9fb9cede258e617e735d7ee6afe6be70204a016fa60645b2ad*
Theory Prop Address:	*TMXKBY1x4AJNkQvcmzLSA5oKynMfanH8j8g*

`setminus_Subq_contra`

Theorem 6.68 $\forall XYZ : \iota.Z \subseteq Y \to X \setminus Y \subseteq X \setminus Z$. *The proposition is identified by the following information:*

Pure Prop Id:	*b1f440be52ba3f0081ae25bcbcaeb0ae2996726d1652a9b43221ba081ebb4046*
Pure Prop Address:	*TML7YKpcn2xkVnB8jzzhMFivx3UtkkeTwEf*
Theory Prop Id:	*9a06b276a8a3609d482bb051c783d938e2b4332e33f0c788b5b1eaa70620ea52*
Theory Prop Address:	*TMYGQFvSYDuhgZUsfQwKDf3uo8fipNdRzPL*

`setminus_nIn_I1`

Theorem 6.69 $\forall XYz.z \notin X \to z \notin X \setminus Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*a34d147d253b8631f71a8ad885d6b07b51174f6adb06f33ff18f2e17cc3a05d8*
Pure Prop Address:	*TMP4Br6DGx2zPHwukCi4NHAb1J6UCTGEUTH*
Theory Prop Id:	*0cf01b8566a440309bf989695975a3eb8c7b5b2b72839f4c5258a27e06a47268*
Theory Prop Address:	*TMKc4mWQxd5nPWm3Twn6HUoZZ3TpTLTY6kk*

`setminus_nIn_I2`

Theorem 6.70 $\forall XYz.z \in Y \to z \notin X \setminus Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*bcb7ba48c815e20d7bee194dfed0cc862dd0e3d301f8c9b1d0280812f816ad6c*
Pure Prop Address:	*TMYK5VS87wrMj21qU9jtK2xRsBEsBv9CAaa*
Theory Prop Id:	*358a53764f4ed435a531d19ce7ac02af9dfb2c98400d9d931a04c2593b2bee95*
Theory Prop Address:	*TMRc5nmsKibAuSoPbYJ7nMiCh26fBzFmE26*

setminus_nIn_E

Theorem 6.71 $\forall XYz.z \notin X \setminus Y \to z \notin X \vee z \in Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*9bc7c60b2c9e62f8b2a9af59f911de6c5d68761f4a2fe77a94c69a7d3eaf26e2*
Pure Prop Address:	*TMW5CoiZXunGqAiTQBjRhmUmvjXoJz3mude*
Theory Prop Id:	*56f952bfe0bd75d292717ecaa4b14fda05ca0cf14fadaaa475598d568bff0ad2*
Theory Prop Address:	*TMLi6vhY79EBfSSmntPk9snhH4VNXepc9wJ*

setminus_selfannih

Theorem 6.72 $\forall X : \iota.(X \setminus X) = \mathsf{Empty}$. *The proposition is identified by the following information:*

Pure Prop Id:	*d3b191afa78022d380048239a2968a37051b62e5c02fa9ac36ee4db31e1083aa*
Pure Prop Address:	*TManQQz76f8CsK8ZetcsbHQhCjjZe659QGn*
Theory Prop Id:	*478fe86072ac5772bd0bc55b172cc272698845d92efc4458128557ac479044a8*
Theory Prop Address:	*TMd6YxJcNSPF58sbPVXKMa7uGjNh2yUe8vf*

setminus_binintersect

Theorem 6.73 $\forall XYZ : \iota.X \setminus Y \cap Z = (X \setminus Y) \cup (X \setminus Z)$. *The proposition is identified by the following information:*

Pure Prop Id:	*2625c6089792c3161e4afb2ac98c688356a04763067fa6803e414d5dcedf8d4f*
Pure Prop Address:	*TMdyYr2Jj41ATEHaGEpR1dQgkGsyZP4FuYL*
Theory Prop Id:	*f764127a273ebffdd46d2090e345b6124d00e4e10a76d5047d199152fb751f21*
Theory Prop Address:	*TMUUoPBVNSzcS2ijLW9PqkQhqPjPTs1Z7MA*

setminus_binunion

Theorem 6.74 $\forall XYZ : \iota.X \setminus Y \cup Z = (X \setminus Y) \setminus Z$. *The proposition is identified by the following information:*

Pure Prop Id:	*0b93177149890035a7cbcbdd8eba497db661ca19eccedc9faeb0ae391b2c8987d*
Pure Prop Address:	*TMFwTVRbH9sP3ic4Jj5K77JHuuDaHidkTTo*
Theory Prop Id:	*651326888d51f7ae0c489bc08dde18dffd32270d616f68516db81047548f94b9*
Theory Prop Address:	*TMT4EsAWT7qGdwNHgiPcC19uNg3gnsu5RfF*

binintersect_setminus

Theorem 6.75 $\forall XYZ : \iota.(X \cap Y) \setminus Z = X \cap (Y \setminus Z)$. *The proposition is identified by the following information:*

Pure Prop Id:	*d1bd304c20aac47c64bd3668e5ee0c534b4dbff3de4e7a34ab27ea65f5a5e53e*
Pure Prop Address:	*TMYXNZDQUqfnGsDS2NpMohAoZPUMeuh4Puw*
Theory Prop Id:	*f50067170c57c9d167d2bbce86fb7bba1d5f359a694a3e8d87c42ea80fc8f4a9*
Theory Prop Address:	*TMXQGHghJkaMcQb53tZ6WfydPdKPLw4Fbx3*

`binunion_setminus`

Theorem 6.76 $\forall XYZ:\iota.X \cup Y \setminus Z = (X \setminus Z) \cup (Y \setminus Z)$. *The proposition is identified by the following information:*

Pure Prop Id:	*99c8981445a26f2a8f3c67db827e6580c563359a4f20df37e41dca73f6393a97*
Pure Prop Address:	*TMbfg9u25oTvirxRNPHRobxdagcb2y9Y1Ez*
Theory Prop Id:	*1c1cfdea82cec4035b3de28d39481169301aa46b0d0912a7f83e9228797f5a48*
Theory Prop Address:	*TMQW2SjLqSWSKT8J2EQPCpxjys4GYkHvDX2*

`setminus_setminus`

Theorem 6.77 $\forall XYZ:\iota.X \setminus (Y \setminus Z) = (X \setminus Y) \cup (X \cap Z)$. *The proposition is identified by the following information:*

Pure Prop Id:	*60e720f6c807ccdae50a934ea56981944603f9d6f18983c678c322715f66a587*
Pure Prop Address:	*TMLXRcQtVsZ2gokZw2jjxkAgJhArzeWbwhF*
Theory Prop Id:	*cab095f0a1f43e4dc50e5489f05911c74f5fd42b4a8dced8ecea3a53e69f1d71*
Theory Prop Address:	*TMEiQhFR4FhRQESPDAccJrCE98eg1tzdAEE*

`setminus_annil`

Theorem 6.78 $\forall X:\iota.\mathsf{Empty} \setminus X = \mathsf{Empty}$. *The proposition is identified by the following information:*

Pure Prop Id:	*fd99d2feff8bf081dd1e98ac3ced977726230ad5af0c70364ed17032b322e958*
Pure Prop Address:	*TMPjszawGe4kxyHBsc9D4AVrRBAh4nHf7YB*
Theory Prop Id:	*3d2f5b9dfe25bba40fc9e8e784f45fff9a4a929712ef764617c8b137da6f2cdb*
Theory Prop Address:	*TMPrnLS8t41sPUjW6B8bNjh2dhMBwTbU9S8*

`setminus_idr`

Theorem 6.79 $\forall X:\iota.X \setminus \mathsf{Empty} = X$. *The proposition is identified by the following information:*

Pure Prop Id:	*20e1086759c8557ab7f7e06736e90912f8684d26fb4e569f3e23fc4960e2a173*
Pure Prop Address:	*TMQi4nBawCq2rp7HipQymKHj8UhkFm1gCeM*
Theory Prop Id:	*ebb8ec6864341bd40d215d011ec32f77f22a7bfdb6d5d8332a0487fa1efbbaf9*
Theory Prop Address:	*TMYkPV9ze3vB5CvjACMPmYU1b8ea1z2jSPg*

`In_irref`

Theorem 6.80 $\forall x.x \notin x$. *The proposition is identified by the following information:*

Pure Prop Id:	*12629a025dcc8a596e030daeffbf6322b3d51631eeebf85bcca0a44b6a7964cd*
Pure Prop Address:	*TMKwkB6s3TXZJG8hL6bLbEGFMysQgpweKpz*
Theory Prop Id:	*7bfd46e967631be2a9098717487f5ae8f88e2867117fd98cce921a47bed1bdbf*
Theory Prop Address:	*TMZagTqZqCeDpgAoeuYZejsuagxzALpQvWL*

`In_no2cycle`

Theorem 6.81 $\forall xy.x \in y \to y \in x \to \mathsf{False}$. *The proposition is identified by the following information:*

Pure Prop Id:	*6d0cb3f4b527344ea21bf482f99b38b72d358ba458097b2ed205e1de6bd2c82f*
Pure Prop Address:	*TMcBgknu3MmmLrKct3NGTcziaJv71unmDbY*
Theory Prop Id:	*344f3082291501cfe856ef475802aea283ae4928899f0eba4e2c3a6556451b82*
Theory Prop Address:	*TMNd1Mv58X5hcYFChdQ93tpgtHdhyahABAg*

`In_no3cycle`

Theorem 6.82 $\forall xyz.x \in y \to y \in z \to z \in x \to \mathsf{False}$. *The proposition is identified by the following information:*

Pure Prop Id:	*660f202c6752b9347ac4d5b7b55e28507e6fdf71c23bf56ce2eae287f04ec6f3*
Pure Prop Address:	*TMQcimvCarrH1MmuAYoNABafQguio3NLZeY*
Theory Prop Id:	*90080b22e6d3c8f5944bbe4d048a7d9fdffe58eac2e3d918ff8ad0d16d572a8e*
Theory Prop Address:	*TMZPDFZYJchoNx1RFvKeMUbtyTaSeQj9LrH*

Chapter 7

Natural Numbers I

Definition 7.1 ordsucc *is the opaque object of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id: *65d8837d7b0172ae830bed36c8407fcd41b7d875033d2284eb2df245b42295a6*
Pure Object Address: *TMavRsMUqs8nxad2sig41TibDMTMy1H1mNn*
Theory Object Id: *1452f7d6b185ccc8dd0cf665789667547457cc1e37f37eeb4542ea13bb0f3b7c*
Theory Object Address: *TMKDT5PUJNLA2gdidecqgknUTRNKvDTHhkr*

`ordsuccI1`

Theorem 7.1 $\forall x : \iota . x \subseteq \mathsf{ordsucc}\ x$. *The proposition is identified by the following information:*

Pure Prop Id: *5a2f15e1e0256bce0e0a36c10194ffcbfe641dbb51c699626b909df04caf5bc8*
Pure Prop Address: *TMPc16mnjiUyzqEV7bqCxpSd6RmykST31RK*
Theory Prop Id: *367d807338b46735c105597b4d7b718fc7262d8a19b606bca8b2fcba8f5b1e15*
Theory Prop Address: *TMPyyF13kHh14HmE9nFU4oA64FKkfCootMK*

`ordsuccI2`

Theorem 7.2 $\forall x : \iota . x \in \mathsf{ordsucc}\ x$. *The proposition is identified by the following information:*

Pure Prop Id: *2f6efdd09ddf2d88f93f929612974d444c064325e7c941dfb1462b672c87b2af*
Pure Prop Address: *TMVqzvRwWJ3SEK7WcdSkCGWDSdUaejyiBbZ*
Theory Prop Id: *2be6be024332289d2aa440c03da38fb1d593f63adf8e2fd3e60d61ed384710cb*
Theory Prop Address: *TMMZ4GfqtzWS5jhaGuWH9wJJD8HBcxQMpbD*

`ordsuccE`

Theorem 7.3 $\forall xy : \iota . y \in \mathsf{ordsucc}\ x \to y \in x \vee y = x$. *The proposition is identified by the following information:*

Pure Prop Id:	*01c4eee053a9b09e095c4f3572245e6c0eab90795b95854b85d38a0087607c4c*
Pure Prop Address:	*TMUfsQNiuSFqHup4XjacKRWBsfxf8BGBqRc*
Theory Prop Id:	*3849f5cb870c46683ed2e33b5b86c0c8d2d347b9f9ef20b2d6c3591e603a785b*
Theory Prop Address:	*TMHz6H3wiAaYh9iPsBhq4bonjGvMxx7bRgr*

Notation. We now use natural numbers $0, 1, 2, \ldots$ as terms where 0 means Empty and $n + 1$ means ordsucc n.

`neq_0_ordsucc`

Theorem 7.4 $\forall a : \iota . 0 \neq \mathsf{ordsucc}\ a$. *The proposition is identified by the following information:*

Pure Prop Id:	*1b20cba417f676cb6dbc9672ee2b03ded8ab6cfed86cd29335c5ffc3734e019f*
Pure Prop Address:	*TMQj2ULKJiSRkfDiw5djhEYAiKFnD8V8ChY*
Theory Prop Id:	*04c4104e681b528cc961597d733a565b395e183dad8c992b88138202d9471b6c*
Theory Prop Address:	*TMSZGeeBC5QqAdFny2AFc64sYMBqNSG7WsX*

`neq_ordsucc_0`

Theorem 7.5 $\forall a : \iota . \mathsf{ordsucc}\ a \neq 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*9b391ea2bb9a775053e83e0216e5088a30439bc1052e445f7e2b049a5e87f59e*
Pure Prop Address:	*TMHpzvw5s6p11s2PhBAUH5tA63dzykXSMV4*
Theory Prop Id:	*346ee0e70829c6afd0125feef2af949dbac67cd5e69584a5150bc6084f37e8e7*
Theory Prop Address:	*TMG4GdZg9J7SSN69pwxvT4GRe1Qx3Dvxqof*

`ordsucc_inj`

Theorem 7.6 $\forall ab : \iota . \mathsf{ordsucc}\ a = \mathsf{ordsucc}\ b \to a = b$. *The proposition is identified by the following information:*

Pure Prop Id:	*40cad67f9c954d4e382390b2aba8222e29db73bba21874edcdab5f1e65bcae9e*
Pure Prop Address:	*TMJhxcvNP3aGv2vSzykSRZBpaD8KiY3FWPU*
Theory Prop Id:	*4869a11c62e30e2d41c7f4524c576be90f3c64f919fc7dfe1442add6e7c4b685*
Theory Prop Address:	*TMNg9uVctRmhnFUUaeZ81Gj1Y7kuPbhrwyc*

`ordsucc_inj_contra`

Theorem 7.7 $\forall ab : \iota . a \neq b \to \mathsf{ordsucc}\ a \neq \mathsf{ordsucc}\ b$. *The proposition is identified by the following information:*

Pure Prop Id:	*79130458153e5721466856a630c586b16f2738bbfc6e0259d8313cb5ade1bfa0*
Pure Prop Address:	*TMa7r3NCaBWkq3wxWzUgF3xxk3pTHF1U4qs*
Theory Prop Id:	*66f4df16ff59481c9056308d94b75bc6bc55adb8b8e510c166184cb6cc83d26c*
Theory Prop Address:	*TMKVLdh2rykBFF1YXiKx1pwSwX7cpuoEp9t*

`In_0_1`

Theorem 7.8 $0 \in 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*e984b0b4b8bd4560157f8d6d3577f303bcbfad9f5202acbc6aafe41c46f5a1e2*
Pure Prop Address:	*TMSTYDAFnhcsxKw7rE1h1KyMkzowN5nMHPH*
Theory Prop Id:	*3be80a55870dc121ca054e5dacdfa1de52caa7939a81bb69beaf6bdbc1246eaf*
Theory Prop Address:	*TMQTUaJ1s8JSg8LytU2CzSFNeMJJfqKmUPn*

`In_0_2`

Theorem 7.9 $0 \in 2$. *The proposition is identified by the following information:*

Pure Prop Id:	*56dc2cbeb7b04a87557e31dd8850f675eda2fcb01d7fe2d95714032d7726ec34*
Pure Prop Address:	*TMXDJP7PuoXtQw49z9dst2KviKHpUhQEeng*
Theory Prop Id:	*5a202e3c8e0f6cac6103c72106cd4d8b5fafa891dddb8c40e68ea9b5e1e4d088*
Theory Prop Address:	*TMVCcMZv24dUjhcz1MQJEf7nxq1pjQVJLEk*

`In_1_2`

Theorem 7.10 $1 \in 2$. *The proposition is identified by the following information:*

Pure Prop Id:	*8704b3e5f3972c8160904d3c95ef653a8d86fd991a0e3e89d6c4ba3a0989cffa*
Pure Prop Address:	*TMcojLEL6YY4Y3YxV2dkpKy8Vbt3vCHJed1*
Theory Prop Id:	*431e9e50b33a518fbfbcb0617273d0749a4068ee4d20068f34f568c8ca0ca98d*
Theory Prop Address:	*TMPAkzgdLgKfuKVB27MKBPmY5KQWhbL2dWE*

Definition 7.2 *We define* nat_p *to be* $\lambda n : \iota.\forall p : \iota \to o.p\ 0 \to (\forall x : \iota.p\ x \to p\ (\mathsf{ordsucc}\ x)) \to p\ n$ *of type* $\iota \to o$ *identified by the following information:*

Pure Object Id:	*458be3a74fef41541068991d6ed4034dc3b5e637179369954ba21f6dff4448e4*
Pure Object Address:	*TMdNqBmjo7vJ5wFYe6UamcKdgnn9WXg7rEJ*
Theory Object Id:	*cfd8eb3ef07509602aa3910d783c30847ee19a1fa94457706e8bcc91da1a0685*
Theory Object Address:	*TMQwsyTWGfzEce8GjWygs11T1pzxgCk15vk*

`nat_0`

Theorem 7.11 nat_p 0. *The proposition is identified by the following information:*

Pure Prop Id:	*1617bca54b26e5228d76be996df23b2acf38a1e8ccb04bf94ec2a951660e6a42*
Pure Prop Address:	*TMUBAZcQKUGLBQhSYCTVsFgkCVP66bx3XMG*
Theory Prop Id:	*457a72a6740767a11460a92c05bf74c2f85ec3d9d68b112eda7a6ccb7585d72a*
Theory Prop Address:	*TMMYbseVvbEE5mCqsfJbV6FkYHcx1cF2uL8*

`nat_ordsucc`

Theorem 7.12 $\forall n : \iota.\mathsf{nat\_p}\ n \to \mathsf{nat\_p}\ (\mathsf{ordsucc}\ n)$. *The proposition is identified by the following information:*

Pure Prop Id:	*87f001f9ea63dd0ca64b98fb5d48ee465b362dbdb42cd8a8108ac9dde660ade1*
Pure Prop Address:	*TMaWAUtmodzvGsvp6q6xHpk4Yuham8jZXcm*
Theory Prop Id:	*57e940f024fda91a7159ab98b6fcc0219d72fb6e7bb86ae7f0b6c4d781dfd58c*
Theory Prop Address:	*TMKZQMyAvFRX9iJG3iXqmXhhgXw3WPru1ww*

nat_1

Theorem 7.13 nat_p 1. *The proposition is identified by the following information:*

Pure Prop Id:	*2f83faeb354b73c788d02a2f05af45d694cfea901b6a7b10dfa230c853b96453*
Pure Prop Address:	*TMKVKW3snRuquEuPq79vsPPwjabNMfZWF9F*
Theory Prop Id:	*672d0d5e7bbed7f2c3c492dd058d40c6f76f54880250fc30060ac1efe4135adc*
Theory Prop Address:	*TMUjC2bRLBWtBtHMGwSpPZY777xJhfnQBRo*

nat_2

Theorem 7.14 nat_p 2. *The proposition is identified by the following information:*

Pure Prop Id:	*a8be1568ff355447f7db7c89da6ae23d1686bd32ec45719d79659417af6f447d*
Pure Prop Address:	*TMWqNeynfSDURZP8Wzc3iZasb4wYMvcrwSj*
Theory Prop Id:	*9112e975c2b7883a953c49c1d03e3a50209fee33f81345ec21f36647283d3373*
Theory Prop Address:	*TMTTG4UrkKbYb6ukW8afwvToLqvgXuokzbh*

nat_3

Theorem 7.15 nat_p 3. *The proposition is identified by the following information:*

Pure Prop Id:	*c0849ff3f73eb686bb761cb6944a4784c9c2d96f7206b9a8024656f2b406317d*
Pure Prop Address:	*TMNxJRiy5Z5ySkwD4bVtViWKE7vD4jrAD15*
Theory Prop Id:	*133bb750b1d962e5d82a3fae627874 79ad909ed8e960846d440b4c68d71ae6da*
Theory Prop Address:	*TMQXcPujARi3ryfse2NkWbvZ4uGumLKcEWS*

nat_4

Theorem 7.16 nat_p 4. *The proposition is identified by the following information:*

Pure Prop Id:	*2c8e1aae548c1357c58dde606cc0aec5ffac22c5e66251a8ce0805caf0f28922*
Pure Prop Address:	*TMH3HZH2XtjuKpCAF7SNvq8oWwWq1gMGf9g*
Theory Prop Id:	*69ffc33641fda2b620e65b8c7a42ab36d1575ef4f6cdffca4419166392954387*
Theory Prop Address:	*TMcTTobfSQSnJ7AbXsvRGCQ7y3HaQnEeeFs*

nat_5

Theorem 7.17 nat_p 5. *The proposition is identified by the following information:*

Pure Prop Id:	*1a2477a480bd21b24c7f26877221e1b8a5a0977ebc3409b63d8b81adc052b208*
Pure Prop Address:	*TMaLjbE49i38tGGYWFzVVSxUtcuU82qvhmk*
Theory Prop Id:	*b84b25e58b844e2ddd85bea48fe48f329f13a25c4c0456681e28158f15b3a5df*
Theory Prop Address:	*TMFj5aVyECSdNXwpDgCbbGDuCSC7gTYNSn4*

`nat_6`

Theorem 7.18 nat_p 6. *The proposition is identified by the following information:*

Pure Prop Id:	*25de88744534dcfb82c0d0379635001cdffce949810f8ad348a12948d32b0af7*
Pure Prop Address:	*TMS5YEXhfPvtcWyPkWtRChML7geomnrXHCd*
Theory Prop Id:	*b4ec6745b4fa46eb645b20d7578dbd15a3c3bd98922e7daf874d00b8ec50fb0c*
Theory Prop Address:	*TMStihyrXSQ2eoCRa882VokNRU7nbYWQD8F*

`nat_0_in_ordsucc`

Theorem 7.19 $\forall n.\mathsf{nat\_p}\ n \to 0 \in \mathsf{ordsucc}\ n$. *The proposition is identified by the following information:*

Pure Prop Id:	*9b50b9ff20796d5e40a96b518b4c6306f3f1fbcdac11d2c3f6be7a95ea734347*
Pure Prop Address:	*TMNGcWtDYoP2Bidne3KeT6KVqrriDbdzHjr*
Theory Prop Id:	*251f3d891ace960a597b3f004c9759fedf5c6738929d3028dba9a61a95fbf120*
Theory Prop Address:	*TMHA18C6v8mXaGqFjk7jf2v9egdfvKKqWuW*

`nat_ordsucc_in_ordsucc`

Theorem 7.20 $\forall n.\mathsf{nat\_p}\ n \to \forall m \in n.\mathsf{ordsucc}\ m \in \mathsf{ordsucc}\ n$. *The proposition is identified by the following information:*

Pure Prop Id:	*819399e48921ef23f6b1409081964e191ac9faf077bcea0b877deb52ebc19088*
Pure Prop Address:	*TMRPbaUhLFQA5eMs4qgHq2eHkAum5RDSzqx*
Theory Prop Id:	*4c2c9521cdcb93eeae5ab0b3bddd72a6ac20699773fda70d1c40ad8369603f79*
Theory Prop Address:	*TMWSu4yCbzbJPdvkEV8EErMS2USSHmDvFoQ*

`nat_ind`

Theorem 7.21 $\forall p : \iota \to o.p\ 0 \to (\forall n.\mathsf{nat\_p}\ n \to p\ n \to p\ (\mathsf{ordsucc}\ n)) \to \forall n.\mathsf{nat\_p}\ n \to p\ n$. *The proposition is identified by the following information:*

Pure Prop Id:	*5d06f5ffa757285d834b9b984ab55693e9c64caee1e15c187462d6b08a91cfb6*
Pure Prop Address:	*TMHLu8uvpDa5T7p3FX9tm1QcSzRDu8EgeoS*
Theory Prop Id:	*ecdce8bd5bb09bae03975edfeb1c1d944e5b0fb2eace3d673428b70400e410f2*
Theory Prop Address:	*TMLKPWPGi2sVCW27Hu2rRaW71Je8eYbfMK9*

`nat_inv`

Theorem 7.22 $\forall n.\mathsf{nat\_p}\ n \to n = 0 \lor \exists x.\mathsf{nat\_p}\ x \land n = \mathsf{ordsucc}\ x$. *The proposition is identified by the following information:*

Pure Prop Id:	*5d4296fccf80fc82c53e9d0330ce9f3d5e1106daedee7d4d25d13d7d4b63ba57*
Pure Prop Address:	*TMa76r1GZgLGs96CTRuUt7foxrtZPG4axsF*
Theory Prop Id:	*4f4167d99f52765043e392f09f9c226ca035e3f1640818079090d6f7ab744d14*
Theory Prop Address:	*TMR4i7cHgevq2znVdMhJMiqjT6wu9k8kJN8*

`nat_complete_ind`

Theorem 7.23 $\forall p : \iota \to o.(\forall n.\mathsf{nat\_p}\ n \to (\forall m \in n.p\ m) \to p\ n) \to \forall n.\mathsf{nat\_p}\ n \to p\ n.$ *The proposition is identified by the following information:*

Pure Prop Id:	*d32e5b24fa29af5ae41e8ad1bdabc16e354331cd6f147252b6051abcd45747b4*
Pure Prop Address:	*TMSAj2j7jwL81oXRAhtY9BFvgyXQUt34jN9*
Theory Prop Id:	*1e16087110e02042e56a9e5c1063925cc2f7ec418bc4e813ba1663102359d49f*
Theory Prop Address:	*TMSj9PFLh5Qr9YAvzYmFtQ9ahr3yMf1DzhX*

`nat_p_trans`

Theorem 7.24 $\forall n.\mathsf{nat\_p}\ n \to \forall m \in n.\mathsf{nat\_p}\ m.$ *The proposition is identified by the following information:*

Pure Prop Id:	*e884494b492bcc0edee39dcf5b1043dc2f35d009618bfad2e00c24a9c042215a*
Pure Prop Address:	*TMHs2RYjZjBVk1upHj4ggqMDExPU6aYRTU5*
Theory Prop Id:	*03b874ce14b10c68c8b003d5521fedd9dbde8f272d9947fada3cf5bee55416fc*
Theory Prop Address:	*TMMK6MMXkYaSfwkm9GkAhh9Lo4bkVBtYWdV*

`nat_trans`

Theorem 7.25 $\forall n.\mathsf{nat\_p}\ n \to \forall m \in n.m \subseteq n.$ *The proposition is identified by the following information:*

Pure Prop Id:	*6bc6e12c22a5d14f7294be0cdc44e66abf13bd33013e1c985c60b88343f6a7e9*
Pure Prop Address:	*TMFwnmbaMpakWwrrmKjohDndW8KqnGgrE2R*
Theory Prop Id:	*cad6fe1daa30ce548334ddc0217dd046070f343d4279370eed3639bb6552015e*
Theory Prop Address:	*TMQmjjPV3zeSxpELyxYbuTXa9bAc7s9QGaw*

`nat_ordsucc_trans`

Theorem 7.26 $\forall n.\mathsf{nat\_p}\ n \to \forall m \in \mathsf{ordsucc}\ n.m \subseteq n.$ *The proposition is identified by the following information:*

Pure Prop Id:	*e42e581777e9b184d838738cd0f2e42b37dbddeaedc030fc6bcbb1017c2c4acf*
Pure Prop Address:	*TMWNTUPr5UHTy5qBphSAPE6eNxNxTmDMM3c*
Theory Prop Id:	*bc868dd13fb819f4f89016d584069c5811ebf3ca5efcc8c1c4d079e962cf85db*
Theory Prop Address:	*TMV35265NGTDZcH9QrCs5zZVseRcm6nuMzu*

`Union_ordsucc_eq`

Theorem 7.27 $\forall n.\mathsf{nat\_p}\ n \to \mathsf{Union}\ (\mathsf{ordsucc}\ n) = n.$ *The proposition is identified by the following information:*

Pure Prop Id:	*87308c20573b24369c99ff4fe9bc8822508ac2939b6f6d5756b6098dda4b527f*
Pure Prop Address:	*TMHVBTxQoQ4YadqkTHbe8MXRCPm3xY51g35*
Theory Prop Id:	*e8cdeba740894b73be89b96729f0c00cde7781b068388e485ec8a7ef82de11cf*
Theory Prop Address:	*TMWEc3LBttyZMJdbQThEZMNmW5T1bNLMb8k*

`In_0_3`

Theorem 7.28 $0 \in 3$. *The proposition is identified by the following information:*

Pure Prop Id:	*5d5d5f868b630108f6d0ef72c022b1a1fdcb3bba9e9ac36ffa44fbd60e4334ae*
Pure Prop Address:	*TMYMGQ2bVW21zWN5CbbTLaasyax3hxdLvqK*
Theory Prop Id:	*313d97e81a38fa191ac39cd36a8d7be9e4da2ad48aa4710d01a9a001638e022b*
Theory Prop Address:	*TMU4Cdfau2fRjQ3n9SbuJ1p1TcVEvxWfhqp*

`In_1_3`

Theorem 7.29 $1 \in 3$. *The proposition is identified by the following information:*

Pure Prop Id:	*85201c4239d9221474ddfc6342817247634f124875f424ae9f7175ce6df17f51*
Pure Prop Address:	*TMGfU5npaRR11H1ymdoVZKTfX7BrhZKDwjf*
Theory Prop Id:	*8f3cec3de25f5229ffbd8a9526581652ba38c8b4ace6f65a6c6b0c34e257b18d*
Theory Prop Address:	*TMZAip8oPziRqgUtYsjaNbkMfq6G8pgm3NH*

`In_2_3`

Theorem 7.30 $2 \in 3$. *The proposition is identified by the following information:*

Pure Prop Id:	*4d67cef397f17c6f96d94a96fb406d790704d539c9dcd26f464a7ede52fd09ea*
Pure Prop Address:	*TMPo1MLQ944guurv3GfbEq47E776WbpEpzr*
Theory Prop Id:	*ebdb5d69cf555fc8c7f6126302fc59f1945db62dc77e65788bb147ab62800823*
Theory Prop Address:	*TMFx1bn63w8vypVgVbQs3uqihXU9xXDRXcb*

`In_0_4`

Theorem 7.31 $0 \in 4$. *The proposition is identified by the following information:*

Pure Prop Id:	*0a2665a34864dd30cb973fd074b11e77208b4a6c95f91a17bc48472090308db8*
Pure Prop Address:	*TMbWh6fxUCtxFfTwjF1FBTop3nZpchLnWC7*
Theory Prop Id:	*2d4164fa05829f0fb01d9e4ae31ab49773fd9055499b85021468859ab970f821*
Theory Prop Address:	*TMG6uSmoWvJMEa12QT4E89xXUiPGnfsLvJb*

`In_1_4`

Theorem 7.32 $1 \in 4$. *The proposition is identified by the following information:*

Pure Prop Id:	*5cabbbcb5f4d3c4366e68c4306207f64e478077557ddbd21b737a9ed99b1bde8*
Pure Prop Address:	*TMFSB8wCuJCgms892RboQ8XaFFohyZ5VoXi*
Theory Prop Id:	*1e2d5bc32b050352877615a028897b5966602f62b83077de259c38244906a955*
Theory Prop Address:	*TMb5w175AXMmikqn3jnFTEuWyxRWW1q9VR1*

In_2_4

Theorem 7.33 $2 \in 4$. *The proposition is identified by the following information:*

Pure Prop Id:	*33c2ff88221f741e21ca67de81295fbf2c5a50f85469edc4c578006445422047*
Pure Prop Address:	*TMSmCUEe8UeCDsTpkgeVsJafexbUrbDtBJT*
Theory Prop Id:	*2cbb94cec93497c80bc1c990383ab0d819918ebba46428b8d07b50c2a8dd3873*
Theory Prop Address:	*TMZ4rUSfGohxvov39V3nn9XfgUBuVUnDP81*

In_3_4

Theorem 7.34 $3 \in 4$. *The proposition is identified by the following information:*

Pure Prop Id:	*f25e446ff25ccb26c9eaf5000bbb167cdc2eee3efcb7a6c5d97b91ed9f586259*
Pure Prop Address:	*TMHhdWYgy6pNRNN33esNvfW6grxMUZbxmQy*
Theory Prop Id:	*22a0af713d24716cae60d0f24b75fa2f7a4806d9e04268f23e037d5bfda1d9ab*
Theory Prop Address:	*TMY5nCeLnTtDAbqPEDMuJXoU1synsTZUbX2*

In_0_5

Theorem 7.35 $0 \in 5$. *The proposition is identified by the following information:*

Pure Prop Id:	*d496b37c7578aa3dd3d6688724973b63a69ab8843dcdd9a9cbf404cee4a9e756*
Pure Prop Address:	*TMWXaxNKNEtuRGm8q1B3GYkjFHYAnyqhofA*
Theory Prop Id:	*965bc2aab360b39c6dcd958d77ad07534cbbe30f14358d204ad4e6b7b7b46d6c*
Theory Prop Address:	*TMK6cfwkMoqpWBimhTSkWz16DVZMkJZvErn*

In_1_5

Theorem 7.36 $1 \in 5$. *The proposition is identified by the following information:*

Pure Prop Id:	*be96a32ec1836ba1630ad994edab9d7936fc2693d7809d868797766ea4190943*
Pure Prop Address:	*TMXHZTLBSjY6b6uscunc7vkkgtZToJhnJx3*
Theory Prop Id:	*34c02d4ab3de796ce3192599f046a83544b80ab964c752f5812cab439ef73dac*
Theory Prop Address:	*TMJSvdJuHEbLX569a7rARyuiAc4TBR5z7jq*

In_2_5

Theorem 7.37 $2 \in 5$. *The proposition is identified by the following information:*

Pure Prop Id:	*ec68b2a5af6caf3d108a9db5a8c18bdba7461c7ae67c3f170c1567638841250e*
Pure Prop Address:	*TMcknqzryhsoUsEHJDCubHX54MtQJ7bamVZ*
Theory Prop Id:	*7ed8e4dcf1f9fa87265224b605b86bd3741e69943d43b50b1e1184851339bc69*
Theory Prop Address:	*TMKyPhdwyzSyueTwxxHt94eVxfRmv1aYr8Z*

`In_3_5`

Theorem 7.38 $3 \in 5$. *The proposition is identified by the following information:*

Pure Prop Id:	*eddd369a108e071365a07eeb0102084ea704a8d55dd011c4fa3a11984c9d03a6*
Pure Prop Address:	*TMFAtiv8XgWv9eW46mFVpYtzxFPKr6Zdm9r*
Theory Prop Id:	*df92fe69df5632294637d1b16ca10e692f827160f76ad0dea688f1cb47248e6d*
Theory Prop Address:	*TMEsH4FBsT68Y7qAu35cdgeMxfncBpZDQGB*

`In_4_5`

Theorem 7.39 $4 \in 5$. *The proposition is identified by the following information:*

Pure Prop Id:	*e72ad7d7edf5bc6c656af02486d73733da6dab698e0c8324a4ef1729c1350b9c*
Pure Prop Address:	*TMM44jRqVsjAWBEjN5MPxHU2crXNCqdxJt9*
Theory Prop Id:	*03b9ff65b44d816e3599756cc6e5dc3b6b50ca8f07749f7adfd9e48f19469c45*
Theory Prop Address:	*TMbtsFeGDXqrFQb2Xe3LnUKy3TXKV8WPAoQ*

`In_0_6`

Theorem 7.40 $0 \in 6$. *The proposition is identified by the following information:*

Pure Prop Id:	*4b3417730a51ee94caf04c6f4de510c62f9f4925400122c58b21177776e3b787*
Pure Prop Address:	*TMLu5nuHNV3AzNorNqbEsf2hWrAxPF519QM*
Theory Prop Id:	*9f78a575d1b7281fbcfa8c95d8e8c97c9bff83b15cc425c99c1cebf91540f0c5*
Theory Prop Address:	*TMVHzfqGiL4fJjTzThd6tjGntRMTKW4S98D*

`In_1_6`

Theorem 7.41 $1 \in 6$. *The proposition is identified by the following information:*

Pure Prop Id:	*4c8428c2d2820e6ddd02f33412f82a456fbb92337a7c24bd2949838775230efa*
Pure Prop Address:	*TMc8kjaqr7JWG5gtEKyEAZGMhS9xsyCNsQu*
Theory Prop Id:	*327244a71fe9738bca8eeac7c0c9b9ec3a4bb249e97ea6275ba6ab2dfd446afe*
Theory Prop Address:	*TMRoR6jxpANVwezSACwWKEdCd4NxVcABGsa*

`In_2_6`

Theorem 7.42 $2 \in 6$. *The proposition is identified by the following information:*

Pure Prop Id:	*a8317784c20b4dc83403cf330adc750c702b00a3cd65c663bc5c901a0367573d*
Pure Prop Address:	*TMXd4ThRkPLZAiyGBMqRWy2aiMvbxmyP3bv*
Theory Prop Id:	*a8f5a1b9820e7d62e90597d5e69f41e69bad70f2d1c754a0ca922ddeed9a53f7*
Theory Prop Address:	*TMGReynZEJe34xYHBotustobc5dLWKRgJyQ*

In_3_6

Theorem 7.43 $3 \in 6$. *The proposition is identified by the following information:*

Pure Prop Id:	*a261e5ada36e036a105a0af5df2c1a6bf280d14dceadb5fb843b1c5b93e6eca9*
Pure Prop Address:	*TMGXG2p1QMD6aexgGew972chtNsckboghZa*
Theory Prop Id:	*9f4635e903ae4d1ec8343f5c6b3c43eb6b2f022214d0c7c82673da42ba715ac7*
Theory Prop Address:	*TMPdrRy79F8aNf9CVxzNbnL4JAAP286vxFk*

In_4_6

Theorem 7.44 $4 \in 6$. *The proposition is identified by the following information:*

Pure Prop Id:	*1b41da8a9246684b8db369a60d5fe5356821065ca71fb292dcb7f664a7828e8c*
Pure Prop Address:	*TMMLyZwuwf8JFrgJFBNjioGMCZyYvyzDqAn*
Theory Prop Id:	*5d38f6e045507f4a93cad11fae8c1540a1eae90b3f52dbbefdcc9ed45b1d3c8e*
Theory Prop Address:	*TMLPcbeA4bjw9wrq33aFxyRQLGSz8g4hz1u*

In_5_6

Theorem 7.45 $5 \in 6$. *The proposition is identified by the following information:*

Pure Prop Id:	*2b492691b7ed3865b84636fa15db3d5054a753686f97633cc4ede78f686ef8d1*
Pure Prop Address:	*TMPEvT5ScMWzFTfTs4L9HbhU3MydujRMzZs*
Theory Prop Id:	*6d1dd13e78e58b97900513a8b85b88143e8f8b02809c1190db86c9dd2790e39f*
Theory Prop Address:	*TMW8GXcL8fcN4SrFvpTeB4P4JkZjrbQxnBM*

cases_1

Theorem 7.46 $\forall i \in 1.\forall p : \iota \to o.p\ 0 \to p\ i$. *The proposition is identified by the following information:*

Pure Prop Id:	*7066058004d5c0b851f77ebf2a55649c2a748e523d3f217177dfd9510fd577ca*
Pure Prop Address:	*TMModSu1YSRUz9bXeP9MgEyHwZQ9newZ6CC*
Theory Prop Id:	*c6f0c0649935f65fab6b1d1d427a6aac062e1ba5d1610667c31ecaa47cb0c54e*
Theory Prop Address:	*TMcp6xnjxks7qVeybXpyew3hEbkn2Y1Yov6*

cases_2

Theorem 7.47 $\forall i \in 2.\forall p : \iota \to o.p\ 0 \to p\ 1 \to p\ i$. *The proposition is identified by the following information:*

Pure Prop Id:	*4693b27abb85322341f1944c565b525391a1ae698d8535be845fd1293fbc3624*
Pure Prop Address:	*TMKetHxomnrco2Vqg88PFWW5AconfvozyEs*
Theory Prop Id:	*74563af5af19787ebf01165e4b1ceb3b061086f68c594ad4c780e78738af0c68*
Theory Prop Address:	*TMZquspohj8HSbBs2iZWfi9rM4tD9PXhYCf*

`cases_3`

Theorem 7.48 $\forall i \in 3.\forall p : \iota \to o.p\ 0 \to p\ 1 \to p\ 2 \to p\ i$. *The proposition is identified by the following information:*

Pure Prop Id:	*2a0ca2e4c94eb26ab5c19412a6da10600e180d42150f6e6127f0717ef3a8c112*
Pure Prop Address:	*TMUXV8fVuZa8oY4GuQa6GtAfrazqQgpJXZJ*
Theory Prop Id:	*1a8a209d9754777b2d79de7e41a0d52b1cb29112b497f6494c04278866122964*
Theory Prop Address:	*TMJNiwboDvoErEPCNUWNtB2URCDsvp59127*

`cases_4`

Theorem 7.49 $\forall i \in 4.\forall p : \iota \to o.p\ 0 \to p\ 1 \to p\ 2 \to p\ 3 \to p\ i$. *The proposition is identified by the following information:*

Pure Prop Id:	*d983698cedfbfb0d50150666cddd3b822dedfadb18caeb579770c029f207cd45*
Pure Prop Address:	*TMa3a2ZsjVQYDT6yBAp8gzu9pDMZmvxMhFX*
Theory Prop Id:	*58ec175e1fd7d83946fa2cbdc271e4619d08449b2fd4b9aa206d25c5c0250722*
Theory Prop Address:	*TMSUJZnbtQsbxnZdQiLbj8KeXys9eLdtKmD*

`cases_5`

Theorem 7.50 $\forall i \in 5.\forall p : \iota \to o.p\ 0 \to p\ 1 \to p\ 2 \to p\ 3 \to p\ 4 \to p\ i$. *The proposition is identified by the following information:*

Pure Prop Id:	*f09c249f31e8eeee27f856932cdd5b636b782b2b3b2e32f82647235920772138*
Pure Prop Address:	*TMaNUYXuGQLR3vViJceHm29PgX2B1DvZRQi*
Theory Prop Id:	*f380534e00e1acdd7956770233333b4d2c419506f8ed1664966fbd5a79325ea4*
Theory Prop Address:	*TMbKWQtmy4GP7grvnKR87i4GqU3qf8y7RHj*

`cases_6`

Theorem 7.51 $\forall i \in 6.\forall p : \iota \to o.p\ 0 \to p\ 1 \to p\ 2 \to p\ 3 \to p\ 4 \to p\ 5 \to p\ i$. *The proposition is identified by the following information:*

Pure Prop Id:	*4dc95970bb9227a3f22f3850c0b8ae6a1fd851594d05dbd5ea64f678a4c05b04*
Pure Prop Address:	*TMTngRfM6nYyAGMJnk3QjkHDpxpz3pM2qJM*
Theory Prop Id:	*ed34a795973972d067788ff2e988cafe8a031b1f5916062b65c812cd42b40e21*
Theory Prop Address:	*TMJ5Xh7NbKFHd1vwDJmsHKLnMiEE6Lffx2L*

`neq_0_1`

Theorem 7.52 $0 \neq 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*9630fda1b35eebeb16a6330ac81eba7219b48d89e756733cef5f05db380572c9*
Pure Prop Address:	*TMby1mMrnJnG3eebgn4AYd1GyHbrhqpL1fH*
Theory Prop Id:	*74867a84ff3fe03d477cec3700c3c0504745b30ba4082bcf6867d3bfce7e06fd*
Theory Prop Address:	*TMW8jqbkDUF7dxtE4b7h7yVNVnwFchv7r4T*

`neq_0_2`

Theorem 7.53 $0 \neq 2$. *The proposition is identified by the following information:*

Pure Prop Id:	*05558045c651b5291665c412a39d06de2be8367ab59466b82289dd5533d9320e*
Pure Prop Address:	*TMZu44mT4xzQgx5223eLUCX58xAjU5Jf5QA*
Theory Prop Id:	*9c9a3f601515a2d7da3b8b00ab6b88f05b63c5fb44c173cd5e868be872f2892a*
Theory Prop Address:	*TMYNJEaA1HL5UDfSf43JCRXZeNbHuYw2YXp*

`neq_1_2`

Theorem 7.54 $1 \neq 2$. *The proposition is identified by the following information:*

Pure Prop Id:	*1d088a05070bdb1abb11db258e0e1a91308bf33b4bfd90fac2b4027e776ab0d7*
Pure Prop Address:	*TMGhpsGUJvMfvoyyDMhyLQnsaoSjoUHHex1*
Theory Prop Id:	*e5542eddbbedca6158ac40fec1d023b24b164c16667074aa41d1e2aa21991e23*
Theory Prop Address:	*TMWMpuvExT7y6Sv9W8iFya7qZnGC8mnKqbz*

`neq_1_0`

Theorem 7.55 $1 \neq 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*3a047842d8ee288fb6cf5794c28a9d57967ac47fed1474610f3d9471590139d4*
Pure Prop Address:	*TMMwBjpCmpUHnTqc64ZdF7on5Eq6WCgDoy8*
Theory Prop Id:	*952fed722f255a691a61f7c092a9f63980b2e4e12b60313b9cdac6dbfd0d38d4*
Theory Prop Address:	*TMVVgmiuwcrrYrrRVLykRqhZy2oz2EkZq5W*

`neq_2_0`

Theorem 7.56 $2 \neq 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*4d54d80a48e261479b57582099a83eaa414e72e5021d582b40a631d6440dcbd7*
Pure Prop Address:	*TMSj4zVM1S7jpaZ1AVkAMacHh7z2a7w1mih*
Theory Prop Id:	*00b357093a27461d60280a55739ba21a89b42f257242ae1a9fb792ab4984b5be*
Theory Prop Address:	*TMSyCWK4mwhWDmvJL9T59TwDeVDzmo5GCZW*

`neq_2_1`

Theorem 7.57 $2 \neq 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*6dd4c5f69b6b5a30c50cc8870ff9c94a49eb2257fc738c81c43eebd10b0b7fc8*
Pure Prop Address:	*TMHNLPWx722xRdb6Lf4hip5kXBPKr3JD1kp*
Theory Prop Id:	*2949ced3d070583d25d9e92478b3bb34672bfefe1c3018361515b55a230cbf19*
Theory Prop Address:	*TMJ4wBoghYfjCp1GwwmbywTP6TCGue5eUGD*

`neq_3_0`

Theorem 7.58 $3 \neq 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*f27effee2c2c94b3616a5e05a191639ff0e04b481a37195eb4c6a1044f82ac79*
Pure Prop Address:	*TMP1HRK27b3SUVqnCvdZEHQAsyFfrJj3ZFT*
Theory Prop Id:	*799b0593a2ee6a2b5ac8d4d9a2bc8d8251c90a17927eb218b54fa15559ab2a26*
Theory Prop Address:	*TMHR2rge7NMrthvfUFGcDrapFUGSLd3GMoR*

`neq_3_1`

Theorem 7.59 $3 \neq 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*7fa352ff4f2f05596173373889b1ff9973ab441dd35cbebda9739720ebb7f5be*
Pure Prop Address:	*TMVVi7vv3wSKb2C8z3cEZ4a9C551bwwE95s*
Theory Prop Id:	*8e4c7a0b39ad3b859373543800077739eb387a05d6213b9129540303af3968aa*
Theory Prop Address:	*TMdwMoiJi93xr8T9C61J9zFJibuUWHxu1oa*

`neq_3_2`

Theorem 7.60 $3 \neq 2$. *The proposition is identified by the following information:*

Pure Prop Id:	*90afb7a36efe45370d8283989fbac866a0e120e2b883d38ea2fe6fad1e2bbb4f*
Pure Prop Address:	*TMRJB22T27bFhKS9ThQj3eqAGQXbT11BEvn*
Theory Prop Id:	*4672768d6a3c4e4a1c04351ed841430bbf66bed9a742ca18ddb5c4ccfef3f93b*
Theory Prop Address:	*TMG3u3ZqBrijhNAwNj1kYTN99h2HYB63pTL*

`neq_4_0`

Theorem 7.61 $4 \neq 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*9bd122b0da951d1af4b4d4c32e237bc4422ce033e491090ee9806587dc9cafb3*
Pure Prop Address:	*TMKZ8uqsY7szUTPxqEmAn695ZoVVu1V891e*
Theory Prop Id:	*dbfe9c5bd8206b487aa4c1e0b115d0543cf6674122a0517ff19c410ec0cd3a44*
Theory Prop Address:	*TMJvPDtd46ULWYjgiP8gUuZNywq5ExcmHBc*

`neq_4_1`

Theorem 7.62 $4 \neq 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*021d04962f43e9033ed2121c6eeb4e5d5e7ebd24e7e42ee06cfef6191229d177*
Pure Prop Address:	*TMQffDs5XzPDL8shLUBqNhq3buem1wiUybw*
Theory Prop Id:	*bbd62921beb6f45383aa837f46086783634 7d2cde92052a3b2f54987f227e5e8*
Theory Prop Address:	*TMHipYaxxuowEA9gh3W7Mg9siWtCc1uwmCj*

neq_4_2

Theorem 7.63 $4 \neq 2$. *The proposition is identified by the following information:*

Pure Prop Id:	*7a3354868dab1b0643c846ebb21f339ebbb411d9da95783a5cd4b5d559f44204*
Pure Prop Address:	*TMRxE4YdWypoUuxdDA1hJAxrZPPnK1xbzyA*
Theory Prop Id:	*0a09eff12034956f5d3fb29bf25815b76d297d68220b9f03eeb417bd726f5609*
Theory Prop Address:	*TMJcWeMVmo9TXWzmFKFkPMhtzFbGJGJ6b8o*

neq_4_3

Theorem 7.64 $4 \neq 3$. *The proposition is identified by the following information:*

Pure Prop Id:	*fdb8798e8cacfe9c22e7a5300ab6ad312c527880e2d6114e93ee6909092b939e*
Pure Prop Address:	*TMWoPuE96Hes4hFCCiWYiKQFfnv8kusTH4Q*
Theory Prop Id:	*2a56945cd215c22b3db025b3a1115bea3f8be9f8cf7e86f135c9d62fe1b0d0ee*
Theory Prop Address:	*TMTJo37vnbVGCAZzhJgVrDjVt3X2aricxbz*

neq_5_0

Theorem 7.65 $5 \neq 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*bf9251e9bb7b6c3fcadb2f0da4a63d6b36fd8576e49f09a3abe2d7b832c72c36*
Pure Prop Address:	*TMEqk6oZYNwmG5wB1ZGKrZ4T3yWfbqHjRqV*
Theory Prop Id:	*29ce2701ae30ad2dbf565d3b4a8bf0d77c130da112d208cdb3edf1b1daf72161*
Theory Prop Address:	*TMFJq3wfqDg8x2vPXgbwmSzud3MUPwdSgcc*

neq_5_1

Theorem 7.66 $5 \neq 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*f97fb195b34610dd92dc1c9a46a8fe29d0b99faddd0ee0cb789d7ff153041aa4*
Pure Prop Address:	*TMLhHuF4WrVXhN5KWBBaWXPp9K7YsCP7XNf*
Theory Prop Id:	*cdb672d8c8dfd8a0236d3cfb650c10ae9a5ce01b958ebc60d0bba75bcd57ab5d*
Theory Prop Address:	*TMahxTsu3gQAQUaNifvwRtxDuvarKz9wQP3*

neq_5_2

Theorem 7.67 $5 \neq 2$. *The proposition is identified by the following information:*

Pure Prop Id:	*25bd00866c407af3c2b8f45d94058aae95d0bd9f06352dd9963408e6745e7f2a*
Pure Prop Address:	*TMLxeQNww6bks2wbWReMUo5Q5HCFEpkctEq*
Theory Prop Id:	*72066ec74ade8ba81c3aefe23451a21df4aca05a08491cc0af4ff4b1131a38f3*
Theory Prop Address:	*TMLSxacJKrd3N21R6mQxy5fSfYVEUaG9E8y*

`neq_5_3`

Theorem 7.68 $5 \neq 3$. *The proposition is identified by the following information:*

Pure Prop Id:	*3372df01e49c94e55fd317e47ba515344be9ed24947b63ae3b7f4648de435eda*
Pure Prop Address:	*TMPpVH6dFjx3FEaJ9sSmDs9wSvDKLg7DmMq*
Theory Prop Id:	*c26bb5b01832d8111735852c7f606f729b39acf9552a1df641404300808a241c*
Theory Prop Address:	*TML4qfcmrd4raBqNGGB7QyhUxV5XtrDK1Q4*

`neq_5_4`

Theorem 7.69 $5 \neq 4$. *The proposition is identified by the following information:*

Pure Prop Id:	*909c7c9d7cea9bcfbcb09a16340b2474d31cd36261d27ffef43b7102b4a4eb71*
Pure Prop Address:	*TMJHtW82A2WHP7ViQ391KAdZ1w1wRKs8HYL*
Theory Prop Id:	*20c3f2d381518f33179709cb5e92c3242267b3069264b9539368291bdf5c269b*
Theory Prop Address:	*TMX5VEFqEbmZM1jXnm6B1dHJJ3UZE1gT5QR*

`ZF_closed_I`

Theorem 7.70 $\forall U.\mathsf{Union\_closed}\ U \to \mathsf{Power\_closed}\ U \to \mathsf{Repl\_closed}\ U \to \mathsf{ZF\_closed}\ U$. *The proposition is identified by the following information:*

Pure Prop Id:	*f643b48ed401d2a1c3f6e0b2127cb23b125acf15c4cb21f77a1c45bb01c85847*
Pure Prop Address:	*TMJPXWCGGsgp2RauzJmcXDkupZ2Bwq8zM8g*
Theory Prop Id:	*6d69881b387db1470aab7b9513ee8d85714e5e34821b8dcdfbe77a49b4958281*
Theory Prop Address:	*TMbg2Nr5QH7Ui64ri9oF1hQu8esX9wKCtSG*

`ZF_closed_E`

Theorem 7.71

$$\forall U.\mathsf{ZF\_closed}\ U \to \forall p : o.$$
$$(\mathsf{Union\_closed}\ U \to \mathsf{Power\_closed}\ U \to \mathsf{Repl\_closed}\ U \to p) \to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*d795d416d63c658dd17353b60159cec522fd5799091016c00532a86a98184a9c*
Pure Prop Address:	*TMFJaZKQXV2BpbnNf3V5PpeaeGyXeZd9p8t*
Theory Prop Id:	*c78c790145214f0f70a0200b6fba47786c8f97e50408def3352d6b836a599b37*
Theory Prop Address:	*TMZSTib6nnUwYgZqRxuRTmGJX2JoBnPeAGv*

`ZF_Union_closed`

Theorem 7.72 $\forall U.\mathsf{ZF\_closed}\ U \to \forall X \in U.\mathsf{Union}\ X \in U$. *The proposition is identified by the following information:*

Pure Prop Id:	*a7cdf13e39c8049cf56acd30a1e889d54edc18ff38a1bbc2e1ed347d9682a5ad*
Pure Prop Address:	*TMUofvcpBARSdnFetm8dQrjhiWg4kWAGFB4*
Theory Prop Id:	*250496cf2e1875d7ec9219017e8d4508f9ca7718cbfd9c968c7bade397f00ea9*
Theory Prop Address:	*TMcGjahmQUapH4bqbAdGBQ6EgJgQDSjPLwm*

`ZF_Power_closed`

Theorem 7.73 $\forall U.\mathsf{ZF\_closed}\ U \to \forall X \in U.\mathsf{Power}\ X \in U$. *The proposition is identified by the following information:*

Pure Prop Id:	*5d781fd8e7019faff9a70275545ea6d69a0eb9adbe24e83a2e06143bffd06746*
Pure Prop Address:	*TMHz75Wp3yULtSkKHqZV4euzQkSQyKevgdT*
Theory Prop Id:	*7411dd8f49dfca8e20b3d4524ebb50b5fb3351ca8588eb34afde921cd3727004*
Theory Prop Address:	*TMFeLtgqxJCBDZ3U5JYcbg9M6iiuFekPu6M*

`ZF_Repl_closed`

Theorem 7.74

$$\forall U.\mathsf{ZF\_closed}\ U \to \forall X \in U.\forall F : \iota \to \iota.$$
$$(\forall x \in X.F\ x \in U) \to \{F\ x | x \in X\} \in U.$$

The proposition is identified by the following information:

Pure Prop Id:	*312268feb1dd5835c81ed94141bfe83ca9dda2b26ecf8263cee15af85f88b8b2*
Pure Prop Address:	*TMPP71S8C6hLepoQ5MX4wJnJZ5iqRp8wjHj*
Theory Prop Id:	*18f8acc434076d8590e9958a09b3287e80e677748bfa68846c9246019587ec5c*
Theory Prop Address:	*TMQuAnjb2usoLHA9bv3uUoCs1dSHiL1SFdi*

`ZF_UPair_closed`

Theorem 7.75 $\forall U.\mathsf{ZF\_closed}\ U \to \forall xy \in U.\{x, y\} \in U$. *The proposition is identified by the following information:*

Pure Prop Id:	*54256a260e0aeb219ec7b9c1f5470e1c879e31656131aab6f98d5191437dc819*
Pure Prop Address:	*TMJ8PnAXtGvJBQ2w7xRg2sQuiEx27bk436v*
Theory Prop Id:	*ef42d3cc50e9cba4432b33a79bbd1192bd239f4d284470ad502781b5fb35ca8e*
Theory Prop Address:	*TMcjrdqKzeo7g2gDjiEsVum3CAuDTSFiFuB*

`ZF_Sing_closed`

Theorem 7.76 $\forall U.\mathsf{ZF\_closed}\ U \to \forall x \in U.\{x\} \in U$. *The proposition is identified by the following information:*

Pure Prop Id:	*5d8a834f1cf303ccfa19193124cf05f94ba5eadb60ba9505bc698fae5beefe9f*
Pure Prop Address:	*TMRHMv7QRuD1cNMZ1DKFBfgHLajcAvhMXXa*
Theory Prop Id:	*d02d66f005b189993512ef90ced927d55db86cbffc2242c0a868958712bdead1*
Theory Prop Address:	*TMQJBe5GNPMz6xkyZp1New6CprvjNNWbVQB*

ZF_binunion_closed

Theorem 7.77 $\forall U.\mathsf{ZF\_closed}\ U \to \forall XY \in U.(X \cup Y) \in U.$ *The proposition is identified by the following information:*

Pure Prop Id:	*8a62402878aa34b70e4993833d0aac7cf60d7b31747f91d9f6bbb966b91e20e1*
Pure Prop Address:	*TMEqKpBN15S7iTeSq7zbuWutkNifE2ingyu*
Theory Prop Id:	*1bacfdb236f4aeb5ec63cb4b7c4d4bb0744883de7265fff6b8b706d8b410cece*
Theory Prop Address:	*TMdzxaBW52KHaW3S2NvEVtisq8WoKg6MZvp*

ZF_ordsucc_closed

Theorem 7.78 $\forall U.\mathsf{ZF\_closed}\ U \to \forall x \in U.\mathsf{ordsucc}\ x \in U.$ *The proposition is identified by the following information:*

Pure Prop Id:	*68a6ba11b0ad56c445b6e8bc81041f5c2cb0db9142f6b147cfcfa7ef1d2f27f3*
Pure Prop Address:	*TMH2dwKRQudFN9m3qx3RNdaBLvqypKizZv5*
Theory Prop Id:	*946e6124c0e8be96a15baa61fedb45d36338fafc05a2f31ce54a95174700bf8f*
Theory Prop Address:	*TMPqDp5W4hUggbHVsbPDuU2ytBMzDWrbM2M*

nat_p_UnivOf_Empty

Theorem 7.79 $\forall n : \iota.\mathsf{nat\_p}\ n \to n \in \mathsf{UnivOf\ Empty}.$ *The proposition is identified by the following information:*

Pure Prop Id:	*c885b475dae13164eaf466c05904664c48aa5e96b64990036d6bbd7ab2b3b01e3*
Pure Prop Address:	*TMTM67QGgM77h8RbmGoR4QhuHU14bxuur8y*
Theory Prop Id:	*645aee297f02d0a7caa06a5efea238cec06af7df69b2643d2adc7c5d9b418911*
Theory Prop Address:	*TMZEUzvGsZE4EoguW8R1ovkAWzhDE4znAr3*

Definition 7.3 omega *is the opaque object of type* ι *identified by the following information:*

Pure Object Id:	*6fc30ac8f2153537e397b9ff2d9c981f80c151a73f96cf9d56ae2ee27dfd1eb2*
Pure Object Address:	*TMb4KgMPXEQBvZWN5yMtrjHJCTxAvdStvip*
Theory Object Id:	*ad9ad81c42d92fd52433645cf2466f4f0179ab10ee124be71bb12991deb2ddf0*
Theory Object Address:	*TMNkXuKe4mYMmqP6e4nkiPeWyEzHKwZy2zC*

omega_nat_p

Theorem 7.80 $\forall n \in \mathsf{omega}.\mathsf{nat\_p}\ n.$ *The proposition is identified by the following information:*

Pure Prop Id:	*f44a309f3fadd69adfefbc11bb8cd5a0e2d204f0f0ccde446b0cf927293e46dd*
Pure Prop Address:	*TMQLBjRkNsbEFRVrBkuh1bmZQqCeUscTWR1*
Theory Prop Id:	*c2c61b35088d7afd198e0d398dabb21f1be9e52d6c73d798f0fa57761801eaa0*
Theory Prop Address:	*TMcBbUsnDN99QLS1dM6hhdDj1W8pvPaMBBH*

`nat_p_omega`

Theorem 7.81 $\forall n : \iota.\mathsf{nat\_p}\ n \to n \in \mathsf{omega}$. *The proposition is identified by the following information:*

Pure Prop Id:	*6a1b445f977fd3fae4e1e1cc46011455e1f0531f1a15c5c25278556928dcee83*
Pure Prop Address:	*TMd8YkcuN2F2evUjAbU6Ts5vNLwKCBES7rW*
Theory Prop Id:	*d5080f911ac6cf73280f9bbcf0edc2218ca24806bdc57695210e6115b9406c38*
Theory Prop Address:	*TMWv1SjyTw4FBL65LJ4Dydky2jqrjzFo4WF*

`omega_ordsucc`

Theorem 7.82 $\forall n \in \mathsf{omega}.\mathsf{ordsucc}\ n \in \mathsf{omega}$. *The proposition is identified by the following information:*

Pure Prop Id:	*246d636d93c78b4e3365422a679abe57556849f5546b6f8e8fbaeeab0e834905*
Pure Prop Address:	*TMYTp6SmJ6Xb74ZB58T5drE74MrfSMJ56Xd*
Theory Prop Id:	*17fb646a22b11762cb7ae21fb49306beac81729c7c3e89992f09e10eff2d8f9e*
Theory Prop Address:	*TMcKRUvCqbX8WtX4XYQZ7VoDqsewDsQmCWB*

Chapter 8

Ordinals

Definition 8.1 *We define* ordinal *to be* $\lambda\alpha : \iota.\mathsf{TransSet}\ \alpha \wedge \forall\beta \in \alpha.\mathsf{TransSet}\ \beta$ *of type* $\iota \to o$ *identified by the following information:*

Pure Object Id:	*ee28d96500ca4c012f9306ed26fad3b20524e7a89f9ac3210c88be4e6a7aed23*
Pure Object Address:	*TMMUuqYqPdrVu6cDtXgARhaXmQ84ZrMXG6P*
Theory Object Id:	*53ee8f616e489e1965c33226035b0df42058ce50fbf01e6172cdf3b427f5a693*
Theory Object Address:	*TMJ4CnbKugQJ97k3pfeF56M78Ug4AubAWjZ*

`ordinal_TransSet`

Theorem 8.1 $\forall\alpha : \iota.\mathsf{ordinal}\ \alpha \to \mathsf{TransSet}\ \alpha$. *The proposition is identified by the following information:*

Pure Prop Id:	*0dbaa3d497ea2fe357d7cdf6de2639b3f4cf815ee1d613eaddc6fb305b46de56*
Pure Prop Address:	*TMdzYsAgm9hgqSggzyEMmMw5w6aRH1fmS1D*
Theory Prop Id:	*c9e708b22b5dc60c895eed22a527c202e52f14066b2f95ce8df1cb630eb1df7e*
Theory Prop Address:	*TMXHSD7yBJJjNhye2R2DtR5uPTVLTRGPMVo*

`ordinal_In_TransSet`

Theorem 8.2 $\forall\alpha : \iota.\mathsf{ordinal}\ \alpha \to \forall\beta \in \alpha.\mathsf{TransSet}\ \beta$. *The proposition is identified by the following information:*

Pure Prop Id:	*f4e77fe3f77ebff8ff08239d5f4175451624803d11b0018e85f316c34806264c*
Pure Prop Address:	*TMYKCKb2mgJ361o53ZDmr4bqCotaiAa9UMT*
Theory Prop Id:	*cc1f55fe1800e237a0d36d0f295ac28bf748b236473d1e1f9caf5515be62a995*
Theory Prop Address:	*TMXH96JbZ5RRa3chjsLwakm9MEVpiN5ygZV*

`ordinal_Empty`

Theorem 8.3 ordinal Empty. *The proposition is identified by the following information:*

Pure Prop Id:	*ce6bea11043bbd03d9af274f7b824c53be0840dd9f324cd7c9afc0d4f811b316*
Pure Prop Address:	*TMHjHTkWmLerJqJCyfCzQ2x3bAbjxupX5rN*
Theory Prop Id:	*7d95d14b163c311c6050bb25900b2f0e05a1fb83c01aacb93a1b6cf03a658823*
Theory Prop Address:	*TMcRLhPoJiCW9i4Gg3Z8GVhyHi3DGQmpBpx*

ordinal_Hered

Theorem 8.4 $\forall \alpha : \iota.\mathsf{ordinal}\ \alpha \to \forall \beta \in \alpha.\mathsf{ordinal}\ \beta$. *The proposition is identified by the following information:*

Pure Prop Id:	*a7e78b55af56fd38debe61d4cf2ccd6512ff509a33690ff7bcb062d67233a307*
Pure Prop Address:	*TMYATDhQjyWwu2denmfYctz9jzc5T6pvxDY*
Theory Prop Id:	*43b0f3ddbec2adab900eae39d5c70d5f8e32937abf8669f79ab503a2cba3c688*
Theory Prop Address:	*TMNMsaVbc4QJAjitEJmAT3A6577bnGUMenX*

TransSet_ordsucc

Theorem 8.5 $\forall X : \iota.\mathsf{TransSet}\ X \to \mathsf{TransSet}\ (\mathsf{ordsucc}\ X)$. *The proposition is identified by the following information:*

Pure Prop Id:	*fc215771da05b7091bde93e8fd1dbf94168ddb5e1c68f7934a03aeb58f1ebd07*
Pure Prop Address:	*TMds6RBqX4EBdbVb2jzpQF4BGu62TbmQW6S*
Theory Prop Id:	*ebf9e34c8261f406aaf9188564b534bc4eee97dba2e5fafb75b1f972cbc9d365*
Theory Prop Address:	*TMTZpuHkdymxGmLoep6QY3LRhQf6Nr8Myhy*

ordinal_ordsucc

Theorem 8.6 $\forall \alpha : \iota.\mathsf{ordinal}\ \alpha \to \mathsf{ordinal}\ (\mathsf{ordsucc}\ \alpha)$. *The proposition is identified by the following information:*

Pure Prop Id:	*0ba97fa745a108db180034344887cefa8fa42aa10e94eb93ee5f6ba8a383902a*
Pure Prop Address:	*TMNpPkbjLapq9BtN9phPiFFNbRzuB8jfTRk*
Theory Prop Id:	*ab0f7f2be575c7979819df5a27b6d7276069f61fd5ec603b15b317dca63b3e76*
Theory Prop Address:	*TMbVvz6WT788KucEJn9rQE17f9WVYDdtwv7*

nat_p_ordinal

Theorem 8.7 $\forall n : \iota.\mathsf{nat\_p}\ n \to \mathsf{ordinal}\ n$. *The proposition is identified by the following information:*

Pure Prop Id:	*274fc409055bdbb382e1defb45f54a49e7f797ab883d97e76722ddad8971f35d*
Pure Prop Address:	*TMKFEu7wFJqZoMviwrZLpLaCP1QjLcwtCwk*
Theory Prop Id:	*47cd49a599f17ace658f9f1d538502f51cde62c8cd2d997a51ce938eecd15d70*
Theory Prop Address:	*TMR9b86LuWgjhZL4m5yEvQhHdYU1YcYRkn3*

ordinal_1

Theorem 8.8 $\mathsf{ordinal}\ 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*c94d41f958c74e02ac9b04ef1997708d28df66c7d56e3a98177bfa3ef526a409*
Pure Prop Address:	*TMT3Q25U1Ux1eVUUm1s1FeitpAvwbXTk1dm*
Theory Prop Id:	*e325a502344d5146d6b51df0408cd431c99b978bd785e7aebd69d0cbd1d3630a*
Theory Prop Address:	*TMLMkFbUXyMVHafQwFnHAFJwHJiUL8uizu8*

`ordinal_2`

Theorem 8.9 ordinal 2. *The proposition is identified by the following information:*

Pure Prop Id:	*a570756c9f8d8069da7f0920bf81bd27397ddff102f6b7444e080f7bed3390ff*
Pure Prop Address:	*TMFLbGodmA7UJer5TihfdxoFBgHSxZjK3cY*
Theory Prop Id:	*e64a0df795b030c5c40d39ceceb101d04488a3dbf090fae286e8f8e284d9727b*
Theory Prop Address:	*TMEjrJXdnsbDexBJZRJysy6hDoZye5ffvEX*

`omega_TransSet`

Theorem 8.10 TransSet omega. *The proposition is identified by the following information:*

Pure Prop Id:	*8e89e66d9788201a152dc0d2e564bf2e428e165c4c240da4dd210f8f102b0b9b*
Pure Prop Address:	*TMNtK2hop4HkuAGDS35C3HQG1M44NKe3USF*
Theory Prop Id:	*c79e752f422afbb22e542dfbd0d6ee1f417aa6fe06a04b74efbecce1e0b036e0*
Theory Prop Address:	*TMKTjT9CEwmv6ZQwdPQWnn1MRx5wxzNZP5R*

`omega_ordinal`

Theorem 8.11 ordinal omega. *The proposition is identified by the following information:*

Pure Prop Id:	*3d621a0d65590ab57c57a1848257b5fa064c0e3a4847bb476e17a549b2bb1179*
Pure Prop Address:	*TMEhcpuAFpHXHFSKrZU9N7ytoAc1KYSKfwJ*
Theory Prop Id:	*177484ec90afd915b52f7e111ce39d29114b2d0f97fd75908b5de1e8b5a405b4*
Theory Prop Address:	*TMdVAnxzLqF7mrAcYCABmsfnfihincw4N1F*

`ordsucc_omega_ordinal`

Theorem 8.12 ordinal (ordsucc omega). *The proposition is identified by the following information:*

Pure Prop Id:	*66659c006c982a0ac99ea4ff01a85f6f4ff5c6737431c0032276450048e05cdf*
Pure Prop Address:	*TMPkd9dTQz5vTvQaDf2GPn1vNYBQR6FCxTM*
Theory Prop Id:	*ecfad7b774257cdf70613958fe3477637a520ac37c9c970c486229116833e51e*
Theory Prop Address:	*TMF6YnnSCpzaZf1K8mSGfwFbrQh9ZW2n7Y9*

`TransSet_ordsucc_In_Subq`

Theorem 8.13 $\forall X : \iota.\mathsf{TransSet}\ X \to \forall x \in X.\mathsf{ordsucc}\ x \subseteq X$. *The proposition is identified by the following information:*

Pure Prop Id: d5d88203bb9e4f58e19b53c1e5a62f15aedc32de8c5bc76e2580367c4174de4d
Pure Prop Address: TMLQBckdoSFHRuVFwGbGbuRrp1fdK4R87ov
Theory Prop Id: 3c2e4593e300f67b32ba028952543ba0dd7e523521087467b0174ba873c292af
Theory Prop Address: TMTRBiP8XXTzUuthK3uDUFLV6ZLsiJDuXee

`ordinal_ordsucc_In_Subq`

Theorem 8.14 $\forall\alpha.\mathsf{ordinal}\ \alpha \to \forall\beta \in \alpha.\mathsf{ordsucc}\ \beta \subseteq \alpha$. *The proposition is identified by the following information:*

Pure Prop Id: 57345e9c93f2215aa2812d32af6e5102ec8faa4224750fe0e5a65c9f1afcd5ff
Pure Prop Address: TMQpZgVSt9a4G3EQn5TNVzeb9wssdHKNW7u
Theory Prop Id: b00304a76db0c730b04afb1cf7ab4e339ad984c2397a5aa9dfcbe8b7ff5fbb77
Theory Prop Address: TMaxmykHWjhaXzn1rWbPuqhxTLTTER9wNAp

`ordinal_trichotomy_or`

Theorem 8.15 $\forall\alpha\beta:\iota.\mathsf{ordinal}\ \alpha \to \mathsf{ordinal}\ \beta \to \alpha \in \beta \vee \alpha = \beta \vee \beta \in \alpha$. *The proposition is identified by the following information:*

Pure Prop Id: 078a34fdf9fecc3740d32b870c8d09ccc0128f7642e5b75098ade0d1089ece30
Pure Prop Address: TMFhw4k9MkF4Xj6fGtXNVWovjCfNEDFgiCV
Theory Prop Id: 21f2d68bdab52d51a33741771dbbe589a47a82c6a747d3f27f2c6e7e7fb46323
Theory Prop Address: TMQiHm6s7fQpsV7SnBFtLZFvEeXFV9nDbmm

`ordinal_In_Or_Subq`

Theorem 8.16 $\forall\alpha\beta.\mathsf{ordinal}\ \alpha \to \mathsf{ordinal}\ \beta \to \alpha \in \beta \vee \beta \subseteq \alpha$. *The proposition is identified by the following information:*

Pure Prop Id: 70fe0df6b00a9a1cdc3d97c6a8ed1db39034d341a16f8a30bddebf8a0a8b8e4f
Pure Prop Address: TMU41PmuGiADNxp6RQqwepi4ZBaznh8t8Dy
Theory Prop Id: 7b99b0def95577474ca6a6371ba472b8ef499bf320c511f3b539db9524a87727
Theory Prop Address: TMUy7ee37ThdPDzbRspVTZUkthgdjtEyQYb

`ordinal_linear`

Theorem 8.17 $\forall\alpha\beta.\mathsf{ordinal}\ \alpha \to \mathsf{ordinal}\ \beta \to \alpha \subseteq \beta \vee \beta \subseteq \alpha$. *The proposition is identified by the following information:*

Pure Prop Id: 3f30a55f5c77831d749f6727a93c5a8bce525eb93957884747af3b16fd2817dc
Pure Prop Address: TMPY4A7R7aYy2invvHTTAywc1usxqJAm5u8
Theory Prop Id: 3b1fce49dbbdca0b438f6709b0e0a85233faa824682dcf5ed3f74df3c3bb6ea0
Theory Prop Address: TMM2dfXndDyhGP7QGpLd2hvEgv48peHsWcx

`ordinal_ordsucc_In_eq`

Theorem 8.18 $\forall\alpha\beta.\mathsf{ordinal}\ \alpha \to \beta \in \alpha \to \mathsf{ordsucc}\ \beta \in \alpha \vee \alpha = \mathsf{ordsucc}\ \beta$. *The proposition is identified by the following information:*

Pure Prop Id: *500378eb85429ae7b33a80121ead8cda752628caffbde51467158317d0e18d74*
Pure Prop Address: *TMHGn9G1ExDEjiH1MPhwk1Vyodkz876yMRi*
Theory Prop Id: *0c19e771defd6f69e7d104285ac72cf1f7b2cf9c9100d4849916879cb6681814*
Theory Prop Address: *TMVF297QZUVVpWRs4i96ncxvbVNynLTkL7q*

ordinal_lim_or_succ

Theorem 8.19 $\forall\alpha.\mathsf{ordinal}\ \alpha \to (\forall\beta \in \alpha.\mathsf{ordsucc}\ \beta \in \alpha) \vee (\exists\beta \in \alpha.\alpha = \mathsf{ordsucc}\ \beta)$. *The proposition is identified by the following information:*

Pure Prop Id: *4450c013030397143dd5b0c0a5fc68ccac288751c8cd230b4f4abcdeec1c5c6d*
Pure Prop Address: *TMankcEMnZXFAP8cvcfDqwi1HBKu5b8txfW*
Theory Prop Id: *a12697d237d0a645eb8cbf1e70cd0b6d46292c225e8a36a42a59dbe9350e27f6*
Theory Prop Address: *TMLRGTTc9BqSSZNBMbxF34StFhkCfsPU8B7*

ordinal_ordsucc_In

Theorem 8.20 $\forall\alpha.\mathsf{ordinal}\ \alpha \to \forall\beta \in \alpha.\mathsf{ordsucc}\ \beta \in \mathsf{ordsucc}\ \alpha$. *The proposition is identified by the following information:*

Pure Prop Id: *f4b1ed473dc1d1af402cb8a1a56b16920553d37b8b365cf4e72a74e2467668f7*
Pure Prop Address: *TMZQdYrrW7LPMeUbwQJGaSaWYZJ342j6TLR*
Theory Prop Id: *8ab1dc0a15cde7e11ff2fda7e5194042b72dc495121dbe98a3cb9d5f23df0650*
Theory Prop Address: *TMF2GA2skuyQZS3EhFZYEwjYb1i5V7CiuYY*

ordinal_Union

Theorem 8.21 $\forall X.(\forall x \in X.\mathsf{ordinal}\ x) \to \mathsf{ordinal}\ (\mathsf{Union}\ X)$. *The proposition is identified by the following information:*

Pure Prop Id: *c733ee1a1d9fd2449e70663578b0382c3bccae896c32e0ec362f1494fda06c3e*
Pure Prop Address: *TMW1Z5hkuor8TPuLNVR1zbiXXaoiAEs63WD*
Theory Prop Id: *8f2519b0a9c7cdd7d42c73a6733e14191e924dccd043f97339e7ea377d746f2b*
Theory Prop Address: *TMJhDobuvDdYqLTuiL9wE7gzst4xAaHHwqt*

ordinal_famunion

Theorem 8.22 $\forall X.\forall F : \iota \to \iota.(\forall x \in X.\mathsf{ordinal}\ (F\ x)) \to \mathsf{ordinal}\ (\bigcup_{x\in X} F\ x)$. *The proposition is identified by the following information:*

Pure Prop Id: *fc035982e0d1f528e70c7a7b191727c9d3afc3eb8cde8f6cf06880f52db2e3d4*
Pure Prop Address: *TMU1RzbEJzVnikGUeRHDEeJKZdU2gv4Lu9U*
Theory Prop Id: *b3f1e1ecfee50fc6903bc622a5d3170a7e8f834e8b8e4cd173ce0c699d33d141*
Theory Prop Address: *TMNBvDL5kWic9wxjPXEkTh7WAf3f2AnJ1NW*

ordinal_binintersect

Theorem 8.23 $\forall\alpha\beta.\mathsf{ordinal}\ \alpha \to \mathsf{ordinal}\ \beta \to \mathsf{ordinal}\ (\alpha \cap \beta)$. *The proposition is identified by the following information:*

Pure Prop Id:	*43d507cd3ab7cd23861b87052726f88b7930ceecde12dab179ead328e6a2cd6*
Pure Prop Address:	*TMGAFLa8mQYJ6L1kp1WKCNwaY3YrMwPqad2*
Theory Prop Id:	*0253282876aaa86e7cea3ea89459e20513307e9d2b9ffe1cc2ba50b357bbb4a9*
Theory Prop Address:	*TMQLySje1TXDy7YnSUR855qqzweodR4BBbP*

`ordinal_binunion`

Theorem 8.24 $\forall \alpha\beta.\mathsf{ordinal}\ \alpha \to \mathsf{ordinal}\ \beta \to \mathsf{ordinal}\ (\alpha \cup \beta)$. *The proposition is identified by the following information:*

Pure Prop Id:	*861ae3bbc499b574608e38df76236c43f8ff5f552de1d70b9002feb31d7edc87*
Pure Prop Address:	*TMWj1DiEDHWbFoRK4tceGziMDSzNMjXvATC*
Theory Prop Id:	*b24989c4f74b8369768af389e0867d9ae88d987aae3abe7414441666572cf91a*
Theory Prop Address:	*TMb8JjtU8NrQYjHW2dN4KBHmc6dZLD8tX2H*

`ordinal_Sep`

Theorem 8.25 $\forall \alpha.\mathsf{ordinal}\ \alpha \to \forall p : \iota \to o.(\forall \beta \in \alpha.\forall \gamma \in \beta.p\ \beta \to p\ \gamma) \to \mathsf{ordinal}\ \{\beta \in \alpha | p\ \beta\}$ *The proposition is identified by the following information:*

Pure Prop Id:	*9c0d4265c1c0802cb20047122fd9d5c097bbab37a60f64d593b6a44e1bd26859*
Pure Prop Address:	*TMMc3fJbFMyUS6zszTmv9rSMmYJS2mMJVFh*
Theory Prop Id:	*b5f69f338e4258431a3a6159a38976e853cda2445044d06e4ed0a660100c1a65*
Theory Prop Address:	*TMHv1MoXTBWSqnddBfw56ZCoB3HkEABQP52*

Chapter 9

Comparing the Sizes of Sets

Definition 9.1 *We define* inj *to be* $\lambda XYf.(\forall u \in X.f\ u \in Y) \land (\forall uv \in X.f\ u = f\ v \to u = v)$ *of type* $\iota \to \iota \to (\iota \to \iota) \to o$ *identified by the following information:*

Pure Object Id:	*264b04a8ece7fd74ec4abb7dd2111104a2e6cde7f392363afe4acca8d5faa416*
Pure Object Address:	*TMQsS5cocdJLxvSrfwFrtyVb2HtH4sHnoSq*
Theory Object Id:	*442958a234e56cda2843c516d8ba0be921c19f4486af20ea18cd78e0b5587a8e*
Theory Object Address:	*TMEtNCh6NfJv35kUGcgFB52zooWEchr4mq3*

Definition 9.2 *We define* surj *to be* $\lambda XYf.(\forall u \in X.f\ u \in Y) \land (\forall w \in Y.\exists u \in X.f\ u = w)$ *of type* $\iota \to \iota \to (\iota \to \iota) \to o$ *identified by the following information:*

Pure Object Id:	*4b68e472ec71f7c8cd275af0606dbbc2f78778d7cbcc9491f000ebf01bcd894c*
Pure Object Address:	*TMcR6n2BpcwoiDwRhTtfhBS2wRexeumPoBF*
Theory Object Id:	*f022f92791e019fecb8ae163b387436a5af8f1fc86e8403db0d2add2d326d2cc*
Theory Object Address:	*TMSbXiP7EjGYNy1Jeo9r2YG6YXA8qunsPXV*

Definition 9.3 *We define* bij *to be*

$$\begin{gathered}\lambda XYf.(\forall u \in X.f\ u \in Y)\\ \land(\forall uv \in X.f\ u = f\ v \to u = v)\\ \land(\forall w \in Y.\exists u \in X.f\ u = w)\end{gathered}$$

of type $\iota \to \iota \to (\iota \to \iota) \to o$ *identified by the following information:*

Pure Object Id:	*76bef6a46d22f680befbd3f5ca179f517fb6d2798abc5cd2ebbbc8753d137948*
Pure Object Address:	*TMWBv3fDiJHG6PbQ5p9tmBynBsCvJmYFGXN*
Theory Object Id:	*f7d93b7e704e3215ae66ae7c330f283bc8b4b71850013429da3d75d7ab6ee432*
Theory Object Address:	*TMWitYCyjoeiXGg1451hhTG6x1DmSshpEsw*

`bijI`

Theorem 9.1

$$\forall XY.\forall f:\iota\to\iota.(\forall u\in X.f\ u\in Y)\to(\forall uv\in X.f\ u=f\ v\to u=v)\\ \to(\forall w\in Y.\exists u\in X.f\ u=w)\to\mathsf{bij}\ X\ Y\ f.$$

The proposition is identified by the following information:

Pure Prop Id:	*a9064fdc226bed9225585cb4f2610b131c0a14aad760bfeb55112ab19f50c269*
Pure Prop Address:	*TMVtA5kA7XycdNhnjcWoD7YH6yuuhkFVjMt*
Theory Prop Id:	*3229f986ac1acb74b9c25702305e679c33fee187075ba5e7e5521402a44407f4*
Theory Prop Address:	*TMHZ6dKwiqgD43Q9CKokQdMMtyUWTqNe73b*

`bijE`

Theorem 9.2

$$\forall XY.\forall f:\iota\to\iota.\mathsf{bij}\ X\ Y\ f\to\forall p:o.\\ ((\forall u\in X.f\ u\in Y)\to(\forall uv\in X.f\ u=f\ v\to u=v)\to(\forall w\in Y.\exists u\in X.f\ u=w)\to p)\\ \to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*2cf2785657b7836bb8cf1988ba517c500841f4597e31341232a8e4ae9a2ac11b*
Pure Prop Address:	*TMPhihHJHHFvCn9A4dYL5TQ6pqHpkEvceX8*
Theory Prop Id:	*cf075922dfef18d341a5a232bb48285fa39e11a63b966fd9557f7bbc707557bc*
Theory Prop Address:	*TMVhokQU6h9PfVvoAJ7Se6F6yDdFRx38gX3*

Definition 9.4 inv *is the opaque object of type* $\iota\to(\iota\to\iota)\to\iota\to\iota$ *identified by the following information:*

Pure Object Id:	*896fa967e973688effc566a01c68f328848acd8b37e857ad23e133fdd4a50463*
Pure Object Address:	*TMYsHRgQQXjN5d3YZ2sLdp9j73Hd4HiYRei*
Theory Object Id:	*f3a0156f444b33934c3c28ed4b3f22eae35078c37c1e0c9f4f931d3c1e1c9266*
Theory Object Address:	*TMNXEs7Lfg3PCmc11RhANqjcTXio2UJSNhw*

`surj_rinv`

Theorem 9.3

$$\forall XY.\forall f:\iota\to\iota.(\forall w\in Y.\exists u\in X.f\ u=w)\to\forall y\in Y.\mathsf{inv}\ X\ f\ y\in X\wedge f\ (\mathsf{inv}\ X\ f\ y)=y.$$

The proposition is identified by the following information:

Pure Prop Id:	*ebf3718d5c90ac32637e5ff7de771588bd899f20728aac1c034cc4575a9f2bd5*
Pure Prop Address:	*TMdfPpzpdtB7VwgaPueTNcMshHsVs117kuo*
Theory Prop Id:	*6489d2e57f0dee884152f4a33ad50d245f1c35dc285ca99ba09b67a783f89c93*
Theory Prop Address:	*TMVFndmdYppMYsHoVKPEwfE1yfNTP6HYsK6*

`inj_linv`

Theorem 9.4

$$\forall XY.\forall f : \iota \to \iota.(\forall uv \in X.f\ u = f\ v \to u = v) \to \forall x \in X.\mathsf{inv}\ X\ f\ (f\ x) = x.$$

The proposition is identified by the following information:

Pure Prop Id:	*f9fc2fb6cfb4eb217cfd90f255fb35b64bd1729aa515ae6c662014a80909baf4*
Pure Prop Address:	*TMEuKQvRXDUoH8sLbTA8MpzpoVNt2Uo41RH*
Theory Prop Id:	*74870e1df65b0cc2ae2c71c585c9735524f602fc82ed42ac4a046496a20db8bd*
Theory Prop Address:	*TMTFjNxnxSQzmggpMN48iCzGFGMZ6u5HUUU*

`bij_inv`

Theorem 9.5 $\forall XY.\forall f : \iota \to \iota.\mathsf{bij}\ X\ Y\ f \to \mathsf{bij}\ Y\ X\ (\mathsf{inv}\ X\ f)$. *The proposition is identified by the following information:*

Pure Prop Id:	*ff1332269c5993981bc337566181eb6a5ac760b35f3e8f90eed0abf70ebc7c17*
Pure Prop Address:	*TMRvnWitWbimHQFYgNtor4w8xmFZq6WVJT5*
Theory Prop Id:	*69c53767bc4248c32eca1d3c840bc6ee0513e90488a4e3d38dba20033986cce4*
Theory Prop Address:	*TMYU31Q8vC6ff6UyuWLEMF43Pms8CHUmb2x*

`bij_comp`

Theorem 9.6 $\forall XYZ : \iota.\forall fg : \iota \to \iota.\mathsf{bij}\ X\ Y\ f \to \mathsf{bij}\ Y\ Z\ g \to \mathsf{bij}\ X\ Z\ (\lambda x.g\ (f\ x))$. *The proposition is identified by the following information:*

Pure Prop Id:	*abf85117d4cc5830f8c4e2302319f45ffb293b1c89044d95312722edcc59ebda*
Pure Prop Address:	*TMbJ4RHULgz1EDJb4kgryxvfAijajbPaBWZ*
Theory Prop Id:	*4c90458f330b4d7cdc780aefbd2a209237607b7b0ce813a26728e290eaf8ae8e*
Theory Prop Address:	*TMMeMmBNYFCGY7zJTLg49As2Zm5US2hziHm*

`bij_id`

Theorem 9.7 $\forall X.\mathsf{bij}\ X\ X\ (\lambda x.x)$. *The proposition is identified by the following information:*

Pure Prop Id:	*a8cefb4071a07310e8c514fb13ecd4fb64481818a12472770c065383dc391c08*
Pure Prop Address:	*TMKDAnvGLshjhnT7DcuJYqVH8EphuBsUUcs*
Theory Prop Id:	*707f370beec6bb62870cc94c9fa8f45ced6eaaf202ba0af81dab213981d7b6a9*
Theory Prop Address:	*TMFbyfnRF5QWxotLuNUgJzCGP2XV6QZ4y1c*

`bij_inj`

Theorem 9.8 $\forall XY.\forall f : \iota \to \iota.\mathsf{bij}\ X\ Y\ f \to \mathsf{inj}\ X\ Y\ f$. *The proposition is identified by the following information:*

Pure Prop Id:	*e3635e28871703e7f8247d1db9057e8a348d2ccd04747dd154febb02a37f9401*
Pure Prop Address:	*TMMrgAbiSR6cnvcK7CVfGRP9kNu67vN98ZP*
Theory Prop Id:	*00097cfdcf04769f3acdf746b6155be23b2782a6c12e002b03fecfc4ed998644*
Theory Prop Address:	*TMRRNL1K8TWBDtgrjqzbQBsxYHZsqGCVu11*

bij_surj

Theorem 9.9 $\forall XY.\forall f:\iota\to\iota.\mathsf{bij}\ X\ Y\ f\to\mathsf{surj}\ X\ Y\ f$. *The proposition is identified by the following information:*

Pure Prop Id:	*e7a049991e8ce73cb9d2cde3714035b3bfd567ffd29a6588380d6afe28d0d422*
Pure Prop Address:	*TMZ1Df2XJMAY4VQtRKyVeYTrjgp9uNrkat1*
Theory Prop Id:	*7fa9a5a19214ed9b46b7c2672858b7198b12a117652c2884ba242b0d3023bb78*
Theory Prop Address:	*TMK69KkYqvqjbC3pgQQ4xrkBTDfGpX9dhLN*

inj_surj_bij

Theorem 9.10 $\forall XY.\forall f:\iota\to\iota.\mathsf{inj}\ X\ Y\ f\to\mathsf{surj}\ X\ Y\ f\to\mathsf{bij}\ X\ Y\ f$. *The proposition is identified by the following information:*

Pure Prop Id:	*ccff0258f3d7f02dbb9c92e1c2c588c500749a613923b0ece6497a2266072218*
Pure Prop Address:	*TMZSLNi3XSR9xpWQPVMJxKdbpuqMGcXGcTa*
Theory Prop Id:	*de895fd8c8d353a1a3e58226dea93e0bcab39207eaea6f1bd63ca76728da5443*
Theory Prop Address:	*TMKfqwemEJSEU94d4EWk9R5Z9o4KFevVF79*

surj_inv_inj

Theorem 9.11 $\forall XY.\forall f:\iota\to\iota.(\forall y\in Y.\exists x\in X.f\ x=y)\to\mathsf{inj}\ Y\ X\ (\mathsf{inv}\ X\ f)$. *The proposition is identified by the following information:*

Pure Prop Id:	*ea4775ce7b91dca8fcab0b476c6da12b3812e7c3bb1c40ac217ef8ec4f7e2421*
Pure Prop Address:	*TMMfpEoFj9egg8ymZiXFBa7mqr6V1aAyR52*
Theory Prop Id:	*dde3b90d9180ac2126f48d3f539c000ff8f515f94358c02d782de14599dfd8e8*
Theory Prop Address:	*TMYyJkfwSTRDidng9k7W8zBoQ7M964PoQu8*

Definition 9.5 *We define* atleastp *to be* $\lambda XY:\iota.\exists f:\iota\to\iota.\mathsf{inj}\ X\ Y\ f$ *of type* $\iota\to\iota\to o$ *identified by the following information:*

Pure Object Id:	*9bb9c2b375cb29534fc7413011613fb9ae940d1603e3c4bebd1b23c164c0c6f7*
Pure Object Address:	*TMdn3jbiRnktYr3yHQk9h3fe2hNUWwmbigq*
Theory Object Id:	*13545c645a339da15781ed2b2b387176e9241a5497e95448eb000efd67f60842*
Theory Object Address:	*TMVULsv3UtPX7zQrujvLPr8j5YcesgYNBcQ*

Definition 9.6 *We define* equip *to be* $\lambda XY:\iota.\exists f:\iota\to\iota.\mathsf{bij}\ X\ Y\ f$ *of type* $\iota\to\iota\to o$ *identified by the following information:*

Pure Object Id:	*eb44199255e899126f4cd0bbf8cf2f5222a90aa4da547822cd6d81c671587877*
Pure Object Address:	*TMQwX3qjReYJVk5GFFSX3Mc3FXLfVZVK9kz*
Theory Object Id:	*7f2bf6cf81f51f17f6ce5c2dbf188c600cd671933bce98176ab19c48014cea47*
Theory Object Address:	*TMLBEa2WvRd6EvHsbywWEHTnGKb6kqvBVtd*

equip_ref

Theorem 9.12 $\forall X.\mathsf{equip}\ X\ X$. *The proposition is identified by the following information:*

Pure Prop Id:	*cc7e804a09044eda808398af78c405e10d681cf2d3ebd1e427405bb3a0f82f4b*
Pure Prop Address:	*TMYk6cm3eVPFH29r4U5oDKnwrrdfovk75q8*
Theory Prop Id:	*c41fb1856e522372cc47cfb80303fc14cb2d9b7b560c6fbb7b1e6aa94caf89c4*
Theory Prop Address:	*TMXscfQQ6JtEo6wQ2rvbuRae4u8ghd3TmK3*

`equip_sym`

Theorem 9.13 $\forall XY.\mathsf{equip}\ X\ Y \to \mathsf{equip}\ Y\ X$. *The proposition is identified by the following information:*

Pure Prop Id:	*f289541b211153f721f213a25c59969aa4873f985ed3dadb265f68134c24c94e*
Pure Prop Address:	*TMY2oL6NVAFGfyVKFHZtMwxnebWdEANu4RE*
Theory Prop Id:	*272f04ef14a36c414e4557b20a0d2296a32dc853b3ed7c4ee2a194386feabcb3*
Theory Prop Address:	*TMP5Vsiypad2yczNmG9eU243SqUyrcsGC6w*

`equip_tra`

Theorem 9.14 $\forall XYZ.\mathsf{equip}\ X\ Y \to \mathsf{equip}\ Y\ Z \to \mathsf{equip}\ X\ Z$. *The proposition is identified by the following information:*

Pure Prop Id:	*322bfbeb779343f7873d6478ed537f6d26e801c3b1dbd29367976be6a6a8cd1e*
Pure Prop Address:	*TMP3Pyh4cMndHz7KEMSUcuThjZ8NUECDxnz*
Theory Prop Id:	*d616c1609dd61f5c209c6cb476fcba79031210d81c19459ab45828203b68f961*
Theory Prop Address:	*TMMPNzj7ZfCkfZFdbzZMRPAohztBSpmcrY9*

Definition 9.7 *We define* finite *to be* $\lambda X.\exists n \in \mathsf{omega}.\mathsf{equip}\ X\ n$ *of type* $\iota \to o$ *identified by the following information:*

Pure Object Id:	*04632dc85b4cfa9259234453a2464a187d6dfd480455b14e1e0793500164a7e3*
Pure Object Address:	*TMTx9uDMYWFe7F4kG1CvRU871EPE4S3LaMV*
Theory Object Id:	*4c54a587d9d6957aacb640a264f675f93a1254e92f4c6f7aad082ab5a4350c68*
Theory Object Address:	*TMFKGVLZA5xtkoBs7Z7ZUYc2RvcSeaNRv3v*

Definition 9.8 *We define* infinite *to be* $\lambda X.\neg\mathsf{finite}\ X$ *of type* $\iota \to o$ *identified by the following information:*

Pure Object Id:	*313e7b5980d710cf22f2429583d0c3b8404586d75758ffcab45219ac373fbc58*
Pure Object Address:	*TMasuVNeN4ZSGDYVEGFs8RGYS2DpDeugNiN*
Theory Object Id:	*fad8eaad083b98f773fe61e7e4c9c8e42ee4b7b5b96a8a831500e0f2097c3ccf*
Theory Object Address:	*TMUDie5EKa9EaiBtjwiXwQDYd6oF2q9cCbx*

`KnasterTarski_set`

Theorem 9.15

$$\forall A.\forall F : \iota \to \iota.(\forall U \in \mathsf{Power}\ A.F\ U \in \mathsf{Power}\ A) \to \\ (\forall UV \in \mathsf{Power}\ A.U \subseteq V \to F\ U \subseteq F\ V) \to \exists Y \in \mathsf{Power}\ A.F\ Y = Y.$$

The proposition is identified by the following information:

Pure Prop Id:	*2ffb9f17db9abba2e99cdc9e9ea64f507ea8b974c232ad295e4047e204a3c2df*
Pure Prop Address:	*TMGYc2cyt576aoAHoJnf5fmecWDttyfoAEA*
Theory Prop Id:	*31ef9df03eb42ad3c6e310449b514b042b286ecbe91fd6d663ff0cb8c38f9bcc*
Theory Prop Address:	*TMVA6dF8zyCJrTc6aSsUANd7jk3viaeNZ4W*

`image_In_Power`

Theorem 9.16

$\forall AB.\forall f : \iota \to \iota.(\forall x \in A.f\ x \in B) \to \forall U \in \mathsf{Power}\ A.\{f\ x|x \in U\} \in \mathsf{Power}\ B.$

The proposition is identified by the following information:

Pure Prop Id:	*6799e88e9e80af6a6d035ad42a185bfe8562bd1a6140b5c7d8848156518b59f1*
Pure Prop Address:	*TMbqTvjYLewp4gH2vMecz4sa5m85ihte7rH*
Theory Prop Id:	*43ed06ac181dbddeb1e0701aeafd6b2b48bdf7da7d513e9c557e99bbc7534b05*
Theory Prop Address:	*TMXRAfJAyZSv2buTnY3cuenqyFKCq8KC47T*

`image_monotone`

Theorem 9.17 $\forall f : \iota \to \iota.\forall UV.U \subseteq V \to \{f\ x|x \in U\} \subseteq \{f\ x|x \in V\}$. *The proposition is identified by the following information:*

Pure Prop Id:	*ac6fe8101f40a432ad82f07e1e2665c922b286ef3145bb1445a03166259e236f*
Pure Prop Address:	*TMP3hoEqEfoP9GJwJnrLJ876qGy57htpeFQ*
Theory Prop Id:	*f923c6e98034954cfdec182ed28c5f254354e531c01861ce39d38a6191a03629*
Theory Prop Address:	*TMHKKD8maX4Ek8ApzVty5uJ6NuWSzXkpFib*

`setminus_In_Power`

Theorem 9.18 $\forall AU.A \setminus U \in \mathsf{Power}\ A$. *The proposition is identified by the following information:*

Pure Prop Id:	*062275a787c47ab2b1e0a2473d1970002748ce7a53f286abe400bbb934df521c*
Pure Prop Address:	*TMS4pSRxLA1Dbq2igDLXVVHw7nhdRCiv3XJ*
Theory Prop Id:	*f503e1c5ae49a7f07828b518f3acefa7e8740424fa3207093249cc6306769e3f*
Theory Prop Address:	*TMPhrNAZaW4G1fJTosqoVHur2BY9wXz7Crc*

`setminus_antimonotone`

Theorem 9.19 $\forall AUV.U \subseteq V \to A \setminus V \subseteq A \setminus U$. *The proposition is identified by the following information:*

Pure Prop Id:	*67bd0e5333f721c6c4ebed67eb48fac3aaa4d7c9e31ecd29eebad8127a29bc85*
Pure Prop Address:	*TMKx1CgcNCpQuzLmEaWfsvEuHkFTPgSbL1D*
Theory Prop Id:	*f9b735ec697563f76b4bf8db58fac77a909244fe1f67ee2c99fdd5259caf0064*
Theory Prop Address:	*TMYUDtXjxfZhhgoCHvxoyj1yVnZ2NwE1zoN*

`SchroederBernstein`

Theorem 9.20 $\forall AB.\forall fg : \iota \to \iota.\mathsf{inj}\ A\ B\ f \to \mathsf{inj}\ B\ A\ g \to \mathsf{equip}\ A\ B$. *The proposition is identified by the following information:*

Pure Prop Id:	*e247a892fd3a3a3c6729106e07939389d6cba1af2c7dab631e4a1a2ad18cba2b*
Pure Prop Address:	*TMNErsk8ZdnQDMxX11dVnP6drZGGGUKRnT7*
Theory Prop Id:	*03a2cd1c8a567976a696d45296506a96202d6ef88ed050e5bd0781df51d7b9b4*
Theory Prop Address:	*TMajcAjnmAvQnwV59Qj54uAcCt6LNS6kuqZ*

Chapter 10

Misc

`f_eq_i`

Theorem 10.1 $\forall f : \iota \to \iota. \forall xy. x = y \to f\ x = f\ y$. *The proposition is identified by the following information:*

Pure Prop Id:	*b0871b523a7b57f1eb5aaeab0fc7f0a48159bfb735912ea5ae95f631f41659b1*
Pure Prop Address:	*TMPVNZYtUqHTp8RYLtZcxXu3DU3ivfR6rAr*
Theory Prop Id:	*2a60fd880e3c5cee57da00ac972ed9116312115366d80d946af04bdedf52ba41*
Theory Prop Address:	*TMcdcpfCVRxKR6sYN1s4kVKEHiMfK814Qv1*

`f_eq_i_i`

Theorem 10.2 $\forall f : \iota \to \iota \to \iota. \forall xyzw. x = y \to z = w \to f\ x\ z = f\ y\ w$. *The proposition is identified by the following information:*

Pure Prop Id:	*99d67d79970e96836845a0ba205f2ae8cf38fbbe7c66edb2e3dd47679e886175*
Pure Prop Address:	*TMVyNZC57hm2JCbUEkm54EFRxyjV7uKHLYV*
Theory Prop Id:	*47e5770eb29a6575bfe0da11c0c722b53eaa9c4f54a6338e54fda222161f1310*
Theory Prop Address:	*TMJfWQ8qLkS2wi6eLaJhtLGmdrxxJvCx3w1*

`eq_i_tra`

Theorem 10.3 $\forall xyz. x = y \to y = z \to x = z$. *The proposition is identified by the following information:*

Pure Prop Id:	*193bbfd16b1f36c82abc18861eb9920fd3ebdab3fa714704e5d9ee842a572cc0*
Pure Prop Address:	*TMMRUBbS4C1QBM5KZBX4P26LywnZaH7iKjP*
Theory Prop Id:	*7ecbc48d1e90e90e475c12e4250d6ed1b153fe65d79663458f1516c1b19ef29e*
Theory Prop Address:	*TMMtqtU6uJfRVq7S9aPMj7up363DpEmoc8j*

Definition 10.1 *We define* nSubq *to be* $\lambda XY. \neg \mathsf{Subq}\ X\ Y$ *of type* $\iota \to \iota \to o$ *identified by the following information:*

Pure Object Id: *ea71e0a1be5437cf5b03cea5fe0e2b3f947897def4697ae7599a73137f27d0ee*
Pure Object Address: *TMZtJ57VpuWkqSBd372tXDYbQV1ru9RQiP6*
Theory Object Id: *1b32290aed0c0437eee0d631ba16b62c8142cad8d60330e8a2c72a430b7453ae*
Theory Object Address: *TMUnveDfk5UPe9zq9bRZ9NGMx9j7NA22YVb*

Notation. We use $\nsubseteq$ as an infix operator corresponding to applying term nSubq.

Sing_inv

Theorem 10.4 $\forall xY.\{x\} = Y \to x \in Y \land \forall y \in Y.y = x$. *The proposition is identified by the following information:*

Pure Prop Id: *57ae4c95e879681858c5ff28d750d4d70e50b072a6d15b2feb91ea9fcef02da8*
Pure Prop Address: *TMGqotQdzdFCuZC8q4vqVRkFBMW4qjArQ3h*
Theory Prop Id: *199c6ccf6b8a2d5af1b102b165811324c6387df31287b0a86242c6611b629fed*
Theory Prop Address: *TMMav2V1grSCKmckxQqmeTmvNxWh6DUx8uL*

TransSet_In_ordsucc_Subq

Theorem 10.5 $\forall xy.\mathsf{TransSet}\ y \to x \in \mathsf{ordsucc}\ y \to x \subseteq y$. *The proposition is identified by the following information:*

Pure Prop Id: *f80c6ef5328d1aebbabe1261cb2a17127aad514383c2c09bbf3354725a9afe82*
Pure Prop Address: *TMPqbNun9yFToi13JVKcoYQrF59XATjGUYz*
Theory Prop Id: *34199340a5562195ace40af331c0a1da26d29faac3847d7b38f9e829ab506bd5*
Theory Prop Address: *TMH3pW2NLQvrgTnYbQVg1dcwdjLcHFcyHCU*

inv_Repl_eq

Theorem 10.6 $\forall X.\forall fg : \iota \to \iota.(\forall x \in X.f\ (g\ x) = x) \to \{f\ y|y \in \{g\ x|x \in X\}\} = X$. *The proposition is identified by the following information:*

Pure Prop Id: *de1759e07c1622dd0cf2ac0a92f57c8d0d379b3d4ee285440b6de212666d9eae*
Pure Prop Address: *TMSStFLLm81AFEdPaJMUoYdRHkYa5ucHRD2*
Theory Prop Id: *5a8fc2fd0b67b701a32a2d1b39179d4975ca830fffb1452d564b9af4e10a99b9*
Theory Prop Address: *TMTeQxUhBU7C2YeyWfVA7p37xMSPpxyofrK*

invol_Repl_eq

Theorem 10.7 $\forall X.\forall f : \iota \to \iota.(\forall x \in X.f\ (f\ x) = x) \to \{f\ y|y \in \{f\ x|x \in X\}\} = X$. *The proposition is identified by the following information:*

Pure Prop Id: *87b0604e91d11a8b4c06ac87cda6148fe5e60301aa91d79609febf82042a8ce3*
Pure Prop Address: *TMQznwJJoHjcdkGoE3rvi8W9R6zhQ68ZwCc*
Theory Prop Id: *393cc6aeb2c74236699aca97e243d3db17d6149941639ffde4f7d58dc26b2e6c*
Theory Prop Address: *TMW5YcT8N18dkwdFihvpKYK9P8scfdxJAhs*

Eps_i_set_R

Theorem 10.8

$$\forall X : \iota.\forall P : \iota \to o.\forall x \in X.P\ x \to \mathsf{Eps\_i}\ (\lambda x.x \in X \wedge P\ x) \in X \wedge P\ (\mathsf{Eps\_i}\ (\lambda x.x \in X \wedge P\ x)).$$

The proposition is identified by the following information:

Pure Prop Id:	*b4f328ffb79aabb0df4389dfe10d26a1a90d7ed192a2b7e062aae20ce483c959*
Pure Prop Address:	*TMFghbYQCmAiS83QkiJceQ1ocqagGS8fdof*
Theory Prop Id:	*a78d0d896dcf99f0838ebc9fbab4b2d7723c3dc6238c649009cf14a169e85421*
Theory Prop Address:	*TMF6oMG8oYpvhQhrGCFXfDFrJYvD2UUnWF3*

`exandE_i`

Theorem 10.9 $\forall PQ : \iota \to o.(\exists x.P\ x \wedge Q\ x) \to \forall r : o.(\forall x.P\ x \to Q\ x \to r) \to r.$
The proposition is identified by the following information:

Pure Prop Id:	*083f822f238e73865a8e53f1b501a327429810344099 6953b5dbe5b9f1c64a49*
Pure Prop Address:	*TMcSPtKUzzFxn3xkTT3qXkbEzgxdcFKD64W*
Theory Prop Id:	*b1b26e0e1db0dccbd32126585f43f1f764c8cd693eeb9898836853ddea6300a1*
Theory Prop Address:	*TMYHN9RveUaKq4szKucPeZQCRGkDxZMkhgC*

`exandE_ii`

Theorem 10.10

$$\forall PQ : (\iota \to \iota) \to o.(\exists x : \iota \to \iota.P\ x \wedge Q\ x) \to \forall p : o.(\forall x : \iota \to \iota.P\ x \to Q\ x \to p) \to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*c3e995324746e718ab713bbf6a225f342b11f85e44af87041c25e67c26eea74b*
Pure Prop Address:	*TMRys3KSWGA2LC5vfb7h5HsWi6UReHYFA4A*
Theory Prop Id:	*ed9d65f783165c1844de5ef6e565738ab285fb760e976e5b2a4a0aaea6db676a*
Theory Prop Address:	*TMJjeW58a6DARdGUxoyLGC7KDSF7e5NyVkz*

`exandE_iii`

Theorem 10.11

$$\forall PQ : (\iota \to \iota \to \iota) \to o.(\exists x : \iota \to \iota \to \iota.P\ x \wedge Q\ x) \to \forall p : o.(\forall x : \iota \to \iota \to \iota.P\ x \to Q\ x \to p) \to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*028a8f7f6f33f8d9071fed86dccba76e2db66b07c2f6b7330daccd1792bb2382*
Pure Prop Address:	*TMNuTg4eH32x4XhwU5Q9dAJWP8zDQ3b8ZXC*
Theory Prop Id:	*80b3b0d1dac86077631ca90e19efac3b726c7697bfe6eeb63b5d74a056a3d3ee*
Theory Prop Address:	*TMKrUpgWDX1Lip64jWzHGPhKdYPQ9L4XevF*

exandE_iiii

Theorem 10.12

$$\forall PQ : (\iota \to \iota \to \iota \to \iota) \to o.(\exists x : \iota \to \iota \to \iota \to \iota.P\ x \wedge Q\ x) \\ \to \forall p : o.(\forall x : \iota \to \iota \to \iota \to \iota.P\ x \to Q\ x \to p) \to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*7807d4121dc1d8d66fac47e62c634706453ae9f2acfb88ce1adf46eedfce6b57*
Pure Prop Address:	*TMPCN5CQwhpCx9SEJcLXFz7VTi2HpDCvNie*
Theory Prop Id:	*d1536a49224a1dfb03981a17133a78f7a7384d730d0fea9ee123ba6b7a9eee92*
Theory Prop Address:	*TMNR9cr6cmoDmVuobFG1uP3pM5DqVkVGcfe*

exandE_iio

Theorem 10.13

$$\forall PQ : (\iota \to \iota \to o) \to o.(\exists x : \iota \to \iota \to o.P\ x \wedge Q\ x) \\ \to \forall p : o.(\forall x : \iota \to \iota \to o.P\ x \to Q\ x \to p) \to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*86782c60904be2e5e01ea093b9564a62c5d7b5c5248e75598e84cda4eaff6db1*
Pure Prop Address:	*TMU81hY5npesCwKcB8HT2xq4ju8aURKMfbk*
Theory Prop Id:	*9e40e24a95644e50b58d83354186c23bf455543eef7af35f3dcfdf02d9c6ef87*
Theory Prop Address:	*TMZrs5GwEKYE1pY5Ctt2Fz9smjaQEZ5RQyx*

exandE_iiio

Theorem 10.14

$$\forall PQ : (\iota \to \iota \to \iota \to o) \to o.(\exists x : \iota \to \iota \to \iota \to o.P\ x \wedge Q\ x) \\ \to \forall p : o.(\forall x : \iota \to \iota \to \iota \to o.P\ x \to Q\ x \to p) \to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*bd6e2d3ff943629d1de39214b2120452cec619ef78043f7c33b48dfd70ae31e3*
Pure Prop Address:	*TMLJQJfu5Yt7gyzMd54gxCSHZYxdRbKTTPQ*
Theory Prop Id:	*ad03be44e266e20551e0c398f53a8d6eb2497ca6b16f6b686368373774c0e421*
Theory Prop Address:	*TMQyqYULvp8WJD8eRf3khDrng9qgVuiu34m*

Chapter 11

Description and If-then-else

11.1 Descr_ii

Let $P : (\iota \to \iota) \to o$ be given.

Definition 11.1 Descr_ii *is the opaque object of type $\iota \to \iota$ identified by the following information:*

Pure Object Id:	*3bae39e9880bbf5d70538d82bbb05caf44c2c11484d80d06dee0589d6f75178c*
Pure Object Address:	*TMYQgJAnkDBZVfPWVEikmdFCVufuxzpsMuR*
Theory Object Id:	*3a4ad2bbdc8560996c770f069c5b0861aa6f677de8c35f7e6825c37a062870ea*
Theory Object Address:	*TMPXygXB3Vatp7ehxXW1SzR7YDxpTfC383X*

Assume the following.

$$\exists f : \iota \to \iota. P\ f \tag{11.1}$$

Assume the following.

$$\forall f g : \iota \to \iota. P\ f \to P\ g \to f = g \tag{11.2}$$

`Descr_ii_prop`

Theorem 11.1 P Descr_ii. *The proposition is identified by the following information:*

Pure Prop Id:	*45bfc0f7e5a41baec35381c982dec98e3138394009ea6479b4734e2b51ee3015*
Pure Prop Address:	*TMRUGkPteutefnmzwHhFudjsxFRU5QyH8LU*
Theory Prop Id:	*e705d1e90646b5936b4d3f399b1ba2f451f5e0bfaa6d4ce06bde1703c2ac6afb*
Theory Prop Address:	*TMdRU9JrS4rFdHpHAP14V6T8N7yZhtAKeUt*

11.2 Descr_iii

Let $P : (\iota \to \iota \to \iota) \to o$ be given.

Definition 11.2 Descr_iii *is the opaque object of type* $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*ca5fc17a582fdd4350456951ccbb37275637ba6c06694213083ed9434fe3d545*
Pure Object Address:	*TMXQVeWJ4sPR355qf1XJWKLuqBobxgZ1f4j*
Theory Object Id:	*877e40e99321711a3798c510f4a5784236e411ebab77a7914b276da43ccba50c*
Theory Object Address:	*TMUoa925kPFT8WA5VCxeG6JDVutAYyuK4A6*

Assume the following.

$$\exists f : \iota \to \iota \to \iota . P\ f \tag{11.3}$$

Assume the following.

$$\forall f g : \iota \to \iota \to \iota . P\ f \to P\ g \to f = g \tag{11.4}$$

`Descr_iii_prop`

Theorem 11.2 P Descr_iii. *The proposition is identified by the following information:*

Pure Prop Id:	*261c3dac795dc9d12a850c45e12570f947cf3e73e7aac1d8b97f4fd8dfcaab74*
Pure Prop Address:	*TMJbcddb8cyABNJEUp19YzaZ62uVYxZcxzw*
Theory Prop Id:	*1604215f70483ff4639dd10934f543d9f25d48553a1e9dbbeae40186c6cf6c06*
Theory Prop Address:	*TMLfZU6SKM2VC18naP9r39PVsCucjtdzWB9*

11.3 Descr_iio

Let $P : (\iota \to \iota \to o) \to o$ be given.

Definition 11.3 Descr_iio *is the opaque object of type* $\iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*e8e5113bb5208434f24bf352985586094222b59a435d2d632e542c768fb9c029*
Pure Object Address:	*TMc7TWyjFWcXBop91LuVowEJbeVve25nK5s*
Theory Object Id:	*0423e66d8558444d27eb1068b080fe14267e388a7434254d29d1c826ce64ac94*
Theory Object Address:	*TMJ6ajRQrfdbCkA3bH4AFazLF71vXbSokSz*

Assume the following.

$$\exists f : \iota \to \iota \to o . P\ f \tag{11.5}$$

Assume the following.

$$\forall f g : \iota \to \iota \to o . P\ f \to P\ g \to f = g \tag{11.6}$$

Descr_iio_prop

Theorem 11.3 P Descr_iio. *The proposition is identified by the following information:*

Pure Prop Id:	*5dd3c7f22282c081da40fb112d8f2d8ffa44620b63f3136e2b179ca8f521ddc5*
Pure Prop Address:	*TMVeGSPy1KHq36b4uHCMZWc5ghcexfzeT12*
Theory Prop Id:	*44dcac73e42284ed3d29fe0e058e46dcdf185b04829a0f7b5929c19617f0f40b*
Theory Prop Address:	*TMK1Bc1JaKqvjwRNdnxP1qds6eXGi9r8UaD*

11.4 Descr_Vo1

From now on we write $Vo\ (n+1)$ to mean the type $Vo\ n \to o$ where $Vo\ 0$ is ι.

Let $P : Vo\ 1 \to o$ be given.

Definition 11.4 Descr_Vo1 *is the opaque object of type* $Vo\ 1$ *identified by the following information:*

Pure Object Id:	*615c0ac7fca2b62596ed34285f29a77b39225d1deed75d43b7ae87c33ba37083*
Pure Object Address:	*TMFQ3dBkT4GNkBxqPW23PKfo5eca5LbGLnM*
Theory Object Id:	*a4ecddac25cdabd614bf297524a2acb05724493b89ab7e7e636cf452a8887b01*
Theory Object Address:	*TMS1y5bGeSZGkV7sivczRgNAQUcrNTaGYAX*

Assume the following.

$$\exists f : Vo\ 1.P\ f \tag{11.7}$$

Assume the following.

$$\forall f g : Vo\ 1.P\ f \to P\ g \to f = g \tag{11.8}$$

Descr_Vo1_prop

Theorem 11.4 P Descr_Vo1. *The proposition is identified by the following information:*

Pure Prop Id:	*f550ef6a927da855d9705aafeb7607fe3931aa8ec0f5f675da6011e035f22458*
Pure Prop Address:	*TMMwzBC9pSvmYAJivGEMP4qt8Mt7WyJqduS*
Theory Prop Id:	*2213e2f6bf896b176ce81e3f28c06d4eda10bd33020cb296513b4b938fcd9056*
Theory Prop Address:	*TMLLpH2SQbA7WJA4grh8eUgfwMWFcLKc8kH*

11.5 Descr_Vo2

Let $P : V o\ 2 \to o$ be given.

Definition 11.5 Descr_Vo2 *is the opaque object of type $V o\ 2$ identified by the following information:*

Pure Object Id:	*a64b5b4306387d52672cb5cdbc1cb423709703e6c04fdd94fa6ffca008f7e1ab*
Pure Object Address:	*TMFZfGbPgS43AdHK51F7w5aNLJzHmKsdsJN*
Theory Object Id:	*546d66811c244b38780425746c3230f23c0a0c78c5d74a314c6ec8eca20e5571*
Theory Object Address:	*TMQQUC9cTGz9T8416CfsGFpzXHbvVhpxAu2*

Assume the following.

$$\exists f : V o\ 2.P\ f \tag{11.9}$$

Assume the following.

$$\forall f g : V o\ 2.P\ f \to P\ g \to f = g \tag{11.10}$$

`Descr_Vo2_prop`

Theorem 11.5 P Descr_Vo2. *The proposition is identified by the following information:*

Pure Prop Id:	*95fc538f5d5098f54c64e376e95b1284227552f9e403c9a68ca16035b2a69c37*
Pure Prop Address:	*TMLM1fK9jLGpTqnThj3QLyRf4RLDCjDXKzv*
Theory Prop Id:	*763c84ad5f1253ac14ad30d1f2765feaf5cc0cdc449dab033ce038bcc35d452c*
Theory Prop Address:	*TMaBE8ZNQAt1zVAJ1G6cs2yjF99pYEBGHwz*

11.6 If_ii

Let $p : o$ be given. Let $f, g : \iota \to \iota$ be given.

Definition 11.6 If_ii *is the opaque object of type $\iota \to \iota$ identified by the following information:*

Pure Object Id:	*17057f3db7be61b4e6bd237e7b37125096af29c45cb784bb9cc29b1d52333779*
Pure Object Address:	*TMJ5CGNSX9XjHHwnu1BSQsVHCKzSFJcwu4b*
Theory Object Id:	*e290152395c5a355ecc6c5012e4cd4507f1895df9d71192a4e860e914c8db43e*
Theory Object Address:	*TMPHWP247Vz8BvmmXSfTeVHxvDsaU8cpPU8*

`If_ii_1`

Theorem 11.6 $p \to \mathsf{If\_ii} = f$. *The proposition is identified by the following information:*

Pure Prop Id:	*31521ec60d1fb026a6fc18736822f20291fbb3d33597bb3efce99a03f03e11d9*
Pure Prop Address:	*TMKosiG2dup6SxL5QSDUpucfR51n9gCyDgV*
Theory Prop Id:	*342a1f3e82c1abf29e708798f7feb58587435d6e872e0fad569a7aa4aff40c55*
Theory Prop Address:	*TMc5keGi4Y146iSqaxLiaYsAZYq1XuEVqNB*

`If_ii_0`

Theorem 11.7 $\neg p \rightarrow \mathsf{If\_ii} = g$. *The proposition is identified by the following information:*

Pure Prop Id:	*6acab05b8c4b66e4263e5b31aaa005d9f082668a8bbace0e6d83fd1b4f2a4e7c*
Pure Prop Address:	*TMNxHhMohQMfeSQ46TzksCAdPGGHded4fQn*
Theory Prop Id:	*36418b196a3dc55851db392b475c08d495d7976847a1bfe1ffa0839c921a9904*
Theory Prop Address:	*TMJAFTnXbqvZwsoNakQckwUkzUMcWByPVDq*

11.7 If_iii

Let $p : o$ be given. Let $f, g : \iota \rightarrow \iota \rightarrow \iota$ be given.

Definition 11.7 If_iii *is the opaque object of type* $\iota \rightarrow \iota \rightarrow \iota$ *identified by the following information:*

Pure Object Id:	*3314762dce5a2e94b7e9e468173b047dd4a9fac6ee2c5f553c6ea15e9c8b7542*
Pure Object Address:	*TMafEMcrDN1G9aE3ciBtqxzAhjPGbGo9wGT*
Theory Object Id:	*dd7e2b57dd8bff886e536f76317306bce1d1c82c8d0c4990736c5435d5194472*
Theory Object Address:	*TMXbifVhYnu2zzxcRpGBEz7yE8Zgdxct2bF*

`If_iii_1`

Theorem 11.8 $p \rightarrow \mathsf{If\_iii} = f$. *The proposition is identified by the following information:*

Pure Prop Id:	*2d62ee998f0e93e8531c4fa56ae24814f441c745adbbf870b6533f8348e4105d*
Pure Prop Address:	*TMXoAktu9YHuqADS1FQhmqPQo88e8Wfe9Tc*
Theory Prop Id:	*48bc10484ec823e29188216c5145c832ea01049091770c743d54872c69c3738d*
Theory Prop Address:	*TMP1NQtm8tugHGMUGtnKEr16wjn5P3HapKj*

`If_iii_0`

Theorem 11.9 $\neg p \rightarrow \mathsf{If\_iii} = g$. *The proposition is identified by the following information:*

Pure Prop Id:	*9b77d90fae20e665d930e198836d49c2f016c9359f15a9036520651d80559a50*
Pure Prop Address:	*TMMxCCtSQ76TqwLa7cLQxLYRL1uvAhxf6G5*
Theory Prop Id:	*d11201a0fa74b1965c04633e2fbf7c186e42c0fbf53635dd329a7bb860c6b930*
Theory Prop Address:	*TMG9gLyxAgRsyCSfKKidGTT9tPyeptZY75x*

11.8 If_Vo1

Let $p : o$ be given. Let $f, g : V o\ 1$ be given.

Definition 11.8 If_Vo1 *is the opaque object of type* $V o\ 1$ *identified by the following information:*

Pure Object Id:	*2bb1d80de996e76ceb61fc1636c53ea4dc6f7ce534bd5caee16a3ba4c8794a58*
Pure Object Address:	*TMJRubZpJkuKGf6tLSqj98TBUniTgSkcx2k*
Theory Object Id:	*c5705984e5c61b59990fd2a143acd737bb2dc7c7db647a62e398dac074967687*
Theory Object Address:	*TMaRPXSn2Fjb29o31JvamGmuMJnp68JPVyZ*

`If_Vo1_1`

Theorem 11.10 $p \to \mathsf{If\_Vo1} = f$. *The proposition is identified by the following information:*

Pure Prop Id:	*bb1012256bb9266025d71663756a5ebfd5e16baf8e93a915bfdd7fcb42d706f3*
Pure Prop Address:	*TMXYuJm2QUHmNybRpkXiJxzCLG25czuz5LQ*
Theory Prop Id:	*62b2b86d0737e9e18346982d5767e5ef03ab4118b70f45a9ff56d1c9e5216feb*
Theory Prop Address:	*TMFtdqVK8VYTrQ6cgnnamxbg6KeHhpZgPcd*

`If_Vo1_0`

Theorem 11.11 $\neg p \to \mathsf{If\_Vo1} = g$. *The proposition is identified by the following information:*

Pure Prop Id:	*59bda37f5c7398ead6c4b3243129fbad85f4f9f3636487f1c51cce4e0e6c8cfd*
Pure Prop Address:	*TMWqGQPgK8uAv1B3geaEm1MSYAnLtEgFcb9*
Theory Prop Id:	*78529c61edfcd02459bc4ec507c773553a628c9c32db5afd8b9669bdeb5624be*
Theory Prop Address:	*TMFdPp5SLYgJtZvzVSfP9YVDKe598dst2rw*

11.9 If_iio

Let $p : o$ be given. Let $f, g : \iota \to \iota \to o$ be given.

Definition 11.9 If_iio *is the opaque object of type* $\iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*bf2fb7b3431387bbd1ede0aa0b590233207130df523e71e36aaebd675479e880*
Pure Object Address:	*TMSVGinFUSHBKXoj7UW42oxwqfA45nW9uYC*
Theory Object Id:	*f0254ba33a025d14997148eaf8b6489b75cbcd0e552813782f2fee4dac52a1ad*
Theory Object Address:	*TMVgQ7ZnQFc9zUqLQEygvRvocriNyw8Urgz*

`If_iio_1`

Theorem 11.12 $p \to \mathsf{If\_iio} = f$. *The proposition is identified by the following information:*

Pure Prop Id:	*6e5b2f9d60ee2b95f684c6c89fac222c87c115edd65ea7faa36260e77030c34b*
Pure Prop Address:	*TMbdPHYFQQVTEPDFuKsr2RTPtLJWVQCkgmo*
Theory Prop Id:	*5c40c9c2a99e6214c0a2e1d4e4d8ffe6b45d1a1be5958c0a0e817a962bcb2e40*
Theory Prop Address:	*TMYBc7MrgDRDwERk16x3XtsYSbpzRT4JfeS*

`If_iio_0`

Theorem 11.13 $\neg p \to \mathsf{If\_iio} = g$. *The proposition is identified by the following information:*

Pure Prop Id:	*9eab66913f2b7839396a2642931e0c4f2b5b96d45aa947c433210d1358ad7ce9*
Pure Prop Address:	*TMLjBZs3dbH9f9eUHYnJdu3t2Pqbht8aDBr*
Theory Prop Id:	*09748b1d13dc8a2265025a9ce3f481495c8a7bccd4d956b5d4fc36dd28dfbd2b*
Theory Prop Address:	*TMbk4ju7Bfvef6aQhpURVB4h7ERGCJXtpm5*

11.10 If_Vo2

Let $p : o$ be given. Let $f, g : V\,o\;2$ be given.

Definition 11.10 If_Vo2 *is the opaque object of type* $V\,o\;2$ *identified by the following information:*

Pure Object Id:	*6cf28b2480e4fa77008de59ed21788efe58b7d6926c3a8b72ec889b0c27b2f2e*
Pure Object Address:	*TMH4op1eDosdCj4CskKLdVdQMoEMPyaKmKz*
Theory Object Id:	*f6760c1a5f26147ef91098a2d633b2f19a2f38b9c1c811244491274cf056b285*
Theory Object Address:	*TMZuq1Mx5McSmUfVdPQ43iubWhx4f9CctV4*

`If_Vo2_1`

Theorem 11.14 $p \to \mathsf{If\_Vo2} = f$. *The proposition is identified by the following information:*

Pure Prop Id:	*0f655bcb4e8ea858e1e47b901188715bcb9443ff29ca227273b5eeab5ccce4fa*
Pure Prop Address:	*TMLf5qJmSHrs6Umik46niap6g6Puq8u38PZ*
Theory Prop Id:	*530c03e594647527928784ed02365579d201bd63d29f76cf2679ec77f9c3a348*
Theory Prop Address:	*TMYAJeyWgmE86YRTxka8iZ3VDvWDz4KFVak*

`If_Vo2_0`

Theorem 11.15 $\neg p \to \mathsf{If\_Vo2} = g$. *The proposition is identified by the following information:*

Pure Prop Id:	*ba20c1835570284aac2a734187c79818c51b3e6b4ef50125ec114563e95c56a3*
Pure Prop Address:	*TMFhCsW5TbyvNPeCPg5q8UZQCNrh7ba7mc7*
Theory Prop Id:	*49a8cfcdec6a427975b6c3b9ac4c13c6b5dbfa0624fe913a360d44ecd3993241*
Theory Prop Address:	*TMRyczcoeSo63tGt7RMgBM4C9d1EHxcvBZi*

Chapter 12

Recursion on Sets

12.1 EpsilonRec_i

Let $F : \iota \to (\iota \to \iota) \to \iota$ be given.

Definition 12.1 $\mathsf{In\_rec\_i}$ *is the opaque object of type $\iota \to \iota$ identified by the following information:*

Pure Object Id:	*fac413e747a57408ad38b3855d3cde8673f86206e95ccdadff2b5babaf0c219e*
Pure Object Address:	*TMNdra6FF88BaWZhLQWoMVGDVkGgU5xZ1Px*
Theory Object Id:	*93aac4fb93be5cc13a42b363fb830917e096fe0d5b269d4cae1a2675fc6fe442*
Theory Object Address:	*TMKBpYHnteFb5wpr6cJ5KfTg3HrzRoSBByu*

Assume the following.

$$\forall X : \iota.\forall gh : \iota \to \iota.(\forall x \in X.g\ x = h\ x) \to F\ X\ g = F\ X\ h \tag{12.1}$$

`In_rec_i_eq`

Theorem 12.1 $\forall X : \iota.\mathsf{In\_rec\_i}\ X = F\ X\ \mathsf{In\_rec\_i}$. *The proposition is identified by the following information:*

Pure Prop Id:	*1106536fa01b17bf5ad01765e95f74fb2d59af6f6540159757750d84c1f699ae*
Pure Prop Address:	*TMS4niVmsvQx9jqComxMJ15nwcNzZqmdhRN*
Theory Prop Id:	*e8653b0bcd75b2d7d06c38f2ac4c66b817c99e8dc96d2647bd2daaee681a632e*
Theory Prop Address:	*TMbKwV8D2cMfE3R9MZ8SdtrUWVk2FfWMq91*

12.2 EpsilonRec_ii

Let $F : \iota \to (\iota \to (\iota \to \iota)) \to (\iota \to \iota)$ be given.

Definition 12.2 In_rec_ii *is the opaque object of type* $\iota \to (\iota \to \iota)$ *identified by the following information:*

Pure Object Id:	*f3c9abbc5074c0d68e744893a170de526469426a5e95400ae7fc81f74f412f7e*
Pure Object Address:	*TMKaBL68miPaU3awFWxavJ9vpffruAuJG6m*
Theory Object Id:	*3841bd9652360e3358d466fb36d771d47d765b56ce11b4a446e1c8ee7792319a*
Theory Object Address:	*TMVfeCfjKozKnhDvd1g6ayuJytZPUtNbSQT*

Assume the following.

$$\forall X : \iota.\forall gh : \iota \to (\iota \to \iota).(\forall x \in X.g\ x = h\ x) \to F\ X\ g = F\ X\ h \tag{12.2}$$

In_rec_ii_eq

Theorem 12.2 $\forall X : \iota.\mathsf{In\_rec\_ii}\ X = F\ X\ \mathsf{In\_rec\_ii}$. *The proposition is identified by the following information:*

Pure Prop Id:	*f7aff626beae77fbb87e16d4b62b6dbf1d4e1fc8b4151ab95e33ff0a7eece1fd*
Pure Prop Address:	*TMc1r8kDpq8yTVrxstWVdBTkGu4bAHa2KqZ*
Theory Prop Id:	*7bb1e907bd4a595013dd5253579a59ded78c5fc86a4aba5c0e6af261615e64e4*
Theory Prop Address:	*TMF2cwdAE8DvbpFHHjfMdf7jfwBbgauH5K1*

12.3 EpsilonRec_iii

Let $F : \iota \to (\iota \to (\iota \to \iota \to \iota)) \to (\iota \to \iota \to \iota)$ be given.

Definition 12.3 In_rec_iii *is the opaque object of type* $\iota \to (\iota \to \iota \to \iota)$ *identified by the following information:*

Pure Object Id:	*9b3a85b85e8269209d0ca8bf18ef658e56f967161bf5b7da5e193d24d345dd06*
Pure Object Address:	*TMGS3HxzKZpy47SjRNptZvfW3hGeTR87kw8*
Theory Object Id:	*651077708696fee3d23e186d6ab9c1805fc9c46eb287174c7c7726c14df5cd25*
Theory Object Address:	*TMNNG5ehc1kjbSr79hs1JoXieVRroM54amr*

Assume the following.

$$\forall X : \iota.\forall gh : \iota \to (\iota \to \iota \to \iota).(\forall x \in X.g\ x = h\ x) \to F\ X\ g = F\ X\ h \tag{12.3}$$

In_rec_iii_eq

Theorem 12.3 $\forall X : \iota.\mathsf{In\_rec\_iii}\ X = F\ X\ \mathsf{In\_rec\_iii}$. *The proposition is identified by the following information:*

Pure Prop Id:	*5f0574ebbeaebb60edd0cf70b9e3649d00e8391b36dc79f8abc40b36f0236f5d*
Pure Prop Address:	*TMLw55kNTBuMX9fEZ9ucafMGa8X9q9Vg5AS*
Theory Prop Id:	*a0709d7e1ee7e89fa5e406d74c01159f7bc4a43f2d6eb3aecf054a5c987acced*
Theory Prop Address:	*TMe1LiqcZFAP9bdmQLTeizcNBRHbFYkJKz8*

12.4 EpsilonRec_iio

Let $F : \iota \to (\iota \to (\iota \to \iota \to o)) \to (\iota \to \iota \to o)$ be given.

Definition 12.4 In_rec_iio *is the opaque object of type* $\iota \to (\iota \to \iota \to o)$ *identified by the following information:*

Pure Object Id:	*8465061e06db87ff5ec9bf4bd2245a29d557f6b265478036bee39419282a5e28*
Pure Object Address:	*TMKaaHZBwQLEGRN8PYDKcEfwZubgSevskVz*
Theory Object Id:	*ebf7a42862e5ae14b0b042ad0dd80452de3ad1c5e8e168f4b1ab3d2fb945f61e*
Theory Object Address:	*TMLjGqbKQQthLSzLhyZ5N8iPmocUeVoLr3N*

Assume the following.

$$\forall X : \iota.\forall gh : \iota \to (\iota \to \iota \to o).(\forall x \in X.g\ x = h\ x) \to F\ X\ g = F\ X\ h \tag{12.4}$$

`In_rec_iio_eq`

Theorem 12.4 $\forall X : \iota.$In_rec_iio $X = F\ X$ In_rec_iio. *The proposition is identified by the following information:*

Pure Prop Id:	*a6bb8bb1ff02720df380edef1d6e1599715ed907c141a8134c85b26a0a245f9b*
Pure Prop Address:	*TMG9Ak4owSMXtiyYkh2BpkesWY7gR3RoFCu*
Theory Prop Id:	*bff321513dbb1d418dd4c5d50b267215cb2c98cc7e1db5de1c7148e57ce1044d*
Theory Prop Address:	*TMZyTR9yto8MPdDZTwmCg5y3krsPUWxxW6u*

12.5 EpsilonRec_Vo1

Let $F : \iota \to (\iota \to V o\ 1) \to V o\ 1$ be given.

Definition 12.5 In_rec_Vo1 *is the opaque object of type* $\iota \to V o\ 1$ *identified by the following information:*

Pure Object Id:	*e9c5f22f769cd18d0d29090a943f66f6006f0d132fafe90f542ee2ee8a3f7b59*
Pure Object Address:	*TMF7xrRrqMjz5Lk6579gDvuGzGVm7S7Xbm3*
Theory Object Id:	*b1f6ea77e17286f0717ab97435a1b23d6c4cd8a5a8d1718c6bbe022c543fe9b5*
Theory Object Address:	*TMRg5yF9XnKxQKRqe5XeJ2UENC7Gurp2pax*

Assume the following.

$$\forall X : \iota.\forall gh : \iota \to V o\ 1.(\forall x \in X.g\ x = h\ x) \to F\ X\ g = F\ X\ h \tag{12.5}$$

`In_rec_Vo1_eq`

Theorem 12.5 $\forall X : \iota.$In_rec_Vo1 $X = F\ X$ In_rec_Vo1. *The proposition is identified by the following information:*

Pure Prop Id:	*fa315696609285c33f1dd6bb13f004c898139dd9c5a7f4d65239606647d55821*
Pure Prop Address:	*TMKzEyM3ak62ud6SUpQnDC1eNXkY9NiztW3*
Theory Prop Id:	*69f9607db0c116ec22508e0bbe3620d5741374de361719968ce5a7a509dfca8c*
Theory Prop Address:	*TMUP4D2yi5VENpkASUcnEs47UeEw4yGYhCw*

12.6 EpsilonRec_Vo2

Let $F : \iota \to (\iota \to V o\ 2) \to V o\ 2$ be given.

Definition 12.6 In_rec_Vo2 *is the opaque object of type* $\iota \to V o\ 2$ *identified by the following information:*

Pure Object Id:	*8bc8d37461c7653ced731399d140c0d164fb9231e77b2824088e534889c31596*
Pure Object Address:	*TMJV2odr76mGWeyBRTv7gyjmWSSMGzGy4vj*
Theory Object Id:	*b67f4be4f70efe3a08c8984e82e3f0ace6bc31b42132285fbf626d533ad4a46e*
Theory Object Address:	*TMMQWnTTmxXXX8n9NmHW2iCY3E2PWon4Gh3*

Assume the following.

$$\forall X : \iota.\forall gh : \iota \to V o\ 2.(\forall x \in X.g\ x = h\ x) \to F\ X\ g = F\ X\ h \qquad (12.6)$$

`In_rec_Vo2_eq`

Theorem 12.6 $\forall X : \iota.$In_rec_Vo2 $X = F\ X$ In_rec_Vo2. *The proposition is identified by the following information:*

Pure Prop Id:	*1386f49a7aeb74994677ac4425fbfe5622be81d9fd2ba5678dc74f1e60cbc108*
Pure Prop Address:	*TMZt6xCkE5e2YHuk9kBiQSM7r1AC1Cp8HNJ*
Theory Prop Id:	*573f9327980a8c0f52e474888757973c754fdd9f151a1b366673a160168ba0cb*
Theory Prop Address:	*TMGqV65crtJCaDQBUgBqHQbtxYEWxrkKgh3*

Chapter 13

If-then-else, Description and Recursion again

13.1 If_Vo3

Let $p : o$ be given. Let $f, g : V\,o\ 3$ be given.

Definition 13.1 If_Vo3 *is the opaque object of type* $V\,o\ 3$ *identified by the following information:*

Pure Object Id:	*73dd2d0fb9a3283dfd7b1d719404da0bf605e7b4c7b714a2b4e2cb1a38d98c6f*
Pure Object Address:	*TMcasq19pTUcLufiu26HYpBfxPmeYk6E8mT*
Theory Object Id:	*333c92c590bdda1d300a193515c5804607be8cb804a38180a59610b36281676f*
Theory Object Address:	*TMKVqTVDwSwbDy1BLiFYKZbD6QhkcDSVJhc*

`If_Vo3_1`

Theorem 13.1 $p \to \mathsf{If\_Vo3} = f$. *The proposition is identified by the following information:*

Pure Prop Id:	*9cf80c77cf895a06b2027489d75da25a65f13d09dbb89106276596f1621e9190*
Pure Prop Address:	*TMJ2NGWhftpmGTYwAy7YTG5gSV15BG4yDC4*
Theory Prop Id:	*da37c739b9dd4956b241bec65be35f2c5b0640a3526959d750cfa4bcd78d0621*
Theory Prop Address:	*TMNrJsMGCaKdKHP7L2e4Nm4EZXFs2uCAAw3*

`If_Vo3_0`

Theorem 13.2 $\neg p \to \mathsf{If\_Vo3} = g$. *The proposition is identified by the following information:*

Pure Prop Id:	*26938a02bd0536ce534ff744512cc88eb86cd2ee1226183b145cae855a142032*
Pure Prop Address:	*TMUNMynXzuArTTrK7p5jFXqihZJb43NBe7x*
Theory Prop Id:	*df49a9a1ae039c4a570658bf925b0179e4b0a63bbabd6021b74d66c7c709f80f*
Theory Prop Address:	*TMb4qEYPVKmSarAqVUz7NwbCdaKVAkyNYPm*

13.2 Descr_Vo3

Let $P : V o\ 3 \to o$ be given.

Definition 13.2 Descr_Vo3 *is the opaque object of type $V o\ 3$ identified by the following information:*

Pure Object Id:	*f25ee4a03f8810e3e5a11b184db6c8f282acaa7ef4bfd93c4b2de131431b977c*
Pure Object Address:	*TMQ2iYFqmoYhwP7TbZqwfTHcnxkgx715jsM*
Theory Object Id:	*d479f5126402f52d9a58899c5731677dad9533674526ab2b066def43094eec28*
Theory Object Address:	*TMSADgxCz6JBunRMJpa7zmWr51aLHrZXZwg*

Assume the following.

$$\exists f : V o\ 3.P\ f \tag{13.1}$$

Assume the following.

$$\forall f g : V o\ 3.P\ f \to P\ g \to f = g \tag{13.2}$$

`Descr_Vo3_prop`

Theorem 13.3 P Descr_Vo3. *The proposition is identified by the following information:*

Pure Prop Id:	*20174ae5a109b906a46115d971515a729f9249f4f20fa5da310e2c4cdeab6249*
Pure Prop Address:	*TMLwRSBien4w3CzFgCJ48yAfqXd7PCssRGE*
Theory Prop Id:	*23c590cb67150ebe43754899c3f2b5d5f092708467626717 3928ecff860e70b5*
Theory Prop Address:	*TMWShaCPNhN2XvW2e4gefY2XgoDY4dWexFY*

13.3 EpsilonRec_Vo3

Let $F : \iota \to (\iota \to V o\ 3) \to V o\ 3$ be given.

Definition 13.3 In_rec_Vo3 *is the opaque object of type $\iota \to V o\ 3$ identified by the following information:*

Pure Object Id:	*80f7da89cc801b8279f42f9e1ed519f72d50d76e88aba5efdb67c4ae1e59aee0*
Pure Object Address:	*TMTrS1bWLxXMW5qTfHXLxP8rqsqDfcSj5it*
Theory Object Id:	*d1066f5fb07b44eb37af262f1b8cbfa415ea64116ce40aa20ad290bbdbba780e*
Theory Object Address:	*TMdJy3eq8ZYfdDZguQUCdxZ6rfJuYZZntCT*

Assume the following.

$$\forall X : \iota.\forall g h : \iota \to V o\ 3.(\forall x \in X.g\ x = h\ x) \to F\ X\ g = F\ X\ h \tag{13.3}$$

`In_rec_Vo3_eq`

Theorem 13.4 $\forall X : \iota.$In_rec_Vo3 $X = F\ X$ In_rec_Vo3. *The proposition is identified by the following information:*

Pure Prop Id:	*2306eb8b31f5ee276553420c5e1f5c8c0651e1258c44c87a9c42e96bd3e79503*
Pure Prop Address:	*TMaEn9SrA4LtLzZLVV3UpDHnxpFiFAqzKBz*
Theory Prop Id:	*4d2789ab957df70c09911021b3cc17f5578b5a09007513d609947d6805b3ca93*
Theory Prop Address:	*TMaSVdip28DZxPPo4ByXDcTwLWpi4sfc89Q*

13.4 If_Vo4

Let $p : o$ be given. Let $f, g : V o\ 4$ be given.

Definition 13.4 If_Vo4 *is the opaque object of type* $V o\ 4$ *identified by the following information:*

Pure Object Id:	*1a8f92ceed76bef818d85515ce73c347dd0e2c0bcfd3fdfc1fcaf4ec26faed04*
Pure Object Address:	*TMFG5MpDVE3nFZV3NJZdM6wd42HBKVgpUkN*
Theory Object Id:	*6f23a6aeaffecd3215c88a1ad6970a5926ecd481bf402e6873321d936dc9a49b*
Theory Object Address:	*TMPj7rvJRkP19dsCJAFArEnBpzMnvjtktEW*

`If_Vo4_1`

Theorem 13.5 $p \to \mathsf{If\_Vo4} = f$. *The proposition is identified by the following information:*

Pure Prop Id:	*06ad5c864f232538e5dcbad7b8a51389f2b7630bb8f0576bcc523d1811805afa*
Pure Prop Address:	*TMLaXf8kJ7EdCg7P6GR8txEnTFCZNLrNGCx*
Theory Prop Id:	*3d1a5d619766f7ca661d7cf070c1c9e481cb4a26a2603723df79b27fbc512fc5*
Theory Prop Address:	*TMTdC39nPta5qDs73eUJZFWgNT3PKvCQPkp*

`If_Vo4_0`

Theorem 13.6 $\neg p \to \mathsf{If\_Vo4} = g$. *The proposition is identified by the following information:*

Pure Prop Id:	*bab37df6d4aabf30ec549e549b8fff3336e2c7eebd629e65365508a0a69be04b*
Pure Prop Address:	*TMTttbgudyrZKJvKhjQTui2Nr8qLhE3y4SX*
Theory Prop Id:	*24dc8a58d4de615eeb9b719733472bdc39bddf577984e0488dbf60ad72aa5b3e*
Theory Prop Address:	*TMdiwt98qwWvCbCfyBMTBrpGjo53TgoPrZM*

13.5 Descr_Vo4

Let $P : V o\ 4 \to o$ be given.

Definition 13.5 Descr_Vo4 *is the opaque object of type* $V o\ 4$ *identified by the following information:*

Pure Object Id:	*8b81abb8b64cec9ea874d5c4dd619a9733a734933a713ef54ed7e7273510b0c3*
Pure Object Address:	*TMLVDACU3jLYPMh9iPh1uhucMrsTaATPeZN*
Theory Object Id:	*85f7ca0492b5dac0db0a41ea793b13fcebe7b788606ad42263186b19feb60364*
Theory Object Address:	*TMd1yKi67f99nixn38zFHJA2mC8ypLfDMrV*

Assume the following.

$$\exists f : V o\ 4. P\ f \tag{13.4}$$

Assume the following.

$$\forall f g : V o\ 4. P\ f \to P\ g \to f = g \tag{13.5}$$

`Descr_Vo4_prop`

Theorem 13.7 *P Descr_Vo4. The proposition is identified by the following information:*

Pure Prop Id:	*a6473b3334fd689271114e94dd8501ed316b996fca678cfd212ee147617f3b63*
Pure Prop Address:	*TMFnuCwCTVMsFiXKonkd3YBQ9HMaKew1LfE*
Theory Prop Id:	*3e862cb55935309dafceacc6dca361c6a70271794a80e68a76ec04b8d5b78104*
Theory Prop Address:	*TMXUspN6Sx6CYJpANQyhMG8CHovhesMrXwZ*

13.6 EpsilonRec_Vo4

Let $F : \iota \to (\iota \to Vo\ 4) \to Vo\ 4$ be given.

Definition 13.6 In_rec_Vo4 *is the opaque object of type $\iota \to Vo\ 4$ identified by the following information:*

Pure Object Id:	*d82c5791815ca8155da516354e8f8024d8b9d43a14ce62e2526e4563ff64e67f*
Pure Object Address:	*TMFrL2oqNRAHoTkwnxvXz88hCQoB8Uoidfq*
Theory Object Id:	*1e5061b04de4514b3a4c4ea251fd26dd589083b04c1276fec71c858ef714657d*
Theory Object Address:	*TMc9WkccuALwugHXmUhucdGnwNEmZuusP9z*

Assume the following.

$$\forall X : \iota.\forall gh : \iota \to Vo\ 4.(\forall x \in X.g\ x = h\ x) \to F\ X\ g = F\ X\ h \tag{13.6}$$

`In_rec_Vo4_eq`

Theorem 13.8 *$\forall X : \iota.$In_rec_Vo4 $X = F\ X$ In_rec_Vo4. The proposition is identified by the following information:*

Pure Prop Id:	*222bdbe9dc303a7fe1b2a2e6e60547f51606b31268e865e7f48ccdfc78fab6dc*
Pure Prop Address:	*TMLZ4TCFwAU7GVexuLeFpen6YpZfwwTmUrz*
Theory Prop Id:	*ab3561ec7f79f59905a72739474d784abea229696e9c83807694ef845230da1b*
Theory Prop Address:	*TMTTyAbVG9aCnaLjwPoFSBiNHFM6ZrztqWx*

Chapter 14

Predicates and Relations

Definition 14.1 *We define* bigintersect *to be*

$$\lambda(D : (\iota \to o) \to o)(x : \iota).\forall P : \iota \to o.D\ P \to P\ x$$

identified by the following information:

Pure Object Id:	*615c0ac7fca2b62596ed34285f29a77b39225d1deed75d43b7ae87c33ba37083*
Pure Object Address:	*TMFQ3dBkT4GNkBxqPW23PKfo5eca5LbGLnM*
Theory Object Id:	*a4ecddac25cdabd614bf297524a2acb05724493b89ab7e7e636cf452a8887b01*
Theory Object Address:	*TMS1y5bGeSZGkV7sivczRgNAQUcrNTaGYAX*

Definition 14.2 *We define* reflexive *to be* $\lambda R.\forall x : \iota.R\ x\ x$ *of type*

$$(\iota \to \iota \to o) \to o$$

identified by the following information:

Pure Object Id:	*6a6dea288859e3f6bb58a32cace37ed96c35bdf6b0275b74982b243122972933*
Pure Object Address:	*TMT7Ad5B4WrSQkBPTK8ixJeatPMtbGf5cdF*
Theory Object Id:	*a8e476299e8d33df6c6811c287e1b9fabc2185a0e461a2c739e7e02c9344377c*
Theory Object Address:	*TMUbRcXexrARoyLrg2fK8SYuYLT1wF4tnjt*

Definition 14.3 *We define* irreflexive *to be* $\lambda R.\forall x : \iota.\neg R\ x\ x$ *of type*

$$(\iota \to \iota \to o) \to o$$

identified by the following information:

Pure Object Id:	*e2ea25bb1daaa6f56c9574c322ff417600093f1fea156f5efdcbe4a9b87e9dcf*
Pure Object Address:	*TMYbVtA5sMrdMd5HyhU7dkL61akED8Yzd4T*
Theory Object Id:	*e3b5275d1d8ea65a249223038cf102aa00ce1f82a9a8ca9a4bde7a257092a89e*
Theory Object Address:	*TMFSJ2T4un5uVsKW7djiQAvN1DAwTnNJUiC*

Definition 14.4 *We define* symmetric *to be* $\lambda R.\forall xy : \iota.R\ x\ y \to R\ y\ x$ *of type* $(\iota \to \iota \to o) \to o$ *identified by the following information:*

Pure Object Id:	*309408a91949b0fa15f2c3f34cdd5ff57cd98bb5f49b64e42eb97b4bf1ab623b*
Pure Object Address:	*TMGcxUiVbTw5fsQKQMYB8qsMsgLdrMBxa7j*
Theory Object Id:	*120a5ebbb3c5e5ef6629806c0a1e8303ae93de2f42212d21a9ae33042777373c*
Theory Object Address:	*TMQbC5GACt2K5MgypgrbgKeZqE3KMVpjpim*

Definition 14.5 *We define* antisymmetric *to be*

$$\lambda R.\forall xy : \iota.R\ x\ y \to R\ y\ x \to x = y$$

of type $(\iota \to \iota \to o) \to o$ *identified by the following information:*

Pure Object Id:	*9a8a2beac3ecd759aebd5e91d3859dec6be35a41ec07f9aba583fb9c33b40fbe*
Pure Object Address:	*TMVbTEZn5mU4ZX9WkJ7WJ2yxJDVpXU5YAUm*
Theory Object Id:	*82374c2974eaa40576881231e8c1e031292e0279528cee22769ae90c0d1ca574*
Theory Object Address:	*TMNn7rfiqnuh8jbUTzq7HQitSyHk6xsoUfm*

Definition 14.6 *We define* transitive *to be*

$$\lambda R.\forall xyz : \iota.R\ x\ y \to R\ y\ z \to R\ x\ z$$

of type $(\iota \to \iota \to o) \to o$ *identified by the following information:*

Pure Object Id:	*b145249bb4b36907159289e3a9066e31e94dfa5bb943fc63ff6d6ca9abab9e02*
Pure Object Address:	*TMFM3TNU7g1EZuVHc24wspMS7Y75XqC8hvN*
Theory Object Id:	*d9f0d1487b5ca94d78a7fe0eb19e9c8bad4aad24dc543be4879ce610d5bd5c53*
Theory Object Address:	*TMW5XFJc1g5eT257Q4ZaCqtxLhMtDWiqqty*

Definition 14.7 *We define* eqreln *to be*

$$\lambda R.\mathsf{reflexive}\ R \land \mathsf{symmetric}\ R \land \mathsf{transitive}\ R$$

of type $(\iota \to \iota \to o) \to o$ *identified by the following information:*

Pure Object Id:	*25d55803878b7ce4c199607568d5250ff00ee63629e243e2a1fd20966a3ee92c*
Pure Object Address:	*TMSuiPhKNmJUrhs9cRrAHLcqoskTttXj1fT*
Theory Object Id:	*c457d55160cd9e8319ea47367df7263cd5e391e71be5fe7a86c761faf2ab6f26*
Theory Object Address:	*TMM8uEsESwkqXBF3DURiTCeRydaN9MdPa2n*

Definition 14.8 *We define* per *to be* $\lambda R.\mathsf{symmetric}\ R \land \mathsf{transitive}\ R$ *of type* $(\iota \to \iota \to o) \to o$ *identified by the following information:*

Pure Object Id:	*5754c55c5ad43b5eaeb1fb67971026c82f41fd52267a380d6f2bb8487e4e959b*
Pure Object Address:	*TMJFMWPiZz3oFCEc6qbqK6ewA1Vd1h48BRr*
Theory Object Id:	*2688d14197e49ace4c9134d31466ccf7c821723681dd3c0a8e19f30828b6e69b*
Theory Object Address:	*TMZ1qY6E5fsXqNMWYcwbbpUKqq24VbUFKZN*

Definition 14.9 *We define* linear *to be* $\lambda R.\forall xy : \iota.R\ x\ y \vee R\ y\ x$ *of type*

$$(\iota \to \iota \to o) \to o$$

identified by the following information:

Pure Object Id:	*9db35ff1491b528184e71402ab4646334daef44c462a5c04ab2c952037a84f3f*
Pure Object Address:	*TMaYw74DvBqB1nULY5YzEwuZZL4wkaUyyxg*
Theory Object Id:	*04b95b0ad1550900eb8cea4a13f9a204f7ee4ade8f078352bfea612b8b08bc4b*
Theory Object Address:	*TMYics8vs8zP2a85EMxEbQ7v2qa88dpRPxV*

Definition 14.10 *We define* trichotomous_or *to be*

$$\lambda R.\forall xy : \iota.R\ x\ y \vee x = y \vee R\ y\ x$$

of type $(\iota \to \iota \to o) \to o$ *identified by the following information:*

Pure Object Id:	*cc2c175bc9d6b3633a1f1084c35556110b83d269ebee07a3df3f71e3af4f8338*
Pure Object Address:	*TMWfJY8TrCuqdBbXjXtyyPgdMaJFFhSYPxa*
Theory Object Id:	*61a1215cd148d58b01ffff5d1f427041f8684c948585f39446780cb2f09332f5*
Theory Object Address:	*TMShCrFwwqitm33uX4Zfg5LP3nxx3wiWms6*

Definition 14.11 *We define* partialorder *to be*

$$\lambda R.\mathsf{reflexive}\ R \wedge \mathsf{antisymmetric}\ R \wedge \mathsf{transitive}\ R$$

of type $(\iota \to \iota \to o) \to o$ *identified by the following information:*

Pure Object Id:	*406b89b059127ed670c6985bd5f9a204a2d3bc3bdc624694c06119811f439421*
Pure Object Address:	*TMW2bk6NEbDRdxmx9iWTp8eEaq5Bqs8wPgv*
Theory Object Id:	*c6fcf7ac241b0cf0da35ccb0b15e5793c002ba2e047a608000d69fd9887f713f*
Theory Object Address:	*TMbQ7jLDSWGqbR63ibKa1g9z7w1VxQwMmpi*

Definition 14.12 *We define* totalorder *to be* $\lambda R.\mathsf{partialorder}\ R \wedge \mathsf{linear}\ R$ *of type* $(\iota \to \iota \to o) \to o$ *identified by the following information:*

Pure Object Id:	*0e5ab00c10f017772d0d2cc2208e71a537fa4599590204be721685b72b5d213f*
Pure Object Address:	*TMKx8GCXERU42pBKUXaDWCBbgA3pQMRP8tN*
Theory Object Id:	*f82fe6af52f2eb7dba22be27f833973c1fc595caf2ed9e8d39761e844f892c7f*
Theory Object Address:	*TMdPhbDwqcJQrVhsNYionDKzTcsQAEnNsVD*

Definition 14.13 *We define* strictpartialorder *to be*

$$\lambda R.\mathsf{irreflexive}\ R \wedge \mathsf{transitive}\ R$$

of type $(\iota \to \iota \to o) \to o$ *identified by the following information:*

Pure Object Id:	*ee5d8d94a19e756967ad78f3921cd249bb4aefc510ede4b798f8aa02c5087dfa*
Pure Object Address:	*TMKCFjFsuHbvaNUnpivHQWRNoRicsnmAMxK*
Theory Object Id:	*c5200681a8c27017c79f5fdf3f12ab2bccac1fae4bdbf8e1fda11162db118256*
Theory Object Address:	*TMUM3ZcpFdPNXn5zGKLbrqdF1jPboSGWNuB*

Definition 14.14 *We define* stricttotalorder *to be*

$$\lambda R.\mathsf{strictpartialorder}\ R \wedge \mathsf{trichotomous\_or}\ R$$

of type $(\iota \to \iota \to o) \to o$ *identified by the following information:*

Pure Object Id:	*42a3bcd22900658b0ec7c4bbc765649ccf2b1b5641760fc62cf8a6a6220cbc47*
Pure Object Address:	*TMUQBjSjcsVDhSC7wkbaDUwYx5frU3mzZYL*
Theory Object Id:	*fc46b3ff515f15dc7d235be5f2ed71d758b31a1c27552cfb9c6a33f42d1a3a4f*
Theory Object Address:	*TMJjLnVKzQjT4WNmmFgDZ2Wjjd3Y3FEBCBr*

`per_sym`

Theorem 14.1 $\forall R : \iota \to \iota \to o.\mathsf{per}\ R \to \mathsf{symmetric}\ R.$ *The proposition is identified by the following information:*

Pure Prop Id:	*79f95ef7f3d535d6717f7ddc757df9b5e552d3ce9b4bdfb2e6a7fe0937c98439*
Pure Prop Address:	*TMHJHazkkUfb6aCs84p71oZHrkAA9SrRF6B*
Theory Prop Id:	*9c47256c1ffdf0d6543ef86bbda771b7500ac846168ef7cd276fefdf6ae54a1f*
Theory Prop Address:	*TMGvYGQrs8TGK53ZwpRhhXKcriX4siG4aWK*

`per_tra`

Theorem 14.2 $\forall R : \iota \to \iota \to o.\mathsf{per}\ R \to \mathsf{transitive}\ R.$ *The proposition is identified by the following information:*

Pure Prop Id:	*607d155b87eb1945d8bd159974a5bd7062c2dea047295c027068baf6d81c8994*
Pure Prop Address:	*TMPq6w6Ar2UhxZJQGGLkL7M9a8YzYHyjaEw*
Theory Prop Id:	*8ce50de71d645606be80665db71c58b8813cd0d19d60d8b763ffc406b2efe6e4*
Theory Prop Address:	*TMXqAeANVNhbDDSZf3ThqJh57d5d8akrVuU*

`per_stra1`

Theorem 14.3 $\forall R : \iota \to \iota \to o.\mathsf{per}\ R \to \forall xyz : \iota.R\ y\ x \to R\ y\ z \to R\ x\ z.$ *The proposition is identified by the following information:*

Pure Prop Id:	*25c6c513d28204a6d67aec12f103a90010140f5c065ee6e81a2a235388669203*
Pure Prop Address:	*TMH8vH9JbJk1oRxfLTaWkXDsdp1tBajpRuU*
Theory Prop Id:	*7450d4de2a0dc28afe52b779285309d8272e4ecc9d18b29694db6d0e88c2c70e*
Theory Prop Address:	*TMWwhnqMzeuBU4txn9M7KbX32G2MYUsYXWX*

`per_stra2`

Theorem 14.4 $\forall R : \iota \to \iota \to o.\mathsf{per}\ R \to \forall xyz : \iota.R\ x\ y \to R\ z\ y \to R\ x\ z.$ *The proposition is identified by the following information:*

Pure Prop Id:	*42041dce9407f5e451a83275d4ed3562d50e979b371ac9cfc9ce2bbf65ffb11b*
Pure Prop Address:	*TMczKpKZoBa6ULrfgZpdRBj5NNSBdUXVrmD*
Theory Prop Id:	*c1c6a191a79810f500d658f41a2be9542a0355f4071cdcddc0e1aca07f3db296*
Theory Prop Address:	*TMYhQJwgjHAdPXsFBVaSDYV1rJZ3eh8ERrF*

per_stra3

Theorem 14.5 $\forall R : \iota \to \iota \to o.\mathsf{per}\ R \to \forall xyz : \iota.R\ y\ x \to R\ z\ y \to R\ x\ z$. *The proposition is identified by the following information:*

Pure Prop Id:	*d7b95260b2573ec1f7c5e60368e6752db7fa181ff7cfcb71c4f345b0bb109dd7*
Pure Prop Address:	*TMGmGZb54Qqh3fp8xP84i6ikrSsAZHtQrK2*
Theory Prop Id:	*21217c9c11e8e6fc8b65cd0e9e3a540900e413d26333d881e5c72b28ed682b59*
Theory Prop Address:	*TMR85C3Y3ZcsxRT7X96TVb5FfvijohyYrhB*

per_ref1

Theorem 14.6 $\forall R : \iota \to \iota \to o.\mathsf{per}\ R \to \forall xy : \iota.R\ x\ y \to R\ x\ x$. *The proposition is identified by the following information:*

Pure Prop Id:	*c9ce8823077b6bac6fe963cb5c95ed848be8455934bbdab468e7fd51a1b26027*
Pure Prop Address:	*TMdPMQFgkvEnLZgXHL1Y1PTzuAK9GLNXFGi*
Theory Prop Id:	*45e4c5e058614a4b8b7d87bd59727e65f215a29fc3061feeb56cde8a17916335*
Theory Prop Address:	*TMTZp74GpF3FnWhQACszYMqVy96d8NVbFXr*

per_ref2

Theorem 14.7 $\forall R : \iota \to \iota \to o.\mathsf{per}\ R \to \forall xy : \iota.R\ x\ y \to R\ y\ y$. *The proposition is identified by the following information:*

Pure Prop Id:	*bbc879269d0205d16beaa6eee6fd30afbcdffa8cec1ac7f6b3b8d26e715d2345*
Pure Prop Address:	*TMbvg6t3f8pBbTLcBciLrre2aCKoFw7zfno*
Theory Prop Id:	*5755e01c047ecd7f625b9e98dc3a6c0651bc16c7c536503e8732f0ecc467f80d*
Theory Prop Address:	*TMLeJkqhNb5gsQPB2bsXFJnDEF5uSEmMFUv*

partialorder_strictpartialorder

Theorem 14.8

$$\forall R : \iota \to \iota \to o.\mathsf{partialorder}\ R \to \mathsf{strictpartialorder}\ (\lambda xy.R\ x\ y \land x \neq y).$$

The proposition is identified by the following information:

Pure Prop Id:	*6d855f41855d31ad7129a125faa7f0b81aed14cfe4b95537164fbea0352b7169*
Pure Prop Address:	*TMGDbQFv1xTKnLt1axDcGD9skkrk1NDjguy*
Theory Prop Id:	*f2be7026be53a9cc86a061447a9845aea99c3af1fd7b0e0a1be19f93eac40826*
Theory Prop Address:	*TMW6QAQXciNiq23bCcarz6nxhYqvHh8imu8*

Definition 14.15 *We define* reflclos *to be* $\lambda Rxy.R\ x\ y \vee x = y$ *of type*

$$(\iota \to \iota \to o) \to (\iota \to \iota \to o)$$

identified by the following information:

Pure Object Id:	*71bff9a6ef1b13bbb1d24f9665cde6df41325f634491be19e3910bc2e308fa29*
Pure Object Address:	*TMT2htU1EHitc1tMVfJYMWKQyBmwmD6rtWn*
Theory Object Id:	*231a7bacb9df61407922150b01fd5a8408982ba28cc53a7b1116373732b564e5*
Theory Object Address:	*TMUaCQBAHyUetsLgqr9moZkwAZ6DHmvevzv*

`reflclos_refl`

Theorem 14.9 $\forall R : \iota \to \iota \to o.$reflexive (reflclos R). *The proposition is identified by the following information:*

Pure Prop Id:	*04de27b34b8ee3720731f1aa3b07e08c79a5f5fe35b989e0e31c85df7d76188b*
Pure Prop Address:	*TMRXSNELTTP8r956vacXYEerEnidnHV3YDx*
Theory Prop Id:	*073a437b4c2e1d6de20d186d3a62ccd4fbe468541fc10b5301ffbd1460aebad2*
Theory Prop Address:	*TMcjRsdk3p8hhg3NaZJTez4aiFcKytNSMzj*

`reflclos_min`

Theorem 14.10 $\forall RS : \iota \to \iota \to o.R \subseteq S \to$ reflexive $S \to$ reflclos $R \subseteq S$. *The proposition is identified by the following information:*

Pure Prop Id:	*3c9bfea39a96499adf86a25b70a01d86dda56919bef77987d48bd540eeb47105*
Pure Prop Address:	*TMdFsDfTaJ75o8jviDeot8gUFggEbgrSQk9*
Theory Prop Id:	*c6c85f19b2d660c314da672cf7ec92b47ac92205119f6d34f6f8d78bae7eb2aa*
Theory Prop Address:	*TMbsP4tFMHwzMtNmcrDFHkzc6XuNP7FvzeF*

`strictpartialorder_partialorder_reflclos`

Theorem 14.11 $\forall R : \iota \to \iota \to o.$strictpartialorder $R \to$ partialorder (reflclos R). *The proposition is identified by the following information:*

Pure Prop Id:	*aae20a4db679340a3b3489212833014ae04ff05454150367d01c07193f0b3f6e*
Pure Prop Address:	*TMRH8GAJvU7QCQVeMCvrdQxYzWTt23ioYvb*
Theory Prop Id:	*8909d872000d95634b270e22a50ab08882a2efd39862ee4dc9372226770395f5*
Theory Prop Address:	*TMMc6BxYynTzGhkvf3wwkqL4XNB328wz9Kc*

`stricttotalorder_totalorder_reflclos`

Theorem 14.12 $\forall R : \iota \to \iota \to o.$stricttotalorder $R \to$ totalorder (reflclos R). *The proposition is identified by the following information:*

Pure Prop Id:	*76031124f76b48f04bc4f4bef1b143e3db985fab98172116e9eb3b62c7a65a54*
Pure Prop Address:	*TMTpzN3QGmt1LjMzKNCWXA5CrTNPPXdsGom*
Theory Prop Id:	*6057db1f3e57d31b555a933f5f17681a6a725f7108faaef6d41f2d8e944fd1ca*
Theory Prop Address:	*TMLi2krY1WNGGUsbxEr2fxHoJC6YZNrmfvS*

Chapter 15

Zermelo's Well-Ordering Theorem

15.1 Zermelo1908

Definition 15.1 ZermeloWO *is the opaque object of type* $\iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*36a362f5d7e56550e98a38468fb4fc4d70ea17f4707cfdd2f69fc2cce37a9643*
Pure Object Address:	*TMYCW3jauecphYdjePotuDSXP7ZXkfAzLaT*
Theory Object Id:	*e62050256ab27fb78004baab27aff9e7e88265df5714b6aeafcd573b2e7015f0*
Theory Object Address:	*TMZVEYfCFAjoTjAywDY2bSVjv6eXC6FdX5Z*

ZermeloWO_Eps

Theorem 15.1 $\forall a : \iota.(\mathsf{Eps\_i}\ (\mathsf{ZermeloWO}\ a)) = a$. *The proposition is identified by the following information:*

Pure Prop Id:	*967f2b509c968b55983160582bb89f4b3b392cdb74bd3014ddadec13cba49a90*
Pure Prop Address:	*TMaCPTLDmtonLabJKS8VNg2KpQHWXLBoNXU*
Theory Prop Id:	*56530eac409ade83907328e28fb11e3478cc0f4637d0fbe97f076524758f297c*
Theory Prop Address:	*TMTxzfNoxPLisqEQk9tTQ8t7JzkFz1bXCit*

ZermeloWO_ref

Theorem 15.2 reflexive ZermeloWO. *The proposition is identified by the following information:*

Pure Prop Id:	*0debc67241d3f1b73ddcdb9242748e16c0a306dd4c5bd3be38285ce4c810b221*
Pure Prop Address:	*TMZLFAPmHBxLwpqpQcE1qZeY2pPcfX3QVi5*
Theory Prop Id:	*3087155d06daaf3f9264297022e0ed58f4b93c8b8bfbbeb282039734efef904e*
Theory Prop Address:	*TMUkjTCw2R4W9Puur76M31DHpHBxtkcez6n*

ZermeloWO_lin

Theorem 15.3 linear ZermeloWO. *The proposition is identified by the following information:*

Pure Prop Id:	*79880f17e3e9a931262c694b18e962541b3131893ba53d44229166cd1edc06ee*
Pure Prop Address:	*TMJkDnfbxikrHuEoUB2KH59QL83E8yZkNdP*
Theory Prop Id:	*feb13fbc27d9aae829703d3a69e71d745f6bbb8008d16029b663316db4a0f0ca*
Theory Prop Address:	*TMMBkNPnE1Xevjyn43F624QPHNDG2eYWp52*

ZermeloWO_tra

Theorem 15.4 transitive ZermeloWO. *The proposition is identified by the following information:*

Pure Prop Id:	*9206e3397ca8b9ed09881d5cb76cc80df1ee726499d1f8f1b5c1780d5e9f59ba*
Pure Prop Address:	*TMSnBoLW5VjgRKasiFgj55RonswXagBwUxF*
Theory Prop Id:	*0cbc46f560bfdfcf23991ad12021c892678d58d3e631d71ff5bab43ce776b5a2*
Theory Prop Address:	*TMSyNvgsZEkSPxH6fbpbuyr29yGLQk1uE2K*

ZermeloWO_antisym

Theorem 15.5 antisymmetric ZermeloWO. *The proposition is identified by the following information:*

Pure Prop Id:	*db152b33b6ab9a13d674d4d926a9099a24633405bb2ac4113622a3bc701b6e86*
Pure Prop Address:	*TMVoZux2uwtDAmhKY3JNavcVyuWpiLWkQ53*
Theory Prop Id:	*6e0efdc62d566eec4426eae46854f865abce0b3abf2038f5022a0c3129893eee*
Theory Prop Address:	*TMdx1hkuD9K2Mfu17Z1gRygw4fnFG42CyZi*

ZermeloWO_partialorder

Theorem 15.6 partialorder ZermeloWO. *The proposition is identified by the following information:*

Pure Prop Id:	*0656f9a90963b7a76e123603eb83d8b92e386a909153edb2d3e6bfed1e801d65*
Pure Prop Address:	*TMYe2evtB7V2D59AgeiBdZxBgksb6og2GHV*
Theory Prop Id:	*3499fde0183fea1cebe20732bc7ad2cb75fe284d9e4d45f292274921f510f343*
Theory Prop Address:	*TMPx6RFHGz7VtFGVUwqZbmtdXG9SYpYHcfG*

ZermeloWO_totalorder

Theorem 15.7 totalorder ZermeloWO. *The proposition is identified by the following information:*

Pure Prop Id:	*88ef7c493983ed7becf7f9e7c5dd717b8b1ad11361f29a5250e8aea67ebb6d56*
Pure Prop Address:	*TMQi6BX2vtRo2UJyxGGrvkYy62BsS4n19XS*
Theory Prop Id:	*d20df0c3bde7031c1b562bdbbd689604af6728a5b9c1130cdd7fd5a07e1cef0d*
Theory Prop Address:	*TMcqRgmAfxGTPiCTYL27ftCtLTjGab61vAE*

ZermeloWO_wo

Theorem 15.8 $\forall p : \iota \to o.(\exists x : \iota.p\ x) \to \exists x : \iota.p\ x \land \forall y : \iota.p\ y \to \mathsf{ZermeloWO}\ x\ y$. *The proposition is identified by the following information:*

Pure Prop Id:	*7e38ffca0702338d27fa602350391a1c15520286ef30a841a14806771c076a95*
Pure Prop Address:	*TMSRd1JSF4kWdSSvoi4w2eq61nrH7PDF95s*
Theory Prop Id:	*321d3a35eac90f0ec080d2b70a9efa74fb4cb4002111c116f5cd2d55be9418e2*
Theory Prop Address:	*TMLKb3jsZ1LNt1giMEWqCcJ48ttSzxEektg*

Definition 15.2 *We define* ZermeloWOstrict *to be* $\lambda ab : \iota.\mathsf{ZermeloWO}\ a\ b \land a \neq b$ *identified by the following information:*

Pure Object Id:	*e5853602d8dd42d777476c1c7de48c7bfc9c98bfa80a1e39744021305a32f462*
Pure Object Address:	*TMNLYaDifZ1BU7nW96xprhzT7ZrQZLkjzMb*
Theory Object Id:	*cef2db95ff08afb8db1745dfd6898d6e39064f4098ca0acb0061eafac129c1b4*
Theory Object Address:	*TMTjf9r8y9UHsnCWkcHJQd5ZgraPrRuvcnA*

`ZermeloWOstrict_trich`

Theorem 15.9 trichotomous_or ZermeloWOstrict. *The proposition is identified by the following information:*

Pure Prop Id:	*157b48f20feb46068f0f2aca7b452a507ead7d12b83628b031a51c5f14ce0e37*
Pure Prop Address:	*TMGSAsnBLMuRws587dr7obynuEWa3KPPuiL*
Theory Prop Id:	*ee31d14ab8f6580d4d2449c878828d85530f8e2efa920c10cbcd5cc992f53766*
Theory Prop Address:	*TMJCGmPXs5syPbHaK8NokgLMNiDtwNKeZYH*

`ZermeloWOstrict_stricttotalorder`

Theorem 15.10 stricttotalorder ZermeloWOstrict. *The proposition is identified by the following information:*

Pure Prop Id:	*56f948bc4585b5f5c01ab14c1def7e85a8b8a620235ac296d35a8d94e34c7b1e*
Pure Prop Address:	*TMYR5UnFMSJLwv3HU9FtTvsT9LpzdoKVbZ3*
Theory Prop Id:	*c2537c16e167c192e2f11d7811816cd1fa923138f2630e88388223d36fa49e70*
Theory Prop Address:	*TMLePQqfgDYUVPqeJYmRyjMFNhntRXwDCMy*

`ZermeloWOstrict_wo`

Theorem 15.11

$$\forall p : \iota \to o.(\exists x : \iota.p\ x) \to$$
$$\exists x : \iota.p\ x \land \forall y : \iota.p\ y \land y \neq x \to \mathsf{ZermeloWOstrict}\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*d7160045949b81cdd599a4103fed7b65d41988baaee1332de79e879bbbfb0dcf*
Pure Prop Address:	*TMVwVvUZQVTnP3hnKczqoHkD8Zxt76L7hq4*
Theory Prop Id:	*458f09e17af7e183e42d050a345d94c93e451920cc3aa0f15d74c38ff9645ca4*
Theory Prop Address:	*TMEvCCPnMvJRotk1TMESMYoU6fL8ARNf8cG*

Zermelo_WO

Theorem 15.12

$$\exists r : \iota \to \iota \to o.\mathsf{totalorder}\ r \wedge$$
$$(\forall p : \iota \to o.(\exists x : \iota.p\ x) \to \exists x : \iota.p\ x \wedge \forall y : \iota.p\ y \to r\ x\ y).$$

The proposition is identified by the following information:

Pure Prop Id:	*d37c8533f52315f30dedfc2fe79f9a6c647286dec8db8cfc0917eb94b01c80f2*
Pure Prop Address:	*TMTrdxnBmgaKrgsRSyEtgvNwTWRhUzG7QEW*
Theory Prop Id:	*3b56530ad7e26682eefc8e7eaf7c2e18d87597abef3ce1d4dd28adb08a1bab16*
Theory Prop Address:	*TMXrj7GDDD1CizwhmhqkHEbXjtQ236xfNTd*

Zermelo_WO_strict

Theorem 15.13

$$\exists r : \iota \to \iota \to o.\mathsf{stricttotalorder}\ r \wedge$$
$$(\forall p : \iota \to o.(\exists x : \iota.p\ x) \to \exists x : \iota.p\ x \wedge \forall y : \iota.p\ y \wedge y \neq x \to r\ x\ y).$$

The proposition is identified by the following information:

Pure Prop Id:	*77cb5b1b3db7a63607bc9d07653bc63fb19f0a5a2226c66935ac1668becd94cf*
Pure Prop Address:	*TMGPaaZebUwDYiYk1ANGBmfqMyyCg7WMmg5*
Theory Prop Id:	*57f61ff6fb48481c18b75895db9d64e93f2f58e90b52ec3ec4e59d1efa68cd83*
Theory Prop Address:	*TMSPtM5ooeGZYBnjiKvH9mKzYtMVNv3LDSW*

Chapter 16

More Logical Properties

`eq_imp_or`

Theorem 16.1 $(\lambda xy : o.(x \to y)) = (\lambda xy : o.(\neg x \lor y))$. *The proposition is identified by the following information:*

Pure Prop Id:	*06c3093aa740f38323ae4d7f0dbad8d508f40a0ca90fa41e9f01516651c3e3c1*
Pure Prop Address:	*TMFQ6hVxLyhKxQEQs23Xv9LC6mgW4iwwgrX*
Theory Prop Id:	*4c9dc0935ef80cbfc0fc55cf8abb066203b42c66e60c4bb6af8b939a9f3dba5a*
Theory Prop Address:	*TMRjkjWby7U8Aw7kefWq3i3PMiD7ahDD43F*

`famunion_Empty`

Theorem 16.2 $\forall F : \iota \to \iota.(\bigcup_{x \in 0} F\ x) = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*fc1bdd286f0d898d898c29f18fbe095090efb65893028ab68d5f98d8caa05ce9*
Pure Prop Address:	*TMZjHxMN3FYwok47JsaP7dDQjKXS6jyzcQx*
Theory Prop Id:	*22bd10cff6495d6e03e6e76619e1426110cc714a9d78d666213a9e4d3b7897b2*
Theory Prop Address:	*TMTAVVZK8aANakE2V9u1NoVueYVDuUeiPWu*

`Empty_or_ex`

Theorem 16.3 $\forall X : \iota.X = \mathsf{Empty} \lor \exists x : \iota.x \in X$. *The proposition is identified by the following information:*

Pure Prop Id:	*ea40ae60ad1226481a72c72991900ab1d75eaab2f2603b30f4021ec9bfa53243*
Pure Prop Address:	*TMN7F3M7BEyv9u1wysCYB4eCnjR17p8fFPJ*
Theory Prop Id:	*16587d445d1c65fd759cdc12b7fec9089c4b514284913be85bb2bfbaa6fe4f57*
Theory Prop Address:	*TMGoeYYeEjatMCpA5EmBZSLRbtsqeYFpDcB*

Chapter 17

Natural Numbers II

`nIn_0_0`

Theorem 17.1 $0 \notin 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*09a099717c75645aead366c3b2036df70c9289907d2755e106f75199e5674813*
Pure Prop Address:	*TMMk6er7mxLyAi5T6t3WpKRXJCMS7xBHwqu*
Theory Prop Id:	*865a398e739b1785dc276dbe8cd4ed7fb4f1d8782c59e0b8e3b09c70fd4034c3*
Theory Prop Address:	*TMSwDSAjn1RkA5QxqBbEUmy3yKyr57g6mWj*

`nIn_1_0`

Theorem 17.2 $1 \notin 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*b197676023d2efaa19e70e356beb7444d5240c7348ab140126a015307c650d50*
Pure Prop Address:	*TMavi1ZRdgBBbDLvwBQchihXtPKTjnNdhsk*
Theory Prop Id:	*2fd3e19cdd9862468dc112216fa972acca941b4de769d35cc59d7ec1577cf0e0*
Theory Prop Address:	*TMdqjwWDUF4ntu7oqJ4gg8mEYbpw158A4BJ*

`nIn_2_0`

Theorem 17.3 $2 \notin 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*135dfc520ebfeb7876e0ee1477daff54f352c678350712bc7747990b981da558*
Pure Prop Address:	*TMXsBVAtddQ17BGoDpSMPvxAsXgE6R4Qm3B*
Theory Prop Id:	*556f07538b44668c0a147b4fc3e95e1df80a4f96ee895c6106b88cb0beddabcf*
Theory Prop Address:	*TMZ7o2p95PaehpRzBsCsB83NrmH4JcRBHJN*

`nIn_1_1`

Theorem 17.4 $1 \notin 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*e668f88d6e9498b6de122df37d7c3f2dc02798e0ebb033a625b6e3b40d880a0d*
Pure Prop Address:	*TMLzAqBPo85Hx5GYyr7xuu18NVjreNUyRSj*
Theory Prop Id:	*7774094463b5d0735559a4bab3714251e4442b5e3727856df52e3ae54b23c64e*
Theory Prop Address:	*TMUrdNWeTXSbGVqA84DoNZn9BGLr6upydqx*

`nIn_2_2`

Theorem 17.5 $2 \notin 2$. *The proposition is identified by the following information:*

Pure Prop Id:	*31c298db536bab1cc362a2d791118533b2cff78953407ece1ed3125867a3beb4*
Pure Prop Address:	*TMXFjntzEU8n1QS25SjFvEdpbj6sq7qbV7d*
Theory Prop Id:	*94398399c4d60eec85399c9cc75054c87b16f94488166ce079063afe25b9d45a*
Theory Prop Address:	*TML9u9hUZHGL9ztWF6S5eMPgi1WMRevw1sY*

`Subq_0_0`

Theorem 17.6 $0 \subseteq 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*e9749df5528d6c773cdda6fe3121311c07ae4197f8e3b399882f384c8541d735*
Pure Prop Address:	*TMLX5CPCFVK66pFehnmXXgUmMn3P7noB8s2*
Theory Prop Id:	*6e655c00e783ba0a41d0029eaa2f71c798229b93c15f02483aafd5566f0a81f7*
Theory Prop Address:	*TMLxnx5doMjojfharrvSwqG5tst7XcCPdHE*

`Subq_0_1`

Theorem 17.7 $0 \subseteq 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*cad7c1cae1ff938a082e5d73d82de4e76f9337136b94ed225e559e21d73e498a*
Pure Prop Address:	*TMbAbdRq1jz52CySGPWWJzvEMuCMw9PQWf2*
Theory Prop Id:	*f75cd0a8e8a447c357f931be8b2e1f23b89f6b92f2086dc3ff43276f2e88d35d*
Theory Prop Address:	*TMH3GTPuv5BQPyQShNEpPrnhYzZUwtmu5ib*

`Subq_0_2`

Theorem 17.8 $0 \subseteq 2$. *The proposition is identified by the following information:*

Pure Prop Id:	*4147baf022e55cf2dd6aacf5e6a00ad38638a106bc08411c05387011d04bb7e3*
Pure Prop Address:	*TMT9Kq6Qfozmik7TG8YKt8eBb884fWgrGUV*
Theory Prop Id:	*e55065b32674ffef3065ed52eb5884c80c49f9fade95b3f25de91a675649b8fe*
Theory Prop Address:	*TMNVFjtQ2Uu5aRqUQUGEuZaFCmJzys2dTcX*

`nSubq_1_0`

Theorem 17.9 $1 \not\subseteq 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*cd48c3a937c072b881fc81c095acf2d3d19992c467934cbf8c623a3e0b31c7b8*
Pure Prop Address:	*TMMWurweEZoERkEgMW94uw2qJFHhYfSdhr1*
Theory Prop Id:	*f57d867ae9fd7ef331bb12d8bbc87507a39c321aa63dccd26566dd794deb29aa*
Theory Prop Address:	*TMJfLY5kzU7711oJKtQgwAjc5L8YVWwNjDY*

`Subq_1_1`

Theorem 17.10 $1 \subseteq 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*6810b4ba8d863b656650594ed3d6b824b91deebd03aaa139e90a032fdc3f761e*
Pure Prop Address:	*TMVoM4uwCGDUnRZZHZKXUcXzt6mXEkzgVcY*
Theory Prop Id:	*786cb1e61bb2545b3c1fa96512a920199016566e4b90be78896f3912039ebdd6*
Theory Prop Address:	*TMcdzMHtEBTXABdHGrdDC4Ly4fpUtxsqxBG*

`Subq_1_2`

Theorem 17.11 $1 \subseteq 2$. *The proposition is identified by the following information:*

Pure Prop Id:	*83e0b593f193b7c5aaa42ebec3fe80fdfec8811d24ad5d1ca621e5e328669b5f*
Pure Prop Address:	*TMKdW2ZiFjAjtmtoqqe1oUa34818gAT8iNr*
Theory Prop Id:	*eb94537ff458732f1b6a217159a18b54524b675d0382a95296b651d3ba8f33c9*
Theory Prop Address:	*TMYt6f8BQGRNnMa17NCKpryL5i39M9NdrbM*

`nSubq_2_0`

Theorem 17.12 $2 \not\subseteq 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*5810f58237fa5ea1a72b1b1a19c59174248b204a444cd813a87d5c24b492efed*
Pure Prop Address:	*TMapUXLAzmMk7EFCn8wdLE9wngohhRx64bJ*
Theory Prop Id:	*3a7512da2ec785465bad179179151f9a8406012ca9652a22c5ee00b1c1e8d549*
Theory Prop Address:	*TMc22TospZpLCRqLYeAzy9Aw2a8eBegKoHu*

`nSubq_2_1`

Theorem 17.13 $2 \not\subseteq 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*63bae94df530a1ce94daf68c68462bbef84a4139da4811189d52312898d4d441*
Pure Prop Address:	*TMSu6yS8YSy2wRfQSYGzR3kr6TiHARnPaqe*
Theory Prop Id:	*0a8588177b5ee4d07cdd286638afa3c070f40cd1a82c6cc63bee5d057c8b5211*
Theory Prop Address:	*TMdVb69DrHqVTcNAtqfbK4id4jxAiXFYfQd*

`Subq_2_2`

Theorem 17.14 $2 \subseteq 2$. *The proposition is identified by the following information:*

Pure Prop Id:	*7b90ad5115eabb6dc9245caeb7cbed2927cbb691b26ae5dcecad88d6115f09fa*
Pure Prop Address:	*TMPuu9Nr9pfr2rJDgk5gScNW6sYP8p19FRM*
Theory Prop Id:	*0a1db4164bb30172507271531b8defd31e8ab3f2d787c5b272ee3bebeb2966f3*
Theory Prop Address:	*TMUA9eA6KznSVoDTCQaVPvd6cuC6M5SRgKy*

`In_0_7`

Theorem 17.15 $0 \in 7$. *The proposition is identified by the following information:*

Pure Prop Id:	*a41b2c54f4d65391d6b32fb603ebffcff201103f2878a4a55f687b12bbe8b91a*
Pure Prop Address:	*TMdFjNSCWXW8Vpvt73BWqM4Zj86eN5S2KNu*
Theory Prop Id:	*4cb36dedf7517e301f2945346202891f39d34107c01ef2e3b86d9e51b259df54*
Theory Prop Address:	*TMUvTmAX6RDWKPVpacNDGZQ9p9sYmuoibQG*

`In_1_7`

Theorem 17.16 $1 \in 7$. *The proposition is identified by the following information:*

Pure Prop Id:	*aa5cdecdaec91fe3a6884b0f09c06631327c30886010ae16f21daddf6ba9c94d*
Pure Prop Address:	*TMJSrd3NEP4iMdJBKZ7F1cS2TZynGhaxyQZ*
Theory Prop Id:	*392ce8e7c3bb5ff3aa4fa822b0258c7f80e76dedcd760af5717a6e916fa61cae*
Theory Prop Address:	*TMTHHZrGZFsevPcmaS6RDEwByfki1EHr2Fo*

`In_2_7`

Theorem 17.17 $2 \in 7$. *The proposition is identified by the following information:*

Pure Prop Id:	*b26cf008f7aa0e7ffcb65a467f1e4f72dd8227f1089ee19c2b649ac8f1118454*
Pure Prop Address:	*TMZoShAAJg9Vr9iuc4kJeMjUubQz3hbFXXd*
Theory Prop Id:	*b47fba676b401c0467d0573bc3d555b8474735646a191dadb139c79d11b2e544*
Theory Prop Address:	*TMUSma8kwGkSuHzU6fZivqqsFZseHSZ6rHq*

`In_3_7`

Theorem 17.18 $3 \in 7$. *The proposition is identified by the following information:*

Pure Prop Id:	*e6779a7f3f395ebcac3bd6e88ecd8088dc9671cd31a2b7cd8763b2d1967a07e0*
Pure Prop Address:	*TMZirYDzJAMpfJGxrd52NTyUfVEWH2SZGzA*
Theory Prop Id:	*38ad3433d50b88a23443fb181777702afa2fd1d0aef7d62ea69949525c1e36c0*
Theory Prop Address:	*TMPbKx6jYxRGDnKufFfM4TW7RyjNLYW4g4p*

`In_4_7`

Theorem 17.19 $4 \in 7$. *The proposition is identified by the following information:*

Pure Prop Id:	*2d470f9c2b92bd61f616a640439618f469a0d784cb0b08a1a90625b00c1d5ba6*
Pure Prop Address:	*TMFfR8LTg8YNKJkWgWwamN1Lgup4j48Whny*
Theory Prop Id:	*07ae6986b58c8e30c6133f6e2016bfeae67ec868d1227fa18a921392adfd8d2a*
Theory Prop Address:	*TMPy9kPwbcVRmzis4rmmQajdnJnWBPWoMWA*

In_5_7

Theorem 17.20 $5 \in 7$. *The proposition is identified by the following information:*

Pure Prop Id:	*685d80ba7d066c28aa1d043f7db6d4b5114f616d882a3bfaf71cd542a148e4ba*
Pure Prop Address:	*TMMku2XXJ9wEw7xtWmoDtdpUqHzpip5x126*
Theory Prop Id:	*81137f00c9dd3d97ce30878bfcd5a44b55d559d1dd83f2f941053dd60b37883f*
Theory Prop Address:	*TMZb8pTSFybxEM3VZWF9cs1v9kTeYLx2Mn3*

In_6_7

Theorem 17.21 $6 \in 7$. *The proposition is identified by the following information:*

Pure Prop Id:	*cb4ca2976fd69ef1a079994c73071be6b55cea8db7238fccc9916cc3dd669ed7*
Pure Prop Address:	*TMMcWExWsKZNzZxK6mn8N5UozNgsLWG22jv*
Theory Prop Id:	*fe9b561a0b2c561827d50aaf6b01fbc0a1fe14ef472c16cc21b1738fdf7693ce*
Theory Prop Address:	*TMa39PQbzrwe6rPaWAdYhM6GqbCbvwB5GFE*

In_0_8

Theorem 17.22 $0 \in 8$. *The proposition is identified by the following information:*

Pure Prop Id:	*0f58323098cf1c7b76cfc3eeea9678f53a42cf23af1f553934572eb5b3fedb4b*
Pure Prop Address:	*TMbVc3Gz8q2ncEs1dxxpVYDJt5maYv7XRYR*
Theory Prop Id:	*7797f1bcb9aa6b1d1d07565f754d8639d16574c1a6b38967e7432403cfbda877*
Theory Prop Address:	*TMQfdecCs6mDoNMAna5TMVPYcgtewWsADLX*

In_1_8

Theorem 17.23 $1 \in 8$. *The proposition is identified by the following information:*

Pure Prop Id:	*fbef0f51f12e7c2ae21b16ba602e7f68892a1dac9c238bf6b4c1f2bed4eb34d0*
Pure Prop Address:	*TMXRwsKNE9viw546hC36BY9U9SuJziTMUDB*
Theory Prop Id:	*0d7bd5f8b87d778142f6aef2fce3ecfd64b37623004ce8979fac7bb6dc71ecc1*
Theory Prop Address:	*TMXL6Tdr6bmwMFVqYqUkidhjWc9dqLm4zuV*

In_2_8

Theorem 17.24 $2 \in 8$. *The proposition is identified by the following information:*

Pure Prop Id:	*a07c8ddbe590e66a92f9257de8f0fd799ef1483c2b0db58b970ebd947d68ede1*
Pure Prop Address:	*TMLXEF3MDk7aJtSHea6Ts5kyRqVvzMTLVMx*
Theory Prop Id:	*a625dbe848c33627c83019ef18fb78feb77869dc01561231f770a2c4534e4fb6*
Theory Prop Address:	*TMbknbBMXvxmm4ESoH75pgyt1UuzokTDX8U*

In_3_8

Theorem 17.25 $3 \in 8$. *The proposition is identified by the following information:*

Pure Prop Id:	*7ecdad34b9fda66b48c56dc6bf6ef24f1dd82edaf5b10a676dcb5b1c71d69d4d*
Pure Prop Address:	*TMKDDRJXEjpy9C7xEE4XcgqVTYixinVGGZq*
Theory Prop Id:	*9b36a6f466e575fb7e2a48f68b7361e970bb6edc56671aeea80a5bed424770b2*
Theory Prop Address:	*TMN2SjRR2yH8LkDiZpxREFxmhDWA1tRrd8y*

In_4_8

Theorem 17.26 $4 \in 8$. *The proposition is identified by the following information:*

Pure Prop Id:	*61791c2e5bf306497941d536e18c90c32dc8db7afd6c98cd73d9bbbf773d2493*
Pure Prop Address:	*TMKmzqKEHSZcjMZkbLYvTdKBAypDeC2vkWw*
Theory Prop Id:	*219ccf028539e884cc6ed244a4235bffac83e454680efdcbe9746634113b96b9*
Theory Prop Address:	*TMapcHhFiw8K5Eyguh2XbYVss7iatNsa3XA*

In_5_8

Theorem 17.27 $5 \in 8$. *The proposition is identified by the following information:*

Pure Prop Id:	*d79eb5b8bab2d0ecce7a5262efe191d5cfa47f1546b64fca19ed3bbb64807bca*
Pure Prop Address:	*TMHdg4GnYxDRPSidyUUEWzVnoASUqBVUVB4*
Theory Prop Id:	*5625eb82bb63b6fa37d0b870f9d8240cc59102ee45b5c516d25f9c467415b0d9*
Theory Prop Address:	*TMZyKWFWemTmDJxiJkpygV8twB2Rtp15Roo*

In_6_8

Theorem 17.28 $6 \in 8$. *The proposition is identified by the following information:*

Pure Prop Id:	*bb5a775918672f1f9754e58316f4d97366beae541ee45c784b2caa7cf74b3d95*
Pure Prop Address:	*TMNL3L2gEnenuX7Xv2E3UpeGxkaNNyZdubb*
Theory Prop Id:	*3c5f2d23bf893b5c57c706c4203ef7618979c66711b505a3e143cd52883986c1*
Theory Prop Address:	*TMGi4xjiJrNb1ynt5cGWncJKwUJnCmsHNBs*

In_7_8

Theorem 17.29 $7 \in 8$. *The proposition is identified by the following information:*

Pure Prop Id:	*126eeac20dfdb85457404f82039e474f2a8eb9e482703a36265ea720baec0a3c*
Pure Prop Address:	*TMEjueeewmbW3RUuC5wTG9v3ube5BAnUEGQ*
Theory Prop Id:	*cc5985ce2772e22df62ec877dd1108f076a8d4ad53bb1b7aa440ceb21c281908*
Theory Prop Address:	*TMGFU5u8dWSnsHseshspGDHDJEYkpW2VFo7*

In_0_9

Theorem 17.30 $0 \in 9$. *The proposition is identified by the following information:*

Pure Prop Id:	*f12d8e4addf556e8ac897b31bd7774ba5f016a114488aa7da47a4982ecf730e0*
Pure Prop Address:	*TMXfAUPidgroRBVsfRFdmxVG3LXaAGCDfc3*
Theory Prop Id:	*5cb21ec1e1dd5e4c549e48ad0c126c468ccd82b46d5eea40b232ccf7c035b032*
Theory Prop Address:	*TMPTV8Q2AeyehU2pLq7PNkwxbmhsacvmwyR*

In_1_9

Theorem 17.31 $1 \in 9$. *The proposition is identified by the following information:*

Pure Prop Id:	*5922261855dc9c35d7a19be6bbe7337de066e405197aa8c3f331b75cb8f166d2*
Pure Prop Address:	*TMFXiJ1JnXuvAUNxeQTQDnCcXESe6geABqf*
Theory Prop Id:	*0d30f02a5d23823a40baed594ac7548d8af020600e5b2a30d8d190892e99ac7f*
Theory Prop Address:	*TMLQmTKRPrhZ6kBwPoMiaYrb5eJ5CspjmFe*

In_2_9

Theorem 17.32 $2 \in 9$. *The proposition is identified by the following information:*

Pure Prop Id:	*024448eff2c81d63335a32519066934b16f60b301c909ac62e1aa2496f034b12*
Pure Prop Address:	*TMSxqdqe4vyyMqhgByQKNDjzpe7hLiYJShy*
Theory Prop Id:	*330a457d080fe85bbe9e6674a86a4ae3f9e863e5ef5d34f3d5fbd1229256ac61*
Theory Prop Address:	*TMGa1rMZX1W4ZGdPLqcVb6bsiTpmjbxmANL*

In_3_9

Theorem 17.33 $3 \in 9$. *The proposition is identified by the following information:*

Pure Prop Id:	*afa2b48908b69981c7ad81b71c4bb9e93fb53c3c337307ba4e9d394be14b4eae*
Pure Prop Address:	*TMa3VWUSB3PNWdYhRYTBKk4sraF2pPhodut*
Theory Prop Id:	*6e40119249c9eff81c846d321343f9181f0fbbae5206bde40c6383b8c0ec20bc*
Theory Prop Address:	*TMP2oP3jaVQRn8x97tPUZDxRbcZQLEPeJ9x*

In_4_9

Theorem 17.34 $4 \in 9$. *The proposition is identified by the following information:*

Pure Prop Id:	*fdadb4647802a0e507d7b6593df90699056f72e47e5888a3135537fb646a3dbc*
Pure Prop Address:	*TMSFuF88nVp1p3DHdC3pEkjfaYLf6goH5vi*
Theory Prop Id:	*caa1f8ed95c772063de3aa6a8b2cbfb5e9d219c3229bb9c5ce31f5eca2f9de0b*
Theory Prop Address:	*TMbbYX4XLfjzv5dH8d76TFx5HmsrMRJ4tm3*

`In_5_9`

Theorem 17.35 $5 \in 9$. *The proposition is identified by the following information:*

Pure Prop Id:	*318ef9258894a50147815073ae5c4f15b2e60e8d262fe4baa2fd4ac0643491bc*
Pure Prop Address:	*TMQXwEo9W8vVJvGC5AHtnroNxcEsfWyG8Kd*
Theory Prop Id:	*f8aae6c774ede7b089d55ed0f0b30d8fa10e7089bcc1eb1db56f40c182c816c0*
Theory Prop Address:	*TMWDNdKoeVzUChM4ATtv7DwzjNjWouiN5u7*

`In_6_9`

Theorem 17.36 $6 \in 9$. *The proposition is identified by the following information:*

Pure Prop Id:	*dc3ff488e971d760fe164604c2ee2de729dd5be38a0bc74d93cedb851d661ab9*
Pure Prop Address:	*TMTGMooeB9YBX55ezejbgrg6SxmcrHwz22C*
Theory Prop Id:	*0c8dd335320b67f918a737c3818ade882eb57d2cecbad2d32baa17a47518ed1d*
Theory Prop Address:	*TMXsYFAkpquNtmfFQAYjQstpLXT7uYETZbU*

`In_7_9`

Theorem 17.37 $7 \in 9$. *The proposition is identified by the following information:*

Pure Prop Id:	*95d3ebc9e30fe58f8ee1d32b8b051ab32276f7f30609b2fdcd7cec1cf2c66cda*
Pure Prop Address:	*TMHUzLX2Z8CF8zE5QGfoumbM8j4vavy7ff5*
Theory Prop Id:	*02d55222d7d63900ce597b6f79802629c35164ffc62e82ce1b5c9351ae90842b*
Theory Prop Address:	*TMZYe552F7mK6w5rhKoUE4iCEyFHkzTaJsC*

`In_8_9`

Theorem 17.38 $8 \in 9$. *The proposition is identified by the following information:*

Pure Prop Id:	*aa9c6bc99164b52c64952a5e4c3f8ab0450b9a32b7a19a40e692e6458352bbdc*
Pure Prop Address:	*TMMVC6pC2fVWYrSnkueRZ7f3obgNuwZK5aW*
Theory Prop Id:	*981f43991092c5fd5e23973a29612ea078b2458ee398f5f4e27b1c669761e9ca*
Theory Prop Address:	*TMdurdeoZjzzmm64ypv6qGYNuFSjkTQnJtF*

17.1 NatRec

Let $z : \iota$ be given. Let $f : \iota \to \iota \to \iota$ be given. Let $F : \iota \to (\iota \to \iota) \to \iota$ be $\lambda ng.\mathsf{if}\ \mathsf{Union}\ n \in n\ \mathsf{then}\ f\ (\mathsf{Union}\ n)\ (g\ (\mathsf{Union}\ n))\ \mathsf{else}\ z$.

Definition 17.1 *We define* nat_primrec *to be* In_rec_i F *of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id: *3be1c5f3e403e02caebeaf6a2b30d019be74b996581a62908316c01702a459df*
Pure Object Address: *TMMruBGibDS8rkGTgYt2TbCgEshTqpc8T9L*
Theory Object Id: *1a64d5d130b189ae403562ffd2a8a2fa63feaf78ed6abca2742cea1af4e1547a*
Theory Object Address: *TMLxfukbsKf284nSh5ByiFX1hp1cggVTAUF*

`nat_primrec_r`

Theorem 17.39 $\forall X : \iota.\forall gh : \iota \to \iota.(\forall x \in X.g\ x = h\ x) \to F\ X\ g = F\ X\ h$. *The proposition is identified by the following information:*

Pure Prop Id: *c955cd8ed0958d2ed269c30204a3cdf992cbaa25660ad46a00085023b9a73746*
Pure Prop Address: *TMV6dvzCjCU8eRTowJftzbC63sPXHppC6uR*
Theory Prop Id: *9ba88707241ec8607e5cec9c0325d12d528f63b0670344631f540a848092b465*
Theory Prop Address: *TMFCUCvpQLtDtMF2PaNT39UWn3VGzEMY9Cz*

`nat_primrec_0`

Theorem 17.40 nat_primrec $0 = z$. *The proposition is identified by the following information:*

Pure Prop Id: *416eafbf1b348bac1c33766729696046e8de834bf29675a0bcbf8e9e2f4a2dc9*
Pure Prop Address: *TMVRM6N2YRqKYHAeVikSGWqmkhKWLLqo61v*
Theory Prop Id: *be11a277d9d20c3312b9592199a4932dcad62cb9aee7679da0a117c5016e5477*
Theory Prop Address: *TMZ6oFEPsAfDfMvyeTRLHoi489W3WuPogFb*

`nat_primrec_S`

Theorem 17.41 $\forall n : \iota.$nat_p $n \to$ nat_primrec (ordsucc n) $= f\ n$ (nat_primrec n). *The proposition is identified by the following information:*

Pure Prop Id: *79718550912e5445000ec2075df6854a99c68fbe7dc1bea4ca955eb813ce143d*
Pure Prop Address: *TMQkJomVG82rUFW7nbsE295nn7dZqExNfUV*
Theory Prop Id: *fe4c88c37a0678c8dd8db722e13c22b2d116aee8f889a70007d0ce07747f3f05*
Theory Prop Address: *TMXGF76gsDqDFEkGMFYsmNF4aDipvSuu3Xj*

17.2 NatArith

Definition 17.2 *We define* add_nat *to be* $\lambda nm : \iota.$nat_primrec n $(\lambda_\ r.$ordsucc $r)$ m *of type* $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*afa8ae66d018824f39cfa44fb10fe583334a7b9375ac09f92d622a4833578d1a*
Pure Object Address:	*TMQX4KdWNNKEeAgNGf2XNUoYDVDzvGnc31d*
Theory Object Id:	*29768b585374d1e88373f200fac530e11d07d4aec939607f546fda4fae32538d*
Theory Object Address:	*TMQ7riv5frn6wUQus29DXdY5DVorPU5wZwh*

Notation. We use + as a right associative infix operator corresponding to applying term add_nat.

`add_nat_0R`

Theorem 17.42 $\forall n : \iota.n + 0 = n$. *The proposition is identified by the following information:*

Pure Prop Id:	*402bcbdce23d41df6234b683ebd9d480b649637a52dbe21deab650c8ceded543*
Pure Prop Address:	*TMYQQZGZaNiuYCPSimRia2Xfn2Q6QWbtqhS*
Theory Prop Id:	*23277f785fd4b6fdf8ebe586a27b9fd4a47086eec22dc99b0d691329fba31a4b*
Theory Prop Address:	*TMXQW56dPBW8mAHqGUEmsjLRYKBrxAAonqY*

`add_nat_SR`

Theorem 17.43 $\forall nm : \iota.$nat_p $m \to n +$ ordsucc $m =$ ordsucc $(n + m)$. *The proposition is identified by the following information:*

Pure Prop Id:	*a53523a8fbb46795673b15c005b9c1c1d054e111ec64c524c3fa3b20ef02e932*
Pure Prop Address:	*TMTGvLHefupPU9uKtSQRk1QWQzFMwgJrrZU*
Theory Prop Id:	*e61e18415fcbe09a2e909f97e3a2d3fe2abd965f61f8aab6a960f324295f0c82*
Theory Prop Address:	*TMZUDpxQFwyy7YPJopzHZxFeJpa5xzNgnSM*

`add_nat_p`

Theorem 17.44 $\forall n : \iota.$nat_p $n \to \forall m : \iota.$nat_p $m \to$ nat_p $(n + m)$. *The proposition is identified by the following information:*

Pure Prop Id:	*a68eaa385d85714b752c87f2a2eee1c568587a1fffda0e97906a41c26003cf88*
Pure Prop Address:	*TMc24JjrtUUsp2EU37UmE4mu7iJ7DKkmhBJ*
Theory Prop Id:	*4c1d5d018a9f1d2b9453690cae8404fa9eb620f367b5eea25a0cb86d97cbe223*
Theory Prop Address:	*TMFY44nBuvZeLpSeXoD777seJQ3FwNH5bHN*

`add_nat_asso`

Theorem 17.45

$$\forall n : \iota.\text{nat\_p}\ n \to \forall m : \iota.\text{nat\_p}\ m \to \forall k : \iota.\text{nat\_p}\ k \to (n + m) + k = n + (m + k).$$

The proposition is identified by the following information:

Pure Prop Id:	*c81c09e2824f3ae69080689a9918f8a8786896c891f60deb4ed02d6f67f3cb62*
Pure Prop Address:	*TMKhPKqB8txYNiBHfmLH1ruc7muR6qocZmM*
Theory Prop Id:	*eec29406f31cd07c1e41a8cfa7dd645a14748dd76d405c653593376354c1801c*
Theory Prop Address:	*TMJ8UyHV8GUb8Vws1rSbV1qG6FtayeCkDeZ*

`add_nat_0L`

Theorem 17.46 $\forall m : \iota.\mathsf{nat\_p}\ m \to 0 + m = m$. *The proposition is identified by the following information:*

Pure Prop Id:	*8ceeb9658771d534867c4e7fb857c194b87b3c269b78a59e437c53340f7a0786*
Pure Prop Address:	*TMLoqyDa4T219PBE9qaSVziSgcNXAuyYZSy*
Theory Prop Id:	*84fbe4456a772afa70e2e251ea34167365bee559c7a5aeb17e73e850db6f7c53*
Theory Prop Address:	*TMYQwEtyoLvmW7jPiZze4QVGPj5DZuaTf5Q*

`add_nat_SL`

Theorem 17.47

$$\forall n : \iota.\mathsf{nat\_p}\ n \to \forall m : \iota.\mathsf{nat\_p}\ m \to \mathsf{ordsucc}\ n + m = \mathsf{ordsucc}\ (n + m).$$

The proposition is identified by the following information:

Pure Prop Id:	*9c442aaec4cf7410aad415dc7f4e04405898410e390d79880d0375c76066e5fa*
Pure Prop Address:	*TMaAYzqo6wqjw8Cyowaj3GfJxJxvbhxYDju*
Theory Prop Id:	*e6efb75f65615ef88f5bde274557d9a8df5606c47b340bdff64b75fde270e0f5*
Theory Prop Address:	*TMHs3GjtfJJMSExofB5jjekwQXt9vQ9JVEp*

`add_nat_com`

Theorem 17.48 $\forall n : \iota.\mathsf{nat\_p}\ n \to \forall m : \iota.\mathsf{nat\_p}\ m \to n + m = m + n$. *The proposition is identified by the following information:*

Pure Prop Id:	*40e9527c8d2ba8c9684c597d041b44dd398a84708d4958ed949bd806aa07a045*
Pure Prop Address:	*TMUKhdGt15R67GHJenCm4tPyhkXRshPnejp*
Theory Prop Id:	*4ab1793481bb6270a606171fd587949f75d9e5d9f76c6cabe92d879d2029efbb*
Theory Prop Address:	*TMTvQoshXSimoWoX7FGXYgBcckAvFxKAzcM*

Definition 17.3 *We define* $\mathsf{mul\_nat}$ *to be* $\lambda nm : \iota.\mathsf{nat\_primrec}\ 0\ (\lambda\_\ r.n + r)\ m$ *of type* $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*35a1ef539d3e67ef8257da2bd992638cf1225c34d637bc7d8b45cf51e0445d80*
Pure Object Address:	*TMQ9VRpXGSTRvGimsfh4aDJ7zMynq4DQkpu*
Theory Object Id:	*80ccc668a68b08f05dbefe4777f8a2954ee878fde2459ea00e9c3f844abba676*
Theory Object Address:	*TMSPdCUvrmtPueTXMGGcZnyt6hALJ9ctyFj*

Notation. We use $*$ as a right associative infix operator corresponding to applying term $\mathsf{mul\_nat}$.

`mul_nat_0R`

Theorem 17.49 $\forall n : \iota . n * 0 = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*b2f56c05dc69c0b009e6ef48273e48c299236dba09369d6470f062d8d4b9f4e0*
Pure Prop Address:	*TMb9Vpd2YbcAwzGuAijMPac1ZZd1JTyv4Xi*
Theory Prop Id:	*1e5b2cd8b1404db3fe789241877d2db676126a3ee22aef833bce50e59717f3b5*
Theory Prop Address:	*TMYKPBEGEsiA6kAm2D5gNMaaoQznS2XzHfH*

`mul_nat_SR`

Theorem 17.50 $\forall nm : \iota . \mathsf{nat\_p}\ m \to n * \mathsf{ordsucc}\ m = n + n * m$. *The proposition is identified by the following information:*

Pure Prop Id:	*37df54fff3ede09b5f27087075ed84f0c3c205aeb237fe98e376929562343633*
Pure Prop Address:	*TMGeZ3mYtjU3qrjXWbppcFZ1rqeDWrfiSqC*
Theory Prop Id:	*6dd6fc3c946630902a37292306a5d4628d0df49b90a16a0b4bde060e327d01ba*
Theory Prop Address:	*TMWCj8uVzNKMcnEEoUyEKaLXqDFHWBQkrus*

`mul_nat_p`

Theorem 17.51 $\forall n : \iota . \mathsf{nat\_p}\ n \to \forall m : \iota . \mathsf{nat\_p}\ m \to \mathsf{nat\_p}\ (n * m)$. *The proposition is identified by the following information:*

Pure Prop Id:	*d63438e3958a2dc77d897cf86bd67bfea42d951dc7cc262804802907e9d2702b*
Pure Prop Address:	*TMG1LZCLEQXdxeijnJMFK24mEb2ZENAEMJe*
Theory Prop Id:	*e910cb7dbed9f807bb2b492fd880072b0fa60ff97bf29504de618c1bb02b9b7b*
Theory Prop Address:	*TMHuRaAVff8dq9SXtxuPNkua68MeuTc6xgX*

`mul_nat_0L`

Theorem 17.52 $\forall m : \iota . \mathsf{nat\_p}\ m \to 0 * m = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*db8a357ff4bbc85955dad0c6561e2f2e46517b67140c8b29f3410a166806d8c9*
Pure Prop Address:	*TMat1oD79SXX8wbvMUfsE7vpuGWs6TV2tvt*
Theory Prop Id:	*6a2dd6eb8d82e1ebdc8d29656a314ba5f0823446cfd1b7e3e023a868e5c576a8*
Theory Prop Address:	*TMKees76uQpkLU55adTSh33A5D4a1pWMJ6o*

`mul_nat_SL`

Theorem 17.53

$$\forall n : \iota . \mathsf{nat\_p}\ n \to \forall m : \iota . \mathsf{nat\_p}\ m \to \mathsf{ordsucc}\ n * m = n * m + m.$$

The proposition is identified by the following information:

Pure Prop Id:	*5cf7ece86a139e3814ee6c6c7d1036b5efd2cb2a7569655a5785267e6b0744c7*
Pure Prop Address:	*TMPvhe9Rvam418EBDU6GUny2MjoZwaH5XpP*
Theory Prop Id:	*24e4ebc5814e4a359b7a34331c431f191c16ea51e3acd3df89dfc39cbb4f5310*
Theory Prop Address:	*TMK29Vjye4s7vokoPSyBUEGUMF7out1xEKo*

`mul_nat_com`

Theorem 17.54 $\forall n : \iota.\mathsf{nat\_p}\ n \to \forall m : \iota.\mathsf{nat\_p}\ m \to n * m = m * n$. *The proposition is identified by the following information:*

Pure Prop Id:	*4525ef58273c984cb807830a9315b6c92ea0e51493bee52c9fdb11b2c032debb*
Pure Prop Address:	*TMR99zvsWSdWrZH1xvD1ZPyyRe2sN3tw9DJ*
Theory Prop Id:	*5d6ce40bd822f0043fa5e91314acb4c38d4657e00036f8292d1c646d61867911*
Theory Prop Address:	*TMK6YYFmjjLN7XjWZngbXtSHGuvLhCNW5sK*

`mul_add_nat_distrL`

Theorem 17.55

$$\forall n : \iota.\mathsf{nat\_p}\ n \to \forall m : \iota.\mathsf{nat\_p}\ m \to \forall k : \iota.\mathsf{nat\_p}\ k \to n * (m + k) = n * m + n * k.$$

The proposition is identified by the following information:

Pure Prop Id:	*7758f10f23cd5bde5f62e6e19176c07070d03250e7ee2f5445ddd169a94ac9cc*
Pure Prop Address:	*TMEg6jVocmXvkpGecshFe76iHzxRPb9JEDU*
Theory Prop Id:	*d57b0261b87e3402ab865d1448a120cf5a64d7df32d00662a23546d0192c5f7f*
Theory Prop Address:	*TMRv9ASbnJeJHuEAVEnAXZtBdyUqKHy7Num*

`mul_add_nat_distrR`

Theorem 17.56

$$\forall n : \iota.\mathsf{nat\_p}\ n \to \forall m : \iota.\mathsf{nat\_p}\ m \to \forall k : \iota.\mathsf{nat\_p}\ k \to (n + m) * k = n * k + m * k.$$

The proposition is identified by the following information:

Pure Prop Id:	*c2db2e2dc62062cd1a587ac9ba967b9afcf30e142bf9bd6067b24d6b2f537912*
Pure Prop Address:	*TManfhaHUANQVLqQTLdBVmdTonLkBBLbBAf*
Theory Prop Id:	*a456bc1368534ecb0130b6b056bce984b8cae31fe686bfe5fa19371ee7bc295d*
Theory Prop Address:	*TMTE1UAeY9QMbiDArVw7LV9YCaXoyHNsWTu*

`mul_nat_asso`

Theorem 17.57

$$\forall n : \iota.\mathsf{nat\_p}\ n \to \forall m : \iota.\mathsf{nat\_p}\ m \to \forall k : \iota.\mathsf{nat\_p}\ k \to (n * m) * k = n * (m * k).$$

The proposition is identified by the following information:

Pure Prop Id:	*564886c3dec6ac8d225ffa511e508fd66f08a374ef9f3441d381346c5fd9da75*
Pure Prop Address:	*TMTj2mzQJp3MhibenRc1qV4e9VBJHGB5LmM*
Theory Prop Id:	*229d2cf885153a28edbff336b15f01f9a69ebb36151922686ca00a59c571593d*
Theory Prop Address:	*TMRAaHnsfviFEtPvtoFHTUV4fNfwKjkAV61*

`add_nat_1_1_2`

Theorem 17.58 $1 + 1 = 2$. *The proposition is identified by the following information:*

Pure Prop Id:	*174b87c6660c5b9369cec9ae0bb6fbfa629fb619d7c5b2abc06dbe96f057f730*
Pure Prop Address:	*TMaJcaiD1zqTNzV6GxQBzYBvpnhKKCh8BgZ*
Theory Prop Id:	*77f23f1fdaa42b143fa225c79a56a1ea865e80541dbf361ba18f855e404b3575*
Theory Prop Address:	*TMWAt7x6EE1xxXg5JkjkfpLoXMkjM4S7GM9*

Definition 17.4 *We define* divides_nat *to be*

$$\lambda mn.m \in \mathsf{omega} \land n \in \mathsf{omega} \land \exists k \in \mathsf{omega}.m * k = n$$

of type $\iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*1b17d5d9c9e2b97e6f95ec4a022074e74b8cde3e5a8d49b62225063d44e67f4f*
Pure Object Address:	*TMFX8yTrN53rRgSLEZohcsRA6EaD3CSANo1*
Theory Object Id:	*355928b746d784c55c7688d88105ecefae6bd0215b4f742280a8381aabba7f6f*
Theory Object Address:	*TMc53M2iKqdhRPHnQpz3E7ib2G4x4BAu8NK*

Definition 17.5 *We define* prime_nat *to be*

$$\lambda n.n \in \mathsf{omega} \land 1 \in n \land \forall k \in \mathsf{omega}.\mathsf{divides\_nat}\ k\ n \to k = 1 \lor k = n$$

of type $\iota \to o$ *identified by the following information:*

Pure Object Id:	*729ee87b361f57aed9acd8e4cdffcb3b80c01378d2410a0dabcf2c08b759e074*
Pure Object Address:	*TMdvXdQkmLZGboSGJXGXRKmLGB5pqxCZpoJ*
Theory Object Id:	*a54ca44017bc4b34632c41638b465b39b14dd8cd1733ed77712e0d245baffed5*
Theory Object Address:	*TMPsbUQYdAiibkkLrzYgRtSGEqJ5zvim61J*

Definition 17.6 *We define* coprime_nat *to be*

$$\lambda ab.a \in \mathsf{omega} \land b \in \mathsf{omega} \land$$
$$\forall x \in \mathsf{omega} \setminus 1.\mathsf{divides\_nat}\ x\ a \to \mathsf{divides\_nat}\ x\ b \to x = 1$$

of type $\iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*73e637446290a3810d64d64adc8380f0b3951e89a2162b4055f67b396b230371*
Pure Object Address:	*TMFmmCt86ZmVJJyYkyLgqnb1fJ56frE48w6*
Theory Object Id:	*5e7a659c94f73f93fcc4101b5280fa38c54cf4b3836afc4acb20cdafe1a91f6c*
Theory Object Address:	*TMHvTwzJu9htnnfJFGzTSVgNXc69bPtJJRj*

Definition 17.7 *We define* equiv_nat_mod *to be*

$$\lambda mkn.m \in \mathsf{omega} \wedge k \in \mathsf{omega} \wedge n \in \mathsf{omega} \wedge \\ ((\exists q \in \mathsf{omega}.m + q * n = k) \vee (\exists q \in \mathsf{omega}.k + q * n = m))$$

of type $\iota \to \iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*23f216cd4975d7a2b1380d30e7b48227ee30bb436b18413678721f3abebc9c6a*
Pure Object Address:	*TMbQxhb4L79WyCM6RdiNH8CwjnACqvJCArJ*
Theory Object Id:	*b3e6279627f6e69ce3e9cae910a6faa1ed2221482dbf9b8cbfff595f2802cb5b*
Theory Object Address:	*TMTkKJpV4fxDM1kVGWKTUL74oaK3t8akX8c*

Definition 17.8 *We define* exp_nat *to be* $\lambda nm : \iota.\mathsf{nat\_primrec}\ 1\ (\lambda\_r.n * r)\ m$ *of type* $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*37c5310c8da5c9f9db9152285b742d487f1a5b1bd7c73a4207d40bcd4f4d13fb*
Pure Object Address:	*TMQbaN33eGzJDYuzCJatBYA9orZ4FEpcRPM*
Theory Object Id:	*8e35a198b6dbafe8b2e95eb0500523f42f6367ca1c5bcf349ca278f24a3a07d7*
Theory Object Address:	*TMEwKZ5cpkx2mJ4qE8BkCSWkbVNAA7Wvn6U*

Definition 17.9 *We define* even_nat *to be*

$$\lambda n.n \in \mathsf{omega} \wedge \exists m \in \mathsf{omega}.n = 2 * m$$

of type $\iota \to o$ *identified by the following information:*

Pure Object Id:	*1725096a59f3309a3f55cd24aaa2e28224213db0f2d4e34a9d3e5f9741529a68*
Pure Object Address:	*TMJGQEurbTgjiciPtBTutT9u7rTkMUN4MWJ*
Theory Object Id:	*c28a39bb677f0bc8b9d085f62ff72d310153afc73e004a5ac360c1cd5397836a*
Theory Object Address:	*TMWJQjHTr9nF6QRD4b8ffhEed2Z8ZMsa6nB*

Definition 17.10 *We define* odd_nat *to be*

$$\lambda n.n \in \mathsf{omega} \wedge \forall m \in \mathsf{omega}.n \neq 2 * m$$

of type $\iota \to o$ *identified by the following information:*

Pure Object Id:	*8358baafa9517c0278f00333f8801296b6383390ea4814caaadcff0dec35238d*
Pure Object Address:	*TMGKNXPQbMNiuCqZnwUEjEUha8fcnFt3Jk2*
Theory Object Id:	*e16741f9a0da9b00388d2994378dc8d14febef4c577e38c02cea2b3600c7b124*
Theory Object Address:	*TMPkkurDzwQcwtKKmG9c9urhgvFf7SduuxD*

Definition 17.11 *We define* nat_factorial *to be*

$$\lambda n.\mathsf{nat\_primrec}\ 1\ (\lambda kr.\mathsf{ordsucc}\ k * r)\ n$$

of type $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*ff333a6e581c2606ed46db99bd40bdd3a1bab9a80526a0741eba5bddd37e1bba*
Pure Object Address:	*TMZjHdCL77axNr9RebheBmYNoJLbcGbbnfG*
Theory Object Id:	*43798425ce2e239eb427ae260e9a7b313bf8bd7df1dab1d082f026f6c879692c*
Theory Object Address:	*TMYeeBw9Gn9bU5FejUUExj87nTNnbgW5gbX*

Chapter 18

Natural Numbers III and Ordinals

PigeonHole_nat

Theorem 18.1

$$\forall n.\mathsf{nat\_p}\ n \to \forall f : \iota \to \iota.(\forall i \in \mathsf{ordsucc}\ n.f\ i \in n) \to \neg(\forall ij \in \mathsf{ordsucc}\ n.f\ i = f\ j \to i = j).$$

The proposition is identified by the following information:

Pure Prop Id:	*d756b7f9b65b31513512c63b940fdfc4ecb49540116753f6c2d340626609ee82*
Pure Prop Address:	*TMLBavCBsSBws7ahi4AVPeESXViLDv31Ak8*
Theory Prop Id:	*604a84ad91cecfb6b42f1126bfec73fd706444dd748ec6c15b12ce0bfb69053a*
Theory Prop Address:	*TMVN7QY12B9fLrGqiNtTGBesGf96NCkiHRC*

PigeonHole_nat_bij

Theorem 18.2

$$\forall n.\mathsf{nat\_p}\ n \to \forall f : \iota \to \iota.(\forall i \in n.f\ i \in n) \to (\forall ij \in n.f\ i = f\ j \to i = j) \to \mathsf{bij}\ n\ n\ f.$$

The proposition is identified by the following information:

Pure Prop Id:	*cd3b8dc5389196c2477610f742fb1ee7dba1dcdf01fea50de9a063945a56cffb*
Pure Prop Address:	*TMV5QRAwtRsRGashmQVHiReREqRjLYeRTHc*
Theory Prop Id:	*44a08f5c7754241cf51e71665e77acf0e4b00fd74a503742d7594661c4883eac*
Theory Prop Address:	*TMQn8ujQbMM2W4yRjHGhzmzMYVZfwKNdhFW*

cases_7

Theorem 18.3

$$\forall i \in 7.\forall p : \iota \to o.p\ 0 \to p\ 1 \to p\ 2 \to p\ 3 \to p\ 4 \to p\ 5 \to p\ 6 \to p\ i.$$

The proposition is identified by the following information:

Pure Prop Id:	*9d65eaceb8856524e55245becb873ac99520db85c1aa6190004c31f201ae565a*
Pure Prop Address:	*TMMGJXuShsSGWBi3CaHZeVWjtSHGMJv1cqn*
Theory Prop Id:	*0b70c477b139fb0134ba4733415ed1d40b67071992567e7df1d98b3fa3ef4f9b*
Theory Prop Address:	*TMXFRbLTTN6ihayKReZVwptm9miwKifcmVm*

cases_8

Theorem 18.4

$$\forall i \in 8.\forall p : \iota \to o.p\ 0 \to p\ 1 \to p\ 2 \to p\ 3 \to p\ 4 \to p\ 5 \to p\ 6 \to p\ 7 \to p\ i.$$

The proposition is identified by the following information:

Pure Prop Id:	*fb834c80b903a21792ab31c32ca79011f1ed7fc1ee605431cab0377c113ce789*
Pure Prop Address:	*TMJ9pan6QGbicHY78ZhcRV5sjSxUpJ9hjAe*
Theory Prop Id:	*148753c82fc2d19e4d0ccfdd34fc8ac73852e40d98c8740e4bbf13f37f1469bc*
Theory Prop Address:	*TMaE8ED9F5UkowTrWVxJoWwkbViqo9kPjoK*

cases_9

Theorem 18.5

$$\forall i \in 9.\forall p : \iota \to o.p\ 0 \to p\ 1 \to p\ 2 \to p\ 3 \to p\ 4 \to p\ 5 \to p\ 6 \to p\ 7 \to p\ 8 \to p\ i.$$

The proposition is identified by the following information:

Pure Prop Id:	*0d0c6a6d27f3197a59eed06051dc8e722f3c73606b083f56fa41970d63f24e12*
Pure Prop Address:	*TMKuRMEvwYcoSa8Gzg1hrtZoBaBSPH2xcsW*
Theory Prop Id:	*cc8c42c160b06cd70b0400e6d0cd711ceb06d2dd8fbbf1468400cdf70a509df5*
Theory Prop Address:	*TMLp2MUsd6cFR8gFchdhfihDsX7oRanJWxR*

nIn_2_1

Theorem 18.6 $2 \notin 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*d4d954c8170695a2939bec3c446c37edaeb27568ad48b1986c6ecaba24070132*
Pure Prop Address:	*TMMJE7KaXfpvgsT5JWqzaMeStdMLFTu4s3R*
Theory Prop Id:	*9f9cbd90053ad69e8dc078d6b9aafbd1e870ff598f884002a52555d6cf96db04*
Theory Prop Address:	*TMSytpFZDKijwdEXRuEoRS26emWEHwyyJn9*

neq_6_0

Theorem 18.7 $6 \neq 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*d94311e2671c987b8b42941206d260e5fad22da17796f0e0af8fd9718168e741*
Pure Prop Address:	*TMJPPVAEoouTBu7DFSenpbm1vf43asWn9A9*
Theory Prop Id:	*f94806573dcb9fa2ec502b75f210c999a07ae981958bf4172d246de9cef66017*
Theory Prop Address:	*TMUnexmnNzMTxwUotBtEVHCSB7LQYjDXGCA*

neq_6_1

Theorem 18.8 $6 \neq 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*c950efa1242fe81c8f031eb108e16f8792592823a5d93bd4f60a0f648062bc05*
Pure Prop Address:	*TMG4GcsPH6pN4zKNadStY83A7KTwr1YmB43*
Theory Prop Id:	*799bdf447bc496543bc3e105fee018f133d24d15ca3c6eb8fa5bb71688379c06*
Theory Prop Address:	*TMNpiWkUsYako2ZQqsmQ8aMpiCajvjtw6WY*

neq_6_2

Theorem 18.9 $6 \neq 2$. *The proposition is identified by the following information:*

Pure Prop Id:	*d5b69e228a453716ca00492b40a6fb313f2f582f72946915bc5cba6815ebd5b4*
Pure Prop Address:	*TMady4g8qST1QcVR9VA5N56ED9kVDm8NbA8*
Theory Prop Id:	*2865cfc1f9a346744761d36b1b8ffe1def60d3f92967f64caf2bdd6621484595*
Theory Prop Address:	*TMYMRVL2d8FLeABGAFvksAq5e8LTs37M9U5*

neq_6_3

Theorem 18.10 $6 \neq 3$. *The proposition is identified by the following information:*

Pure Prop Id:	*03fcc4b85dfade6c906248f8f17bb4e90da92e6bc4bd7dae07dc87b1c793cafc*
Pure Prop Address:	*TMZhLzSJKhPJ6LW7S8RESbikoubnNwygd8E*
Theory Prop Id:	*1ffa21cb3a06b988a3518b1986f09cebcda356923bb17b459190071df400c1fd*
Theory Prop Address:	*TMdSxgcJr8YTuZy9uTgG9ajdHdm6pAfiETZ*

neq_6_4

Theorem 18.11 $6 \neq 4$. *The proposition is identified by the following information:*

Pure Prop Id:	*0ebd5de52c3bf03fa06ad9a52177c94376ac6d6d21e4f7eec2ae524c20eb865a*
Pure Prop Address:	*TMYRPQnnTxR6J8VzzoFmHbqoXkGa3GgD775*
Theory Prop Id:	*b3e48dff2905f986f3b5caced5a64dcd7234bfbc66ef0306f80725cd1ece61c7*
Theory Prop Address:	*TMbKF48GnbjUBGJE9sw73TPmMcB8d19vTHv*

neq_6_5

Theorem 18.12 $6 \neq 5$. *The proposition is identified by the following information:*

Pure Prop Id:	*e3c546719f53e598755a05b44ff1592522513e88eab553a3118e5486a81ed067*
Pure Prop Address:	*TMYY56rp3pu1Md2tx7cYLemwdM1mR5gcmwn*
Theory Prop Id:	*f44366e0c31269b36c404d0c744fe0d1b9ff6021d8c4b58406adb81f5b1c2657*
Theory Prop Address:	*TMdM2SjU46csQY7mCoKywbHX9m8H2pRDGvc*

neq_7_0

Theorem 18.13 $7 \neq 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*f1aca13486b782dc00a51c22a046ddf10b9e5b6024aa2d8487df4668d134ad21*
Pure Prop Address:	*TMTpLW3ApNNyU9zRQ4a5vLvH6GvaG3hDZsw*
Theory Prop Id:	*04777e1121acde4051c1076405353442ecc7c841d8f5b1e282a7bbc610a446dd*
Theory Prop Address:	*TMakKfA3Sv9UTsXeCRpShgJQ4dUnFkNenjx*

neq_7_1

Theorem 18.14 $7 \neq 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*7f68ac2aa17e9e283bde82aaa6848151f8aae61ca586307fee893ae0535713a5*
Pure Prop Address:	*TMQ5xCGH582yDx55yDN51RKxK9oZ7i3pmXr*
Theory Prop Id:	*c517bd071cddfea2ce1c10f9a4334e27e7bb4cf04db904a41ccb36e001fe703d*
Theory Prop Address:	*TMMMZYiwPgt5qwiuqAdeUwY2qzRX4uwcTBy*

neq_7_2

Theorem 18.15 $7 \neq 2$. *The proposition is identified by the following information:*

Pure Prop Id:	*e915b85aabc6ff55b599e8b765d9d05c01d9dda3408fd4fa0a9833c8f62024b5*
Pure Prop Address:	*TMbBFKirdwXXTLhAyJKum1V8XdNj5BBozBE*
Theory Prop Id:	*719b90b534d821a74d146a517ec28b80f23cdcc8ebf4522af7dfd48863cb4fa8*
Theory Prop Address:	*TMFfMwPwBr11dsiDFnzdKdva5FgSwooFhKZ*

neq_7_3

Theorem 18.16 $7 \neq 3$. *The proposition is identified by the following information:*

Pure Prop Id:	*bc468ab5a647c84bfe1cbf2177bddf3cd9dcbed388c41930e8429925a62165e5*
Pure Prop Address:	*TMLWnYbD25mqPFYupL6Lc7hitrjdJcfniuS*
Theory Prop Id:	*7c0147f7e0c5d264d821868156432da25efcb90102666d5c3d25e4e52d14e374*
Theory Prop Address:	*TMLUE8BbNrawrU1x36jKypY1Hc7Dh9sTeBx*

neq_7_4

Theorem 18.17 $7 \neq 4$. *The proposition is identified by the following information:*

Pure Prop Id:	*0241dc3875c580942490118be9d0c699ec0514fe9d772ea7a904789477d54238*
Pure Prop Address:	*TMGN51cm5R9iSyp1WjfR5ejM1X6VgXfxqwX*
Theory Prop Id:	*b83dd2a8b6844c39918736d4bb65dae6cac7b407b3e1d7f856124a1b8c8b1bbb*
Theory Prop Address:	*TMczss8yk3mckgdGRz59sXdSgkJJAmtAG5j*

neq_7_5

Theorem 18.18 $7 \neq 5$. *The proposition is identified by the following information:*

Pure Prop Id:	*0d6c1682fbca7a87f757e77ddd09c5bb907565625c9bb3bd4d4106f080d57fc6*
Pure Prop Address:	*TMRiUm9T8u2HUSVqbxBbzUpwb8M5kpBY8y9*
Theory Prop Id:	*2c37cdd7efc305b90d6607a132b9b9dd842317e03383654b67242cd93c5a9abe*
Theory Prop Address:	*TMNyRaG3YG1Vk3iDDYuv3o2c1VCAw8DviHL*

neq_7_6

Theorem 18.19 $7 \neq 6$. *The proposition is identified by the following information:*

Pure Prop Id:	*0d1fd098069d41630b89611eb93f1f08bce1c36d9c813d078d3d146904cdd42f*
Pure Prop Address:	*TMdEyrS1cfBLmsvCe812P4oyjUa3LvHoT5m*
Theory Prop Id:	*7b8e224d3ff812c0454afa567fe7c3949fb0d82d46543a26c9e7ca9d51dd332e*
Theory Prop Address:	*TMdRvdFMDC1LW6RXkFfr25AGkBNkYkv7Mt1*

neq_8_0

Theorem 18.20 $8 \neq 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*e3931d2d8a8848d0d9ee64433a011a9aa4a51737fbdb5b7ff75eda148be9a459*
Pure Prop Address:	*TMMdAqero4SC35AeBiqqtuQEkyo6rgyk4ti*
Theory Prop Id:	*30657128da01d2a236a2d6ddcc73e1f8049084468c98dd69a2605d082c526bb1*
Theory Prop Address:	*TMSNjV8GGDiVXMRQBxvQkSYEkGBVdZjVjFm*

neq_8_1

Theorem 18.21 $8 \neq 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*6a16eeb33a53075e488dca90100eb48cca90e3c02804aed2943df9da34c76b07*
Pure Prop Address:	*TMU2yPgAvkUoBCYpDgKoFG8xJAzksaqUjrV*
Theory Prop Id:	*3fab0e9aef535966602b629d8b325e7d4fcc22fddae36170ebb822bab8fe78f2*
Theory Prop Address:	*TMPHMdwsKTGixUr9ovcL92TPpNn2DZ59xR3*

`neq_8_2`

Theorem 18.22 $8 \neq 2$. *The proposition is identified by the following information:*

Pure Prop Id:	*9f7e3d7052052f039493deaa72664346f98b44644d5ce8e1d6b29f702756dc87*
Pure Prop Address:	*TMHiEy2g7ytqLkCShdFDkQrHwAxrHLJ4ewz*
Theory Prop Id:	*e34cd1607418d341f87f72d6c64acebcadb8c4c22064d081a9a312fa6ea07e26*
Theory Prop Address:	*TMKa84jucjzdciZ6ZrGg7jbnwmESXpiuuyX*

`neq_8_3`

Theorem 18.23 $8 \neq 3$. *The proposition is identified by the following information:*

Pure Prop Id:	*338f0e75ea4c6be85d54aac86da1fd615b97b42931170925c246bdb4ead8b0c7*
Pure Prop Address:	*TMXDqEySQWNgPDXaxytYNRxHDU8nnVJBQmE*
Theory Prop Id:	*c0304effdb21f61184bffd799d5816aa2c736176a0f60054faf868b3e0667412*
Theory Prop Address:	*TMcMdW4UMw76zq4HEXEdz38xzdN3uEfNvfU*

`neq_8_4`

Theorem 18.24 $8 \neq 4$. *The proposition is identified by the following information:*

Pure Prop Id:	*d95ab6bf4885a62645e050411d3dc149b0c30553a1b11777fb9626b5056dd31a*
Pure Prop Address:	*TMZ1C39y5oktaYiujD5mMcSng5NPSLJeEQz*
Theory Prop Id:	*626b967685fe040cf728abce59c8526d52b9320642a11332b64fb176654bcba1*
Theory Prop Address:	*TMdUEjMQYbenfRooZ94bsLXZekKQw7AwjpY*

`neq_8_5`

Theorem 18.25 $8 \neq 5$. *The proposition is identified by the following information:*

Pure Prop Id:	*3582f2a69391c993ecc9217f73540497ab0af13613233acff03d6d05999a68b9*
Pure Prop Address:	*TMNQpkxEPyJb6YgZiPPBGZQ9GbC7g7hb3TA*
Theory Prop Id:	*f8515d4c2c16dac9f1318ebd500262d8f6382c1822da217b3713dd322b505844*
Theory Prop Address:	*TMK22UzEck1xp8U1jNqX9wkEh8eNg2kiLgv*

`neq_8_6`

Theorem 18.26 $8 \neq 6$. *The proposition is identified by the following information:*

Pure Prop Id:	*15c47f6efea919c6469a2b32df126c3a593df8a81ab4ebc5df17a98087ebf8e6*
Pure Prop Address:	*TMcBgaEvvTaTaTDaifgkFsjxZQsqU4WzYoY*
Theory Prop Id:	*f5b7bf37bd7435e07a0b28d0a188937c24f605630a543a4e882d91a080499e2a*
Theory Prop Address:	*TMM574yv9Ufaw1F3N1jvDPpcg9GNzrTpYD6*

neq_8_7

Theorem 18.27 $8 \neq 7$. *The proposition is identified by the following information:*

Pure Prop Id:	*8d3267526667c8bc389f97552b72f511aec9f5f758b6ffb5677e384888680a6f*
Pure Prop Address:	*TMSmpMULYdUBC4xMvaX48vBU8PW3G6mmegc*
Theory Prop Id:	*4f0658fb5c5f798e5218782be81c0fc926d764645d937e30bf2af1ae4d64d8f2*
Theory Prop Address:	*TMLSfV5KwoLtB3AyRu1AQrfTfXfUQMcVAmW*

neq_9_0

Theorem 18.28 $9 \neq 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*7fad6a4021cfaad3cd966f9db7cbc322601a15b27f7bebd190db92d9227d7d9d*
Pure Prop Address:	*TMYqJFCaupzUjNDmU955uhVqnu4ij9Mkuyn*
Theory Prop Id:	*6e3ac6edcc976b5d4163e1e746d46f422f0f0916bd84e45ef0045db9d55d4c19*
Theory Prop Address:	*TMKrgJnjKmPUDz3Ei1FeB8qAqPZq38H7jTb*

neq_9_1

Theorem 18.29 $9 \neq 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*d87ea84c9c5771eec45a1fd14ad9153c2ddcdc39dae8e258585d43f60d51fc0d*
Pure Prop Address:	*TMJggtzHrkHQuUoShHgUHjqXcPh3YDT7pN1*
Theory Prop Id:	*80c63319a6d1e1304573edb4dae25f062780f4a278368ab2d11ffeb00f49b7c6*
Theory Prop Address:	*TMcbAZhdMU2jbnR5osTzLG3UcaHdNK1HoSu*

neq_9_2

Theorem 18.30 $9 \neq 2$. *The proposition is identified by the following information:*

Pure Prop Id:	*e2f4a18541452ebc1ff6a767768646cff4511a43dacb70dc53eb524adbacf071*
Pure Prop Address:	*TMW6fNB5sUi4nGQoEwhzSpZyo1Y1LXDN1Ax*
Theory Prop Id:	*906ccd2680d9bcd663a9b598d0e0b69b8c88fbeaa778ec48beaa22ea36b6f5b6*
Theory Prop Address:	*TMWS25pQtSvRUgzYu1ZiDeYaCbk35nSzVQb*

neq_9_3

Theorem 18.31 $9 \neq 3$. *The proposition is identified by the following information:*

Pure Prop Id:	*c17c1de4ec497af0fe4297a258d7a7c3d135e22ec81c834e7c9a96add5cccf55*
Pure Prop Address:	*TMcK89BgzJhDbNna2LzUzWNiyoYx6z5zftc*
Theory Prop Id:	*d46f51d9a1c08f9230e9eb18a81e934945d4012ad2dbe5e8e979e80e9c2d0a15*
Theory Prop Address:	*TML3NJwKxkRE2ZwqumY1ptX5orP6hMULsaL*

`neq_9_4`

Theorem 18.32 $9 \neq 4$. *The proposition is identified by the following information:*

Pure Prop Id:	*b2be6a2b14079f391b61f0f2c994e6c1f83f55f33e48888f5714fe2885e624f7*
Pure Prop Address:	*TMG2u9oAHq7WMiN9fonJks9eiAQRK26GM6C*
Theory Prop Id:	*b136fba6a8b8fe9273ffb6386834de99d159f7c0ebb273ae030c99bdb951d513*
Theory Prop Address:	*TMUecGRuCAeSZwXfRoh3G38RVvj1529JShy*

`neq_9_5`

Theorem 18.33 $9 \neq 5$. *The proposition is identified by the following information:*

Pure Prop Id:	*b1aaed7edfc0b04b990d933a7c324285c4e528683b5444ae62b5f6a6a1fea631*
Pure Prop Address:	*TMWqKkpLGp13Tno4pYWFNtmRzmAH8WgKrKw*
Theory Prop Id:	*0ae3b202241c8b6489ff2afc286f400f7db9f53d731d7c5c96fbc5157b8f2546*
Theory Prop Address:	*TMUn176msQ6tprqPwqSfnayaBTxnWVJfjxL*

`neq_9_6`

Theorem 18.34 $9 \neq 6$. *The proposition is identified by the following information:*

Pure Prop Id:	*b0286fd9ca576d16fe9d5933787a03791678ce168ecfef6360c216512ecbe419*
Pure Prop Address:	*TMYwbNuQhDPUxShjx7H2boyojGDg8rAw8MU*
Theory Prop Id:	*19a02fe5d8602e59495deff9213cd8d10a5cc30204dadfbad1f34d1f0fad400f*
Theory Prop Address:	*TMbZ1StXWmE3GzbtLteGPhs8Bh7HEHWxABd*

`neq_9_7`

Theorem 18.35 $9 \neq 7$. *The proposition is identified by the following information:*

Pure Prop Id:	*aa22ad31d3f91e04f56851635a9322d13c23e3854cf45c973dea5a3cd0990b88*
Pure Prop Address:	*TMXRgW5zUyKotMqQa3YaGwUZ6vFLUNEzUV9*
Theory Prop Id:	*ff267fe87e45aca713d50f6e46c11d032357b90dc7acb206e3eefd8d01d36e82*
Theory Prop Address:	*TMRwAzgMNqVuBgS6WGFdfKaAwYzx2HWCZgu*

`neq_9_8`

Theorem 18.36 $9 \neq 8$. *The proposition is identified by the following information:*

Pure Prop Id:	*552eeb72bea997741bbc3cfc4807d1e73d2669cb5a06966cd70231206647a992*
Pure Prop Address:	*TMRjxiyZgHvxgNtxTc3rKqKfBHHQ3fpUnty*
Theory Prop Id:	*e47272afbed4b7d1b148ec0b93566130bd83c0c213facce0dfa529c59ee5d182*
Theory Prop Address:	*TMXoTBB5rYLbYrboKc4G2FzdBcPPXa5C9uF*

Subq_1_Sing0

Theorem 18.37 $1 \subseteq \{0\}$. *The proposition is identified by the following information:*

Pure Prop Id:	*aeae9a39918acd52e99e4648ef2ce22082b14cdb7f056833b3c9b6c7b08071bb*
Pure Prop Address:	*TMd1acsG4T47vanN7AeZN9GBKVvieBkvmww*
Theory Prop Id:	*abab2d4437158699c98b6dc7b4a1d462e539047454145838ea1260d68fc49944*
Theory Prop Address:	*TMbUJrpN4q3Hv8rt9wMEJobfB2VW2EKtT8g*

Subq_Sing0_1

Theorem 18.38 $\{0\} \subseteq 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*cd786497c06a79d7f8391817754ef67063326d18ddbf160ab5ed1c22c0a322eb*
Pure Prop Address:	*TMZDF4AA5AbafCq1V1cfLd6D4iDMcgeXjr5*
Theory Prop Id:	*1806bf9a09183656922b1ceb29490f6cc9898d3b353ad377a7e13edd309c7791*
Theory Prop Address:	*TMHq3pPKYpd4KhjT9PUsaM9T7pKh9PDKCDK*

eq_1_Sing0

Theorem 18.39 $1 = \{0\}$. *The proposition is identified by the following information:*

Pure Prop Id:	*5e59965f8dea2376b44a4645087aa7b0fb447c5e0b32fdfa0a419f8c6730821f*
Pure Prop Address:	*TMM87nnYFzft3WcRjLweo3mTQXoJfb1o2yT*
Theory Prop Id:	*264f38ba2201bb27cd34ae54a896eda9cfb3191811b417511fd59a42a1aad583*
Theory Prop Address:	*TMJZ4wN5mJDvtTGG8T4dkZwXYestYUa7nSE*

Subq_2_UPair01

Theorem 18.40 $2 \subseteq \{0, 1\}$. *The proposition is identified by the following information:*

Pure Prop Id:	*d908841bebc314441f7259cb1ae1e06c95ce753fde5f00fc59b6dde11d4d93c1*
Pure Prop Address:	*TMHVo8N2YAFhqLdCesUKneKkycAMCjufnxX*
Theory Prop Id:	*94203e1d2b2c0faaa1b53b57d6e02377ba6909bbad4345113cca6c3b30ffc6c2*
Theory Prop Address:	*TMdUkY7ub8vSPAiJMtCVLRMpJmEeZvULcKe*

Subq_UPair01_2

Theorem 18.41 $\{0, 1\} \subseteq 2$. *The proposition is identified by the following information:*

Pure Prop Id:	*a855482b832c8ea776aa01316555b8b83b59d1577d45ef4b5d123674420d0e01*
Pure Prop Address:	*TMGfNJyPsWS2KxUk5KVgkcMQB4XboikFQcH*
Theory Prop Id:	*0095605b59f5d2bbd6d2ac9dc6c80902a1f7530a558319eac4bd5e1225e3059a*
Theory Prop Address:	*TMYm6neMj3cdSovA76Sn3Rzr22vfiLnJrMu*

eq_2_UPair01

Theorem 18.42 $2 = \{0, 1\}$. *The proposition is identified by the following information:*

Pure Prop Id:	*da80075027dc7a68c335dc91d0d1e2008733d49f594a4654f4808a72f790fe18*
Pure Prop Address:	*TMTAkWeRWfntW6KcrRUzppdf93RSZBsj2kq*
Theory Prop Id:	*7ae5a1ab159f51b92a97a85e8382349a9ddca06dc5c2f387f58ab6cada7ddc8d*
Theory Prop Address:	*TMd33UnmSrxCctoLEbFMBCqkF1i4pvM11uq*

ordinal_ind

Theorem 18.43

$$\forall p : \iota \to o.(\forall \alpha.\mathsf{ordinal}\ \alpha \to (\forall \beta \in \alpha.p\ \beta) \to p\ \alpha) \to \forall \alpha.\mathsf{ordinal}\ \alpha \to p\ \alpha.$$

The proposition is identified by the following information:

Pure Prop Id:	*bb557547876580348d1b27707e8329c129045f75384f6cfee3b6b3660d679f76*
Pure Prop Address:	*TMMjF9N5BddMz7dZjRy3jb2edCyq6JxUnjh*
Theory Prop Id:	*973cd8ebf285141bb206385be0589deb9b0d2b22f4291d4f8935767e2c8d72f7*
Theory Prop Address:	*TMHyu4a19t1LKmHrnT3eDCrBexiXHK9SKmm*

least_ordinal_ex

Theorem 18.44

$$\forall p : \iota \to o.(\exists \alpha.\mathsf{ordinal}\ \alpha \wedge p\ \alpha) \to \exists \alpha.\mathsf{ordinal}\ \alpha \wedge p\ \alpha \wedge \forall \beta \in \alpha.\neg p\ \beta.$$

The proposition is identified by the following information:

Pure Prop Id:	*df6c7763ab1d598ac956e39f705674757a597eb43301218c4a0cf210679e8487*
Pure Prop Address:	*TMR2LGsWB47Dd2V32aJMtRmhkcuHkb32B5c*
Theory Prop Id:	*ab6fe147dcaf15113c3482aa2b52e2418ef973d6f5f9ad0df1e81a91ac34fd0f*
Theory Prop Address:	*TMWujpqPswYN98tJhWJajsNjSmNit3C1wxB*

ordinal_trichotomy_or_impred

Theorem 18.45

$$\forall \alpha \beta.\mathsf{ordinal}\ \alpha \to \mathsf{ordinal}\ \beta \to$$
$$\forall p : o.(\alpha \in \beta \to p) \to (\alpha = \beta \to p) \to (\beta \in \alpha \to p) \to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*7c0f0bd46025b48ff67a7143c3e4a076e5e080c3c720416d50d3fda07959571a*
Pure Prop Address:	*TMbyFsACnhcNuinkiWbTRsyq9Xw7dG3yxen*
Theory Prop Id:	*bb6d2d91d4ebfae062a8c9b008250ab6b380ac9e09ebf6e8ec9dfd01afcfc71b*
Theory Prop Address:	*TMX8gEh9epaBjfv8Rid5iBE5GnLWmimvYwE*

`ordinal_trichotomy`

Theorem 18.46

$$\forall \alpha \beta : \iota . \textsf{ordinal}\ \alpha \to \textsf{ordinal}\ \beta \to \textsf{exactly1of3}\ (\alpha \in \beta)\ (\alpha = \beta)\ (\beta \in \alpha).$$

The proposition is identified by the following information:

Pure Prop Id:	*2230cfc37c714b9145baabb918487c4c4cb9582cb14de24bf562f395dbffef9e*
Pure Prop Address:	*TMUJTrRxNbofUZ7cPQahpGQK3QFoP7UEpr5*
Theory Prop Id:	*72667378ef8d960e362a49147b9f82d2da940398bc2f4f035524457ddae3b011*
Theory Prop Address:	*TMadvrwG2zGWUTUNaw1gN8KJ6gPMX3d1LQJ*

Chapter 19

Disjoint Unions

Definition 19.1 *We define* Inj1 *to be* In_rec_i $(\lambda X f.\{0\} \cup \{f\ x | x \in X\})$ *of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*8f0026627bca968c807e91fff0fdc318bc60691e5ae497542f92c8e85be9fd7d*
Pure Object Address:	*TMHfHa5vujJQpf428B9wNXuDE5s6mwdPL2X*
Theory Object Id:	*7736ca8991b680139f61982df3af1ac650f0f26acdc0d8f0eee357ccb025fa14*
Theory Object Address:	*TMG73kjDALbhSPSDyKjUdTNFaHJFKmvRPR3*

Inj1_eq

Theorem 19.1 $\forall X : \iota.\mathsf{Inj1}\ X = \{0\} \cup \{\mathsf{Inj1}\ x | x \in X\}$. *The proposition is identified by the following information:*

Pure Prop Id:	*73d189eaaf79f7e70a5868626e8599a06905bffe21fe665a8b1af2a7532ef363*
Pure Prop Address:	*TMVWdgZGFsyibGkrWpy2DrjXjAj4ZHnCcnv*
Theory Prop Id:	*b804575f2f48fe85cc3e16f46d28aedf5c9e1bd2db40240cdc402976b31678dc*
Theory Prop Address:	*TMc2FdiN6hAsJo3duwEKeNeMQK18PyUEFsh*

Inj1I1

Theorem 19.2 $\forall X : \iota.0 \in \mathsf{Inj1}\ X$. *The proposition is identified by the following information:*

Pure Prop Id:	*3c2cc4eebcc7df71902a867c8e10780084c122974b276dfc2e42b3636fcb7f48*
Pure Prop Address:	*TMUKupGQMJTgBLEF8gUr11TxXdBgWPczFDF*
Theory Prop Id:	*4ccac3180e2819292f9b0007f95830075776538ef5094469c5b9112c6324d5ae5*
Theory Prop Address:	*TMFoqv2T2HTXaw58WBHGfN3Yg1LLmHDXTZY*

Inj1I2

Theorem 19.3 $\forall X x : \iota.x \in X \to \mathsf{Inj1}\ x \in \mathsf{Inj1}\ X$. *The proposition is identified by the following information:*

Pure Prop Id:	*b6f23b38b854e4764eb026a61fb5de29c88c81746a47fcf866ef0b179e8cceb7*
Pure Prop Address:	*TMZjKduCZEmbUN9ZBjsaBnKgkfw7LN2UDHT*
Theory Prop Id:	*086475da641652d7d705bb3a628b3be4360161ede19a126c9e4a25189e1af347*
Theory Prop Address:	*TMcYPGZrNfjZ1uwVn5pPjvCKQnbnj8Fb9jY*

`Inj1E`

Theorem 19.4 $\forall X y : \iota . y \in \mathsf{Inj1}\ X \to y = 0 \vee \exists x \in X . y = \mathsf{Inj1}\ x$. *The proposition is identified by the following information:*

Pure Prop Id:	*d4177b1a2defe4e7a1dc9a2f0cdd24121f05df2353248c4fbce54a7813e386aa*
Pure Prop Address:	*TMRaNkUGFqMNGWjPSkyqhVQ1u7CuZYnE44m*
Theory Prop Id:	*259217a7917e6f724d1d01957b59b045b53d464a75ffd225751fab998674d1eb*
Theory Prop Address:	*TMKgb5K1zBg9yUsASeSHAXS5dXzMAp6g4mC*

`Inj1NE1`

Theorem 19.5 $\forall x : \iota . \mathsf{Inj1}\ x \neq 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*5e3197f060ee011e661df22a9f6fa83fd21ce6975b1e8fffe8bf822020036e6e*
Pure Prop Address:	*TMZWFtRyxiZnniPvM2zyYdwdjemD8ju9G9q*
Theory Prop Id:	*09bd10c733e446cd49301e4db4b5b278f8d986e3543f97f02967ca0c21b264ad*
Theory Prop Address:	*TMSEKm63FLbyd37zrxFC3ESBmnwSacXbajE*

`Inj1NE2`

Theorem 19.6 $\forall x : \iota . \mathsf{Inj1}\ x \notin \{0\}$. *The proposition is identified by the following information:*

Pure Prop Id:	*9e94b180d7b2cb52448d59e85f15103666357701440f2f18d5af02589fcb09ff*
Pure Prop Address:	*TMaoFDrWfevDLaLmviNwM8qU5VfCbfQNyxm*
Theory Prop Id:	*3c4dd0ee065b1e59cfafc01a250811bc2c5b00c5d2b523db2b6fb82b36158b54*
Theory Prop Address:	*TMdfQzzFY8EHNJ3UScdjLaKoU7T5kaYk9Gg*

Definition 19.2 *We define* $\mathsf{Inj0}$ *to be* $\lambda X . \{\mathsf{Inj1}\ x | x \in X\}$ *of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*8143218ffde429ff34b20ee5c938914c75e40d59cd52cc5db4114971d231ca73*
Pure Object Address:	*TMNyr2ri5LN86eqRzW8u1M6rvw78xBZoUjT*
Theory Object Id:	*7d54bea1f00d9595006e5596af5681c32a7d5291e967d47e9c1f618685b8067e*
Theory Object Address:	*TMM4pcVYzZz9VCeFoN6ah22KyBbZKKNTDAc*

`Inj0I`

Theorem 19.7 $\forall X x : \iota . x \in X \to \mathsf{Inj1}\ x \in \mathsf{Inj0}\ X$. *The proposition is identified by the following information:*

Pure Prop Id:	*7fc2e938ecb986314c840c9ebc715b8faa04b3a118675b2f97af4d5a6475aa9f*
Pure Prop Address:	*TMPfdYfdnemz7bK4pdryqUiJ88tNURoednk*
Theory Prop Id:	*5653780aa993662d9f130c83c987c41047896e81076539a071e059bef75c3a85*
Theory Prop Address:	*TMWQMDFSDmKyL2MMoQEMXjXRGQkvWtCHUuQ*

`Inj0E`

Theorem 19.8 $\forall X y : \iota . y \in \mathsf{Inj0}\ X \to \exists x : \iota . x \in X \wedge y = \mathsf{Inj1}\ x$. *The proposition is identified by the following information:*

Pure Prop Id:	*41668460a96da96670eda3373b14c44145fecc8b7543d1eb48cc87d1a03b5873*
Pure Prop Address:	*TMR3n3H2ExAyH57HwSFYymEuxUDc7FLsyoo*
Theory Prop Id:	*855213768c213323d014713167da61d832ba2c1f0e4bac38154d846f4887467d*
Theory Prop Address:	*TMGnC4VJWDhLev8VPHUK3NMMFBEpWB7UKzh*

Definition 19.3 *We define* Unj *to be* $\mathsf{In\_rec\_i}\ (\lambda X f . \{f\ x | x \in X \setminus \{0\}\})$ *of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*f202c1f30355001f3665c854acb4fdae117f5ac555d2616236548c8309e59026*
Pure Object Address:	*TMWjsgx6D3647vX6CZ7FACDq76sirVgNudL*
Theory Object Id:	*2e16753b23b9f054a644cd2c352a33cf3885b3e69791a426da64d5a3ff83d6ee*
Theory Object Address:	*TMUQznLHs487u6czgXR1tuqBrKovEsSFuwB*

`Unj_eq`

Theorem 19.9 $\forall X : \iota . \mathsf{Unj}\ X = \{\mathsf{Unj}\ x | x \in X \setminus \{0\}\}$. *The proposition is identified by the following information:*

Pure Prop Id:	*3b1aa65e273ab9e8836c2bc74542816dbc66db6da358e79281baa68c1eac4c51*
Pure Prop Address:	*TMUzG5DSxEUo14WoVVEZHRUadq2y9fEMq9D*
Theory Prop Id:	*b4c8525077a8bd9a2f4b5fe9b1db873fe4209bd38a98f300f580351d5d18d7d3*
Theory Prop Address:	*TMLCXNWg3jeqjg6spuSQMWMiiuoNrDLSqsX*

`Unj_Inj1_eq`

Theorem 19.10 $\forall X : \iota . \mathsf{Unj}\ (\mathsf{Inj1}\ X) = X$. *The proposition is identified by the following information:*

Pure Prop Id:	*e2cdfc6f275aeef110524d267044d56c4438d2db1293e2867fb46d60760a0f45*
Pure Prop Address:	*TMRKw89oG3aDYwZKbJ2YCSPZoxR6oQrXVAM*
Theory Prop Id:	*ae00e1714271d7aa2be5da6b28ba3d58bf1dcb1e86e45bd075db78fb8e268271*
Theory Prop Address:	*TMMx1iggqKMgEC5fpswYPb7SMn6mXdV3Gd6*

`Inj1_inj`

Theorem 19.11 $\forall X Y : \iota . \mathsf{Inj1}\ X = \mathsf{Inj1}\ Y \to X = Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*ce8b6635f1785f75c7b5b16d1f61111f3895e4072cc06a71b7f2548024ab9fcb*
Pure Prop Address:	*TMPRTGLAeftK68qCgSftNHQAt5ZoPyRmGsh*
Theory Prop Id:	*3e918a17810ff35edf26ed66220d1d7a2ce50e44383669fb3e82f49186cdee06*
Theory Prop Address:	*TMaXPPDHYrLmw3vrDF8cSrSzcALUJ1fr2vQ*

`Unj_Inj0_eq`

Theorem 19.12 $\forall X : \iota.\mathsf{Unj}\ (\mathsf{Inj0}\ X) = X$. *The proposition is identified by the following information:*

Pure Prop Id:	*f3e39c79aeba9f53d5600670c3217bc096389fccc95b8a119918ee61bbd2ba1f*
Pure Prop Address:	*TMZUKGCMAbV3pSZqA94UEu6kCWXx8F95cVU*
Theory Prop Id:	*3808ad5876773f87048df2f1763c5b4749fa9feb633aabecd3c5bc64e8bea8ba*
Theory Prop Address:	*TMWJtkfszm1k5JmP9LnmZB5GUL6tyPJE95H*

`Inj0_inj`

Theorem 19.13 $\forall XY : \iota.\mathsf{Inj0}\ X = \mathsf{Inj0}\ Y \to X = Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*662f53ab21221e41310d9ac6c9389a4c91b5f8a8c92bc0e7c16500877a0ee8d3*
Pure Prop Address:	*TMc3veqimj5NLmacT7fJhju75JwAWeR5tRC*
Theory Prop Id:	*29f9450d88499de2d9169079cdcb356767832e57ed9a0111ca120c9eb731b8d1*
Theory Prop Address:	*TMV8xsSpddijAdXaoJ58C6Ds3FGDXzmngXz*

`Inj0_0`

Theorem 19.14 $\mathsf{Inj0}\ 0 = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*171f471b1ee40b5e9e23d33ae47d474ea49798216a6286e99f7505fd596e0b1d*
Pure Prop Address:	*TML5YcJHULJ6j353CwJ1qM8742qPffm73wi*
Theory Prop Id:	*57f5af3069e244db75ffa5bfd8014bb81b0c95ace6e267ef286f6f4f11003544*
Theory Prop Address:	*TMbQhddJBnbci3edhLUCUA3LAWeGzxZF6vQ*

`Inj0_Inj1_neq`

Theorem 19.15 $\forall XY : \iota.\mathsf{Inj0}\ X \neq \mathsf{Inj1}\ Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*21bb03ac63365d37ed4ef00915506a97c5a5b79e468a1019f8db539f0520ead3*
Pure Prop Address:	*TMNaUd4a4oqrV7GxRGw17zvy3w3U9tCHd4W*
Theory Prop Id:	*2b14d886759a5ff3af07d29a955feeebe2511955316846bdd6b6a0516947b7e*
Theory Prop Address:	*TMNgqTkR1PJCufRmpkGQmktjoL6ruuRLg22*

Definition 19.4 *We define* setsum *to be* $\lambda XY.\{\mathsf{Inj0}\ x | x \in X\} \cup \{\mathsf{Inj1}\ y | y \in Y\}$ *of type* $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*afe37e3e9df1ab0b3a1c5daa589ec6a68c18bf14b3d81364ac41e1812672537a*
Pure Object Address:	*TMLfysEtzMCWeSWegZfR794mvNom3FuyKrm*
Theory Object Id:	*5b7e09a6e30fb5cf9e9ae6e183db0d299ed352490f06c4381c56c42c8e75cced*
Theory Object Address:	*TMctw4uoy129ZSr7Njfx1xj2Bd38ipPNaKu*

Notation. We use $\oplus$ as a left associative infix operator corresponding to applying term setsum.

`Inj0_setsum`

Theorem 19.16 $\forall X Y x : \iota . x \in X \to \mathsf{Inj0}\ x \in X \oplus Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*1c640838de3fb39c7e9ba79c5409e04f12c37645baf67043e9f7f4278de39039*
Pure Prop Address:	*TMKTYzxjjWdrWetApF9Sj67Z6edBVD8EBTo*
Theory Prop Id:	*246ac5b8e5010b12d563e2f045e2abdb51d794611ea8dd590bc0fe11e5a7ad33*
Theory Prop Address:	*TMTNL3J3wEnAzxucGw8kUWBscSaUn4BLgiu*

`Inj1_setsum`

Theorem 19.17 $\forall X Y y : \iota . y \in Y \to \mathsf{Inj1}\ y \in X \oplus Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*d47a0f2a013d89032cb64812b92012dc27b1d5fcc204fbc9a9c86638237204e2*
Pure Prop Address:	*TMVgD1dS7zmrgJcgocmjP99ka8Dpg62ZCtL*
Theory Prop Id:	*2b4f9491906d1ef49131201b69a9b8bd7d2f1ebf95fe8d5c0244a559215334ac*
Theory Prop Address:	*TMVk4zcLngHpsDrazL9rzQjzJ6jxFvj2f6c*

`setsum_Inj_inv`

Theorem 19.18 $\forall X Y z : \iota . z \in X \oplus Y \to (\exists x \in X . z = \mathsf{Inj0}\ x) \vee (\exists y \in Y . z = \mathsf{Inj1}\ y)$. *The proposition is identified by the following information:*

Pure Prop Id:	*645a6ce0a1cd5ad2ce3ac83a7b4d74f5954bae61db680341db619c0b10612a16*
Pure Prop Address:	*TMGJozk41a81Kp4SygTYFZeKcuHrdPrC32q*
Theory Prop Id:	*a36400c28b0f8902008e9de3432e7367f347381ef074614db8802940be43cbc6*
Theory Prop Address:	*TMHiPKm9hQiccXFfb4skC57UHenaqznn87b*

`Inj0_setsum_0L`

Theorem 19.19 $\forall X : \iota . 0 \oplus X = \mathsf{Inj0}\ X$. *The proposition is identified by the following information:*

Pure Prop Id:	*0be1e755f689c575a9eda08885f30343eeee939fbebeab4f3fa42931ed3bf82f*
Pure Prop Address:	*TMK8zdDs3kr6c2kZDhfjxDumvkoP5SgQ9RW*
Theory Prop Id:	*36fabd108b18f5565ca33eddd506b2b62395333c16b3399474bae3abea3b0ada*
Theory Prop Address:	*TMbUYG9UEmYDqfxYZUKHNBKT4Y75njU2mbq*

`Inj1_setsum_1L`

Theorem 19.20 $\forall X : \iota.1 \oplus X = \mathsf{Inj1}\ X$. *The proposition is identified by the following information:*

Pure Prop Id:	*f8d72132080ef3e20ece3ff94da26f46176e6715a7f569d277aefa88c73c3774*
Pure Prop Address:	*TMVZEiZNntaDr4daY1DmuXvonFq9uNTdmig*
Theory Prop Id:	*45b1c8dba7f1e6bb6910722f4bbf2ac5cec44dc5e8ebd74f245640488f2aa566*
Theory Prop Address:	*TMME8xmRrBtE5QcayY5Ek96RbNEE8zSo5C2*

nat_setsum1_ordsucc

Theorem 19.21 $\forall n : \iota.\mathsf{nat\_p}\ n \to 1 \oplus n = \mathsf{ordsucc}\ n$. *The proposition is identified by the following information:*

Pure Prop Id:	*3be3cce904aadda436daf1b9679086129be4811093362608100e6f068bdd4039*
Pure Prop Address:	*TMHYS9eCHQRks9avhfkoeHGSrbHGL8uaBv1*
Theory Prop Id:	*8fbdda58ab70c8d36db764f1f86d169ce58ba6cc946bd4cb21cc5936808f1183*
Theory Prop Address:	*TMUdCytedFWrC4iKskiF2SDYbA8djETMgpn*

setsum_0_0

Theorem 19.22 $0 \oplus 0 = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*5dcaebebb574103d0edf890d85e23a5fa97b59db661795af9daeb7aa65f336a4*
Pure Prop Address:	*TMaVr4aCWi6koabmAQVJ2wQdpL7mC7foUu9*
Theory Prop Id:	*4f3651b8f623daa8b048a8735ce8791d53d3c1d1b0f2aa681c537176ab51a619*
Theory Prop Address:	*TMHtDbLm7fFXE2PBw9a9TaKFqUT1oKq4DFj*

setsum_1_0_1

Theorem 19.23 $1 \oplus 0 = 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*c2107f17a63765673ccaf223da15ffd6fe6fcb31499a0a5fc5415779192120e5*
Pure Prop Address:	*TMXomoBYE3zuVe9e7kyBAEnPduKtPVYL79A*
Theory Prop Id:	*e8609a742bc15d0823294b218d9d7e9f9d926e9cb69df0e589c81a5f60f5366e*
Theory Prop Address:	*TMGKXxfahnZuNyLnFqGHv1d446PYGT5n9MX*

setsum_1_1_2

Theorem 19.24 $1 \oplus 1 = 2$. *The proposition is identified by the following information:*

Pure Prop Id:	*9c89c04ece2b66ca83b8a9b210a7f94ab137834db0b093883a5ab955eb6ef743*
Pure Prop Address:	*TMSMi29gtadSxAPeBb5CCz27pw11Gt1r5Md*
Theory Prop Id:	*ef3f41b81000348e190c1253edc936248dee43fc698ee536fa6d9f1c6e97c1aa*
Theory Prop Address:	*TMH43YDo7oYg8zrgf9FjkpTTpmMyXLKMTTt*

setsum_mon

Theorem 19.25 $\forall XYWZ.X \subseteq W \to Y \subseteq Z \to X \oplus Y \subseteq W \oplus Z$. *The proposition is identified by the following information:*

Pure Prop Id:	*6e30186eb9483aaffe4c265cfe38dc213a2cf9faf3f5aa66b43cdfe7c9a5d2ca*
Pure Prop Address:	*TMUfRBKC5kCVDQmt7f2Q83pWENdCggpCtEM*
Theory Prop Id:	*838d561df06bc2aee2d92713cb2b2d1c726613db9da9186614c6563f51c2bf86*
Theory Prop Address:	*TMW7Tpt1vt99hraFFFRzz5pjENcaXQ9VV51*

Definition 19.5 *We define* combine_funcs *to be*

$$\lambda XYfgz.\mathsf{if}\ z = \mathsf{Inj0}\ (\mathsf{Unj}\ z)\ \mathsf{then}\ f\ (\mathsf{Unj}\ z)\ \mathsf{else}\ g\ (\mathsf{Unj}\ z)$$

of type $\iota \to \iota \to (\iota \to \iota) \to (\iota \to \iota) \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*ccac4354446ce449bb1c967fa332cdf48b070fc032d9733e4c1305fb864cb72a*
Pure Object Address:	*TMKSa978XJEMycuWYBhLHN2QGXRqqHaF4Bb*
Theory Object Id:	*aa7bb3bbd2ec8e8fd8ba27aa97a6fc35a6ab1e3003dbbf6136a6110f1823bb93*
Theory Object Address:	*TMJtkvSshA8XZqXDCbMyvjzjiyzZfNwnBom*

`combine_funcs_eq1`

Theorem 19.26

$$\forall XY.\forall fg : \iota \to \iota.\forall x.\mathsf{combine\_funcs}\ X\ Y\ f\ g\ (\mathsf{Inj0}\ x) = f\ x.$$

The proposition is identified by the following information:

Pure Prop Id:	*19f032786d18528530a8e52c17dfc6a4417805e17b1a6ee6ed1364c06b749913*
Pure Prop Address:	*TMXtLgamyXaY4YgLpaWoWvf8a28MSYTmXZ7*
Theory Prop Id:	*a0730ef167ec6039eaf3084667d145e2e26662596942c8973bd0711b71f18ad3*
Theory Prop Address:	*TMa8SMHnUqoCuZw2Kk78KMtZVfqopa2somA*

`combine_funcs_eq2`

Theorem 19.27

$$\forall XY.\forall fg : \iota \to \iota.\forall y.\mathsf{combine\_funcs}\ X\ Y\ f\ g\ (\mathsf{Inj1}\ y) = g\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*709f34af1e90fda80a83b8965675935d92fb44648d48274e89b23dc20e8ebff5*
Pure Prop Address:	*TMLVrnvc3Z1Pjpg39KChTrX1Jjknz3bF3ri*
Theory Prop Id:	*3933870bb4bccaad7d50c4972cba92adb92a7d7ff41c00d8ef4c83b8c60fad83*
Theory Prop Address:	*TMTpr4M3N39vULCLJKgNMnuiESzd162fkqo*

Chapter 20

Pairs, Sums, Functions and Products

20.1 pair_setsum

Let *pair* be setsum.

`pair_0_0`

Theorem 20.1 *pair* $0\ 0 = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*5dcaebebb574103d0edf890d85e23a5fa97b59db661795af9daeb7aa65f336a4*
Pure Prop Address:	*TMaVr4aCWi6koabmAQVJ2wQdpL7mC7foUu9*
Theory Prop Id:	*4f3651b8f623daa8b048a8735ce8791d53d3c1d1b0f2aa681c537176ab51a619*
Theory Prop Address:	*TMHtDbLm7fFXE2PBw9a9TaKFqUT1oKq4DFj*

`pair_1_0_1`

Theorem 20.2 *pair* $1\ 0 = 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*c2107f17a63765673ccaf223da15ffd6fe6fcb31499a0a5fc5415779192120e5*
Pure Prop Address:	*TMXomoBYE3zuVe9e7kyBAEnPduKtPVYL79A*
Theory Prop Id:	*e8609a742bc15d0823294b218d9d7e9f9d926e9cb69df0e589c81a5f60f5366e*
Theory Prop Address:	*TMGKXxfahnZuNyLnFqGHv1d446PYGT5n9MX*

`pair_1_1_2`

Theorem 20.3 *pair* $1\ 1 = 2$. *The proposition is identified by the following information:*

Pure Prop Id:	*9c89c04ece2b66ca83b8a9b210a7f94ab137834db0b093883a5ab955eb6ef743*
Pure Prop Address:	*TMSMi29gtadSxAPeBb5CCz27pw11Gt1r5Md*
Theory Prop Id:	*ef3f41b81000348e190c1253edc936248dee43fc698ee536fa6d9f1c6e97c1aa*
Theory Prop Address:	*TMH43YDo7oYg8zrgf9FjkpTTpmMyXLKMTTt*

`nat_pair1_ordsucc`

Theorem 20.4 $\forall n : \iota.\mathsf{nat\_p}\ n \to pair\ 1\ n = \mathsf{ordsucc}\ n$. *The proposition is identified by the following information:*

Pure Prop Id:	*3be3cce904aadda436daf1b9679086129be4811093362608100e6f068bdd4039*
Pure Prop Address:	*TMHYS9eCHQRks9avhfkoeHGSrbHGL8uaBv1*
Theory Prop Id:	*8fbdda58ab70c8d36db764f1f86d169ce58ba6cc946bd4cb21cc5936808f1183*
Theory Prop Address:	*TMUdCytedFWrC4iKskiF2SDYbA8djETMgpn*

Definition 20.1 *We define* proj0 *to be* $\lambda Z.\{\mathsf{Unj}\ z | z \in Z, \exists x : \iota.\mathsf{Inj0}\ x = z\}$ *of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*21a4888a1d196a26c9a88401c9f2b73d991cc569a069532cb5b119c4538a99d7*
Pure Object Address:	*TMJAVsjow6vGiHhjkrvQ3hFGbgbvS9xcWyV*
Theory Object Id:	*9c5e4e6be64531b8ea143c51d4eae82a384c81ad95f7a944e1c670a8a30da3b9*
Theory Object Address:	*TMaHY5KSgmVdGVqgzTcQgf4sHUsLLKmvAny*

Definition 20.2 *We define* proj1 *to be* $\lambda Z.\{\mathsf{Unj}\ z | z \in Z, \exists y : \iota.\mathsf{Inj1}\ y = z\}$ *of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*7aba21bd28932bfdd0b1a640f09b1b836264d10ccbba50d568dea8389d2f8c9d*
Pure Object Address:	*TMaSYWTJtnhHMswCbUGaheomcukAYucQua4*
Theory Object Id:	*7fd81e3c066cdbca377abd70b13843fd894f77ad69e67cf26842b8bc147db710*
Theory Object Address:	*TMPJv5Do7FeQVBbPTL9ZY9H9YGMGpRV652o*

`Inj0_pair_0_eq`

Theorem 20.5 $\mathsf{Inj0} = pair\ 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*0fe60101f9b90226a3183f7eee66d492a07dd05cb531e7baf46d3ab4302ca05f*
Pure Prop Address:	*TMVm4GBpHVCYV2pJB5HL2QNkHnuSGWZ9qAh*
Theory Prop Id:	*490c2a25a7dca847f40f638f694a872a3f6f47d42aea7867c086f2c4cf3f6988*
Theory Prop Address:	*TMNdMHTR5f8Y73jbhAeJgaesKTmGkru7yzB*

`Inj1_pair_1_eq`

Theorem 20.6 $\mathsf{Inj1} = pair\ 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*0bbde85d4f9a32c3b689bb6a75b3954d57abb296cbd0aa2745de678af1df9a59*
Pure Prop Address:	*TMPgPyPUzWL2JW9oss4msqgSCZ6zCy2Ztc1*
Theory Prop Id:	*8272d79377fa78cbfc8589d848b8f5e3862c8d19e10dd47a58c6bea9d16d8447*
Theory Prop Address:	*TMNjuSVZSzPL5H7zv2hTWCGkBLDksUvykJN*

`pairI0`

Theorem 20.7 $\forall XYx.x \in X \to pair\ 0\ x \in pair\ X\ Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*21a4141f1e8cac366335c6139b8e6f6c143e493b4b78e1106e78e213967caf29*
Pure Prop Address:	*TMPWuPuKr4fNYduR2p8k1ib9xWvBLuRs4wX*
Theory Prop Id:	*34813b26053e164b7ed43057b6f8ee6ff0cd96d01f5b6401b42ceae658468e01*
Theory Prop Address:	*TMNQzaX8ZiddhSn6QTSqc1McYEwNG1Fe8wT*

`pairI1`

Theorem 20.8 $\forall XYy.y \in Y \to pair\ 1\ y \in pair\ X\ Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*bdaab525a50b9c2d070e4f67b9e4b3b931b06622695aa3c0f0b51683e1905bc7*
Pure Prop Address:	*TMbhSedKYd5RyPq2mELrEkTMb7JkqLqrGWh*
Theory Prop Id:	*b5248b96fbd2dbc9c65f96e27bfae42798a39bc110034aa9dcc82afc9fe0056a*
Theory Prop Address:	*TMM3oDhF32usrf43sjdNDwEZbSftUuY8cjh*

`pairE`

Theorem 20.9

$$\forall XYz.z \in pair\ X\ Y \to (\exists x \in X.z = pair\ 0\ x) \lor (\exists y \in Y.z = pair\ 1\ y).$$

The proposition is identified by the following information:

Pure Prop Id:	*aac6108d214baf821b35d72218cb0b4f9e63fc00aecbbb7a4ce9bcbb8a21b941*
Pure Prop Address:	*TMUqfs3rToFZZEUcGSfnRKkNmLL7jyk7GqJ*
Theory Prop Id:	*0afdeb4f80887eb047d9b590fad88a119cbb419efa9edfb46ab666e1282c8fd4*
Theory Prop Address:	*TMJLCz7nFpUma4iUHHjfnc85LCDmaCUqtN2*

`pairE0`

Theorem 20.10 $\forall XYx.pair\ 0\ x \in pair\ X\ Y \to x \in X$. *The proposition is identified by the following information:*

Pure Prop Id:	*042f308329163b02cbbc55bb0027f0352880a98c06f8ab271eadf3ec5e9f04e3*
Pure Prop Address:	*TMJeqsKVgs3VYVeCG9rGS4ZNKAdR8nTs38m*
Theory Prop Id:	*f50b255d63e6dbed7324927c7b1afe6317eea22dbd269f2a5b27d6f6565c5445*
Theory Prop Address:	*TMLQ2VeBqCmuyodmpqK5dmQM5s98wqPSfk2*

`pairE1`

Theorem 20.11 $\forall XYy.pair\ 1\ y \in pair\ X\ Y \to y \in Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*505cfcb3d7610169f4bfa8ba33596724c69138cb7d875cad9a309a91e2402eb6*
Pure Prop Address:	*TMcH1uWQFgoCUsZbTGyxzwk17PtdC9cvCA6*
Theory Prop Id:	*30a181bf14bf2566fdcf937c9b42f3512a8a40f569e90dc2aef4aa7ae0db9e24*
Theory Prop Address:	*TMQgeZWRp9TKUwYNKVDSo4jYfniQaLPJzRY*

`pairEq`

Theorem 20.12

$$\forall XYz.z \in pair\ X\ Y \Leftrightarrow (\exists x \in X.z = pair\ 0\ x) \vee (\exists y \in Y.z = pair\ 1\ y).$$

The proposition is identified by the following information:

Pure Prop Id:	*98d332f11ba6fc7f438c81b60a59777921748d69f214c8a1788cc4374ca7f53b*
Pure Prop Address:	*TMVrjFeVrNRrLEwuGQPfHQJgGH3nN34D2yd*
Theory Prop Id:	*65c2d317407da943881beedf59fac794f29f668a2676e85d7c1d17f0f0f1c0ca*
Theory Prop Address:	*TMRyemtQaJ4VgBEH4b2NuAukq6NEzdWuaof*

`pairSubq`

Theorem 20.13 $\forall XYWZ.X \subseteq W \to Y \subseteq Z \to pair\ X\ Y \subseteq pair\ W\ Z$. *The proposition is identified by the following information:*

Pure Prop Id:	*6e30186eb9483aaffe4c265cfe38dc213a2cf9faf3f5aa66b43cdfe7c9a5d2ca*
Pure Prop Address:	*TMUfRBKC5kCVDQmt7f2Q83pWENdCggpCtEM*
Theory Prop Id:	*838d561df06bc2aee2d92713cb2b2d1c726613db9da9186614c6563f51c2bf86*
Theory Prop Address:	*TMW7Tpt1vt99hraFFFRzz5pjENcaXQ9VV51*

`proj0I`

Theorem 20.14 $\forall wu : \iota.pair\ 0\ u \in w \to u \in \mathsf{proj0}\ w$. *The proposition is identified by the following information:*

Pure Prop Id:	*be55b9a07966119a7062e99cbd0ce90614937b1321ad2f7982157399ef7cd51b*
Pure Prop Address:	*TMMvCe8JzDEJMj9cXvrha66rpRYMz5CKGJ1*
Theory Prop Id:	*840c8df1f99f4da23ec51cac3d8dc3d97931d9d67121464862c6504eec69245a*
Theory Prop Address:	*TMGtD6G4wVv2RV84KASZp4iaX1TDhDn2QMG*

`proj0E`

Theorem 20.15 $\forall wu : \iota.u \in \mathsf{proj0}\ w \to pair\ 0\ u \in w$. *The proposition is identified by the following information:*

Pure Prop Id:	*b9afa9f543fc43d88abece7f6e50c4eb30c889faceb03f4deb53230d9fb540ba*
Pure Prop Address:	*TMQYzQmm1NFjQJzmtbAGu6ymzj9LF6Gx53A*
Theory Prop Id:	*5b7f586d6ff9e7dda36e45b26ab54d9324d38494897913660be264154c751908*
Theory Prop Address:	*TMKRt7H9e4QrANP7FuuuVFh5G963n269NGA*

`proj1I`

Theorem 20.16 $\forall wu : \iota.pair\ 1\ u \in w \to u \in \mathsf{proj1}\ w$. *The proposition is identified by the following information:*

Pure Prop Id:	*fbddb75c7d6c8d452a3849e8b97ab824ad9f6385bfaee6fab47eb8e903c72ff0*
Pure Prop Address:	*TMMhHn9rASEBErsJ8RZqUMcdtgkQZHiTtjS*
Theory Prop Id:	*595b3cca1eb4a87a06e1fac1e30ea2a8d161b87216b075cfab0ff0066f3025c6*
Theory Prop Address:	*TMPGXMep9UXBiUhuNpo5MxyTNz4GnkUrTMz*

`proj1E`

Theorem 20.17 $\forall wu : \iota.u \in \mathsf{proj1}\ w \to pair\ 1\ u \in w$. *The proposition is identified by the following information:*

Pure Prop Id:	*06e32e59aafc4113d0d25461bbdd08d7d93a870dec775e19a9e48a763362b709*
Pure Prop Address:	*TMTjqJpi2rtJW9qijp4JKmBpdTppjv1rP6f*
Theory Prop Id:	*c3c3e6c3e94f9e03aeb2b4f93e33744decc142ead482a1fa50a8767e7fc99638*
Theory Prop Address:	*TMcN2J3tSopfuEz9NRL1jfvCCu5kxE2v2No*

`proj0_pair_eq`

Theorem 20.18 $\forall XY : \iota.\mathsf{proj0}\ (pair\ X\ Y) = X$. *The proposition is identified by the following information:*

Pure Prop Id:	*d546f32a1c10eae38a9af6397e8d2af2759b1d0646a5f3d4868c51100c9e2395*
Pure Prop Address:	*TMbPLyxLXKPkMnkdo72LQeKTjebPKPLSJmk*
Theory Prop Id:	*0c12fbce603161ada7fcfdceb916aa8184711c821cd1e1b2326d5152dfe84499*
Theory Prop Address:	*TMHWPP4mLNCpHEx7SiPwPaQi8HiEHdVrm9b*

`proj1_pair_eq`

Theorem 20.19 $\forall XY : \iota.\mathsf{proj1}\ (pair\ X\ Y) = Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*9f98a8d2797cf5bb49a476c118564c127ff5edcaea373b1ca114e096c594679f*
Pure Prop Address:	*TMLxPAuDd6Zdjkv16vcwxEZoxAdBz8adaQM*
Theory Prop Id:	*e0cb0f860b00c8c1dea60e21e3bab80b7213e7c2da3750ce479bd67ef1eee60d*
Theory Prop Address:	*TMM7LPbGJRfCrTrhXupSUxTM5PSBW4xxwqf*

`pair_inj`

Theorem 20.20 $\forall xywz : \iota.pair\ x\ y = pair\ w\ z \to x = w \wedge y = z$. *The proposition is identified by the following information:*

Pure Prop Id:	*c7ef9fe33c7c1572d541f409e082fd4867a3959389d1246ddb8c6e790550ad23*
Pure Prop Address:	*TMEvDEchwftSTVQExszQAystHF4dFnXXiiV*
Theory Prop Id:	*972ddf22ac0ad35ef54fbe2f14d3eb51f3932d2eb7c90de81cea04fd839a85b6*
Theory Prop Address:	*TMTqqAmny9TT4MKV73TgCn5hxdYCZV3qB4j*

`pair_eta_Subq_proj`

Theorem 20.21 $\forall w.pair\ (\mathsf{proj0}\ w)\ (\mathsf{proj1}\ w) \subseteq w$. *The proposition is identified by the following information:*

Pure Prop Id:	*a868f0131174256073ca2e69f0629c02909a5e8cddbddc8bf755c9d11a4fc8df*
Pure Prop Address:	*TMSF6AUrFFqfp7WqPsWnmZrPyBmHXMFBof5*
Theory Prop Id:	*9afd8b29c6f5307394a6a98ed81326ea8d0a1e58965e3886e0fcd87155aba943*
Theory Prop Address:	*TMNSTW7wqZuPxSL1kuFRRghsCCu3eRTUUC1*

20.2 Dependent Sums

Definition 20.3 *We define* Sigma *to be* $\lambda XY.\bigcup_{x\in X}\{pair\ x\ y|y\in Y\ x\}$ *of type* $\iota\to(\iota\to\iota)\to\iota$ *identified by the following information:*

Pure Object Id:	*8acbb2b5de4ab265344d713b111d82b0048c8a4bf732a67ad35f1054a4eb4642*
Pure Object Address:	*TMMDLyvQS3eoiP32KBXjSQyuvc4keN5yMmX*
Theory Object Id:	*d64bcbbefe13f9fc0035d2317f017d9ee3aeb3de90b836ed7cbe11d46f478799*
Theory Object Address:	*TMS2X4GvB3DZiVdQjy5ZeDZQVVCDuJBZvgW*

Notation. We use $\Sigma x,\ldots,y.$ as a binder notation corresponding to a term constructed using Sigma.

pair_Sigma

Theorem 20.22 $\forall X:\iota.\forall Y:\iota\to\iota.\forall x\in X.\forall y\in Y\ x.pair\ x\ y\in\Sigma x\in X.Y\ x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*0d8645a51ab956ff15d92813d2cab140e7352947463636774795 8b8e18bab905*
Pure Prop Address:	*TML8ffdR3C51f2E9BMf7H2BS9S1rFurh827*
Theory Prop Id:	*1456cb5b576a7c97a2a8bc57c2896584b0d55dd30ca4212f1733b4c7ebe4f919*
Theory Prop Address:	*TMaVj4zKhXkoLhyH11SzjuosBXYALhD49xN*

Sigma_eta_proj0_proj1

Theorem 20.23

$$\forall X:\iota.\forall Y:\iota\to\iota.\forall z\in(\Sigma x\in X.Y\ x).$$
$$pair\ (\mathsf{proj0}\ z)\ (\mathsf{proj1}\ z)=z\wedge\mathsf{proj0}\ z\in X\wedge\mathsf{proj1}\ z\in Y\ (\mathsf{proj0}\ z).$$

The proposition is identified by the following information:

Pure Prop Id:	*afe96271dba741ef6022853ac10aa26abecb9b32d05c44f71e7c5089f782d778*
Pure Prop Address:	*TMWpztGVWdAFQ1ovvNdLFx6cps617NMUj8f*
Theory Prop Id:	*77ba8e9a2110bd246cbd0efc93c8437da7906392890ad5f4156a4e526a76a950*
Theory Prop Address:	*TMY1GRBXj483wdHtvtfTD9uZNgUp8CAiaRD*

proj_Sigma_eta

Theorem 20.24

$$\forall X:\iota.\forall Y:\iota\to\iota.\forall z\in(\Sigma x\in X.Y\ x).pair\ (\mathsf{proj0}\ z)\ (\mathsf{proj1}\ z)=z.$$

The proposition is identified by the following information:

Pure Prop Id:	*8b31cdfbbf34fe7c3b0423c153108eed30fc19b3ff82132f32fc21939416a2ef*
Pure Prop Address:	*TMJH9UNCBubGXwZc46iPvjZm3iBB1y6mfHu*
Theory Prop Id:	*f30f76cb2aef08be1838ea268e49d550fefb9694a95470f51dd35304cd46da86*
Theory Prop Address:	*TMX4ChFaKVytLhSzjfKyAapz8j6LrbpcHM5*

`proj0_Sigma`

Theorem 20.25

$$\forall X : \iota.\forall Y : \iota \to \iota.\forall z : \iota.z \in (\Sigma x \in X.Y\ x) \to \mathsf{proj0}\ z \in X.$$

The proposition is identified by the following information:

Pure Prop Id:	*ae12a7b526c0a13cce84f529e93ced0f678ce1c9b63337256f2952f31af9eb47*
Pure Prop Address:	*TMGDaF1fWHGqSd5NBBSVmU2WGERM8Yb3k9o*
Theory Prop Id:	*50686996f47a43fc8b07a9886de2c8d44fb1f23e5ebe1c95ea71b5d7a06d274a*
Theory Prop Address:	*TMWkPywgVwGwhQpBQMdXhZTDgiXWekNhrCd*

`proj1_Sigma`

Theorem 20.26

$$\forall X : \iota.\forall Y : \iota \to \iota.\forall z : \iota.z \in (\Sigma x \in X.Y\ x) \to \mathsf{proj1}\ z \in Y\ (\mathsf{proj0}\ z).$$

The proposition is identified by the following information:

Pure Prop Id:	*553db6938c6f87c354d6c9209f682b09c73f8cbc2eb21ab6513fee7dd2092d35*
Pure Prop Address:	*TMaYkGoHBN1iZiAYUVzJRZLn3EgRSQWrx1H*
Theory Prop Id:	*d79917b7b626dcc0902b44780a05f036fcf9f4445434650e2ea129fa0f955938*
Theory Prop Address:	*TMFMACi6zbuVxKdSyevDC9cG2fhuphqdLvD*

`pair_Sigma_E0`

Theorem 20.27 $\forall X : \iota.\forall Y : \iota \to \iota.\forall xy : \iota.pair\ x\ y \in (\Sigma x \in X.Y\ x) \to x \in X.$ *The proposition is identified by the following information:*

Pure Prop Id:	*c4b4172cb06b68610331df0bff40a80e131c5fe7a0f37012432d5896d5e2c4a8*
Pure Prop Address:	*TMa5xp6pY52DfG6pfd7VzRGhjHUWXTk87nR*
Theory Prop Id:	*481fddcf7b10f918784064dcf83451596b1a8fec3423f6e7215608a7dccefb43*
Theory Prop Address:	*TMRJQf5xNbV6uehrGdTPq6X4eyNfy3irJAQ*

`pair_Sigma_E1`

Theorem 20.28 $\forall X : \iota.\forall Y : \iota \to \iota.\forall xy : \iota.pair\ x\ y \in (\Sigma x \in X.Y\ x) \to y \in Y\ x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*c294a7d73a371c2ac828dbda009fa9e74a8b85adfe24c4731eb12086f8913b12*
Pure Prop Address:	*TMMukS2p563YcLJwvCEj2kjfqr8z23JmFUE*
Theory Prop Id:	*81d8b0d6136a1b697d4b5753d87fb42789104c2db7a89e6023100fcb32516c3f*
Theory Prop Address:	*TMWdjWXycEquBp15ycxwyeRHKb8YEFjrntf*

`Sigma_E`

Theorem 20.29

$$\forall X : \iota.\forall Y : \iota \to \iota.\forall z : \iota.z \in (\Sigma x \in X.Y\ x) \to \exists x \in X.\exists y \in Y\ x.z = pair\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*5b1bd2e14ea3dbf36ed03b2bca51d8d6329ffb7924a6198502c84e472877864e*
Pure Prop Address:	*TMKZhCQwWmLxNmtRGpKgTrmnpUbXzpUqcC6*
Theory Prop Id:	*8ecfabfede2e6f041d7ca7aa6d8ddc7d93e1d4a8f77164e302775838b170ef2f*
Theory Prop Address:	*TMHvzRUPwTDz6sNjr1Xu75jPe97kiKyiFRf*

`Sigma_Eq`

Theorem 20.30

$$\forall X : \iota.\forall Y : \iota \to \iota.\forall z : \iota.z \in (\Sigma x \in X.Y\ x) \Leftrightarrow \exists x \in X.\exists y \in Y\ x.z = pair\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*49886b5979c9429dfa4b9590d2e5c308bccd95f87e7cc32f24dc682e35e497f8*
Pure Prop Address:	*TMXKo25CKqXa5jWu42vnAgmVtNvhqv9e29Q*
Theory Prop Id:	*e7bf483e07224467cd16dcad77572201131ce4f5471414967c469948e6ef38dd*
Theory Prop Address:	*TMUV6V5qcBof6dwE3ebfhwBLiWuk2pD9W8w*

`Sigma_mon`

Theorem 20.31

$$\forall XY : \iota.X \subseteq Y \to \forall ZW : \iota \to \iota.(\forall x \in X.Z\ x \subseteq W\ x) \to (\Sigma x \in X.Z\ x) \subseteq \Sigma y \in Y.W\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*425a20091aa7362e407182389fe24965433da14fc0de3e08e13b5905910b2208*
Pure Prop Address:	*TMYSaxNB5bpwjKeSA3Sf2F7RCAM2gnN5zrb*
Theory Prop Id:	*a8addf8ad27e68150e96f0d2dbd7dd38f2d552c5eb34a1bd96b1975a2139fa44*
Theory Prop Address:	*TMS4jg3iXSAoEt25M7crKBhYaShmP982iYR*

`Sigma_mon0`

Theorem 20.32

$$\forall XY : \iota.X \subseteq Y \to \forall Z : \iota \to \iota.(\Sigma x \in X.Z\ x) \subseteq \Sigma y \in Y.Z\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*9b3fbfc10d8cb198135b15ef9120823b40c83a7048721a2a8f5bd592dccd207e*
Pure Prop Address:	*TMHvdHmzNtjNYsphs5AwquVMExTDwRKzC6i*
Theory Prop Id:	*0c39e5d90a03c8b53d1f91b86500a8426061994e891ba8d4f0abf6ed55874388*
Theory Prop Address:	*TMG1irPWX7hSKZCJSoz5U9WvofwpYiLcPrD*

`Sigma_mon1`

Theorem 20.33

$\forall X : \iota.\forall Z W : \iota \to \iota.(\forall x.x \in X \to Z\ x \subseteq W\ x) \to (\Sigma x \in X.Z\ x) \subseteq \Sigma x \in X.W\ x.$

The proposition is identified by the following information:

Pure Prop Id:	*8f6c670d9cf58964547a2c8f5ecb2655c51fed679a89fcbaf042a9a98576b22c*
Pure Prop Address:	*TMHxEXAifEfqBZatTVRBvvzdAKBzqA8xJC8*
Theory Prop Id:	*8a20d4273c3bd449f68d0446ea954be5016f26a33d1b32bc491ace72b24bfce8*
Theory Prop Address:	*TMVezD2kkz73gqZqGBDvxeirmP7cryxcrP2*

`Sigma_Power_1`

Theorem 20.34

$\forall X : \iota.X \in \mathsf{Power}\ 1 \to \forall Y : \iota \to \iota.(\forall x \in X.Y\ x \in \mathsf{Power}\ 1) \to (\Sigma x \in X.Y\ x) \in \mathsf{Power}\ 1.$

The proposition is identified by the following information:

Pure Prop Id:	*7a3ae673c65b20d28f4b80b127c87fe8effc6abadb514348178f01786f8f687a*
Pure Prop Address:	*TMTUPmfM2Mi7ztuVrQGCeVRgWXipKh9vVXb*
Theory Prop Id:	*889ba9237de623f9140e05e0574c4f3e91b2c3c0df705ac3109723a3a09dda78*
Theory Prop Address:	*TMZYULyhhi7GEPAnwEARVDsEJgpr2MpVTbG*

Definition 20.4 *We define* setprod *to be* $\lambda X Y : \iota.\Sigma x \in X.Y$ *of type* $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*fc0b600a21afd76820f1fb74092d9aa81687f3b0199e9b493dafd3e283c6e8ca*
Pure Object Address:	*TMX5iqj6cvWjd5eoMqqTEF1367VHDQkj6Yq*
Theory Object Id:	*107b6bb74ab2c1972d7258ba373ae94a01269f3d05e17f604d541e405e9ab93f*
Theory Object Address:	*TMFyVvPCFYsHfsACwhMzeFYF8jZqyyWje7X*

Notation. We use $\times$ as a left associative infix operator corresponding to applying term setprod.

`pair_setprod`

Theorem 20.35 $\forall X Y : \iota.\forall (x \in X)(y \in Y).pair\ x\ y \in X \times Y.$ *The proposition is identified by the following information:*

Pure Prop Id:	*f55ea3828e02273317f0b400d2105e5f6809d31a9ac1326fe0fea71657f5048d*
Pure Prop Address:	*TMP1cFf4SMJvCYXWGuPAbdAT3rrBv9oFfUd*
Theory Prop Id:	*1dd4994dec4372a29bd9c00f15aa916beb53d6a66b15c1d25e72219e6a2d006c*
Theory Prop Address:	*TMJCXgkH2PkoAWeAmeW98Fvbhtn48QfWyEd*

`proj0_setprod`

Theorem 20.36 $\forall XY : \iota.\forall z \in X \times Y.\mathsf{proj0}\ z \in X$. *The proposition is identified by the following information:*

Pure Prop Id:	*510623d6d9fee58fe36195a65b68d9e88774298fd789c76bb947e43ba0e9ddd3*
Pure Prop Address:	*TMbvsu8j8H6CMFRSSMgZzBoz1STTTdmJCQv*
Theory Prop Id:	*f7a519336a9ebe4633840fe128d350ab70461bc5396fe75659857496a4df6c8e*
Theory Prop Address:	*TMQGsuKAnuZCk7jn3UYPfyZCtmqNjDLEXm9*

`proj1_setprod`

Theorem 20.37 $\forall XY : \iota.\forall z \in X \times Y.\mathsf{proj1}\ z \in Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*f6765c051b33e8891d8407887b98ac6204877cdb39b1c5336a548c49859ce225*
Pure Prop Address:	*TMMHAkHerqWaT2pFe3dt7C1yXqqaPT3Lotj*
Theory Prop Id:	*afe85e979f6864648f01f74dec8776c350283f9b19d37ad3b00f2558dc5da050*
Theory Prop Address:	*TMVQEexDvYgcxvr3ptPV72JSsdGheT6ge7G*

`pair_setprod_E0`

Theorem 20.38 $\forall XYxy : \iota.pair\ x\ y \in X \times Y \to x \in X$. *The proposition is identified by the following information:*

Pure Prop Id:	*107e92452bde569e548509981b8ad6b3760e221a2995bd8b8b11748c06428f6e*
Pure Prop Address:	*TMKaLir23aqUncRog77yHWNUteSZpVWVbf5*
Theory Prop Id:	*92b4c14b744c79aa7e9d294133d7983d9ba8f30c9cc011c9d079a5f451447047*
Theory Prop Address:	*TMSWuEEd13oCxUA81Augy7MtxEyUeafaiQQ*

`pair_setprod_E1`

Theorem 20.39 $\forall XYxy : \iota.pair\ x\ y \in X \times Y \to y \in Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*50179b781626f48b09be181df9f551b18fbc72192d119884122eea0b4427b916*
Pure Prop Address:	*TMTFx9j8XQY7mMx8nXnVLkS1ReCnJvoc5VR*
Theory Prop Id:	*4e83a82fa29d4046c8ca4bd9154e06e2992eba3ed652c3aa954d141cccef49be*
Theory Prop Address:	*TMYMex5ZPQm8KKBnWBPkZD51i9QL5Ao4tHE*

20.3 Functions

Let $lam : \iota \to (\iota \to \iota) \to \iota$ be Sigma. We write $\lambda x \in X.t$ for $lam\ X\ (\lambda x.t)$. Note that $\Sigma_{x \in X}.t$ and $\lambda x \in X.t$ are two notations for the same term.

Definition 20.5 *We define* ap *to be* $\lambda f x.\{\mathsf{proj1}\ z | z \in f, \exists y : \iota.z = pair\ x\ y\}$ *of type* $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*c7aaa670ef9b6f678b8cf10b13b2571e4dc1e6fde061d1f641a5fa6c30c09d46*
Pure Object Address:	*TMRkKHvzv8Tta9AhRXsVE4k7dQ68XWymCoD*
Theory Object Id:	*a4d82c5a3643a7e174676c313ef995e0307b3b56f57e8ae761b7c6033bbc40db*
Theory Object Address:	*TMULGKR83zvLzsaT5wtaLqEme8A8TV9uVcG*

Notation. When x and y are of type ι, then xy means `ap` x y.

`lamI`

Theorem 20.40 $\forall X : \iota.\forall F : \iota \to \iota.\forall x \in X.\forall y \in F\ x.pair\ x\ y \in \lambda x \in X.F\ x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*0d8645a51ab956ff15d92813d2cab140e7352947463636774 7958b8e18bab905*
Pure Prop Address:	*TML8ffdR3C51f2E9BMf7H2BS9S1rFurh827*
Theory Prop Id:	*1456cb5b576a7c97a2a8bc57c2896584b0d55dd30ca4212f1733b4c7ebe4f919*
Theory Prop Address:	*TMaVj4zKhXkoLhyH11SzjuosBXYALhD49xN*

`lamE`

Theorem 20.41

$$\forall X : \iota.\forall F : \iota \to \iota.\forall z : \iota.z \in (\lambda x \in X.F\ x) \to \exists x \in X.\exists y \in F\ x.z = pair\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*5b1bd2e14ea3dbf36ed03b2bca51d8d6329ffb7924a6198502c84e472877864e*
Pure Prop Address:	*TMKZhCQwWmLxNmtRGpKgTrmnpUbXzpUqcC6*
Theory Prop Id:	*8ecfabfede2e6f041d7ca7aa6d8ddc7d93e1d4a8f77164e302775838b170ef2f*
Theory Prop Address:	*TMHvzRUPwTDz6sNjr1Xu75jPe97kiKyiFRf*

`lamEq`

Theorem 20.42

$$\forall X : \iota.\forall F : \iota \to \iota.\forall z.z \in (\lambda x \in X.F\ x) \Leftrightarrow \exists x \in X.\exists y \in F\ x.z = pair\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*49886b5979c9429dfa4b9590d2e5c308bccd95f87e7cc32f24dc682e35e497f8*
Pure Prop Address:	*TMXKo25CKqXa5jWu42vnAgmVtNvhqv9e29Q*
Theory Prop Id:	*e7bf483e07224467cd16dcad77572201131ce4f5471414967c469948e6ef38dd*
Theory Prop Address:	*TMUV6V5qcBof6dwE3ebfhwBLiWuk2pD9W8w*

`apI`

Theorem 20.43 $\forall fxy.pair\ x\ y \in f \to y \in f\ x.$ *The proposition is identified by the following information:*

Pure Prop Id: 64667c3e12b9f3ae61ec356833747e367523a8c0ce6c2a9d93f4c38b0783bde5
Pure Prop Address: TMZLPpY9KNgV39N2yHL65R4C438SdAntCJF
Theory Prop Id: 7708163781ef5b94401f65bdb2a5b080ff257a5930faf05dd1bbe0b4789e9537
Theory Prop Address: TMPak8MUV4CmmTFj6jaPQ3fYN19JukFbBVX

apE

Theorem 20.44 $\forall fxy.y \in f\ x \to pair\ x\ y \in f$. *The proposition is identified by the following information:*

Pure Prop Id: 85693a617f5d7231598e57242e17d01ba19426804680415415d83c36ca14dcf3
Pure Prop Address: TMM471VfyY9L9ihJZoGVZy3MTJucpLkPz71
Theory Prop Id: c08565940adcaebbb0bd8ea7dca3ebc52f096c6a861e7e3eb1dfcc522c057448
Theory Prop Address: TMNq9VhD4hU2LQs2wbRmvWrApYxACcfdNyd

apEq

Theorem 20.45 $\forall fxy.y \in f\ x \Leftrightarrow pair\ x\ y \in f$. *The proposition is identified by the following information:*

Pure Prop Id: 4c9d67e97abd0fc158c7647a05ca09ff2992ddd126c6b8d0b452e70e8cbcc406
Pure Prop Address: TMZQ7PrqUw2JPS363w3buW9bffJsBTNttxy
Theory Prop Id: bd396ee22cc900202a9db5ef15b2b6dc151ca2285723ce93f3f245def051fbe2
Theory Prop Address: TMSiUYhpZzUqv5JfXdvkMd9Vyn2Xj1gRY8z

beta

Theorem 20.46 $\forall X : \iota.\forall F : \iota \to \iota.\forall x : \iota.x \in X \to (\lambda x \in X.F\ x)\ x = F\ x$. *The proposition is identified by the following information:*

Pure Prop Id: 95991de564f22218049f5c01a2054c9f531e2a0e7f89348f4cd3cce56269ee6a
Pure Prop Address: TMW5SkUgFpNvxZtwNs5tigJKy4uVF4aLEX4
Theory Prop Id: b5182dff18aa9200707f42f128e18ba010c3c5b0a3a598682cfd9fe3e36def5d
Theory Prop Address: TMUK7trdHToYkqBA9FA7nyBfnGCJVnzMrdt

beta0

Theorem 20.47 $\forall X : \iota.\forall F : \iota \to \iota.\forall x : \iota.x \notin X \to (\lambda x \in X.F\ x)\ x = 0$. *The proposition is identified by the following information:*

Pure Prop Id: 3bb4348d3945f138f08291a2d198487854d211b0b272222fe4f9e932e577522b
Pure Prop Address: TMc9Mv7ukGPvjkLta2ci5R66AEtkDgNuCSM
Theory Prop Id: 475c212b14b3b0ca0e09cf957e6ba095fef5db59bc5b0dcdbe6adbb59a281cce
Theory Prop Address: TMFBwjzjE5yzpGy66RMUx1PdJ9mHVfc6dHb

proj0_ap_0

Theorem 20.48 $\forall u.\mathsf{proj0}\ u = u\ 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*588f379b68a9458a540fcdf77dc5d2f4783c00f4cce95dc51c95471cc943eb03*
Pure Prop Address:	*TMYCXMFZ4HzrBKRUDMYgmo4eHSvQVMQ7p31*
Theory Prop Id:	*d4f3513ee04ff36d85b2db23be3080385b47ffa013ac715ce8658accd5d66b38*
Theory Prop Address:	*TMSgBry8vyDWLoFEMZhop4n2xDSn5k839xF*

`proj1_ap_1`

Theorem 20.49 $\forall u.\mathsf{proj1}\ u = u\ 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*d116b3ec53d3345a265357a353929b0e25b62670c0a9b2a3d51a8c73ef732f19*
Pure Prop Address:	*TMNoHiTHocDKAmUg9hNSv1FxD2Amd3S3tUP*
Theory Prop Id:	*2328b0e51e8e97743e5baf0897eb65244f631c1e1667f87ea13b28f83b8939b1*
Theory Prop Address:	*TMRaYAR6WpaEp6hPTm43iVAHVqRrJwCb9di*

`pair_ap_0`

Theorem 20.50 $\forall xy : \iota.(pair\ x\ y)\ 0 = x$. *The proposition is identified by the following information:*

Pure Prop Id:	*ac34b9cde0ee2208ea8e422cba572e3fb7982b46ebfdd714251b2dfcb897193e*
Pure Prop Address:	*TMVVcCiRogXNQekjrhetLEc1AwKsjAoaCUb*
Theory Prop Id:	*bb047796b50fbb719f3e14fe10b517fd118f2911a01dac2dc1972ece4948275e*
Theory Prop Address:	*TMPNWnANeRPomJ9tENqayKNjmwfidUynXLZ*

`pair_ap_1`

Theorem 20.51 $\forall xy : \iota.(pair\ x\ y)\ 1 = y$. *The proposition is identified by the following information:*

Pure Prop Id:	*2274fc6211c808c3b1604a439df9779ba0f1ddad055df119366dc394c2fb97f6*
Pure Prop Address:	*TMSs4jYzqSwfDX666rdcAD6CeBDVaoMRoyc*
Theory Prop Id:	*cce3d21c0d689ca2584d628eb0fdd682ec793195238268afe769ce51f75bb164*
Theory Prop Address:	*TMFmHmqub8mZKhdbCxJVhYYkdtHahTZ7UAh*

`pair_ap_n2`

Theorem 20.52 $\forall xyi : \iota.i \notin 2 \to (pair\ x\ y)\ i = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*aa4c5b4688eb6b4f059f6d4ecc867417bf44703f3f25ae2c2a4bb84351e4918a*
Pure Prop Address:	*TMJvhJVjoT8tymCLYAEd8KMpMSd7c5dzvrs*
Theory Prop Id:	*6a36da546596b4c9b0cab2739118ca96148377b2acb8a70b37d0a4aa0d404b22*
Theory Prop Address:	*TMapLv3NXF9X9vWbexn1gRz4mSdux6SZs6w*

`ap0_Sigma`

Theorem 20.53 $\forall X : \iota.\forall Y : \iota \to \iota.\forall z : \iota.z \in (\Sigma x \in X.Y\ x) \to (z\ 0) \in X$. *The proposition is identified by the following information:*

Pure Prop Id:	*861a795017a942802f4443c95b5a36332a33f167ce1441dfc6a24503f6268b3e*
Pure Prop Address:	*TMKRf5LGKqydnVhB8vpAzDvodiXpqNvrMxB*
Theory Prop Id:	*1affcc338ae841e0e000cf9be970918006435331f595cedc951a44209748026c*
Theory Prop Address:	*TMUBP4wFdxgzoA73DJqdvnWDxq8D4ZPGmSV*

ap1_Sigma

Theorem 20.54

$$\forall X : \iota.\forall Y : \iota \to \iota.\forall z : \iota.z \in (\Sigma x \in X.Y\ x) \to (z\ 1) \in (Y\ (z\ 0)).$$

The proposition is identified by the following information:

Pure Prop Id:	*ad3bfe036fa895ae325d64498611356bb49df8ebd34b91be258534581bb43fb7*
Pure Prop Address:	*TMQ3NDTgxp9a1VuLPqoNmEGA5MjKwt6QJLH*
Theory Prop Id:	*520af5f2f80e3d8411b1460e91384a058099174706d3029c6f03f1ab389a8f32*
Theory Prop Address:	*TMKB8YXKTKjDb1T1UTPbMwrfxTLy3KBSoJv*

Definition 20.6 *We define* pair_p *to be* $\lambda u : \iota.pair\ (u\ 0)\ (u\ 1) = u$ *of type* $\iota \to o$ *identified by the following information:*

Pure Object Id:	*ade2bb455de320054265fc5dccac77731c4aa2464b054286194d634707d2e120*
Pure Object Address:	*TMSvXNQFUtikJWHkJ8EmjpLPVE6eVHHPye3*
Theory Object Id:	*5c8a35346b94416c8921f9496e68b956c0e80aa2f5b966805dd30a7cc9bdb03e*
Theory Object Address:	*TMbuqZXCg75fnX39GezV6NmULFhCWCyVQLX*

pair_p_I

Theorem 20.55 $\forall xy.$pair_p $(pair\ x\ y)$. *The proposition is identified by the following information:*

Pure Prop Id:	*06d9a28b2c025648a5c4c367e18ca07d131c2483cbec5ef5ed9b8c412a2959e3*
Pure Prop Address:	*TMJoDcS8DRqyUSNGb82nnjwSRKn9hqLmFdN*
Theory Prop Id:	*8f2fdee4c2a4b4668940264d97e6a4d34887ed1cb2b9462e5a68ea86f3267c92*
Theory Prop Address:	*TMPcM8MemhXia2UuEVstbB2UDdYPEQwK1R1*

pair_p_I2

Theorem 20.56 $\forall w.(\forall u \in w.$pair_p $u \wedge u\ 0 \in 2) \to$ pair_p w. *The proposition is identified by the following information:*

Pure Prop Id:	*e4b35e6cad38e54d2a79aadbe2af0245099589e5c7fade68162ef8871c460979*
Pure Prop Address:	*TMJUphhHJCGQn6H3JsbDsYQ2vhJYQpGSSq6*
Theory Prop Id:	*f66937108389bce047e36e45cd53689abfbd69e81d439e978b5d6f03a84804fd*
Theory Prop Address:	*TMUhxxoG6nPHQGfqpNzk4M5VX4pepkB3W7b*

pair_p_In_ap

Theorem 20.57 $\forall wf.$pair_p $w \to w \in f \to w\ 1 \in$ ap $f\ (w\ 0)$. *The proposition is identified by the following information:*

Pure Prop Id:	*e93eb76f4ff4537fc952c7e2663dd348eba0a1eda133a280fa73a98aaed1afd7*
Pure Prop Address:	*TMMpbcE9Nf47Qr4V5rA9WJeQ3beYck2RvoP*
Theory Prop Id:	*cda5c674d061ba7e438ebe2318a287bc128928b90a50c0bbd5f76eb976eda17a*
Theory Prop Address:	*TMVJ3KJ5wJShTfj2JebzJWxoqicyYhHmzhf*

20.4 Tuples as Functions on Naturals

Definition 20.7 *We define* tuple_p *to be*

$$\lambda nu.\forall z \in u.\exists i \in n.\exists x : \iota.z = pair\ i\ x$$

of type $\iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*83d48a514448966e1b5b259c5f356357f149bf0184a37feae52fc80d5e8083e5*
Pure Object Address:	*TMPQRFB8kv9Mi3hJcbAVVFkKaCjpTiKn5DP*
Theory Object Id:	*7b913e689b741f5e3d1a5c19e7580fd46ffd026760b86242807b35879571cbe3*
Theory Object Address:	*TMSwuuEUBwMbLNhY2oHRChGyRGNSMgRmKA1*

pair_p_tuple2

Theorem 20.58 pair_p = tuple_p 2. *The proposition is identified by the following information:*

Pure Prop Id:	*edb4f2daac705fc5fde5fb625306e4ce3b70b5c324811dcfe9286a3352609130*
Pure Prop Address:	*TMa7Ph4F792pWejWQxMqTYL67K6y3Atz3GB*
Theory Prop Id:	*4ab8c71265b072dffa2255ec379c857d2480a9f4cc0b5722bf710c7b2e3aaa21*
Theory Prop Address:	*TMPDLkLax6xryGqZWfjiAzsWiueQwkeoq77*

Notation. When $x_0, \ldots, x_{n-1}$ are of type ι, we write $(x_0, \ldots, x_{n-1})$ for $\lambda i \in n.$if $i = 0$ then x_0 else $\cdots$ if $i = n-1$ then x_{n-1} else 0.

tuple_p_2_tuple

Theorem 20.59 $\forall xy : \iota.$tuple_p 2 (x, y). *The proposition is identified by the following information:*

Pure Prop Id:	*982145ec24f8189cb7ff562fa9a82c89405822de093f290a7c78ff1b6560c79b*
Pure Prop Address:	*TMRXKTaQZ2mMDiGAZrvuMg3ozPJcAZAgWGP*
Theory Prop Id:	*f8f972a715455be65b85075f8f7f738798be5894513ebfc46853439a15150d79*
Theory Prop Address:	*TMX5VxQzTL4dEJu18K3FBfoBgfDf59XiNpV*

tuple_pair

Theorem 20.60 $\forall xy : \iota.pair\ x\ y = (x, y)$. *The proposition is identified by the following information:*

Pure Prop Id:	*c122f6c45d6ae2b29699ba19714006844818fe6aed0b78c730fa61d541f8f655*
Pure Prop Address:	*TMNGGwosAVCCgzqRpMz6fsnenJVoBPWXYUc*
Theory Prop Id:	*18198e977ce2f209097f879f8fb820813712427886 07f8a3fa8da991d443ee60*
Theory Prop Address:	*TMUTEozs7LjRCKnvt77suPHQfR7gNvzEi7v*

20.5 Dependent Products

Definition 20.8 *We define* Pi *to be*

$$\lambda XY.\{f \in \mathsf{Power}\ (\Sigma x \in X.\mathsf{Union}\ (Y\ x)) | \forall x \in X.f\ x \in Y\ x\}$$

of type $\iota \to (\iota \to \iota) \to \iota$ *identified by the following information:*

Pure Object Id:	*40f1c38f975e69e2ae664b3395551df5820b0ba6a93a228eba771fc2a4551293*
Pure Object Address:	*TMMmsr8jojnNQnfEu32oXr8oHW6b2JucDYn*
Theory Object Id:	*a7b1b09cb152a467eba4c7154e043aede9f10dd364de00ce2145c5178d3ad12b*
Theory Object Address:	*TMb8zzbN5gj4sGhAcwYC7Re7qCi97Rtgo7h*

Notation. We use $\Pi x, \ldots, y.$ as a binder notation corresponding to a term constructed using Pi.

`PiI`

Theorem 20.61

$$\forall X : \iota.\forall Y : \iota \to \iota.\forall f : \iota.(\forall u \in f.\mathsf{pair\_p}\ u \wedge u\ 0 \in X) \to (\forall x \in X.f\ x \in Y\ x) \to f \in \Pi x \in X.Y\ x.$$

The proposition is identified by the following information:

Pure Prop Id:	*2b7ef42e911f5c9d245f0e1650bdf7a017dc07c61643b180beacd902afa937ac*
Pure Prop Address:	*TMHGz8t2EnwQDiuH6KDQpg8EnyxGgR8dS7e*
Theory Prop Id:	*1f8f6f437998cacea340ec1ea9cc9798e39298aa0d85498c0c20643140f46942*
Theory Prop Address:	*TMWx4ti6JwTdu8xygg15tK2YVvzLMBHLWpP*

`PiE`

Theorem 20.62

$$\forall X : \iota.\forall Y : \iota \to \iota.\forall f : \iota.f \in (\Pi x \in X.Y\ x) \to (\forall u \in f.\mathsf{pair\_p}\ u \wedge u\ 0 \in X) \wedge (\forall x \in X.f\ x \in Y\ x).$$

The proposition is identified by the following information:

Pure Prop Id:	*e900d056f38f163f77287b64052d5d9e602edf6c698bce849b83205e4fecd08a*
Pure Prop Address:	*TMQswaTQQ8v2NGGPZrcYHSjahdLxdVqq8Bc*
Theory Prop Id:	*c6a19abb37631cc1754b1e349cb516006e88cf3a4a1da2d0f9de9154571fabdc*
Theory Prop Address:	*TMNmPoYTgXJ1oQ5bHuJoN3MvGSuMrhGxKb8*

`PiEq`

Theorem 20.63

$$\forall X:\iota.\forall Y:\iota\to\iota.\forall f:\iota.f\in \mathsf{Pi}\ X\ Y \Leftrightarrow$$
$$(\forall u\in f.\mathsf{pair\_p}\ u\wedge u\ 0\in X)\wedge(\forall x\in X.f\ x\in Y\ x).$$

The proposition is identified by the following information:

Pure Prop Id:	*aee68f00e9f9b4dfa4f93208b50b84f39e34d3143871795718db72cf160eceba*
Pure Prop Address:	*TMLoctKVTPvcs1L1sQJmpzmMB7ZGefxweHF*
Theory Prop Id:	*e80f95c3f8e7d257d04588c36cf7c2cbcef829f184e76311e443137e0d9bf708*
Theory Prop Address:	*TMJq3SLpMrtKWAEFsc6WRLhigo1buRia2SA*

`lam_Pi`

Theorem 20.64

$$\forall X:\iota.\forall Y:\iota\to\iota.\forall F:\iota\to\iota.(\forall x\in X.F\ x\in Y\ x)\to$$
$$(\lambda x\in X.F\ x)\in(\Pi x\in X.Y\ x).$$

The proposition is identified by the following information:

Pure Prop Id:	*c0b88f5343d69ece7a5c1776f75f883b1ea441d9f54b23c0f2f9bfd3d63291ce*
Pure Prop Address:	*TMK1tiobRV7Rk8oLJDiqRt9emhAiARd65fd*
Theory Prop Id:	*3733f8d8b8cfb744a472ecadfd3dd8ec6a02c94d9f4ba9cb8fe6fa05f899b65e*
Theory Prop Address:	*TMR7xMjyWg42N9C62nDT2fP29rt63NXuwrb*

`ap_Pi`

Theorem 20.65

$$\forall X:\iota.\forall Y:\iota\to\iota.\forall f:\iota.\forall x:\iota.f\in(\Pi x\in X.Y\ x)\to x\in X\to f\ x\in Y\ x.$$

The proposition is identified by the following information:

Pure Prop Id:	*7c36179a82d8644940659a1f02b440b62c39cf8ece8fc0d2538fccfd980eaa68*
Pure Prop Address:	*TMXNoSqzT5Lu24RbFYcKt5N8JRNvmf1MDSm*
Theory Prop Id:	*cb962ef0bbe77996198527ec24108318b6142c9b6d3b1cacdc185efa044e94f6*
Theory Prop Address:	*TMFXJjxwrkNdrbgK4hwdYsRNhanGr8DEELP*

`Pi_ext_Subq`

Theorem 20.66

$$\forall X:\iota.\forall Y:\iota\to\iota.\forall fg\in(\Pi x\in X.Y\ x).(\forall x\in X.f\ x\subseteq g\ x)\to f\subseteq g.$$

The proposition is identified by the following information:

Pure Prop Id:	*895d76cfaedb480895437 91c77f82e5ff8402576798d94cc12c928b42720b3c7*
Pure Prop Address:	*TMK7g7rTpdWeWhZFQAosbFDbAB6K9oH1vVL*
Theory Prop Id:	*fcbe2211dda51b2c7c056b9eb958563051102a56a57c22caef81a1ef81d5955c*
Theory Prop Address:	*TMQb6BZWimWiUUD2TKu7WGhQkSQpf98JfKA*

Pi_ext

Theorem 20.67

$$\forall X : \iota.\forall Y : \iota \to \iota.\forall f g \in (\Pi x \in X.Y\ x).(\forall x \in X.f\ x = g\ x) \to f = g.$$

The proposition is identified by the following information:

Pure Prop Id:	*afabf60d2f0c584af57bc85ddf915b13c2173fba3ae62a67a3862a472418d979*
Pure Prop Address:	*TMZQyqEpgM1nAy5SpShxXSu4QC8bRbftnW5*
Theory Prop Id:	*08bc8829905fa1c9330919448051750a09ed9872e5fe47a6c8f242ef8905a6e1*
Theory Prop Address:	*TMFnU1vnCxgdDuPv6P9CoEak4jkUTjULRYT*

Pi_eta

Theorem 20.68

$$\forall X : \iota.\forall Y : \iota \to \iota.\forall f : \iota.f \in (\Pi x \in X.Y\ x) \to (\lambda x \in X.f\ x) = f.$$

The proposition is identified by the following information:

Pure Prop Id:	*9ca3eb4ba8300315de6cc6aa4bc0569bf6750d0a55b1e27929bafce0326878c2*
Pure Prop Address:	*TMZ1A4m5suKjS6q8ujjemybzpezfVbf3Ja5*
Theory Prop Id:	*9bd10ac78e1b9dd36b301ed8e06904d4553c0bd53f28e2dd52662fee5e5fd680*
Theory Prop Address:	*TMXWQh8ieRCGD99r9bmA7JmXyAU4rDWeass*

Definition 20.9 *We define* setexp *to be* $\lambda XY : \iota.\Pi y \in Y.X$ *of type* $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*1de7fcfb8d27df15536f5567084f73d70d2b8526ee5d05012e7c9722ec76a8a6*
Pure Object Address:	*TMdMKJiLvRmU1utdH9i5YSFhSMeAkFKRH1a*
Theory Object Id:	*021ae079aee80b7f24175c08d98ddd174bcabd841750953bdefaa681a83714e7*
Theory Object Address:	*TMMDiZSCEZ1dDZ4PWaQHcXQkH4ocT26aDjJ*

Notation. We use superscripts as notation for applying the term setexp.

pair_tuple_fun

Theorem 20.69 *pair* $= (\lambda xy.(x, y))$. *The proposition is identified by the following information:*

Pure Prop Id:	*b02dfae335ec8db39552072294 1b8dc3a0d6514527797822554c3261ab3f1cb8*
Pure Prop Address:	*TMQLyvPjAxztmV8UGpQ822nUAH8ziqxafSL*
Theory Prop Id:	*ed37d0c53ab39329962382f39457e2117aa8fbbe349e0cb167d2c2d76af2a96e*
Theory Prop Address:	*TMQyWvvFdsJND6mF8ATSUatvvo9p2RAUqY3*

lamI2

Theorem 20.70 $\forall X.\forall F : \iota \to \iota.\forall x \in X.\forall y \in F\ x.(x, y) \in \lambda x \in X.F\ x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*d4a9d90bcc14a1607b933d2b6324f74a12e560f1ee23d66bbef65f0ab2300245*
Pure Prop Address:	*TMTxDywb49ok9weqR5PA1YkH5HYVVQgfmLa*
Theory Prop Id:	*de381fee5dd742f3e056c75fdc7abdebc389a9d213f325ce23fff6b0562da398*
Theory Prop Address:	*TMYB1uFbN7KQg4EMgzCmiEUJxJcauCrH6gi*

lamE2

Theorem 20.71

$$\forall X.\forall F : \iota \to \iota.\forall z : \iota.z \in (\lambda x \in X.F\ x) \to \exists x \in X.\exists y \in F\ x.z = (x, y).$$

The proposition is identified by the following information:

Pure Prop Id:	*2907d7e677f8d705f7bf5bfe75cf312a139a28537d4ee131f50b8e62731177d7*
Pure Prop Address:	*TMMNkCJJRFygiBFaXLzPkZ9eYzm8gs5REwb*
Theory Prop Id:	*c69df21f0c5d7582f84f032104dcbf0cdde6b5a1e91fbe54831893e9ab547722*
Theory Prop Address:	*TMZ49ENvtTqA2Tv7yJYdvGtiYB5BtFdTeVV*

tuple_2_inj

Theorem 20.72 $\forall xywz : \iota.(x, y) = (w, z) \to x = w \land y = z.$ *The proposition is identified by the following information:*

Pure Prop Id:	*5a5be1cedd8243deb6c455ce395e463e404fca87c52b0a3a277a9b3855cb78e9*
Pure Prop Address:	*TMFbqwXKr8H1bgcdquem9YDZJMmsUJmHeKC*
Theory Prop Id:	*8feaceee2c95bc779292a107468e26dcc3aeefca4f4cec066bd23102d06d83ad*
Theory Prop Address:	*TMNPCfqHk5t1KqeT14ns1V9EhdHeBU9wLyc*

20.6 Pairs as Tuples

Let $x0, x1 : \iota$ be given.

tuple_2_0_eq

Theorem 20.73 $(x0, x1)\ 0 = x0.$ *The proposition is identified by the following information:*

Pure Prop Id:	*42d4527df30bd5915ecca74999921e236618128fee94e9018ab8452c7d825590*
Pure Prop Address:	*TMQ8nhtfV6W3JDU3mscgLR5QMXHfDZgZYK4*
Theory Prop Id:	*ad903742c17c3fe99f8b82cdfafdc7c72f5443fcb7032eb137490fe2367b8749*
Theory Prop Address:	*TMMmXMGjkWqC3h5qQxyBwLzymNx8CZr1RZA*

tuple_2_1_eq

Theorem 20.74 $(x0, x1)\ 1 = x1.$ *The proposition is identified by the following information:*

Pure Prop Id:	*596569b2cc94a7411cd00ed3dcc85f68340782a2dea77d8454b30f81c7eb6661*
Pure Prop Address:	*TMJ9iszZSDm4Q8D744Pr5ok8cAppzRJapau*
Theory Prop Id:	*2c0ad17788686eabbd5f29d00be65516a9112c392a4d4466c10b9df595bb100e*
Theory Prop Address:	*TMNHajqbPTVi7TYeCTT1YBrSfZg2Srt6YhV*

20.6.1 Abstracting with Two Variables

Definition 20.10 *We define* Sep2 *to be*

$$\lambda XYR.\{u \in \Sigma x \in X.Y\ x | R\ (u\ 0)\ (u\ 1)\}$$

of type $\iota \to (\iota \to \iota) \to (\iota \to \iota \to o) \to \iota$ *identified by the following information:*

Pure Object Id:	*b86d403403efb36530cd2adfbcacc86b2c6986a54e80eb4305bfdd0e0b457810*
Pure Object Address:	*TMJBzUyWurQtsdPSJTbqaZih7trGyeYoC3m*
Theory Object Id:	*fc03e85944df87e6a24b2bdf2798d99cf3faa3ada8ae8ebef483367bb485a195*
Theory Object Address:	*TMYXWf8P7ybU2LjpjrRcGd9ZaYEfsgynmzt*

`Sep2I`

Theorem 20.75

$$\forall X.\forall Y : \iota \to \iota.\forall R : \iota \to \iota \to o.\forall x \in X.\forall y \in Y\ x.$$
$$R\ x\ y \to (x, y) \in \mathsf{Sep2}\ X\ Y\ R.$$

The proposition is identified by the following information:

Pure Prop Id:	*f2dd7d7d0de46ec460fe30813842b2da107da11fbc9608d96eea869888811cca*
Pure Prop Address:	*TMZdgPdsrcEjsnAmQm7foGQpRLeMQ4R8i64*
Theory Prop Id:	*35493133c295677bdfd5c4e9ab9236a9a50b7d9c1c379734567bf32375fc9508*
Theory Prop Address:	*TMYNAkcLqkkyU7u7qUwEAZxTvqQnKYD6FxK*

`Sep2E`

Theorem 20.76

$$\forall X.\forall Y : \iota \to \iota.\forall R : \iota \to \iota \to o.\forall u \in \mathsf{Sep2}\ X\ Y\ R.$$
$$\exists x \in X.\exists y \in Y\ x.u = (x, y) \wedge R\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*aca0dd1d7527fc7cb76890b26390b9351092342fe8f0f530c801862f12edf753*
Pure Prop Address:	*TMMMFQtAKedKiQ38BC5pn49TBQDJSeKoTYo*
Theory Prop Id:	*f094fb5ccc800bcb94dec37b2867363bf66db0ad9e299e07454e901889561056*
Theory Prop Address:	*TMYo6UtpLGR595GYoBTpBoawMbFG3BTJ6aH*

`Sep2E'`

Theorem 20.77

$$\forall X.\forall Y : \iota \to \iota.\forall R : \iota \to \iota \to o.$$
$$\forall xy.(x, y) \in \mathsf{Sep2}\ X\ Y\ R \to x \in X \wedge y \in Y\ x \wedge R\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*c5ad5457a7903751d537c31ccfbac1f4b84637cc2029ba26617d81ccecc36758*
Pure Prop Address:	*TMbEYwrs85BumegRS4Nue1VQK9pgGsjNjk6*
Theory Prop Id:	*f89326bba190a5f12b5139594f24a9ffae64a5b1b99d9ad4ab4e87f02434d466*
Theory Prop Address:	*TMEw2C6gFcC7Dv9bdaWQaJELJTrEwgTQ5p6*

`Sep2E'1`

Theorem 20.78

$$\forall X.\forall Y : \iota \to \iota.\forall R : \iota \to \iota \to o.\forall xy.(x, y) \in \mathsf{Sep2}\ X\ Y\ R \to x \in X.$$

The proposition is identified by the following information:

Pure Prop Id:	*d38eaf92e4d64f247f4a780f3acacb7e2b72c2c3161acddce4ff56153a152346*
Pure Prop Address:	*TMKEJf9zJKiTAqodB3HUgFWJktNYut6txng*
Theory Prop Id:	*0df0e0988ad65ec90d05105e87f7530d402ff762879f9f56fd2cea42ba0e3868*
Theory Prop Address:	*TMQamUmivkZALac37sX35Qy2VnbAqqLtDUR*

`Sep2E'2`

Theorem 20.79

$$\forall X.\forall Y : \iota \to \iota.\forall R : \iota \to \iota \to o.\forall xy.(x, y) \in \mathsf{Sep2}\ X\ Y\ R \to y \in Y\ x.$$

The proposition is identified by the following information:

Pure Prop Id:	*aa8d9b9df491ed7a1a582cbb2433626cfd9178aed409e00cfb68e55d7be32deb*
Pure Prop Address:	*TMTb1KVCtVPHStwUTCiwdLPEPhorfx9R1DH*
Theory Prop Id:	*b637cd9c32289ddb25bdd962484739bdb1e70ff50694648a27ce6bcbbd48aadf*
Theory Prop Address:	*TMRNNqqH7CLCLvFwcFthUaz6sHcYuKw3TTR*

`Sep2E'3`

Theorem 20.80

$$\forall X.\forall Y : \iota \to \iota.\forall R : \iota \to \iota \to o.\forall xy.(x, y) \in \mathsf{Sep2}\ X\ Y\ R \to R\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*5bc2e18996dfa5f943f7598be61adad327423e248744c541671fbe4a34dbcdb9*
Pure Prop Address:	*TMJnp3Ji7AtjsMbmW8cAVsi53FTqYVAGgdL*
Theory Prop Id:	*379abcf237632ff104c4aa456b6ade54b471ea851766aabd28e15e516d1cbc2d*
Theory Prop Address:	*TMS5WEGW72CGY6U9AD8kjGtvBWaHnyN25En*

Definition 20.11 *We define* set_of_pairs *to be* $\lambda X.\forall x \in X.\exists yz.x = (y, z)$ *of type* $\iota \to o$ *identified by the following information:*

Pure Object Id:	*f0b1c15b686d1aa6492d7e76874c7afc5cd026a88ebdb352e9d7f0867f39d916*
Pure Object Address:	*TMUjSWaE3xf1N5QcPC5311QNAmCHwv9P5Wh*
Theory Object Id:	*fd6f81f636454aafecfc9ce1aff42ca28607f59856bf0029f61a1c9a3db299fd*
Theory Object Address:	*TMGXX5uaVkMUzQ1LmgBVrBHf1qxYoxXsvza*

set_of_pairs_ext

Theorem 20.81

$$\forall XY.\mathsf{set\_of\_pairs}\ X \to \mathsf{set\_of\_pairs}\ Y \to (\forall vw.(v,w) \in X \Leftrightarrow (v,w) \in Y) \to X = Y.$$

The proposition is identified by the following information:

Pure Prop Id:	*a6a9b8a124fce5adc383130f08213682d41360dcd1f274f71b33e154a055cc62*
Pure Prop Address:	*TMJxKznQHdHfEU5fnQZwkd5AQ1SGfmDPbwC*
Theory Prop Id:	*4367f3d59340f9ed670f3765311f00f3089bb7fe2b92d37317898a53a96fbced*
Theory Prop Address:	*TMJd1FMb8pL4Bst6TsZEDp74LjsFRSbxwiC*

Sep2_set_of_pairs

Theorem 20.82 $\forall X.\forall Y : \iota \to \iota.\forall R : \iota \to \iota \to o.\mathsf{set\_of\_pairs}\ (\mathsf{Sep2}\ X\ Y\ R).$
The proposition is identified by the following information:

Pure Prop Id:	*a60553fc65df7d42bd360fe1cd8b1f9fc8ccc08be19e4177f1ba046a7532ceca*
Pure Prop Address:	*TMTsAuBK39fZ9F9TNnMkeSTj8FGHfLzf6dj*
Theory Prop Id:	*405a7213163c862c29aeb2e09d4e10989b3c7e06b28850ebc7378a653fbe4fdc*
Theory Prop Address:	*TMH6hRBLodwvqfQVE5ky49zcXJLCo6tRMY5*

Sep2_ext

Theorem 20.83

$$\forall X.\forall Y : \iota \to \iota.\forall RR' : \iota \to \iota \to o.(\forall x \in X.\forall y \in Y\ x.R\ x\ y \Leftrightarrow R'\ x\ y) \to \mathsf{Sep2}\ X\ Y\ R = \mathsf{Sep2}\ X\ Y\ R'.$$

The proposition is identified by the following information:

Pure Prop Id:	*0d9958e6cc16ac9372dc8846011178fe201c3d9b19cdc001f26e2e162b34d938*
Pure Prop Address:	*TMJACniaRX5HaiHuCE6bbMAHvHDcUhXZ2pH*
Theory Prop Id:	*94f11b19e5200e39967696f4804cf43ac713a68f6e15ca6369756da68d2d967b*
Theory Prop Address:	*TMRjiAegumXRHuqWAazF89LwDqZTyyMSddG*

lam_ext_sub

Theorem 20.84

$$\forall X.\forall FG : \iota \to \iota.(\forall x \in X.F\ x = G\ x) \to (\lambda x \in X.F\ x) \subseteq (\lambda x \in X.G\ x).$$

The proposition is identified by the following information:

Pure Prop Id:	*b2e26f98453afefcb6f3f49b69c601bd95524ea1ad1932d9fcbd17ae8e3732d6*
Pure Prop Address:	*TMTQkRNn8PD1f7MWUC1oifnJZfbsstmDxEp*
Theory Prop Id:	*79c629a63ae9a61f24fd859e86bcc2c5d7619d2e635836bd41d82acd839a0640*
Theory Prop Address:	*TMGxXnao732sc8NYXxP9KnXrb3jXwMstnZZ*

`lam_ext`

Theorem 20.85

$$\forall X.\forall FG : \iota \to \iota.(\forall x \in X.F\ x = G\ x) \to (\lambda x \in X.F\ x) = (\lambda x \in X.G\ x).$$

The proposition is identified by the following information:

Pure Prop Id:	*2a894ae4a36b338440c9116b7d1b5a156eee0387a8f218922dbee9a1525dd98f*
Pure Prop Address:	*TMczyezqH3mj8KqKko9PzTFrNnQbEq2hbYj*
Theory Prop Id:	*15b7126fd92862295ccc8704bc2c31eecaf79f06db2c59eff164b6dc9e9b9d46*
Theory Prop Address:	*TMVLMDciL8LE8HAPbirRFgBN1S6reWhpgHE*

`lam_eta`

Theorem 20.86 $\forall X.\forall F : \iota \to \iota.(\lambda x \in X.(\lambda x \in X.F\ x)\ x) = (\lambda x \in X.F\ x)$. *The proposition is identified by the following information:*

Pure Prop Id:	*19cd9ab03cf4c749ed30f30ed2ed505f627543fbe9a6a2e5eb19aa3c7b0d1b1b*
Pure Prop Address:	*TMTTDo5Up8FsTSMC9bQsVxqiR8BLjPdvHGD*
Theory Prop Id:	*9103bc442270a76ca14900757db09b5b74bccb4b76e1e17b7ef9f7dd674fc8f9*
Theory Prop Address:	*TMMRnP58fZgQnvj38xKhFcWRj7vE5fPQZAr*

`tuple_2_eta`

Theorem 20.87 $\forall xy.(\lambda i \in 2.(x, y)\ i) = (x, y)$. *The proposition is identified by the following information:*

Pure Prop Id:	*87a4aae7ea69d72da1affd33a6e6fdff7bb4f0d789de45c146aa1dfd53b9aeda*
Pure Prop Address:	*TMLTia4kgENRM1HdxJL6n2c4optHfFxVxdR*
Theory Prop Id:	*29558e38c4342ce7594d397f1a575f3b2c5f24e1032c87cf9c8a8a79b6832c83*
Theory Prop Address:	*TMGGubBp6qMsgQrvqroY5peGfyYATfj7oYx*

Definition 20.12 *We define* lam2 *to be* $\lambda XYF.\lambda x \in X.\lambda y \in Y\ x.F\ x\ y$ *of type* $\iota \to (\iota \to \iota) \to (\iota \to \iota \to \iota) \to \iota$ *identified by the following information:*

Pure Object Id:	*f73bdcf4eeb9bdb2b6f83670b8a2e4309a4526044d548f7bfef687cb949027eb*
Pure Object Address:	*TMYPta1FoiMuP1GN6PBin1Xx2bXh9F4jZu5*
Theory Object Id:	*1af26ca6648a0a5f2f504ed4d407dacb2d1295021bfab70e16a756e32019be58*
Theory Object Address:	*TMFfLmxP1LgSqtznbFPEPY5Ne2h6kro5fyD*

`beta2`

Theorem 20.88

$$\forall X.\forall Y:\iota\to\iota.\forall F:\iota\to\iota\to\iota.\forall x\in X.\forall y\in Y\ x.\mathsf{lam2}\ X\ Y\ F\ x\ y = F\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*2de6dedd220a397af6370f72b7a1335bcfa1475e0d9eb3d9cc7ff55075b114b9*
Pure Prop Address:	*TMagrn8mKPRRmZwMivWkUiffvhd5tm79Win*
Theory Prop Id:	*4bb51dcd166eb4dab05760eec9ee7a9c07acd7688f394f84fc37ce9a362ea7f7*
Theory Prop Address:	*TMGMZXJkR32EUzV9izhKBE2MEoS3MKPLbXm*

`lam2_ext`

Theorem 20.89

$$\forall X.\forall Y:\iota\to\iota.\forall FG:\iota\to\iota\to\iota.(\forall x\in X.\forall y\in Y\ x.F\ x\ y = G\ x\ y)\to \mathsf{lam2}\ X\ Y\ F = \mathsf{lam2}\ X\ Y\ G.$$

The proposition is identified by the following information:

Pure Prop Id:	*2fcf8329eb8970256dd334aa201b65fa25e730316c3d42fb8e24bbfb8baa1da6*
Pure Prop Address:	*TMWA1B5WDTBTsPxXTq3a4ASUuADPsARiKwo*
Theory Prop Id:	*36ee1989db22297f701d46985e3f5b8484beb0cc98783f705be675646c00e858*
Theory Prop Address:	*TMQRLHS3cYoRzXTpwWXkAqwX3mPwkxYT46Y*

20.7 Encodings of Functions and Predicates as Sets

Definition 20.13 *We define* encode_u *to be lam of type* $\iota\to(\iota\to\iota)\to\iota$ *identified by the following information:*

Pure Object Id:	*8acbb2b5de4ab265344d713b111d82b0048c8a4bf732a67ad35f1054a4eb4642*
Pure Object Address:	*TMMDLyvQS3eoiP32KBXjSQyuvc4keN5yMmX*
Theory Object Id:	*d64bcbbefe13f9fc0035d2317f017d9ee3aeb3de90b836ed7cbe11d46f478799*
Theory Object Address:	*TMS2X4GvB3DZiVdQjy5ZeDZQVVCDuJBZvgW*

Definition 20.14 *We define* decode_u *to be* ap *of type* $\iota\to\iota\to\iota$ *identified by the following information:*

Pure Object Id:	*c7aaa670ef9b6f678b8cf10b13b2571e4dc1e6fde061d1f641a5fa6c30c09d46*
Pure Object Address:	*TMRkKHvzv8Tta9AhRXsVE4k7dQ68XWymCoD*
Theory Object Id:	*a4d82c5a3643a7e174676c313ef995e0307b3b56f57e8ae761b7c6033bbc40db*
Theory Object Address:	*TMULGKR83zvLzsaT5wtaLqEme8A8TV9uVcG*

Definition 20.15 *We define* encode_b *to be* $\lambda XF.\mathsf{lam2}\ X\ (\lambda\_.X)\ F$ *of type* $\iota\to(\iota\to\iota\to\iota)\to\iota$ *identified by the following information:*

Pure Object Id:	*4c89a6c736b15453d749c576f63559855d72931c3c7513c44e12ce14882d2fa7*
Pure Object Address:	*TMEnk3YrwKupP9z6sTwXokwBz2J6VVGp5Zh*
Theory Object Id:	*2857218138584f5287cfe4ade2be20096822070b538af7d77aac47d32304f2b6*
Theory Object Address:	*TMEomjpi2NLe2mLYiaW5hqvmjvtSXDYVCri*

Definition 20.16 *We define* decode_b *to be* $\lambda Fxy.F\ x\ y$ *of type* $\iota \to \iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*1024fb6c1c39aaae4a36f455b998b6ed0405d12e967bf5eae211141970ffa4fa*
Pure Object Address:	*TMLJXUAwxDCzt7HeMEhmMtFjPFH82BuahcJ*
Theory Object Id:	*3b93ea7cf603f82962cfaa76e280e5218df781b7ff8b501b4dea38f2c76f0ce4*
Theory Object Address:	*TMdai21pMbN5LhRjuzG8fbyq6jLXZp8xvQD*

Definition 20.17 *We define* encode_p *to be* $\lambda XP.\mathsf{Sep}\ X\ P$ *of type* $\iota \to (\iota \to o) \to \iota$ *identified by the following information:*

Pure Object Id:	*f336a4ec8d55185095e45a638507748bac5384e04e0c48d008e4f6a9653e9c44*
Pure Object Address:	*TMUtfXxDDLj9Tk47vMVXxn7X4YsuGxPH9iG*
Theory Object Id:	*7008ec0ee19d11cc67c5f11635c82852ccb27360b8bd8431676b0c7e03219428*
Theory Object Address:	*TMU5xEoVJ4JK7LsxiB8DMm5ouqBu3Rgy3CL*

Definition 20.18 *We define* decode_p *to be* $\lambda Px.x \in P$ *of type* $\iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*02231a320843b92b212e53844c7e20e84a5114f2609e0ccf1fe173603ec3af98*
Pure Object Address:	*TMNmYnbPvxg781LaE377972CwGGRP6A4t2c*
Theory Object Id:	*e9507346effb55f2b66cdfb801523074014fb64eeccb8e226d8fdc9359337830*
Theory Object Address:	*TMEnXjKVptqvfEYytFygN49BmBFhyNV8YJ8*

Definition 20.19 *We define* encode_r *to be* $\lambda XR.\mathsf{Sep2}\ X\ (\lambda\_.X)\ R$ *of type* $\iota \to (\iota \to \iota \to o) \to \iota$ *identified by the following information:*

Pure Object Id:	*17bc9f7081d26ba5939127ec0eef23162cf5bbe34ceeb18f41b091639dd2d108*
Pure Object Address:	*TMVAvkv75vEbx8iL4LYfx77aKBqGpyF2x3q*
Theory Object Id:	*e6e31b91e856a7c568a0d85625bf50833d9a6ba3696590b494c9be80206757f7*
Theory Object Address:	*TMaAgBFCWYRYoi83L9vFaz2ee8AkoYAQ7dn*

Definition 20.20 *We define* decode_r *to be* $\lambda Rxy.(x, y) \in R$ *of type* $\iota \to \iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*f2f91589fb6488dd2bad3cebb9f65a57b7d7f3818091dcc789edd28f5d0ef2af*
Pure Object Address:	*TMUxUtfWjzRHvsq1Hh2nzZmnFZn4W4k5yZ1*
Theory Object Id:	*b93a65581af63e84497eb9a1ce7d86376ff948ca78630d50cb4fb879fe8b35a6*
Theory Object Address:	*TMGH5mJWrbQCiFMwfcAubZ4xHfJdMSJxuWk*

Definition 20.21 *We define* encode_c *to be*

$$\lambda XC.\mathsf{Sep}\ (\mathsf{Power}\ X)\ (\lambda U.(C\ (\lambda x.x \in U)))$$

of type $\iota \to ((\iota \to o) \to o) \to \iota$ *identified by the following information:*

Pure Object Id: *02824c8d211e3e78d6ae93d4f25c198a734a5de114367ff490b2821787a620fc*
Pure Object Address: *TMPoUwriPcHMpXKUSPr2n5eeCVHPwVTvJjh*
Theory Object Id: *abcb4f0b23ce219b19f08deedb28f9946fc52a200cd5ab9c101ffc24d294a926*
Theory Object Address: *TMdFVb6jHzinhqHU8R1E9GETd9GtqL67GFs*

Definition 20.22 *We define* decode_c *to be*

$$\lambda CU.\exists V.(\forall x.U\ x \Leftrightarrow x \in V) \wedge V \in C$$

of type $\iota \to (\iota \to o) \to o$ *identified by the following information:*

Pure Object Id: *47a1eff65bbadf7400d8f2532141a437990ed7a8581fea1db023c7edd06be32c*
Pure Object Address: *TMGRAT91AwN8Jcefxc45YTgTK8w2jHVUmwH*
Theory Object Id: *3edd4a21f511f37d2fa9b701d6bceefc3ec4979b0aa937da2224b9712a600388*
Theory Object Address: *TMKMbEUcTZ35ASSvnJwrdeRpZrpSFw3akQn*

decode_encode_u

Theorem 20.90

$$\forall X.\forall F : \iota \to \iota.\forall x \in X.\mathsf{decode\_u}\ (\mathsf{encode\_u}\ X\ F)\ x = F\ x.$$

The proposition is identified by the following information:

Pure Prop Id: *95991de564f22218049f5c01a2054c9f531e2a0e7f89348f4cd3cce56269ee6a*
Pure Prop Address: *TMW5SkUgFpNvxZtwNs5tigJKy4uVF4aLEX4*
Theory Prop Id: *b5182dff18aa9200707f42f128e18ba010c3c5b0a3a598682cfd9fe3e36def5d*
Theory Prop Address: *TMUK7trdHToYkqBA9FA7nyBfnGCJVnzMrdt*

encode_u_ext

Theorem 20.91

$$\forall X.\forall FF' : \iota \to \iota.(\forall x \in X.F\ x = F'\ x) \to \mathsf{encode\_u}\ X\ F = \mathsf{encode\_u}\ X\ F'.$$

The proposition is identified by the following information:

Pure Prop Id: *2a894ae4a36b338440c9116b7d1b5a156eee0387a8f218922dbee9a1525dd98f*
Pure Prop Address: *TMczyezqH3mj8KqKko9PzTFrNnQbEq2hbYj*
Theory Prop Id: *15b7126fd92862295ccc8704bc2c31eecaf79f06db2c59eff164b6dc9e9b9d46*
Theory Prop Address: *TMVLMDciL8LE8HAPbirRFgBN1S6reWhpgHE*

decode_encode_b

Theorem 20.92

$$\forall X.\forall F : \iota \to \iota \to \iota.\forall xy \in X.\mathsf{decode\_b}\ (\mathsf{encode\_b}\ X\ F)\ x\ y = F\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*3ee73039a869eb8092f936deea2d3411e72305d82a14a6e988c4a1bf8d16ca83*
Pure Prop Address:	*TMMn46yDXgo1kjEyPcuewexTo1Z9Ay42Wje*
Theory Prop Id:	*8d10ce96d1c3a115ca4a3c97096a0ef9b1241c7944342a034d52676ec1576ca9*
Theory Prop Address:	*TMNxzXmNthWVuFuJrEbR2rZdFYrRPUUpZEg*

`encode_b_ext`

Theorem 20.93

$$\forall X.\forall F F' : \iota \to \iota \to \iota.(\forall xy \in X.F\ x\ y = F'\ x\ y) \to \textsf{encode\_b}\ X\ F = \textsf{encode\_b}\ X\ F'.$$

The proposition is identified by the following information:

Pure Prop Id:	*332cea87dd4a05957a0e32b3f6ad9a7a65f7c156b384ab1468dabd57ed690e07*
Pure Prop Address:	*TMF4RHp7VrPNSphZfAnfvL4aSLfmrbynJJb*
Theory Prop Id:	*d010291fc325252d96b8617d5107ca33d0bd9906a978a831ac853d04850ba94b*
Theory Prop Address:	*TMMdZbVStJuaWiLad5uZeQNMN7wid9XW5qy*

`decode_encode_p`

Theorem 20.94

$$\forall X.\forall P : \iota \to o.\forall x \in X.(\textsf{decode\_p}\ (\textsf{encode\_p}\ X\ P)\ x) = (P\ x).$$

The proposition is identified by the following information:

Pure Prop Id:	*76e0c4faba69952ec67742eeb94d927de40a5f192c9d6aecccd1035a84de0723*
Pure Prop Address:	*TML6EPefmfw97vntVGgeVFz8HyFXfeVpHzC*
Theory Prop Id:	*d58325ff2359f69c9e2bd75a02be4ef6854835056c014507b6c92d4e2cf7ceb3*
Theory Prop Address:	*TMcADFusuPHtjARgdAGwzLTRfoHmoLdpnbx*

`encode_p_ext`

Theorem 20.95

$$\forall X.\forall P P' : \iota \to o.(\forall x \in X.P\ x \Leftrightarrow P'\ x) \to \textsf{encode\_p}\ X\ P = \textsf{encode\_p}\ X\ P'.$$

The proposition is identified by the following information:

Pure Prop Id:	*3b7abeffd741d27cffec5971d6a6a069502f5125137259639ec8c7fe9e08137e*
Pure Prop Address:	*TMJ9Y8Nubbgp1Z251WmGmhd5ZqmvRxaibVa*
Theory Prop Id:	*c103040e5a72e28e42a8b370fe0915a43c227b83d6405d91b72df648a194fbe7*
Theory Prop Address:	*TMaHzJHmzSrfHeEaV6hRXwCRYPY5P2JHZy1*

`decode_encode_r`

Theorem 20.96

$$\forall X.\forall R : \iota \to \iota \to o.\forall xy \in X.(\mathsf{decode\_r}\ (\mathsf{encode\_r}\ X\ R)\ x\ y) = (R\ x\ y).$$

The proposition is identified by the following information:

Pure Prop Id:	*1014a9bcf05c5f44d84ef0e62da543b87727aca195ad8496cd18d175889c170e*
Pure Prop Address:	*TMGVNLSr6YarTJyvLJtRUNqkQhdbWL1MPdw*
Theory Prop Id:	*08a8a48da6a92bd5fc4224fbd113b9d97621ed1e4dd0440c1782a73cdced5b82*
Theory Prop Address:	*TMKQ1xev2H6uLTz75vjFPQTwWGXdKuTWAw4*

`encode_r_ext`

Theorem 20.97

$$\forall X.\forall RR' : \iota \to \iota \to o.(\forall xy \in X.R\ x\ y \Leftrightarrow R'\ x\ y) \to \mathsf{encode\_r}\ X\ R = \mathsf{encode\_r}\ X\ R'.$$

The proposition is identified by the following information:

Pure Prop Id:	*7e28f4c2e174003767040fb758e5a7abd8bf6f6859c428b0cba2d3531a22173d*
Pure Prop Address:	*TManVioQo3HkeWcxBTPLk7ZWsNqo8qG2Y11*
Theory Prop Id:	*fa3b122d8b29e30bf435481e40c1cf6b59f126107a670fcc9198128c41d97b0c*
Theory Prop Address:	*TMEyPUz1f5trR77TrCy5T6ALTtFw5QB8hNw*

`decode_encode_c`

Theorem 20.98

$$\forall X.\forall C : (\iota \to o) \to o.\forall U : \iota \to o.(\forall x.U\ x \to x \in X) \to \\ (\mathsf{decode\_c}\ (\mathsf{encode\_c}\ X\ C)\ U) = (C\ U).$$

The proposition is identified by the following information:

Pure Prop Id:	*7a75630d7138237a35850129ec0899748c1ef7c52bef0bde27d23694b7dea800*
Pure Prop Address:	*TMYedqhNDgqDPXcTzM355Rp53xYHrH9uyDf*
Theory Prop Id:	*880bfbe387f844851c05145a1db503fb36936d2294f75af51c5ce2785d96bf21*
Theory Prop Address:	*TMG3Jf332EdHAKZvGPSKqfyZgKCYjmL7sQ3*

`encode_c_ext`

Theorem 20.99

$$\forall X.\forall CC' : (\iota \to o) \to o.(\forall U : \iota \to o.(\forall x.U\ x \to x \in X) \to (C\ U \Leftrightarrow C'\ U)) \to \\ \mathsf{encode\_c}\ X\ C = \mathsf{encode\_c}\ X\ C'.$$

The proposition is identified by the following information:

Pure Prop Id:	*baacc5826835bdf9ff6b14cd7397faa14ac4deec11a936d577ef7808babe0837*
Pure Prop Address:	*TMKwAfPDG9pG12Fvp5oGJz7NXs7naikE7x8*
Theory Prop Id:	*0c5483759fc520ad24416714aabea399eb7cb84016cdcbc605ffec9d6504a0b0*
Theory Prop Address:	*TMTNaV2fXyXXwapDqmZgoh9Bwtevm9wRREw*

`setprod_mon`

Theorem 20.100 $\forall XY : \iota.X \subseteq Y \to \forall ZW : \iota.Z \subseteq W \to X \times Z \subseteq Y \times W.$ *The proposition is identified by the following information:*

Pure Prop Id:	*63d72a5736b6dad618fb3fe72b9bce7bd5535acd2498fd7efcd60df85fe65ef0*
Pure Prop Address:	*TMa6kynUKR5kseqtucWFdECHsZBBaPS7mrj*
Theory Prop Id:	*a74bcaef7267fce98f2a364fb6c5734d406f80cb027560a59947536a21433041*
Theory Prop Address:	*TMRkBMu4VTR26BPV4YUTQHzEAUt27bx9EXj*

`setprod_mon0`

Theorem 20.101 $\forall XY : \iota.X \subseteq Y \to \forall Z : \iota.X \times Z \subseteq Y \times Z.$ *The proposition is identified by the following information:*

Pure Prop Id:	*7bd32d8dd36d89ace87f57f3e3d5d041a6b193856d3cabf95317325c73b3df3f*
Pure Prop Address:	*TMP3EP5PaqbZ1cxhh8WmVQQLcsFCXTW7rCy*
Theory Prop Id:	*455a4284ff6e2959b9e8a657510f722883ee919b1b17703dac1942a7edbdda0c*
Theory Prop Address:	*TMVWdnQ7qpSHvYcKK6yidsz39oPTDKBkq8r*

`setprod_mon1`

Theorem 20.102 $\forall X : \iota.\forall ZW : \iota.Z \subseteq W \to X \times Z \subseteq X \times W.$ *The proposition is identified by the following information:*

Pure Prop Id:	*6e2e07464345fa4f1a371f688df924851a665ecd94aaed51223be1ae886bc23e*
Pure Prop Address:	*TMSnywK9u9fk9Ed6jbYvBs1ZUC6LKFAgpKS*
Theory Prop Id:	*a90d34c038dd8dbab84e007a80008f440fca47232d9907f639d52500513f5a4f*
Theory Prop Address:	*TMPygssHhnaE9kaN6NAnhRiDZooV9D1npXA*

`pair_eta_Subq`

Theorem 20.103 $\forall w.pair\ (w\ 0)\ (w\ 1) \subseteq w.$ *The proposition is identified by the following information:*

Pure Prop Id:	*4f3ebfd1a2003335ce7adcc7c9b187a22011f65ae156fa1445a702cf001c3927*
Pure Prop Address:	*TMVusjsTB3ex7nLq2jbiMtM6jhKF3AJBbiD*
Theory Prop Id:	*74e00933da8b7173ceedb24a78d36aedd8271dc642ef0da29ab7608538b9bcb4*
Theory Prop Address:	*TMFvqxkSHYjcmeNXXjANHaP3wFYajKhHUMA*

`Sigma_eta`

Theorem 20.104 $\forall X : \iota.\forall Y : \iota \to \iota.\forall z \in (\Sigma x \in X.Y\ x).pair\ (z\ 0)\ (z\ 1) = z.$ *The proposition is identified by the following information:*

Pure Prop Id:	*49ff5151b49302ac99558664cc438f0a4d40498c14aadb3e399282edcabda3fb*
Pure Prop Address:	*TMSCgjHPA66KEFpj15NkePtPi8MUx8TeT4Z*
Theory Prop Id:	*8049133218c094e36a326d0302747b6faed7820279c4ca43b4aaaba59da8d86f*
Theory Prop Address:	*TMZAcyQRmA4RUYzXLLDvy9X9mq3N9qQySRD*

`ReplEq_setprod_ext`

Theorem 20.105

$$\forall XY.\forall FG : \iota \to \iota \to \iota.(\forall x \in X.\forall y \in Y.F\ x\ y = G\ x\ y) \to \{F\ (w\ 0)\ (w\ 1)|w \in X \times Y\} = \{G\ (w\ 0)\ (w\ 1)|w \in X \times Y\}.$$

The proposition is identified by the following information:

Pure Prop Id:	*c486ab09b581a313204dc303b15e3611e31121022211233d7418fa02f6cc7299*
Pure Prop Address:	*TMWwkDTMCqBE34kbrXdyyAhqVm2sbfTQjce*
Theory Prop Id:	*f689a07de22b0aec4db3cba58ed366ef7f1c7bf74bbf6b2e727fd32a368b68eb*
Theory Prop Address:	*TMTDNNDk6TRtimfgiJDfHre2Zghrr4cfc5k*

`tuple_p_3_tuple`

Theorem 20.106 $\forall xyz : \iota.\mathsf{tuple\_p}\ 3\ (x, y, z)$. *The proposition is identified by the following information:*

Pure Prop Id:	*cf993987b431467487357706a7857f006cbd148697f781826b45bb9c65766a0d4*
Pure Prop Address:	*TMF2RMvq3PMYzBY3dD3WXb7CLziCt6USMZD*
Theory Prop Id:	*77393568486dac4f68311171dad84d01e9b6df6e1d91b218fcdd823e87841c937*
Theory Prop Address:	*TMR9fqGfQKrPPzAcNDvk14YN5xCHfLnGWnB*

`tuple_p_4_tuple`

Theorem 20.107 $\forall xyzw : \iota.\mathsf{tuple\_p}\ 4\ (x, y, z, w)$. *The proposition is identified by the following information:*

Pure Prop Id:	*5cb9dd366dfb4bd3932546b2b6ce146f154c0ab4a373c38ab6422bfba5679085*
Pure Prop Address:	*TMdpGdQnb2pjcxbUxX2XUCUMgbpPFfAHvzv*
Theory Prop Id:	*58c3b26de9e903a7f82ccf8e0237aeba24a5fc22ffdc521a5446cbca1243b942*
Theory Prop Address:	*TMMXKVHZmRa3TWvKaoU4TWL4zLf8ocjRiox*

`Pi_Power_1`

Theorem 20.108

$$\forall X : \iota.\forall Y : \iota \to \iota.(\forall x \in X.Y\ x \in \mathsf{Power}\ 1) \to (\Pi x \in X.Y\ x) \in \mathsf{Power}\ 1.$$

The proposition is identified by the following information:

Pure Prop Id: *749337506bbff9c2bed1261a0faa5ea2bf92d0442dacccd5c665ddabd12a5e41*
Pure Prop Address: *TMSQqTJzh57zKzeJmXEtHFgtRKNkkoRZ5Yg*
Theory Prop Id: *53185f562ff7551b58214aa425c4679c294ce17583608a17fded6fd0251b725e*
Theory Prop Address: *TMQ6JF9rZDAhvvypZvKH3NTaGvdGWRexq52*

`Pi_0_dom_mon`

Theorem 20.109

$$\forall XY:\iota.\forall A:\iota\to\iota.X\subseteq Y\to$$
$$(\forall y\in Y.y\notin X\to 0\in A\ y)\to(\Pi x\in X.A\ x)\subseteq\Pi y\in Y.A\ y.$$

The proposition is identified by the following information:

Pure Prop Id: *afd68d2c59d656bb239ccbebabc9bd0a0bd90439f78158e146e8fc748463f7d5*
Pure Prop Address: *TMbgKzF8UHU8QuRUAoJPZtfPZpKdMV6fH4X*
Theory Prop Id: *8b265dd368d5ae9935219b30f9ab92687d8f287fc776d2c4ff902bdf59efc063*
Theory Prop Address: *TMbVdN1MHHnjdbTQaqcsa3c43EUJLPRyXxf*

`Pi_cod_mon`

Theorem 20.110

$$\forall X:\iota.\forall AB:\iota\to\iota.(\forall x\in X.A\ x\subseteq B\ x)\to(\Pi x\in X.A\ x)\subseteq\Pi x\in X.B\ x.$$

The proposition is identified by the following information:

Pure Prop Id: *3f6023959e78603c118adb6a4ff1b02d56955380be26e0747fae716b2cd95d97*
Pure Prop Address: *TMaeWxiHTomfFysYznzJfEDq1N349zj13nT*
Theory Prop Id: *c65ee3b2fffcc8ba8c7c4d2fef3d6aee20f82c17cf3c09dc7a7e72f8a251d98a*
Theory Prop Address: *TMZud88tiwxGZkbhtpgTUFSV8FPmskRGQHU*

`Pi_0_mon`

Theorem 20.111

$$\forall XY:\iota.\forall AB:\iota\to\iota.(\forall x\in X.A\ x\subseteq B\ x)\to X\subseteq Y\to$$
$$(\forall y\in Y.y\notin X\to 0\in B\ y)\to(\Pi x\in X.A\ x)\subseteq\Pi y\in Y.B\ y.$$

The proposition is identified by the following information:

Pure Prop Id: *74a4777899bd48bce73a6950d5c843aa0ef4895865b762f7d6e9504173de2d2a*
Pure Prop Address: *TMQsGPc8N3Ubv5n8RJVHF7c8JKNSvH4P3Mv*
Theory Prop Id: *d876ad3e474910e13beae3e36658e1cba27e007339b5c367e122cea96ba5a530*
Theory Prop Address: *TMJpkgSLqGH7HNbMBRcohiUdmHzCQFD3orZ*

`setexp_2_eq`

Theorem 20.112 $\forall X : \iota . X \times X = X^2$. *The proposition is identified by the following information:*

Pure Prop Id:	*ffa8ca03579eb26597d40ec371bf1e68b77c9e00e70e22aa75cb005b599928fa*
Pure Prop Address:	*TMQo8m7mTFS5ULGe5RFn9ewAV6USEFmXGEe*
Theory Prop Id:	*97b6d296a52705b137946c7684a42f90e6dcdc16ccd27fcc6f843903334d6124*
Theory Prop Address:	*TMU8ZQ356N7f1ur9PqrcyhFvUrEU8Uw3pCd*

setexp_0_dom_mon

Theorem 20.113 $\forall A : \iota . 0 \in A \to \forall X Y : \iota . X \subseteq Y \to A^X \subseteq A^Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*c82a77bea63c3045dd2d0390019427d36b3850e817a7c7d81c9d44dd83da9951*
Pure Prop Address:	*TMQY2MPGRLH7DstKxCBvTt7ycuHKzG3Egta*
Theory Prop Id:	*320220221d2ff027b0b203b874e4695ae660ced55ac0790f282597f5e9613e9f*
Theory Prop Address:	*TMdJC2UYVneWjpMTvmnbkFG2QrY6sZa7dzC*

setexp_0_mon

Theorem 20.114 $\forall X Y A B : \iota . 0 \in B \to A \subseteq B \to X \subseteq Y \to A^X \subseteq B^Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*9626c863679499ee9efb3a97f6d2c94d448186ea2212527485c1494342277958*
Pure Prop Address:	*TMSVKPxofDcRCRvbvF6CiPxy7w1U9GV3abU*
Theory Prop Id:	*90c509fbef3ca7822c8b20ec7e5a62fd62a043b9500aaa739b5118a3c994d408*
Theory Prop Address:	*TMG4aTTfVaxVUnsqSgBoVVNATLoR7h38zjG*

nat_in_setexp_mon

Theorem 20.115 $\forall A : \iota . 0 \in A \to \forall n . \mathsf{nat\_p}\ n \to \forall m \in n . A^m \subseteq A^n$. *The proposition is identified by the following information:*

Pure Prop Id:	*d4aff1269d336f4267535d4b630549f976211672cd2893c8f0d5f93ccc2a2c01*
Pure Prop Address:	*TMHmDB4dHXevKqyYjk66EaYCALxGuURyksG*
Theory Prop Id:	*325485fa49bc41efd250c31eb18bded07de4228f53ab976ea68b483631dc46be*
Theory Prop Address:	*TMX7YsW1eVF8qPSQpQudbRa9xp2Q6wGVEfD*

tupleI0

Theorem 20.116 $\forall X Y x . x \in X \to (0, x) \in (X, Y)$. *The proposition is identified by the following information:*

Pure Prop Id:	*a5d9e60bb578510eba8f8727ca81d4ad6c6ea416e3b1dec75e56736f2c7fbf28*
Pure Prop Address:	*TMbWA1F32KHzES5haJKo6Ltd6Myvmdja6kj*
Theory Prop Id:	*7bad4e2f646b78cc8e4ab2dde15f4dad2279a633db69af2d130325d2df660497*
Theory Prop Address:	*TMYRiZH6QeEcQSbTsa8ff35vULKtFPjDWGZ*

tupleI1

Theorem 20.117 $\forall XYy.y \in Y \rightarrow (1, y) \in (X, Y)$. *The proposition is identified by the following information:*

Pure Prop Id:	*5c347a6088d371c6a7be279b519d891f8b247a1dc86c7f1a5e4e91182d2cd741*
Pure Prop Address:	*TMR457F4pvZZp7RcTd3TbZY6m59BdcrF2rh*
Theory Prop Id:	*5e3cb317730d8131182f7c240578e9b710c06861aa264924c853babd66ad53f1*
Theory Prop Address:	*TMQKbRaumSCTKfCNkFDrzfYb7gjLPbdz9bg*

`tupleE`

Theorem 20.118

$$\forall XYz.z \in (X, Y) \rightarrow (\exists x \in X.z = (0, x)) \vee (\exists y \in Y.z = (1, y)).$$

The proposition is identified by the following information:

Pure Prop Id:	*8b6d364ac254012dfb266e48ccd8017c4df7092dc82d2f6356aa98f2c113f819*
Pure Prop Address:	*TMaevMuU9VPUjmzk93asQnRSf6s5giQjEGz*
Theory Prop Id:	*226332642373ce44237e3431b4e96d902d54149ae33bde4bedb64b30fb8d0fa2*
Theory Prop Address:	*TMdn86EVVTVBkyXsXj9Vmq1oLxSWWr4xxfb*

`tuple_2_Sigma`

Theorem 20.119 $\forall X : \iota.\forall Y : \iota \rightarrow \iota.\forall x \in X.\forall y \in Y\ x.(x, y) \in \Sigma x \in X.Y\ x$. *The proposition is identified by the following information:*

Pure Prop Id:	*d4a9d90bcc14a1607b933d2b6324f74a12e560f1ee23d66bbef65f0ab2300245*
Pure Prop Address:	*TMTxDywb49ok9weqR5PA1YkH5HYVVQgfmLa*
Theory Prop Id:	*de381fee5dd742f3e056c75fdc7abdebc389a9d213f325ce23fff6b0562da398*
Theory Prop Address:	*TMYB1uFbN7KQg4EMgzCmiEUJxJcauCrH6gi*

`tuple_2_setprod`

Theorem 20.120 $\forall X : \iota.\forall Y : \iota.\forall x \in X.\forall y \in Y.(x, y) \in X \times Y$. *The proposition is identified by the following information:*

Pure Prop Id:	*e907b80c4db444253db900e46741dbeb4c373f9357cc51990287b6b4bc4e0111*
Pure Prop Address:	*TMU5XWQo6K7rWoGntpngjoB7dZXxPoSYoxC*
Theory Prop Id:	*1f9d7fcc7017d00b96708fc1b8cd4c299aede7855c9c051ad9ed1f17edf0696a*
Theory Prop Address:	*TMQby1erMukDCSudbBu6DXLdSmpJnUwy4bN*

`tuple_Sigma_eta`

Theorem 20.121 $\forall X : \iota.\forall Y : \iota \rightarrow \iota.\forall z \in (\Sigma x \in X.Y\ x).(z\ 0, z\ 1) = z$. *The proposition is identified by the following information:*

Pure Prop Id:	*7cc266621e49b9c431179e0045a15c6c2bd49d0bb6d49aefbb923becb4f3b9ee*
Pure Prop Address:	*TMSgpyjZ757izmXroHBFU7jiFn39AxWAcPa*
Theory Prop Id:	*d965e01dd42fa888cdd06e063200196d2578c975d5c706f981c47060524c38ec*
Theory Prop Address:	*TMXQjnReFQQDxxkiWBRLot9EZBBJmxctC5k*

`apI2`

Theorem 20.122 $\forall fxy.(x,y) \in f \to y \in f\ x$. *The proposition is identified by the following information:*

Pure Prop Id:	*334f53713d549322405aec659458426a25e4aa7e1dcba4fbfad411bfe0628835*
Pure Prop Address:	*TMcAcmb8L7yHFzTFqsQ3a4kWhFMbRKovMNC*
Theory Prop Id:	*6f716abb65c7606ae6b5749d37dda052e685a4d4f7519d400bedcc1fcee23320*
Theory Prop Address:	*TMTjJZzRce6va39d2z3pncuj6jrfGtJ3eoy*

`apE2`

Theorem 20.123 $\forall fxy.y \in f\ x \to (x,y) \in f$. *The proposition is identified by the following information:*

Pure Prop Id:	*2a2c24116214dee00936cfdd3231d8a70a4ff6d3efda38a95a724e0e4cd155a1*
Pure Prop Address:	*TMWSE1naRk6JPjqHbwkoVtBu4k9tumZgTyX*
Theory Prop Id:	*57702acf505aa564187ee95d1b240f6ea6befe178029989924da2df6cea2a42f*
Theory Prop Address:	*TMGiJF872peSuqfKRH9jn9RcqbjuUAS8qbD*

`ap_const_0`

Theorem 20.124 $\forall x.0\ x = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*a4ea444af8365c1c911ed81357b063cf1823fcc07b1e9fa2848a6e26d14a5f23*
Pure Prop Address:	*TMZQjyRKodf7sfVuPrUmzZLt88BXcuEKsxk*
Theory Prop Id:	*f66b937fdac9351c50455abd7b2f8c4008f271f243b5a551b8020099b6ea59d9*
Theory Prop Address:	*TMYDg1Trrx3692oVX9Md1Ss8h7pKgzipsXy*

`tuple_2_in_A_2`

Theorem 20.125 $\forall xyA.x \in A \to y \in A \to (x,y) \in A^2$. *The proposition is identified by the following information:*

Pure Prop Id:	*1fa999eab25ebfb7f69caf608dacd0faa4724522bf176de884702e54c99b0dfc*
Pure Prop Address:	*TMTdPZ59TT37q3Gv7R8XAXdt6EShALVC2bx*
Theory Prop Id:	*234a4a6a1e3d22203eab3e75c58b409a92981de21f60b36b7bacdbe94c2f695e*
Theory Prop Address:	*TMEgxar8usGXDWMn2dQrfqcaTBMsQmobYQL*

`tuple_2_bij_2`

Theorem 20.126 $\forall xy.x \in 2 \to y \in 2 \to x \neq y \to \mathsf{bij}\ 2\ 2\ (\lambda i.(x,y)\ i)$. *The proposition is identified by the following information:*

Pure Prop Id:	*1458e4c76e00e5e5ab03f1e51a59e8b2db6f5aedb0ec7b7437f01c606e57f61c*
Pure Prop Address:	*TML944PLcXEhMqhaj1bSJ66jCqyeP9nidJV*
Theory Prop Id:	*5fae0dbc2404ab825ce386605b9b51bc9aeac0b473aaf79cb2ca0d2802ca5c3d*
Theory Prop Address:	*TMTfcLL44Zqzsq91secQcUt9S5DKUtpL89s*

`tuple_3_eta`

Theorem 20.127 $\forall xyz.(\lambda i \in 3.(x, y, z)\ i) = (x, y, z)$. *The proposition is identified by the following information:*

Pure Prop Id:	*557b0ee79b2a52e8ce18d61ad94a7f15864471688ec9c794f1d5f24c9783005c*
Pure Prop Address:	*TMXagwe8TteWaGYj5KgUBttDC1wW28KphHN*
Theory Prop Id:	*b1962f9a855cfebb9323ea0e424ab44c88fc28f2f25e542d11602c8562b09544*
Theory Prop Address:	*TMNJCyieXsgj2hu8gCm37y28P14rCCpCUyg*

`tuple_4_eta`

Theorem 20.128 $\forall xyzw.(\lambda i \in 4.(x, y, z, w)\ i) = (x, y, z, w)$. *The proposition is identified by the following information:*

Pure Prop Id:	*774296b519bf68aacdfe0445662dcdccfc6c74dcc7d3898d0507c5079d63d8f8*
Pure Prop Address:	*TMQxXPgmA8bowKXArUpmiz87siaeSw2Q5MW*
Theory Prop Id:	*e04fd23dfc3d7c5f1a127aba4c6547e3b393dfb6545e480f9953e26ca3c3fc0d*
Theory Prop Address:	*TMYx47haX7KQ8PtiUHHnWXRWY2AfpDb5mNZ*

20.7.1 Tuples with More Than 2 Components

Let $x0, x1, x2 : \iota$ be given.

`tuple_3_0_eq`

Theorem 20.129 $(x0, x1, x2)\ 0 = x0$. *The proposition is identified by the following information:*

Pure Prop Id:	*2299dd127ed9d00d183b81051f52579a3638dd91f46ef778aa50c442abc6b092*
Pure Prop Address:	*TMFhwi4ZC3VWuBy42eEzHkLCbNwo3Ev8HEf*
Theory Prop Id:	*0a4f57728c3c5c761b448d44960ca6a828c0a440fd63d1aa6d9c2e57c0243787*
Theory Prop Address:	*TMW5V1hAcqqC4skHEBzYR3ZiJGL8pvjwYWK*

`tuple_3_1_eq`

Theorem 20.130 $(x0, x1, x2)\ 1 = x1$. *The proposition is identified by the following information:*

Pure Prop Id:	*414e8bd1880695006637be5415198e0c43b53e9609ce62bb325610bdc38977f9*
Pure Prop Address:	*TMX5TRv8W5gSMK3DnrehU1GjFmEMuBgMie9*
Theory Prop Id:	*14ea6626db4405895dcefd0542273be1017929dc5a35662b5fc7930a86d2bb05*
Theory Prop Address:	*TMNBfanXBAVa8NNCyyTX1KCDoX1oZugTZeK*

`tuple_3_2_eq`

Theorem 20.131 $(x0, x1, x2)\ 2 = x2$. *The proposition is identified by the following information:*

Pure Prop Id:	*6b6b08fb7649ca4a49e5f256d73d38901cade1dbab78712081d4c92f5f63a44f*
Pure Prop Address:	*TMUzP7TDpQtCwif8aAwohSj3JNWVX6TEdjc*
Theory Prop Id:	*f69c06f91e4e8a435b4ac8fb4653343c5430c0f76186ed2c6b5a8285be49dd75*
Theory Prop Address:	*TMQ7XSB36vKMD44joDM3WQNXQ5hqU7Zeu4r*

Let $x3 : \iota$ be given.

`tuple_4_0_eq`

Theorem 20.132 $(x0, x1, x2, x3)\ 0 = x0$. *The proposition is identified by the following information:*

Pure Prop Id:	*a6e3de0ca7e59eaaf3122d630187610de4188d6b0275524d8b75d2bc27abda32*
Pure Prop Address:	*TMF5etreB3YRde69S2FeY8M1oVAWjjxQ6v5*
Theory Prop Id:	*fe562773787449 4b543777e809941dce8d7e9a8013c7963f2809078137e82277*
Theory Prop Address:	*TMYGbLVThnvCWxfpjrsDHdPixQYq8AwiQAz*

`tuple_4_1_eq`

Theorem 20.133 $(x0, x1, x2, x3)\ 1 = x1$. *The proposition is identified by the following information:*

Pure Prop Id:	*733dff0a1d011b3d6a42f3265c3727b7801c71ce1aae4ed97fa0c7086d2a3fc6*
Pure Prop Address:	*TMSYE1DSLMRdPHth8BZAHKadTikLP9Hp1dN*
Theory Prop Id:	*bf73bc8d27290ee55ba023d0c035f7daaf4cf11c12a380d059c86f22816eb234*
Theory Prop Address:	*TMX1ctd3QUcXQ7ntu5MvE9xqwY9fi3maxEg*

`tuple_4_2_eq`

Theorem 20.134 $(x0, x1, x2, x3)\ 2 = x2$. *The proposition is identified by the following information:*

Pure Prop Id:	*ed4aa73355586a09081629dab3b75a5dec8a417c17ed0d84b8443443fb2785a4*
Pure Prop Address:	*TMExys3qMfkBQ43ojYUvQ8d1LY1vZw1enfP*
Theory Prop Id:	*7d4422df851f3d7c9b2b10bc33820c35b9f3567e7a28a65da668a14c240d5d55*
Theory Prop Address:	*TMNiHJsEoF9pvnK73fnpVv3rnvdPVJKtB7P*

`tuple_4_3_eq`

Theorem 20.135 $(x0, x1, x2, x3)\ 3 = x3$. *The proposition is identified by the following information:*

Pure Prop Id:	*6a238391dd9d73ec2a175b97bcd4c846967958d9fb539afaec96724c51bf7e9f*
Pure Prop Address:	*TMTCp2qqDTin3jc37q6q6PLfMoJcCzaq48h*
Theory Prop Id:	*1a82d64dd6e2e5694e2d044a818489902c8025fada8fb5f3561aa926882f5af2*
Theory Prop Address:	*TMHfo157hjhQzHLRL8e3AAL7NmnK4YXNyFv*

Let $x4 : \iota$ be given.

`tuple_5_0_eq`

Theorem 20.136 $(x0, x1, x2, x3, x4)\ 0 = x0$. *The proposition is identified by the following information:*

Pure Prop Id:	*f808b62bbfe0261c431302a1f9d628c853f68d912aa2f152472c2dfa90557d18*
Pure Prop Address:	*TMGre8WKqnRD2NJPkoaA7jbLWUipgYm5jog*
Theory Prop Id:	*2eb5ddc5a16b3a7d9a1280b24d3e509bd011ce52296873af2d6879fb547774ea*
Theory Prop Address:	*TMe1KWBCo857cwBq4Uy3dWDEAzMuSo7LxQm*

`tuple_5_1_eq`

Theorem 20.137 $(x0, x1, x2, x3, x4)\ 1 = x1$. *The proposition is identified by the following information:*

Pure Prop Id:	*5714821964a45eae8276b9e96c0d7f3dc70c4a421256104663fe4622cb6dda9d*
Pure Prop Address:	*TMQcHzma6cxkMaJgGHj1SX9kiVwhL66i5aW*
Theory Prop Id:	*49b93f2b2cb80b89ed8e5dba6af36b0110694d89f4106c7b23eb3abdee76c4f2*
Theory Prop Address:	*TMachUcuahLvMtFu7FZn7cYP4oDkK8ciBkE*

`tuple_5_2_eq`

Theorem 20.138 $(x0, x1, x2, x3, x4)\ 2 = x2$. *The proposition is identified by the following information:*

Pure Prop Id:	*cfa53ec600e79400f6f667fa9b058c813cc71e4bebcbef4760d5e26acfb75b75*
Pure Prop Address:	*TMFYhqnKU46a4x3LZyhSt4RajqBX6cEKbFx*
Theory Prop Id:	*3bb23a1a7686e9d933ead58faf0abbc08a769032e8988fd111a8d08413d02d1f*
Theory Prop Address:	*TMWnPwZQBGvW4pee3GLW8LkfkhmmfRtVKmF*

`tuple_5_3_eq`

Theorem 20.139 $(x0, x1, x2, x3, x4)\ 3 = x3$. *The proposition is identified by the following information:*

Pure Prop Id:	*1b1587958694c5a5e88c0848a23a4e473c235eefdc4ef28de59de0e840fdd536*
Pure Prop Address:	*TMaXA4j4fzMqVPAEs32Lkx3LxUCXkwLbpMV*
Theory Prop Id:	*527729cf29653b87109e805ea351973493674baa2ed19682f7fbb67c753e4707*
Theory Prop Address:	*TMHJVjGa5dY6E8B16YUobHUQV7gxBZGTtkQ*

`tuple_5_4_eq`

Theorem 20.140 $(x0, x1, x2, x3, x4)\ 4 = x4$. *The proposition is identified by the following information:*

Pure Prop Id:	*4b873a60b686c40372a054db2e8bc63d7af0fe67ef2f26f88f9a9279bb14648f*
Pure Prop Address:	*TMQuKuBaYQaVUjVyxJUQ247CUXS7YEvahrM*
Theory Prop Id:	*39fb6373be9c1ed3946575a5f4f3666171d7e8bfe812b3f8d57f6a9a6c5dafec*
Theory Prop Address:	*TMG8m35MwfUjvACH7KLAnZSxxnizHeeC6py*

Let $x5 : \iota$ be given.

`tuple_6_0_eq`

Theorem 20.141 $(x0, x1, x2, x3, x4, x5)\ 0 = x0$. *The proposition is identified by the following information:*

Pure Prop Id:	*09a916f7cab6d7d54eeebdf48131510af7e2286ceae22ad4b167cef732ec5024*
Pure Prop Address:	*TMHkcrA1nz7EC1iKBwwwMHDsTRdph4WGCMe*
Theory Prop Id:	*431823a5e23f06055a6b8d3de1cd5baaf82ffaca75dc198d13b368e15c356496*
Theory Prop Address:	*TMZfq1XThKjcGE26mvQn13MnXXuNnXLjT91*

`tuple_6_1_eq`

Theorem 20.142 $(x0, x1, x2, x3, x4, x5)\ 1 = x1$. *The proposition is identified by the following information:*

Pure Prop Id:	*29da6a2f4d3d0147e6f18d6c08aac5b7c54b66d6227fec45e9b37962c74efd62*
Pure Prop Address:	*TMLK9MjtbDbJHdSXq3UFqNrBCpsMatyNCUJ*
Theory Prop Id:	*649eac8b3e9c6e791fc56cac42ca9787ec30ec00c1f8ecc5b68831e978a459c8*
Theory Prop Address:	*TMcCvMHVhFKscbxwpj6BsdrkD4ne1ELkvgn*

`tuple_6_2_eq`

Theorem 20.143 $(x0, x1, x2, x3, x4, x5)\ 2 = x2$. *The proposition is identified by the following information:*

Pure Prop Id:	*8dde2d93ac41664c4b1687e280531801c7c9b7502d9ef12722abd166d16050e0*
Pure Prop Address:	*TMaHcLc1izgmwaWPEBG57z2qAT17g26H4L7*
Theory Prop Id:	*f750917ac06e451a6eecda8fca53336730cd931fdf493afb63413e3fe3aea0f9*
Theory Prop Address:	*TMUHUCWa6x5Cs3iENyWQFdVm4DWTTXfjXKk*

`tuple_6_3_eq`

Theorem 20.144 $(x0, x1, x2, x3, x4, x5)\ 3 = x3$. *The proposition is identified by the following information:*

Pure Prop Id:	*a5391e373cc4ee39be40eb3722e01b208d2935f48e4a422581e36b530e106751*
Pure Prop Address:	*TMWnuyCP9Xhcj8WXFvvmExAgQv5q8FLMf87*
Theory Prop Id:	*9c6fc47f25c0dfa31aaf9660010b55a370cf936d297c0550428e6361c3846acb*
Theory Prop Address:	*TMR4FTBgD9zS7EN9XzWafwrqhmFsUjwH57g*

`tuple_6_4_eq`

Theorem 20.145 $(x0, x1, x2, x3, x4, x5)\ 4 = x4$. *The proposition is identified by the following information:*

Pure Prop Id: *454d77053132c71f1d965c99fa38da3d894908d33f7b7e16eaf6821ccc7266f8*
Pure Prop Address: *TMS87PUFZMv9WtLqYWCHZds6nUvxfnbUYRh*
Theory Prop Id: *59ae63f950d447046bb6bae0eeeed28c4a7cf3348434b3fe2b85ed366aaccc5a*
Theory Prop Address: *TMYgmDm3q8F4Mm3PxvLRJqjhm53p6v4SckS*

`tuple_6_5_eq`

Theorem 20.146 $(x0, x1, x2, x3, x4, x5)\ 5 = x5$. *The proposition is identified by the following information:*

Pure Prop Id: *e53f7e86153a20747dc21bbf79ab7de024febeef898b88b7ca1b8a25f1dbe489*
Pure Prop Address: *TMdTTWmw4pKJNhk1pcAsNTBNcpR4rYNZdWx*
Theory Prop Id: *3948d3857fe77f9ea2e9fa5db183819280b873fc0ff58efec01627914174b604*
Theory Prop Address: *TMcBoE6Vfa7iKDQFescgG8sF1w6NkoWVsRP*

Let $x6 : \iota$ be given.

`tuple_7_0_eq`

Theorem 20.147 $(x0, x1, x2, x3, x4, x5, x6)\ 0 = x0$. *The proposition is identified by the following information:*

Pure Prop Id: *2e6014a9fce0b27e920adaf5d4ce5cd70132df4f7186fe3ee541fbb88d55e5ff*
Pure Prop Address: *TMSw7zxqpUofUXu1iw13sy9eGNZFkJ83G8m*
Theory Prop Id: *1dd20a0f28040bbf9f665874e170b2a7507504613edf15bfdaeccac483fe8d57*
Theory Prop Address: *TMFmBTCrMA6APNwYVkmVE5BR4uSANNxuCnr*

`tuple_7_1_eq`

Theorem 20.148 $(x0, x1, x2, x3, x4, x5, x6)\ 1 = x1$. *The proposition is identified by the following information:*

Pure Prop Id: *50757afcd46384f164088eecd0890ea20e41d6faabc75520acbbcbc8caded1f9*
Pure Prop Address: *TMGz5g964HWbsbR8EdvqGJX38rYAsa8B6A4*
Theory Prop Id: *df1371c7546e44515aa1062d1787c0276481ea613ed0b0c96d52df80ee45960b*
Theory Prop Address: *TMJrw6pMykGM46pQg8E3npd2RCSbU1sxJdZ*

`tuple_7_2_eq`

Theorem 20.149 $(x0, x1, x2, x3, x4, x5, x6)\ 2 = x2$. *The proposition is identified by the following information:*

Pure Prop Id: *42d292c18db7c0a80c38bd4ad63c9931011a99c7f28f1d3f98fc7be5ba02f6fa*
Pure Prop Address: *TMaDjeg9UWvnEZBxqcyDSjZqA9PdDXB6FdL*
Theory Prop Id: *f1f4ec953eb39d38b739d1cb8eb86ec8cb64944b9e496c426a73c3e9e8247f1e*
Theory Prop Address: *TMQmpwNMGPPbdwGTDnpPL4Bofnunn3sUWup*

`tuple_7_3_eq`

Theorem 20.150 $(x0, x1, x2, x3, x4, x5, x6)\ 3 = x3$. *The proposition is identified by the following information:*

Pure Prop Id:	*9bf8cc6fcb3bbff2fb13bd77a32b81fc0c78e82ac38fba8ecf898116272a76c6*
Pure Prop Address:	*TMUya4w7djLwbukHFDyKwD1eFvHneTmmPTn*
Theory Prop Id:	*0ee6c00d403225b610cf3cbdcb67fe69f1b99a36ed115272a676b3ff5895ea20*
Theory Prop Address:	*TMJdFNXdzogDAxgPCDUbrEfUH5qS2fodYvz*

`tuple_7_4_eq`

Theorem 20.151 $(x0, x1, x2, x3, x4, x5, x6)\ 4 = x4$. *The proposition is identified by the following information:*

Pure Prop Id:	*8462f55255364b442133232a99948ac976e6a6fb1fcad9753fd37e423292b072*
Pure Prop Address:	*TMbPipbzrrjCW5jfNhEUnUim1TDD5mFRKh2*
Theory Prop Id:	*a1e2a4c78071f6df819ba47d9b0dfd266b20490f5cfd23e24c39c68d7733ab6c*
Theory Prop Address:	*TMSvveckTP4ZbhKhqsjNNf6HyqQpZtmtqeN*

`tuple_7_5_eq`

Theorem 20.152 $(x0, x1, x2, x3, x4, x5, x6)\ 5 = x5$. *The proposition is identified by the following information:*

Pure Prop Id:	*d72f29ba20ccd83f14385aee069bd57c87c90fff8d2a86cf9c56d81df57ccf7a*
Pure Prop Address:	*TMYnZE8WXPd7Lz6nNYaNwd53r9FLYVqbHot*
Theory Prop Id:	*b3bff055493211cb1eaee1162d67c798ed21ac3305b1e4f92aea7ef0a84f21a3*
Theory Prop Address:	*TMGq56172uFcBvMUViudP5XBkrjMQZDLoWW*

`tuple_7_6_eq`

Theorem 20.153 $(x0, x1, x2, x3, x4, x5, x6)\ 6 = x6$. *The proposition is identified by the following information:*

Pure Prop Id:	*6a6e0703f1e5cdb6bb4cdfe2feaf16914b1e19e04acbf2ec6f83392a9ba32159*
Pure Prop Address:	*TMRnWwZbYhS2xiXxEBDwKjrHtL82AeLyj3v*
Theory Prop Id:	*3fd34b4049502dd532ee544c178b866077b61510f73540a979b4cb630631a075*
Theory Prop Address:	*TMUYHfZMdobjQfNUAjkyHkmaeYViwqtCndh*

Let $x7 : \iota$ be given.

`tuple_8_0_eq`

Theorem 20.154 $(x0, x1, x2, x3, x4, x5, x6, x7)\ 0 = x0$. *The proposition is identified by the following information:*

Pure Prop Id:	*4d531e28345c633c2002654dcc743a093af9ee8b7f68ff77c4f7267261a19d71*
Pure Prop Address:	*TMckKDatqdQwJzLKWCavwMR6tjSPWLVCMWD*
Theory Prop Id:	*1b9fca46dbafdb59c9e774e473a1cbe61e9e6f78e7eaf15a1d24c3f6c61b8b52*
Theory Prop Address:	*TMSNRtL24tamygfhPna4TbgXg6e1xm7U4SF*

`tuple_8_1_eq`

Theorem 20.155 $(x0, x1, x2, x3, x4, x5, x6, x7)\ 1 = x1$. *The proposition is identified by the following information:*

Pure Prop Id:	*a89a9e6dadd13a5330864d0051afdedff37a8d6c55b3991826097d7fd9e2a431*
Pure Prop Address:	*TMcfnBEJnMxoVVovoBVi4dwC2PvUfkqu3eZ*
Theory Prop Id:	*fafd80409a2da6a08db5bda00f3225829870d378adfb385c57fde0e89b0b986e*
Theory Prop Address:	*TMPoe6wCmvUNubyq1MbpHDtwHtVKon77Awk*

`tuple_8_2_eq`

Theorem 20.156 $(x0, x1, x2, x3, x4, x5, x6, x7)\ 2 = x2$. *The proposition is identified by the following information:*

Pure Prop Id:	*49e82c53acd392cd8e7fdd61b280cc3953c508ee664a1617ecdc5a378d0dbc31*
Pure Prop Address:	*TMduvfU8rksxh1TRJ5nqEAuDZZfnJ7orBPX*
Theory Prop Id:	*c4f8325d16f1f37c6c1b8f3611beacf3e5027e0b4a964af75dd596b2fd3f3964*
Theory Prop Address:	*TMYtGYjHksBMoXSBuCV2L7EZPAK41CBNa5B*

`tuple_8_3_eq`

Theorem 20.157 $(x0, x1, x2, x3, x4, x5, x6, x7)\ 3 = x3$. *The proposition is identified by the following information:*

Pure Prop Id:	*165a4eea8749fd9e02498a1ebda5ee12fe020e0fd7c18820b62a240cee4313ed*
Pure Prop Address:	*TMVio5C24nLLNCMHHwF2Hj74cLzmmxeD1Vf*
Theory Prop Id:	*813d2d46172e67e30343892ae6f61699148d68c69c7af2ea5359c272e75dc1f6*
Theory Prop Address:	*TMYr323e2X9sRx49CLZrkYn4vV85NQRyoRT*

`tuple_8_4_eq`

Theorem 20.158 $(x0, x1, x2, x3, x4, x5, x6, x7)\ 4 = x4$. *The proposition is identified by the following information:*

Pure Prop Id:	*4e1b495a5072ad1d3b879ed62eff2c100fb8f975c0ad2a6f54d301c81de26c44*
Pure Prop Address:	*TMY5bJDeHJshTEqZj17jxPqa8JntKZiRsCe*
Theory Prop Id:	*4437b79893da40005550d241d6c12c7063f25afb64d237b52b0242a035218898*
Theory Prop Address:	*TMMVNvpvjzLdjgRF292vzfCgLN1tRcuXph8*

`tuple_8_5_eq`

Theorem 20.159 $(x0, x1, x2, x3, x4, x5, x6, x7)\ 5 = x5$. *The proposition is identified by the following information:*

Pure Prop Id:	*563677657a81d01f8e8dd06ec4032e620ce92956af773ea886d10e8a18a0e37e*
Pure Prop Address:	*TMP7vfkqYAkgGnygvPp1DhbmEUEpX2vcTC7*
Theory Prop Id:	*5078953fb1b78cf7ee83f76004c34c338e15e1b22a251c44b4c22a7e75b3e49d*
Theory Prop Address:	*TMK3bws6NYHpsM12NM3kXJordSrkVZiwXdS*

tuple_8_6_eq

Theorem 20.160 $(x0, x1, x2, x3, x4, x5, x6, x7)\ 6 = x6$. *The proposition is identified by the following information:*

Pure Prop Id:	*8a30e81feba75452c73351f490aecb9e61529a2fb4beee2aea3f106abbec6783*
Pure Prop Address:	*TMGVN5oXmWa1ZAx5JDJzTAxZNkAPUL18MWj*
Theory Prop Id:	*2e35ad0124fc4b59017241f14ed8cadd718350ce99b31f1f8010bd4433d2fc10*
Theory Prop Address:	*TMVH2QvTzDv8bXohG5W1at4o2xEqMR3wTiN*

tuple_8_7_eq

Theorem 20.161 $(x0, x1, x2, x3, x4, x5, x6, x7)\ 7 = x7$. *The proposition is identified by the following information:*

Pure Prop Id:	*57dbf4f7ff71f1e9c347c26193ffe394feb41601e047c06c67f8288012465ab2*
Pure Prop Address:	*TMULadmzcV4DX8JBCi7ogRsVnsU97NMsnBA*
Theory Prop Id:	*96777da9be70f7524c8eb35e7801fe9f0e20108d7bd0be0f5dc289bb821dc659*
Theory Prop Address:	*TMcoMGrJAhGkKy8Louj6kpH3a7yNkTuMNYw*

Let $x8 : \iota$ be given.

tuple_9_0_eq

Theorem 20.162 $(x0, x1, x2, x3, x4, x5, x6, x7, x8)\ 0 = x0$. *The proposition is identified by the following information:*

Pure Prop Id:	*26d3d48c9c93b7ec957763149ffa832050df3100f14d665f9a1c7a61f59e9925*
Pure Prop Address:	*TMJMTn7StT21vzeC8qwYcWigwJ4kq5aY9oo*
Theory Prop Id:	*29c132099b2f6c53ca2f04ce3ca16cc8de2ea951e11fb4ac5dbd7fc184921f8c*
Theory Prop Address:	*TMXgk5r2Wc1ku9pdXFHc89zP32xZ8hwx4kt*

tuple_9_1_eq

Theorem 20.163 $(x0, x1, x2, x3, x4, x5, x6, x7, x8)\ 1 = x1$. *The proposition is identified by the following information:*

Pure Prop Id:	*3a3c1cb789c9f600657400101f3562fb437f82776a00c2a32c686549a63f8e42*
Pure Prop Address:	*TMHkZbR5acwXro73RegaXWPvpdK2cEUGU44*
Theory Prop Id:	*6919b89ef581bce0dccbc7b8e0d02d831b4094ed9862095c50cb82f7fce1f138*
Theory Prop Address:	*TMQE3e1rPeJMLKx8ndXckY8TL8GUjATgfDx*

tuple_9_2_eq

Theorem 20.164 $(x0, x1, x2, x3, x4, x5, x6, x7, x8)\ 2 = x2$. *The proposition is identified by the following information:*

Pure Prop Id:	*62a1ca05201c6214162f1b3abb7aac30f56939908278fa886379a9d4b8b82e80*
Pure Prop Address:	*TMR3ZPXHQu2ywtSwNsvBLZjBq5B3bXZt1gr*
Theory Prop Id:	*cbc94460959b85613108686618927 9bf769c522b562c6627b9f28426cbd337d6*
Theory Prop Address:	*TMcHUnvscx8DjAsJLn8qhtXg6wp1Uwae864*

tuple_9_3_eq

Theorem 20.165 $(x0, x1, x2, x3, x4, x5, x6, x7, x8)\ 3 = x3$. *The proposition is identified by the following information:*

Pure Prop Id:	*ff32780d6432c453f8f483ee108e60697671b297f3692720ed857d111dfb71ff*
Pure Prop Address:	*TMM8Wc2najKFrxPLc9rLMDHAPijUVkEZqgE*
Theory Prop Id:	*1c8999f1f6c222289e7f742f2e89913886228a37f06d003c45a34a2f39454710*
Theory Prop Address:	*TMPrj3JT9vGnAt3EdTZh4hNCBJWF9jSUnCc*

tuple_9_4_eq

Theorem 20.166 $(x0, x1, x2, x3, x4, x5, x6, x7, x8)\ 4 = x4$. *The proposition is identified by the following information:*

Pure Prop Id:	*a6eff8bb384773eb50520928a17869c5a362f5a8aad62aefdbc76289e9a6d79f*
Pure Prop Address:	*TMVD2U7o4KhyJ5vRSXBuCtRYNqiDUPGGdvV*
Theory Prop Id:	*a8ebb556022b919699ce1585f44fe5382a8f90cbfb906757e003fee89ef48dc3*
Theory Prop Address:	*TMYBL14qJDzq5tT7Kafcbuu2kZuud4Kxisj*

tuple_9_5_eq

Theorem 20.167 $(x0, x1, x2, x3, x4, x5, x6, x7, x8)\ 5 = x5$. *The proposition is identified by the following information:*

Pure Prop Id:	*c78a9b1e8f5a59a13fac52e14833a96eaca2cfa10c50958e33b15229b119fe88*
Pure Prop Address:	*TMZwGSJLxkX6PKAVCtFGmZZJrSr2BqfJGTX*
Theory Prop Id:	*d77f4e4c53456c2e683d10d9701b4877455bca6113ce67c8044a027158f9bda4*
Theory Prop Address:	*TMYxcJb6i94M3w3oNmuZV5FueX4QnERLxfS*

tuple_9_6_eq

Theorem 20.168 $(x0, x1, x2, x3, x4, x5, x6, x7, x8)\ 6 = x6$. *The proposition is identified by the following information:*

Pure Prop Id:	*6e88afe0e1aee7023c9f258a6001f4b275581eec0fa26c41b702c8ca1ff82aac*
Pure Prop Address:	*TMWrYTvycmEwE7477Npqu6z4QaWkaJsxrK1*
Theory Prop Id:	*14cf05c1356df0137d6e6227fa3cf268b8268ba41d288eb04289088d348ca7fc*
Theory Prop Address:	*TMbPUqUn32XqEuCscjPeu2f4QjNwqVJq1Vv*

tuple_9_7_eq

Theorem 20.169 $(x0, x1, x2, x3, x4, x5, x6, x7, x8)\ 7 = x7$. *The proposition is identified by the following information:*

Pure Prop Id:	*810f4de4b017dc83221bcc8ff7116206998e3957bec3786295c9a0429ce531eb*
Pure Prop Address:	*TMdA5ixqphhBmefBSSzHsamLt8GAzvT9yfo*
Theory Prop Id:	*6cad9319962321f56f773887d81f42aa4ad5b8eedcdcf7236252cca9d9f647ea*
Theory Prop Address:	*TMbt5vtmatV53yku7m11vPWF23UU2d79D6y*

`tuple_9_8_eq`

Theorem 20.170 $(x0, x1, x2, x3, x4, x5, x6, x7, x8)\ 8 = x8$. *The proposition is identified by the following information:*

Pure Prop Id:	*8486de1f45eebeac976e64b40817e81d801358e1aa3eb4db3f60660662253a51*
Pure Prop Address:	*TMbcVQrFnbaUtBNZg7rEVeHjsbRYhwsse9z*
Theory Prop Id:	*011a8763916b9f748447c0172bb5ffdb73d7eda2528b2a21e183952258975e18*
Theory Prop Address:	*TMNGCEjuYZhPiHizVLu725KogvvA3BCGiiK*

20.8 Notation for Sums and Products

Notation. We use $\Sigma x, \ldots, y.$ as a binder notation corresponding to a term constructed using Sigma. We use $\times$ as a left associative infix operator corresponding to applying term setprod. We use $\Pi x, \ldots, y.$ as a binder notation corresponding to a term constructed using Pi. We use superscripts as notation for applying the term setexp.

`tuple_3_in_A_3`

Theorem 20.171 $\forall xyzA.x \in A \to y \in A \to z \in A \to (x, y, z) \in A^3$. *The proposition is identified by the following information:*

Pure Prop Id:	*b6cd63778e5c307591e09f0dbe6709f3e2a0a25002362cc8bc25c497de01c8af*
Pure Prop Address:	*TMJv4GJ6yA6E4zrTys8Fvo4pAxneFHykrEo*
Theory Prop Id:	*039f9b74048094fcf8f6a563d8ade3c5c8fae05574e684d2f7a1119e2bb00978*
Theory Prop Address:	*TMcqM5egc3mW237ChXvW4EschnVxw2CaHsw*

`tuple_3_bij_3`

Theorem 20.172

$$\forall xyz.x \in 3 \to y \in 3 \to z \in 3 \to x \neq y \to x \neq z \to y \neq z \to \\ \mathsf{bij}\ 3\ 3\ (\lambda i.(x, y, z)\ i).$$

The proposition is identified by the following information:

Pure Prop Id:	*8fc73d4725ec781165c998d0b4398e6ba4fcf725b30d828af8a3419c83437f62*
Pure Prop Address:	*TMHDdHVHUS4jN4k7phZYZXRAa2d6TcLAbNa*
Theory Prop Id:	*921a459779265cfc541794e752d792d1a82c2eaf3b4d1acdafdd92cbf66dab3e*
Theory Prop Address:	*TMXTMvgV3GghUjq7QBj1XncxXwtTjahmJVW*

`tuple_4_in_A_4`

Theorem 20.173

$$\forall xyzwA.x \in A \to y \in A \to z \in A \to w \in A \to (x, y, z, w) \in A^4.$$

The proposition is identified by the following information:

Pure Prop Id:	*9dfa39b843ce4aacc1ee090d1483a1708c5dc6df066ae816e7a9ecee8013a642*
Pure Prop Address:	*TMUtkQ3zWCfss9WPT6D8cb1egDHRfo1MVkn*
Theory Prop Id:	*c23e38f69ea81031bbd8a5d922a52882ec9ad56393a4fac4d35a44bfa890dd02*
Theory Prop Address:	*TMPzwDRXgLuZdfX4s8JwnPpH2jBU25hsWEm*

`tuple_4_bij_4`

Theorem 20.174

$$\begin{gathered}\forall xyzw.x \in 4 \to y \in 4 \to z \in 4 \to w \in 4 \to \\ x \neq y \to x \neq z \to x \neq w \to y \neq z \to y \neq w \to z \neq w \to \\ \mathsf{bij}\ 4\ 4\ (\lambda i.(x, y, z, w)\ i).\end{gathered}$$

The proposition is identified by the following information:

Pure Prop Id:	*8d7a7d5ba57f64b10810f6a1009fb528d3588de74399da873640c14afb5cf725*
Pure Prop Address:	*TMXDjtm9zxy9LdmBcvD1u1oaVds6NRkYyfJ*
Theory Prop Id:	*d8db975b63b69912e055c4b2c24308cac5831ad7491c3f8bea7b38fe6767e7bc*
Theory Prop Address:	*TMdGBeJemPTJ6p1TyGocFh5ejMH1SY1thm4*

`iff_refl`

Theorem 20.175 $\forall A : o.A \Leftrightarrow A$. *The proposition is identified by the following information:*

Pure Prop Id:	*01338dd4902fcd80dd89af75713f451fe1b1e4804229f05955b962fe6def6250*
Pure Prop Address:	*TMSKz5gXPFakcgFRBJ5RJYfDQC2q8ocrFi1*
Theory Prop Id:	*7b71b52f0f8ece18ad34af695ce60729d9853085b816941378947b88e44cbf0a*
Theory Prop Address:	*TML3Gcc3WbCE559uqmKxvpDahXs8iDJjURL*

`iff_sym`

Theorem 20.176 $\forall AB : o.(A \Leftrightarrow B) \to (B \Leftrightarrow A)$. *The proposition is identified by the following information:*

Pure Prop Id:	*00bac1cbdb77d159cd3e6b357772b77cfbf5fd08ad17791fd058a31c0475a3f6*
Pure Prop Address:	*TMWtfJhtiuKuNHV2k6qcVXL3USFSvbCNETm*
Theory Prop Id:	*11f51b14cc9177b5bc57ce79c943b2dcdfe3d83f93b1a6336d2174ba4e9df0b7*
Theory Prop Address:	*TMEgLJHQtw8TKqiFHfyA8RQk6wHyjMKHyrr*

`iff_trans`

Theorem 20.177 $\forall ABC : o.(A \Leftrightarrow B) \to (B \Leftrightarrow C) \to (A \Leftrightarrow C)$. *The proposition is identified by the following information:*

Pure Prop Id:	*363285ef27e2778afb838fe5fa3dd62807d7a90305f7def9e4fde9ad6aee1383*
Pure Prop Address:	*TMZye8CSrfjLgw85BDLDes4EZvTCzY9U9wh*
Theory Prop Id:	*c3b8f10058ec7ec17c37e487120493fd766e40d5ebea182f66ee1a52c306ddac*
Theory Prop Address:	*TMGt6JXemXxv9o18nreYuNrNEaPaXkqRn3V*

`not_or_and_demorgan`

Theorem 20.178 $\forall AB : o.\neg(A \vee B) \to \neg A \wedge \neg B$. *The proposition is identified by the following information:*

Pure Prop Id:	*8733b0a9ab0e00cdbd3a98177e398aa4cdb186d6fc7dc3377ad202c7a9841879*
Pure Prop Address:	*TMRcU714fZk34v4xKvog8EUFH1iBy9go9dP*
Theory Prop Id:	*27db911b3442575d6174ef608b04eba2264a9ff43956cb9f48038590759a688b*
Theory Prop Address:	*TMVWiPV5LBpzWhx13DG3yh1p7aPM8pSt1F5*

`and_not_or_demorgan`

Theorem 20.179 $\forall AB : o.\neg A \wedge \neg B \to \neg(A \vee B)$. *The proposition is identified by the following information:*

Pure Prop Id:	*d4efd56c095b9a01570df661ec52e9fc1d3acf99dbb90a82e8047f781b21c525*
Pure Prop Address:	*TMKRSDh9GPJPyX9ZtDqiV2KvjiUnNipaWCc*
Theory Prop Id:	*1b5ff58044cbc333c55074f5e5983bf4a0ac161bc254b95d032435918a88edba*
Theory Prop Address:	*TMZ7VzgbGyQBuJB8KTDGonc6PtcMRxd2VSQ*

`not_ex_all_demorgan_i`

Theorem 20.180 $\forall P : \iota \to o.(\neg\exists x.P\ x) \to \forall x.\neg P\ x$. *The proposition is identified by the following information:*

Pure Prop Id:	*73523ea5da5dea92107627d5a291d82f8297690e14be35d05c010105c296529b*
Pure Prop Address:	*TMMTCJ7GAVwzZ3Kw9d5Cbo8nEoqaWP6dBZs*
Theory Prop Id:	*075ddf1e62156846e54afd9c4c7a78cb61fa7d76482304fb50d121bd59d57fd5*
Theory Prop Address:	*TMNGgYHHcc31AsZAqJEYPmSY4HMBaLLKAHS*

`not_all_ex_demorgan_i`

Theorem 20.181 $\forall P : \iota \to o.\neg(\forall x.P\ x) \to \exists x.\neg P\ x$. *The proposition is identified by the following information:*

Pure Prop Id:	*9bfa06e2e338dc6635e2549a3e17611c80217077a8b944a0478de74cd035b7e7*
Pure Prop Address:	*TMdvAZELU6wRxcmyDehPqHUGeECff7oXd9k*
Theory Prop Id:	*4b147405ce7450475ec000fd6756e31ec6f80a467321e8614a031eb4efa82ae9*
Theory Prop Address:	*TMReccPGDm1kZKJBaUcrWSWSEbaxPa45Qos*

eq_or_nand

Theorem 20.182 or $= (\lambda xy : o.\neg(\neg x \wedge \neg y))$. *The proposition is identified by the following information:*

Pure Prop Id:	*24ac29991a11e5ff752e6de9a587fd6a214a662220d9d009904454a69ff0f25a*
Pure Prop Address:	*TMaWRCFfz4Q1tpK7DeCGe2qubLQxDVTPHUN*
Theory Prop Id:	*d212ef38daafdab7978d40905362a79d7c00ead847e06d6f47e2921708ce316e*
Theory Prop Address:	*TMd6MEzYCjYvnk16kTfPmz5gduojxUSzocf*

Definition 20.23 EpsR_i_i_1 *is the opaque object of type* $(\iota \to \iota \to o) \to \iota$ *identified by the following information:*

Pure Object Id:	*20c61c861cf1a0ec40aa6c975b43cd41a1479be2503a10765e97a8492374fbb0*
Pure Object Address:	*TMdTiwgMp6N6HYyfybe9XUGgVm2baQ7rg6M*
Theory Object Id:	*aced991b30f594da8af7187f84a33d664ee2260df28fd11d86b6d8b6281bc768*
Theory Object Address:	*TMUkEyWxAV2ZVYdDpQxADf9KP7MV4bVGB3m*

Definition 20.24 EpsR_i_i_2 *is the opaque object of type* $(\iota \to \iota \to o) \to \iota$ *identified by the following information:*

Pure Object Id:	*eced574473e7bc0629a71e0b88779fd6c752d24e0ef24f1e40d37c12fedf400a*
Pure Object Address:	*TMFeweZzkVerYMn5MZFuEXjq9udK79AoNif*
Theory Object Id:	*9087d981ba2801ae343ebf3a94c2e492b130dcee92d602d81eb4fa743b836009*
Theory Object Address:	*TMG4jTkPFvT4pZQYm7XyoTAQ2rCoJWUGyE8*

EpsR_i_i_12

Theorem 20.183

$$\forall R : \iota \to \iota \to o.(\exists xy.R\ x\ y) \to R\ (\mathsf{EpsR\_i\_i\_1}\ R)\ (\mathsf{EpsR\_i\_i\_2}\ R).$$

The proposition is identified by the following information:

Pure Prop Id:	*95a215a34ac047e6487e5f941e77b1193b239ef381850705e74d3f067c0bd706*
Pure Prop Address:	*TMHei6pp5iZeH81iwXzRtjmAKAWBcbScwk3*
Theory Prop Id:	*444c78c63c5fa77e04db6105dd5977e47fec752e9fdc8aa48314397572063a6d*
Theory Prop Address:	*TMM2ecoMZfXHpCELLyzJSaxKGa3nTPfMSpY*

Definition 20.25 DescrR_i_io_1 *is the opaque object of type*

$$(\iota \to (\iota \to o) \to o) \to \iota$$

identified by the following information:

Pure Object Id:	*1d3fd4a14ef05bd43f5c147d7966cf05fd2fed808eea94f56380454b9a6044b2*
Pure Object Address:	*TMYtxWX7Bmit31XQaJHqNRjz54fNehWemQx*
Theory Object Id:	*82aeccfa6a8d109c25ec362683dff6b3c19f69eb033bc53476eebc1307c2f73a*
Theory Object Address:	*TMG2AhxUeoPURmtyqa7z6V67bTzk9kjG2Gb*

Definition 20.26 DescrR_i_io_2 *is the opaque object of type*

$$(\iota \to (\iota \to o) \to o) \to \iota \to o$$

identified by the following information:

Pure Object Id:	*768eb2ad186988375e6055394e36e90c81323954b8a44eb08816fb7a84db2272*
Pure Object Address:	*TMQ5YqiemA8Nk8YhMLn6FVrHVdoZLZYhFMU*
Theory Object Id:	*3f82bc7f6e996e5dd88799a9921cab56f8d25ed0bcfad1f9f0e2b2a2968fcab4*
Theory Object Address:	*TMSA1F1Xty3y6u3fgqcMyVysuyvX9HKs1vN*

DescrR_i_io_12

Theorem 20.184

$$\forall R : \iota \to (\iota \to o) \to o.$$
$$(\exists x.(\exists y : \iota \to o.R\ x\ y) \land (\forall yz : \iota \to o.R\ x\ y \to R\ x\ z \to y = z)) \to$$
$$R\ (\mathsf{DescrR\_i\_io\_1}\ R)\ (\mathsf{DescrR\_i\_io\_2}\ R).$$

The proposition is identified by the following information:

Pure Prop Id:	*a4fae2546427685ef36418f88478e6af3ac2e79bb359fbb0c2ffab0ae113217f*
Pure Prop Address:	*TMXTFXfZpFuoocq9aqaPeoZwN93KkV2Yp2R*
Theory Prop Id:	*da4f023fa8a6db67051918ac52618aa28cd3cd54056987f781a8085ae23273a1*
Theory Prop Address:	*TMZVTm3MugdHj2HeAWLwWA24D1RpC1Kq1TU*

Chapter 21

Surreal Numbers I

21.1 Surreal Numbers as Predicates on Ordinals

Definition 21.1 *We define* PNoEq_ *to be* $\lambda \alpha p q . \forall \beta \in \alpha . p\ \beta \Leftrightarrow q\ \beta$ *of type* $\iota \to (\iota \to o) \to (\iota \to o) \to o$ *identified by the following information:*

Pure Object Id:	*d7d95919a06c44c69c724884cd5a474ea8f909ef85aae19ffe4a0ce816fa65fd*
Pure Object Address:	*TMWJ8MJ9pcvgRi55Dv3T4ZDyt4PLqinJMsL*
Theory Object Id:	*144b6539b9fe7bd5fe6597bae5a8aafdcf9b0b0be449f56a38d2ec86b003c9fa*
Theory Object Address:	*TMFreHQ2BuPpMHdDn38YhqZKp88uZF4gkiP*

PNoEq_ref_

Theorem 21.1 $\forall \alpha . \forall p : \iota \to o.$PNoEq_ $\alpha\ p\ p$. *The proposition is identified by the following information:*

Pure Prop Id:	*c27ebb17a66d8faafb90c6e2c087edacd9723fae48be3b2096fcbfb163cae3ba*
Pure Prop Address:	*TMR5Xh2p5AFCBWLLpLnHCLb42vf2t6zTXDM*
Theory Prop Id:	*df4cf66649d1fd1d26f29aadca2801400f850f3b4eafdc3be31d69bde8bff283*
Theory Prop Address:	*TMVv8oi3PpMpj4sDpqjfj6YKhg5rdMeRTKn*

PNoEq_sym_

Theorem 21.2

$$\forall \alpha . \forall p q : \iota \to o . \text{PNoEq\_}\ \alpha\ p\ q \to \text{PNoEq\_}\ \alpha\ q\ p.$$

The proposition is identified by the following information:

Pure Prop Id:	*b813f3d183ae4292b34de6ea5ae07acbc9fd890e0ca37ba1d70d462219b6b8bc*
Pure Prop Address:	*TMdRTKSCoAPR1yxWCt8NqSNPsCeHURA3SC1*
Theory Prop Id:	*bfda0b2058ffa91747bd1ac3030674a6cc496315b46c8308434fba9aea3f2657*
Theory Prop Address:	*TMLYYwg1Kxo3WZ71E4yBpKXxAE1s3vDazeX*

PNoEq_tra_

Theorem 21.3

$$\forall \alpha.\forall pqr:\iota\to o.\mathsf{PNoEq\_}\ \alpha\ p\ q\to \mathsf{PNoEq\_}\ \alpha\ q\ r\to \mathsf{PNoEq\_}\ \alpha\ p\ r.$$

The proposition is identified by the following information:

Pure Prop Id:	*dded4c9b3c97511b828af400470ae4f1a1e6f19deb142324786a79e7a5e93f13*
Pure Prop Address:	*TMHc7CPnyGLonvb5EfKN1tAwcVMJrF3RQEa*
Theory Prop Id:	*fc92f3d0615e8bb0c739baceeb42a209768755f252f7c3eec934cebe529e6569*
Theory Prop Address:	*TMdXdRjndh2ZZ4bnALH7m92PyqRv7hAdFp5*

PNoEq_antimon_

Theorem 21.4

$$\forall pq:\iota\to o.\forall \alpha.\mathsf{ordinal}\ \alpha\to \forall \beta\in\alpha.\mathsf{PNoEq\_}\ \alpha\ p\ q\to \mathsf{PNoEq\_}\ \beta\ p\ q.$$

The proposition is identified by the following information:

Pure Prop Id:	*8868e6589767bf4a8cd7788af6bd77faeab57b18ffdcecbe07aba3ed7624aee9*
Pure Prop Address:	*TMT6Y9Nfrocaqa6thWmudf8Un3qLyLduStm*
Theory Prop Id:	*094ae21814e156023eeffb5f103f193c98ceca6be42e5d08f1b636e235fc167a*
Theory Prop Address:	*TMcxE6t2A2eCrodDrECpNG7uGeMtfeS2zeN*

Definition 21.2 *We define* PNoLt_ *to be*

$$\lambda \alpha pq.\exists \beta\in\alpha.\mathsf{PNoEq\_}\ \beta\ p\ q\wedge\neg p\ \beta\wedge q\ \beta$$

of type $\iota\to(\iota\to o)\to(\iota\to o)\to o$ *identified by the following information:*

Pure Object Id:	*34de6890338f9d9b31248bc435dc2a49c6bfcd181e0b5e3a42fbb5ad769ab00d*
Pure Object Address:	*TMZD4o1Ego5dxANfVMw5J8RL7t5naNG9U6L*
Theory Object Id:	*c95964975192a20b94bb694b4b9a7c55de7fce1d4a56cd9d0ed8d67ae58fb60e*
Theory Object Address:	*TMG3oBNXZHp4KrZAdKCT2kf3DvDUqGNax9b*

PNoLt_E_

Theorem 21.5

$$\forall \alpha.\forall pq:\iota\to o.\mathsf{PNoLt\_}\ \alpha\ p\ q\to\forall R:o.$$
$$(\forall \beta.\beta\in\alpha\to\mathsf{PNoEq\_}\ \beta\ p\ q\to\neg p\ \beta\to q\ \beta\to R)\to R.$$

The proposition is identified by the following information:

Pure Prop Id: *9ba5433aba8ff79cf5b29265ee6f028e168d178699edf447c85bbc8deeac2a4e*
Pure Prop Address: *TMPQB13m96524sAUWzy2GhLxsUb6akgizyg*
Theory Prop Id: *45aec844f872697c4972ac0ebc6a05b529ce6188709968bbc3161616d02ce8ae*
Theory Prop Address: *TMNravm4aL128MRxYXGMQuCHaauCV7HVCoo*

PNoLt_irref_

Theorem 21.6 $\forall \alpha . \forall p : \iota \to o . \neg \mathsf{PNoLt\_}\ \alpha\ p\ p$. *The proposition is identified by the following information:*

Pure Prop Id: *fcb502c9cc3805a7a66fa875754cf8bf23ca05874f5dbdc9fa408a45bf6b32cd*
Pure Prop Address: *TMQ9CemPxrwhC7R2QenNg6KpFZuCjjKb8x2*
Theory Prop Id: *987dff18ff1b92d203fd612706b6d76ae7437c60c295931c75665e68331e9fff*
Theory Prop Address: *TMYjm5nDdDXhStiue7yDo3jqogooZAR36uB*

PNoLt_mon_

Theorem 21.7

$$\forall pq : \iota \to o . \forall \alpha . \mathsf{ordinal}\ \alpha \to \forall \beta \in \alpha . \mathsf{PNoLt\_}\ \beta\ p\ q \to \mathsf{PNoLt\_}\ \alpha\ p\ q.$$

The proposition is identified by the following information:

Pure Prop Id: *b1c3cf515eec97e9e1b0860995940355d51e18df35bc3f66115f683ebee76539*
Pure Prop Address: *TMJDSNwcnhwkzoXr3Z8kRHFuHXmWJPaX3Ce*
Theory Prop Id: *d8d0f010ecea2f707f8d719d67b8d74d77b42bb87c9317a35e1a8a3e812312c5*
Theory Prop Address: *TMdFZnNNsv6frZTGhJdUhF23hzzvniYV8hv*

PNoLt_trichotomy_or_

Theorem 21.8

$$\forall pq : \iota \to o . \forall \alpha . \mathsf{ordinal}\ \alpha \to \mathsf{PNoLt\_}\ \alpha\ p\ q \lor \mathsf{PNoEq\_}\ \alpha\ p\ q \lor \mathsf{PNoLt\_}\ \alpha\ q\ p.$$

The proposition is identified by the following information:

Pure Prop Id: *6644084c8b8082d13eb4f918c0458a6a57e0e4c038c596a1395d3eb99cd50e79*
Pure Prop Address: *TMcyZEERpexRUZhwpuM9GSuX3Sk5rPkRLNj*
Theory Prop Id: *8d3824c21ec2e23224adc53da96078403c04f5984cb1b5363e6033afb3557ba8*
Theory Prop Address: *TMdzU9ADKUux6zhuXTRcw9NBL5GrWiZFnL5*

PNoLt_tra_

Theorem 21.9

$$\forall \alpha . \mathsf{ordinal}\ \alpha \to \forall pqr : \iota \to o . \mathsf{PNoLt\_}\ \alpha\ p\ q \to \mathsf{PNoLt\_}\ \alpha\ q\ r \to \mathsf{PNoLt\_}\ \alpha\ p\ r.$$

The proposition is identified by the following information:

Pure Prop Id: *3d7d91c93d4e62413c6d9dd981bb8b73fe7cceb7d203f0798d848121d5071de2*
Pure Prop Address: *TMLwvVmBwmxcKpTr11BfuohRJkbqDNeYgkK*
Theory Prop Id: *44767a86f732b5f6b0f1ea4f2fd902fb4c657cd5b93d818dccf9adaf334006e4*
Theory Prop Address: *TMXUBoU9EyL1j4brgyEDkPSkDKW8vBUKaAk*

Definition 21.3 PNoLt *is the opaque object of type*

$$\iota \to (\iota \to o) \to \iota \to (\iota \to o) \to o$$

identified by the following information:

Pure Object Id: *8f57a05ce4764eff8bc94b278352b6755f1a46566cd7220a5488a4a595a47189*
Pure Object Address: *TMQBApBYGx5gde2jLgpjV5KGX8BPSrspNY9*
Theory Object Id: *f2094ca61fcaf27eecff3b6b119f415eaf83537511fa1030a24f9fa1658338a9*
Theory Object Address: *TMUcNNa6THZvpai86WFEVkAxVdKBFzgWMLt*

PNoLtI1

Theorem 21.10

$$\forall \alpha\beta.\forall pq : \iota \to o.\mathsf{PNoLt\_}\ (\alpha \cap \beta)\ p\ q \to \mathsf{PNoLt}\ \alpha\ p\ \beta\ q.$$

The proposition is identified by the following information:

Pure Prop Id: *c5735c7eb7f3308228886e03ec77196c4e828c0817e59a83eb6ea91b420edb3e*
Pure Prop Address: *TMS9TizXPog5PcwVnGnsKDk5q9DPz7TbhoH*
Theory Prop Id: *ecbc7189a71dd70627c550c8d9fdc53c6bd779fdc38db26b5b96dd819de399c8*
Theory Prop Address: *TMdBwxsaeBCn1Cz8duce8N385NuVba9M1iz*

PNoLtI2

Theorem 21.11

$$\forall \alpha\beta.\forall pq : \iota \to o.\alpha \in \beta \to \mathsf{PNoEq\_}\ \alpha\ p\ q \to q\ \alpha \to \mathsf{PNoLt}\ \alpha\ p\ \beta\ q.$$

The proposition is identified by the following information:

Pure Prop Id: *419b53c5b7f090cd6916ecca9011b3d8d19ae95bc60020cad45d8c92803c2005*
Pure Prop Address: *TMLoQH9MBcB6zQZDo5PkbRneUZBXQQRzDr8*
Theory Prop Id: *03009382079584527 4dac213f6f565752cfe3de0e2652c9c76ca161494618286*
Theory Prop Address: *TMGXNx56uxbqYFaCe1PxneJjhYYGcLCyE8T*

PNoLtI3

Theorem 21.12

$$\forall \alpha\beta.\forall pq : \iota \to o.\beta \in \alpha \to \mathsf{PNoEq\_}\ \beta\ p\ q \to \neg p\ \beta \to \mathsf{PNoLt}\ \alpha\ p\ \beta\ q.$$

The proposition is identified by the following information:

Pure Prop Id:	*c392cb53f2ad2f2d109882679fc4b8a23c27eeb0ff9a5fef31347a885ba891da*
Pure Prop Address:	*TMNZw7Cj3j1w5e5hoebGCVirnCRB3RXaNvW*
Theory Prop Id:	*4104429992b8f35592b8ceba357f6dc0dc7f2ba8a5f6b06451be23cdd6fb161b*
Theory Prop Address:	*TMFhY7wPRcF4NknzzkXMPbsPrBbMZ2PZQb8*

`PNoLtE`

Theorem 21.13

$$\forall \alpha \beta . \forall p q : \iota \to o . \mathsf{PNoLt}\ \alpha\ p\ \beta\ q \to \forall R : o . \\ (\mathsf{PNoLt\_}\ (\alpha \cap \beta)\ p\ q \to R) \to \\ (\alpha \in \beta \to \mathsf{PNoEq\_}\ \alpha\ p\ q \to q\ \alpha \to R) \to \\ (\beta \in \alpha \to \mathsf{PNoEq\_}\ \beta\ p\ q \to \neg p\ \beta \to R) \to R.$$

The proposition is identified by the following information:

Pure Prop Id:	*f7e6e3e5f56759996f8e71b71ecc1cfeba2385a738e3535fb9f45c5f05a7ba7a*
Pure Prop Address:	*TMPLU4MZci1RZyu9YajBNzBvAn2625YmMhz*
Theory Prop Id:	*af36ceaf250752d7f1c85964b80e0b0df1acb7a0538c6e38f8cd6b431c530c5e*
Theory Prop Address:	*TMbpk33jSr7sb7xpz9amLj1PFYcwRPKWBzJ*

`PNoLtE2`

Theorem 21.14

$$\forall \alpha . \forall p q : \iota \to o . \mathsf{PNoLt}\ \alpha\ p\ \alpha\ q \to \mathsf{PNoLt\_}\ \alpha\ p\ q.$$

The proposition is identified by the following information:

Pure Prop Id:	*ad392c5c3af8700314707c033e9816397448a5fcd60aab607c4fa7af5a143745*
Pure Prop Address:	*TMX9jn8QF6K3SxcZrS4opjiQZ9yxZ4mP3ju*
Theory Prop Id:	*7ef10485fe210dd748b752318940f7a2ccbe8a44d685b1525c931e391a9d5662*
Theory Prop Address:	*TMX4uzbNMJMGDv9jTKuePDdqiWegaum41fi*

`PNoLt_irref`

Theorem 21.15 $\forall \alpha . \forall p : \iota \to o . \neg \mathsf{PNoLt}\ \alpha\ p\ \alpha\ p$. *The proposition is identified by the following information:*

Pure Prop Id:	*ff1a80a33f6f089b5a40873efe47b5c845fd624b4250c5c7bb6d8c4de6bc398d*
Pure Prop Address:	*TMYgpDGCHGRHDBZdPr12n2a5kcKFbuHdceA*
Theory Prop Id:	*d4c3797f7ec820194be1429644bfb16565fc442db2d2480bac78b9f481bb2a97*
Theory Prop Address:	*TMdk8u8CYrFAFVf8jmy3pdMCDAMd2SPuATi*

`PNoLt_trichotomy_or`

Theorem 21.16

$$\forall \alpha \beta . \forall p q : \iota \to o . \textsf{ordinal}\ \alpha \to \textsf{ordinal}\ \beta \to$$
$$\textsf{PNoLt}\ \alpha\ p\ \beta\ q \lor \alpha = \beta \land \textsf{PNoEq\_}\ \ \alpha\ p\ q \lor \textsf{PNoLt}\ \beta\ q\ \alpha\ p.$$

The proposition is identified by the following information:

Pure Prop Id:	*4f39e49b96938924ecee0c62b18b8054abc06386845a55af32ecbc5c07a60f7d*
Pure Prop Address:	*TMcxeDW7jxsMf6iBBZyA1rMm28N9LGxrxAe*
Theory Prop Id:	*90b38722517f98c1ebbb09338d5ac216a2660c649969e817ce4af46864ab1174*
Theory Prop Address:	*TMKtLGK85iJ16Tv3NbRNth1Uc13sd3VWcEC*

`PNoLtEq_tra`

Theorem 21.17

$$\forall \alpha \beta . \textsf{ordinal}\ \alpha \to \textsf{ordinal}\ \beta \to$$
$$\forall p q r : \iota \to o . \textsf{PNoLt}\ \alpha\ p\ \beta\ q \to \textsf{PNoEq\_}\ \ \beta\ q\ r \to \textsf{PNoLt}\ \alpha\ p\ \beta\ r.$$

The proposition is identified by the following information:

Pure Prop Id:	*2e082ab6742a9e96182b14bc401ef1691287ea931ad64b2d9f8143dd38e72381*
Pure Prop Address:	*TMKVFaHQYU7RMkic7ehXR1Hz5AxwJLND1Xs*
Theory Prop Id:	*2f2a828f72a2bc454fe6d454ef8a36f6aad539b02f33d5546be1b804baa1f49b*
Theory Prop Address:	*TMVam3H7JTsW4GptAT9oJpfx8ncfn8EV3XV*

`PNoEqLt_tra`

Theorem 21.18

$$\forall \alpha \beta . \textsf{ordinal}\ \alpha \to \textsf{ordinal}\ \beta \to$$
$$\forall p q r : \iota \to o . \textsf{PNoEq\_}\ \ \alpha\ p\ q \to \textsf{PNoLt}\ \alpha\ q\ \beta\ r \to \textsf{PNoLt}\ \alpha\ p\ \beta\ r.$$

The proposition is identified by the following information:

Pure Prop Id:	*fe144b0ff77f85ded00481bd97ae472212a28995d89fa3667ab8889dff197bb5*
Pure Prop Address:	*TMVXYVUmzv2xFHz12tNLMKuyKAeKQiURGoM*
Theory Prop Id:	*f076541f250b71cb6126517531b079c63b10d79f85685776f781bf746959df68*
Theory Prop Address:	*TMRsHUfrwX3yw5eXZcC3S8S9yJKYP2vG94Z*

`PNoLt_tra`

Theorem 21.19

$$\forall \alpha \beta \gamma . \textsf{ordinal}\ \alpha \to \textsf{ordinal}\ \beta \to \textsf{ordinal}\ \gamma \to$$
$$\forall p q r : \iota \to o . \textsf{PNoLt}\ \alpha\ p\ \beta\ q \to \textsf{PNoLt}\ \beta\ q\ \gamma\ r \to \textsf{PNoLt}\ \alpha\ p\ \gamma\ r.$$

The proposition is identified by the following information:

Pure Prop Id: *b72127d429a420ed218c832bea2f3aaf8a09149da0276d6333614f557fff90a7*
Pure Prop Address: *TMV9P8LWZv2DWhZZm536XdWjUpfNXSoY83p*
Theory Prop Id: *8eff85e4785607b84b3c70850a18baea404f11fd03cc04cfd5afa1cb189a0c5f*
Theory Prop Address: *TMGtkttdVbaT9NVAgziSNR2tFzasudosPWy*

Definition 21.4 *We define* PNoLe *to be*

$$\lambda \alpha p \beta q.\mathsf{PNoLt}\ \alpha\ p\ \beta\ q \vee \alpha = \beta \wedge \mathsf{PNoEq\_}\ \alpha\ p\ q$$

of type $\iota \to (\iota \to o) \to \iota \to (\iota \to o) \to o$ *identified by the following information:*

Pure Object Id: *ac6311a95e065816a2b72527ee3a36906afc6b6afb8e67199361afdfc9fe02e2*
Pure Object Address: *TMa2eyC7pjLoLi2qJAssPFeBwpm49h4vYjE*
Theory Object Id: *770cf7219dc1ea3a0f1a555c8be6a16fbc83df72a88b338d5a9b6c5282933d86*
Theory Object Address: *TMHvhi4cZx4BpvWATQS4A5YGwuJKHFHnqZ8*

`PNoLeI1`

Theorem 21.20

$$\forall \alpha \beta.\forall p q : \iota \to o.\mathsf{PNoLt}\ \alpha\ p\ \beta\ q \to \mathsf{PNoLe}\ \alpha\ p\ \beta\ q.$$

The proposition is identified by the following information:

Pure Prop Id: *1f0528aa090a84e44d3bcd21d1f5c364665fee6e39da73097b72aa626f61a067*
Pure Prop Address: *TMMU61EKjZQ56vwqnq9EJcT3RZD4kTLXmEj*
Theory Prop Id: *674a2cb5c37edfb9cea0bddf54687b384d613f51ce354dd54fc543730f626cc9*
Theory Prop Address: *TMHGpziusAgRkcNhfxaVjHNxAiJnaP9Aq4m*

`PNoLeI2`

Theorem 21.21

$$\forall \alpha.\forall p q : \iota \to o.\mathsf{PNoEq\_}\ \alpha\ p\ q \to \mathsf{PNoLe}\ \alpha\ p\ \alpha\ q.$$

The proposition is identified by the following information:

Pure Prop Id: *b453329022d5a4973fd4979e8276c334bd9855b0fcef38a490d0d3832d17f295*
Pure Prop Address: *TMRnggtbHEorcGeqrrjsVRdcj87SoZCNfE9*
Theory Prop Id: *0b2157420981c9fb11d49eda5ed224af6f8a3582031030525c97a2cb5d7f2065*
Theory Prop Address: *TMEuEJtVZKAPqoRvcrAuffF91gTB2F6p1ur*

`PNoLe_ref`

Theorem 21.22 $\forall \alpha.\forall p : \iota \to o.\mathsf{PNoLe}\ \alpha\ p\ \alpha\ p.$ *The proposition is identified by the following information:*

Pure Prop Id:	*1b91bdb4dd7268a37d5bc57b8815bd8867df7f57b837c68a5a3cecba131ffe39*
Pure Prop Address:	*TMZFLxhVzuJSDerZWEiaV9vKjwvZbajaqne*
Theory Prop Id:	*613386d1779902b3b59aa4f008e13853ef42f333df28e697121c9469f82e8a8d*
Theory Prop Address:	*TMNTwN9qruHGfPxKb1SjuvQHFwhXyGnA1sj*

PNoLe_antisym

Theorem 21.23

$$\forall \alpha \beta . \mathsf{ordinal}\ \alpha \to \mathsf{ordinal}\ \beta \to \\ \forall p q : \iota \to o . \mathsf{PNoLe}\ \alpha\ p\ \beta\ q \to \mathsf{PNoLe}\ \beta\ q\ \alpha\ p \to \alpha = \beta \land \mathsf{PNoEq\_}\ \alpha\ p\ q.$$

The proposition is identified by the following information:

Pure Prop Id:	*934d7a99d54f73bf9a9e6317bc9decff59c4047034ee26980b50587b2dd95801*
Pure Prop Address:	*TMNTPG6YJEwCxRs1V24gTDCmTzHzRvRc6gk*
Theory Prop Id:	*4e35944f68e317a6041fb50a04c326a498b0c1bc1e0ed7e8064b1a16614489aa*
Theory Prop Address:	*TMLEM4Wo7kALZKsWnk4Wd5KjwhRhR5uC1HT*

PNoLtLe_tra

Theorem 21.24

$$\forall \alpha \beta \gamma . \mathsf{ordinal}\ \alpha \to \mathsf{ordinal}\ \beta \to \mathsf{ordinal}\ \gamma \to \\ \forall p q r : \iota \to o . \mathsf{PNoLt}\ \alpha\ p\ \beta\ q \to \mathsf{PNoLe}\ \beta\ q\ \gamma\ r \to \mathsf{PNoLt}\ \alpha\ p\ \gamma\ r.$$

The proposition is identified by the following information:

Pure Prop Id:	*1565862dfbb792dc415a313525571261e4fc5597f958d0f6839b147404b16497*
Pure Prop Address:	*TMS5q2iXwJmrbrvpyaPEgKeFc4yT8unvrtd*
Theory Prop Id:	*09246b78705144b6b5679fa64a57f406c40b20b13b935e7a3c426c016425ec52*
Theory Prop Address:	*TMP8YWTicgVUQEKtiP6hLWexaY1AtmqKF7Y*

PNoLeLt_tra

Theorem 21.25

$$\forall \alpha \beta \gamma . \mathsf{ordinal}\ \alpha \to \mathsf{ordinal}\ \beta \to \mathsf{ordinal}\ \gamma \to \\ \forall p q r : \iota \to o . \mathsf{PNoLe}\ \alpha\ p\ \beta\ q \to \mathsf{PNoLt}\ \beta\ q\ \gamma\ r \to \mathsf{PNoLt}\ \alpha\ p\ \gamma\ r.$$

The proposition is identified by the following information:

Pure Prop Id:	*a260ca2dcf87c48e05a6f7916c83ca227958075cb681e56d4f97f095c7890142*
Pure Prop Address:	*TMEugnvG1Bt7rUt9psKYMG4B3eXfMrHtrVq*
Theory Prop Id:	*ad6971ea2f07aa1305f0c6bc480a834ada5728150db9419ad7ccd521ebe1ce04*
Theory Prop Address:	*TMT7tULGek9mdvm2GJBcURRtAYaQxeEmHmE*

PNoEqLe_tra

Theorem 21.26

$$\forall \alpha \beta.\mathsf{ordinal}\ \alpha \to \mathsf{ordinal}\ \beta \to$$
$$\forall pqr : \iota \to o.\mathsf{PNoEq\_}\ \alpha\ p\ q \to \mathsf{PNoLe}\ \alpha\ q\ \beta\ r \to \mathsf{PNoLe}\ \alpha\ p\ \beta\ r.$$

The proposition is identified by the following information:

Pure Prop Id:	*7d3ff80df17ec96e504b4964272829064a97b36a1e75174ea67fe631b2aa92ab*
Pure Prop Address:	*TMWEkTHjED61EVaMYyFDrbsBT3uuPxsd2WS*
Theory Prop Id:	*002ebf808b3010e1129d1a3d0f3318345316603ff5e9f8a4752963619b88ad30*
Theory Prop Address:	*TMX3rf1dBjD7HFURU7toJLUuPtwumdeBSD4*

`PNoLeEq_tra`

Theorem 21.27

$$\forall \alpha \beta.\mathsf{ordinal}\ \alpha \to \mathsf{ordinal}\ \beta \to$$
$$\forall pqr : \iota \to o.\mathsf{PNoLe}\ \alpha\ p\ \beta\ q \to \mathsf{PNoEq\_}\ \beta\ q\ r \to \mathsf{PNoLe}\ \alpha\ p\ \beta\ r.$$

The proposition is identified by the following information:

Pure Prop Id:	*2def03362104b85520032b762fc8d4c1d8df8e5ec52650b7c3b2e85cae9c2fcc*
Pure Prop Address:	*TMJmHGCtRPZGoKvdPGHCWSTaoKXedqvvXcT*
Theory Prop Id:	*ed30aa19eb8fe59fe9c61636687168d8be8ac512c5d98ff2581121ba1ffd0765*
Theory Prop Address:	*TMRHBJiUqPVNK9rBz9VWXHRyyHjTuJD3RGU*

`PNoLe_tra`

Theorem 21.28

$$\forall \alpha \beta \gamma.\mathsf{ordinal}\ \alpha \to \mathsf{ordinal}\ \beta \to \mathsf{ordinal}\ \gamma \to$$
$$\forall pqr : \iota \to o.\mathsf{PNoLe}\ \alpha\ p\ \beta\ q \to \mathsf{PNoLe}\ \beta\ q\ \gamma\ r \to \mathsf{PNoLe}\ \alpha\ p\ \gamma\ r.$$

The proposition is identified by the following information:

Pure Prop Id:	*70b92b6662709f7dc5822fd97e53303a3a2d055ae004b41c4e645dce93c651bc*
Pure Prop Address:	*TMHzFcB1asSRVnHkPCarNFE3z9eXcp4jGKN*
Theory Prop Id:	*4b557dc79b8f761dcdf6d3964db848fe773041a6fb41c452623328cec43d42d1*
Theory Prop Address:	*TMRqHazfJDQBeT8YDQifbd4FBKaDUPSHefM*

Definition 21.5 *We define* $\mathsf{PNo\_downc}$ *to be*

$$\lambda L \alpha p.\exists \beta.\mathsf{ordinal}\ \beta \land \exists q : \iota \to o.L\ \beta\ q \land \mathsf{PNoLe}\ \alpha\ p\ \beta\ q$$

of type $(\iota \to (\iota \to o) \to o) \to \iota \to (\iota \to o) \to o$ *identified by the following information:*

Pure Object Id: *93dab1759b565d776b57c189469214464808cc9addcaa28b2dbde0d0a447f94d*
Pure Object Address: *TMFFjJ65YM9kJCjR5s8uzZawGiUXuQM1tjB*
Theory Object Id: *f6c96279232d9e9cca23ffe49c0c29166fada636c1195de7ffbd7fabd07ac1c7*
Theory Object Address: *TMbMkrzAgHfL7RAFKvYtv8vLTv3CdoSjov9*

Definition 21.6 *We define* PNo_upc *to be*

$$\lambda R \alpha p. \exists \beta. \mathsf{ordinal}\ \beta \wedge \exists q : \iota \to o. R\ \beta\ q \wedge \mathsf{PNoLe}\ \beta\ q\ \alpha\ p$$

of type $(\iota \to (\iota \to o) \to o) \to \iota \to (\iota \to o) \to o$ *identified by the following information:*

Pure Object Id: *5437127f164b188246374ad81d6b61421533c41b1be66692fb93fdb0130e8664*
Pure Object Address: *TMaAkmpSZzSfoMddDbQnVgH1TnDuodvBF4s*
Theory Object Id: *b97935067aeb5829f0c939603d3342813777d67c23748ee242158f33ac42ed50*
Theory Object Address: *TMdz4JctWDDKNbEgwhwYLZAiTPcFhWgefwa*

`PNoLe_downc`

Theorem 21.29

$$\forall L : \iota \to (\iota \to o) \to o. \forall \alpha \beta. \forall p q : \iota \to o. \mathsf{ordinal}\ \alpha \to \mathsf{ordinal}\ \beta \to \mathsf{PNo\_downc}\ L\ \alpha\ p \to \mathsf{PNoLe}\ \beta\ q\ \alpha\ p \to \mathsf{PNo\_downc}\ L\ \beta\ q.$$

The proposition is identified by the following information:

Pure Prop Id: *6b1f0d64dd061a5eba7c0342d7f3676187349916ec76342278678c84b205b24f*
Pure Prop Address: *TMWAyyBEGMhjikXgumQbCvdi5WRdMG4XswZ*
Theory Prop Id: *cd130e6e61b7fe7f0bfe1dd262ffaa5f86165a48fa9f622a9e0d11503c799ff3*
Theory Prop Address: *TMM4WeADC271qKWhMRuk4GonoM9GLTxaXM9*

`PNo_downc_ref`

Theorem 21.30

$$\forall L : \iota \to (\iota \to o) \to o. \forall \alpha. \mathsf{ordinal}\ \alpha \to \forall p : \iota \to o. L\ \alpha\ p \to \mathsf{PNo\_downc}\ L\ \alpha\ p.$$

The proposition is identified by the following information:

Pure Prop Id: *347aa9ec835b739b3314c47b28ca0cd2ebf6b08b10135cc7cb91d5cdf39bed56*
Pure Prop Address: *TMT5dDg6pVSTQmj3sYdZvHj975JFuQSEf3R*
Theory Prop Id: *69dfcb8516ec52d7595fe7c4aaf6805468aa6d2634d29ea2b8226ad567420a27*
Theory Prop Address: *TMUEbPAgDX5aYyR8xXjQWGAsxbj8SwH4QN4*

`PNo_upc_ref`

Theorem 21.31

$$\forall R : \iota \to (\iota \to o) \to o. \forall \alpha. \mathsf{ordinal}\ \alpha \to \forall p : \iota \to o.$$
$$R\ \alpha\ p \to \mathsf{PNo\_upc}\ R\ \alpha\ p.$$

The proposition is identified by the following information:

Pure Prop Id:	*4039424bd88802176a6cbb56b76abcf8b8a1f616a3d0721d28e58f9f096aa816*
Pure Prop Address:	*TMRis4sdu68KHLQb77r68nGDtFxyhKXKcBM*
Theory Prop Id:	*c4bf71e4c9e3609364a3a0c2ce3ae0a4befd6f3181cea4eec8656173a1b32b72*
Theory Prop Address:	*TMFX2E2HrfER46NsW7mPNvJqqJZFcAuJx1i*

`PNoLe_upc`

Theorem 21.32

$$\forall R : \iota \to (\iota \to o) \to o. \forall \alpha \beta. \forall p q : \iota \to o. \mathsf{ordinal}\ \alpha \to \mathsf{ordinal}\ \beta \to$$
$$\mathsf{PNo\_upc}\ R\ \alpha\ p \to \mathsf{PNoLe}\ \alpha\ p\ \beta\ q \to \mathsf{PNo\_upc}\ R\ \beta\ q.$$

The proposition is identified by the following information:

Pure Prop Id:	*111b74916dd3b9d72613111538f033f3bb8b5dfe2d40082b76d45a031efb7cc7*
Pure Prop Address:	*TMcxmbHaZztM29ZiLWZpgrqw4nZNUzfJYeS*
Theory Prop Id:	*eb9ed5adca244da4ff7ae93d024a4823361cb4695e55f8fa465e7e14c460eae2*
Theory Prop Address:	*TMJYddW3A9sz7atvMHDox6oK3Fz5ZXhVgw5*

Definition 21.7 *We define* $\mathsf{PNoLt\_pwise}$ *to be*

$$\lambda L R.$$
$$\forall \gamma. \mathsf{ordinal}\ \gamma \to \forall p : \iota \to o. L\ \gamma\ p \to$$
$$\forall \delta. \mathsf{ordinal}\ \delta \to \forall q : \iota \to o. R\ \delta\ q \to$$
$$\mathsf{PNoLt}\ \gamma\ p\ \delta\ q$$

of type $(\iota \to (\iota \to o) \to o) \to (\iota \to (\iota \to o) \to o) \to o$ *identified by the following information:*

Pure Object Id:	*ae448bdfa570b11d4a656e4d758bc6703d7f329c6769a116440eb5cde2a99f6f*
Pure Object Address:	*TMc96g2JPpBeTmDVeFvvSZTGX5Tb5262YGw*
Theory Object Id:	*12380893a7dd9882f429d48a7961599b1aa9e328a364d33120d9b2b3f2b04f70*
Theory Object Address:	*TMQdLYLNu2LJgg1svc2g1R5FdDAS2iXTH2E*

`PNoLt_pwise_downc_upc`

Theorem 21.33

$$\forall L R : \iota \to (\iota \to o) \to o. \mathsf{PNoLt\_pwise}\ L\ R \to$$
$$\mathsf{PNoLt\_pwise}\ (\mathsf{PNo\_downc}\ L)\ (\mathsf{PNo\_upc}\ R).$$

The proposition is identified by the following information:

Pure Prop Id:	*2b8e3bd1c2f83410acc3eb16745b3c189b91f3d88eff48c21bc1fe2da62df2a9*
Pure Prop Address:	*TMaupVE3uvrBXnhCNWJG4uidezoNueB9Pzn*
Theory Prop Id:	*2f244c622c329868222671ac992535b2ec8a107cab98a55451d89fc062755198*
Theory Prop Address:	*TMSZ221fidpj3oVVEjh7AxtKFSvU4SKPRwZ*

Definition 21.8 *We define* PNo_rel_strict_upperbd *to be*

$$\lambda L\alpha p.\forall\beta\in\alpha.\forall q:\iota\to o.\mathsf{PNo\_downc}\ L\ \beta\ q\to\mathsf{PNoLt}\ \beta\ q\ \alpha\ p$$

of type $(\iota\to(\iota\to o)\to o)\to\iota\to(\iota\to o)\to o$ *identified by the following information:*

Pure Object Id:	*dc1665d67ed9f420c2da68afc3e3be02ee1563d2525cbd6ab7412038cd80aaec*
Pure Object Address:	*TMPiEDqG64KxZbAF6RkpdrzFu2A2HmgQJ6p*
Theory Object Id:	*3e1663abe9ff987d905d14ab1ac2db49036bac313db730a8f2e8fbb4db756c76*
Theory Object Address:	*TMGu9LKc1Ew275sc36SoMWGV6V7yydJCxoa*

Definition 21.9 *We define* PNo_rel_strict_lowerbd *to be*

$$\lambda R\alpha p.\forall\beta\in\alpha.\forall q:\iota\to o.\mathsf{PNo\_upc}\ R\ \beta\ q\to\mathsf{PNoLt}\ \alpha\ p\ \beta\ q$$

of type $(\iota\to(\iota\to o)\to o)\to\iota\to(\iota\to o)\to o$ *identified by the following information:*

Pure Object Id:	*e5a4b6b2a117c6031cd7d2972902a228ec0cc0ab2de468df11b22948daf56892*
Pure Object Address:	*TMMtUywXSfQieSYmZXKhRaMVMvNBJ4mv5n1*
Theory Object Id:	*ff3150c8fe3f147f482d147d4948a3adfa0f23616d03c877213e009957f55a8e*
Theory Object Address:	*TMUamWcr8Mykvg8ZyWAHjTGbXtZ7LdvyWix*

Definition 21.10 *We define* PNo_rel_strict_imv *to be*

$$\lambda LR\alpha p.\mathsf{PNo\_rel\_strict\_upperbd}\ L\ \alpha\ p\wedge\mathsf{PNo\_rel\_strict\_lowerbd}\ R\ \alpha\ p$$

of type $(\iota\to(\iota\to o)\to o)\to(\iota\to(\iota\to o)\to o)\to\iota\to(\iota\to o)\to o$ *identified by the following information:*

Pure Object Id:	*64ce9962d0a17b3b694666ef1fda22a8c5635c61382ad7ae5a322890710bc5f8*
Pure Object Address:	*TMFES6j6egEfn4hHem6CQT3DsNYsaxQYweU*
Theory Object Id:	*5b35fbff3a2443e9ad06778c49bcebadc3d05d5d8c2da1fb0fad3c9047e95755*
Theory Object Address:	*TMXkoajkux97senphT28NAy8HKer2u4kPUc*

`PNoEq_rel_strict_upperbd`

Theorem 21.34

$$\forall L:\iota\to(\iota\to o)\to o.\forall\alpha.\mathsf{ordinal}\ \alpha\to\forall pq:\iota\to o.\mathsf{PNoEq\_}\ \alpha\ p\ q\to \mathsf{PNo\_rel\_strict\_upperbd}\ L\ \alpha\ p\to \mathsf{PNo\_rel\_strict\_upperbd}\ L\ \alpha\ q.$$

The proposition is identified by the following information:

Pure Prop Id: *bb65cf435cd1184f19969d40225b30f85ec7629c2a8b3845828f9cae4b8218cd*
Pure Prop Address: *TMbeVMWawJhuN1ZSdk7xfgpCpASxk2FoVZv*
Theory Prop Id: *db4b8216b0bda52674a1b262e30b3b493a88c2284eb09e35892c6464171da698*
Theory Prop Address: *TMR2fFPGsUbreZFxGSKzs2Ut4aBjCbtG8FU*

PNo_rel_strict_upperbd_antimon

Theorem 21.35

$$\forall L : \iota \to (\iota \to o) \to o. \forall \alpha. \mathsf{ordinal}\ \alpha \to \forall p : \iota \to o. \forall \beta \in \alpha. \mathsf{PNo\_rel\_strict\_upperbd}\ L\ \alpha\ p \to \mathsf{PNo\_rel\_strict\_upperbd}\ L\ \beta\ p.$$

The proposition is identified by the following information:

Pure Prop Id: *9986afc591c8630816ae93586b1d747b9e6a7d43426e319ea6699264ca845b9c*
Pure Prop Address: *TMFfTTpCGWMgKptEhmq4H5u6A1pxWeCHZWr*
Theory Prop Id: *ed53da6a8a942b087f817d785b79f257de9cb0f526d77897b831913b7bfb6d4f*
Theory Prop Address: *TMdLwNGovgqDXAVmQmmx97gnpUHqbCcATFy*

PNoEq_rel_strict_lowerbd

Theorem 21.36

$$\forall R : \iota \to (\iota \to o) \to o. \forall \alpha. \mathsf{ordinal}\ \alpha \to \forall p q : \iota \to o. \mathsf{PNoEq\_}\ \alpha\ p\ q \to \mathsf{PNo\_rel\_strict\_lowerbd}\ R\ \alpha\ p \to \mathsf{PNo\_rel\_strict\_lowerbd}\ R\ \alpha\ q.$$

The proposition is identified by the following information:

Pure Prop Id: *76080ee1804cd6f07353c102ecc22dbe6f88819fba1748db34678b3a3205ce51*
Pure Prop Address: *TMRaaoK9aPyvyCQEyS6u7CuxJeMRarcqdua*
Theory Prop Id: *614e589383386bf6bffd56988d8ed052f11fa7717c9bfc928aa218b49ba14436*
Theory Prop Address: *TMN9uHHDyxEb7wZLV54NYgKUBnfgd8Syeiz*

PNo_rel_strict_lowerbd_antimon

Theorem 21.37

$$\forall R : \iota \to (\iota \to o) \to o. \forall \alpha. \mathsf{ordinal}\ \alpha \to \forall p : \iota \to o. \forall \beta \in \alpha. \mathsf{PNo\_rel\_strict\_lowerbd}\ R\ \alpha\ p \to \mathsf{PNo\_rel\_strict\_lowerbd}\ R\ \beta\ p.$$

The proposition is identified by the following information:

Pure Prop Id: *27f253b3d87846cda9c59a20ef82f3175140815d3b9625c2b5dfe1a332086817*
Pure Prop Address: *TMQ6kfs1zMTGEcR3v2oD9G2cmxvPeeGcygk*
Theory Prop Id: *344b63ede0823903787534 76ed8b39a6cdcbc2522581fb25d8b3e2dbcd366fd5*
Theory Prop Address: *TMVV2kTQbx2zWgUFoViphDG4PSygzxibjvw*

`PNoEq_rel_strict_imv`

Theorem 21.38

$$\forall LR : \iota \to (\iota \to o) \to o. \forall \alpha. \mathsf{ordinal}\ \alpha \to \forall pq : \iota \to o. \mathsf{PNoEq\_}\ \alpha\ p\ q \to \mathsf{PNo\_rel\_strict\_imv}\ L\ R\ \alpha\ p \to \mathsf{PNo\_rel\_strict\_imv}\ L\ R\ \alpha\ q.$$

The proposition is identified by the following information:

Pure Prop Id:	*e3f251299cd1135da075622be47bde557bbba6cbac1e28d78bb914fdda99a216*
Pure Prop Address:	*TMX9qa29bL95NMwjxy6sv4Ubp6sbR8ZXSAy*
Theory Prop Id:	*7ae13df8bdc5ae35409ee041f3bcf55387bae17d0117e5e6491d3cb5cb89d4f9*
Theory Prop Address:	*TMWRDDHJba821Tbd4gG36AUGcRoP4wgdEtZ*

`PNo_rel_strict_imv_antimon`

Theorem 21.39

$$\forall LR : \iota \to (\iota \to o) \to o. \forall \alpha. \mathsf{ordinal}\ \alpha \to \forall p : \iota \to o. \forall \beta \in \alpha. \mathsf{PNo\_rel\_strict\_imv}\ L\ R\ \alpha\ p \to \mathsf{PNo\_rel\_strict\_imv}\ L\ R\ \beta\ p.$$

The proposition is identified by the following information:

Pure Prop Id:	*2434dc0ac445139024cddcec7dd72380f70b6c60cbf7e3eea162062ef8ef30b5*
Pure Prop Address:	*TMTzSVmWpr6G9xpGB9xRz9GHutphxoFHpm5*
Theory Prop Id:	*87e0ce7b41f81ec7cd5cb539c8d29a4ee84162503f0111c69851aa5cb6250f67*
Theory Prop Address:	*TMdHPiqFbrqqnJZvUEgGaaNprnFuirYL2XJ*

Definition 21.11 *We define* $\mathsf{PNo\_rel\_strict\_uniq\_imv}$ *to be*

$$\lambda LR\alpha p. \mathsf{PNo\_rel\_strict\_imv}\ L\ R\ \alpha\ p \land \forall q : \iota \to o. \mathsf{PNo\_rel\_strict\_imv}\ L\ R\ \alpha\ q \to \mathsf{PNoEq\_}\ \alpha\ p\ q$$

of type $(\iota \to (\iota \to o) \to o) \to (\iota \to (\iota \to o) \to o) \to \iota \to (\iota \to o) \to o$ *identified by the following information:*

Pure Object Id:	*23c9d918742fceee39b9ee6e272d5bdd5f6dd40c5c6f75acb478f608b6795625*
Pure Object Address:	*TMcVvujMz8D899TFXW92xtjqAjrJ7xZ5wns*
Theory Object Id:	*56ed1b924504c7b2800fa12637e1e0a522196c5a1a82a00af10c643a8a5e9e6f*
Theory Object Address:	*TMJg15q9kSMojiP1K5gjvfgbDB2SFT15THF*

Definition 21.12 *We define* $\mathsf{PNo\_rel\_strict\_split\_imv}$ *to be*

$$\lambda LR\alpha p. \mathsf{PNo\_rel\_strict\_imv}\ L\ R\ (\mathsf{ordsucc}\ \alpha)\ (\lambda \delta. p\ \delta \land \delta \neq \alpha) \land \mathsf{PNo\_rel\_strict\_imv}\ L\ R\ (\mathsf{ordsucc}\ \alpha)\ (\lambda \delta. p\ \delta \lor \delta = \alpha)$$

of type $(\iota \to (\iota \to o) \to o) \to (\iota \to (\iota \to o) \to o) \to \iota \to (\iota \to o) \to o$ *identified by the following information:*

Pure Object Id:	*217d179d323c5488315ded19cb0c9352416c9344bfdfbb19613a3ee29afb980d*
Pure Object Address:	*TMcNiZGAEhYLnFaHRVtxA3RMJ6rQRD68crC*
Theory Object Id:	*dd9ceee1f10e584685cd8a5d181d107a00ffeb879bddd77bb943edc9d869a262*
Theory Object Address:	*TMaSwGSK92j2THPci27ww2urWPVb7Y8PaRX*

`PNo_extend0_eq`

Theorem 21.40 $\forall \alpha . \forall p : \iota \to o . \mathsf{PNoEq\_}\ \alpha\ p\ (\lambda \delta . p\ \delta \wedge \delta \neq \alpha)$. *The proposition is identified by the following information:*

Pure Prop Id:	*2d441ed2f6faaa3204d9c074c159ce10040fd6680f3c4f69726e8dbc60efc1b8*
Pure Prop Address:	*TMWBZZ2LzEnHQnhQ2EUjYaKDpHL6urz6cT8*
Theory Prop Id:	*5ed08e400c3976db41fd77c3d882770ae3d2bc85093f84c0060820655b4328c3*
Theory Prop Address:	*TMPWoqhrnorzigPXmr4GmgWiGibdw73sjz8*

`PNo_extend1_eq`

Theorem 21.41 $\forall \alpha . \forall p : \iota \to o . \mathsf{PNoEq\_}\ \alpha\ p\ (\lambda \delta . p\ \delta \vee \delta = \alpha)$. *The proposition is identified by the following information:*

Pure Prop Id:	*8eb99c36aa09b197a65897ed80cdf2570eba4e719d286f36fc4bc09afd4f3689*
Pure Prop Address:	*TMQEFMAonmrWaLGhniLUme237vwrY6AwX3L*
Theory Prop Id:	*4cac1a7bb678742b042b74050de5270cd3a104cca4e451080d85e20b33bff7dc*
Theory Prop Address:	*TMGkMWYd35nspxAzshHYLLW93mSrEQa177W*

`PNo_rel_imv_ex`

Theorem 21.42

$$\begin{gathered}\forall L R : \iota \to (\iota \to o) \to o . \mathsf{PNoLt\_pwise}\ L\ R \to \forall \alpha . \mathsf{ordinal}\ \alpha \to \\ (\exists p : \iota \to o . \mathsf{PNo\_rel\_strict\_uniq\_imv}\ L\ R\ \alpha\ p) \\ \vee (\exists \tau \in \alpha . \exists p : \iota \to o . \mathsf{PNo\_rel\_strict\_split\_imv}\ L\ R\ tau\ p).\end{gathered}$$

The proposition is identified by the following information:

Pure Prop Id:	*3e3224100da33bad94d660a0cfa8843fa7519fd5ff75e4c23da58e5b68707714*
Pure Prop Address:	*TMb7LjGHosQp5Zu2ZyhehFbFtpyZL2yxrkB*
Theory Prop Id:	*749b63b90f39e599387d66934130ae6ac3e08e4f9989ca927eec36e855e09fc8*
Theory Prop Address:	*TMMKoMsquCSh1xogZaGrGcBP6CoCXQLGM1t*

Definition 21.13 *We define* $\mathsf{PNo\_lenbdd}$ *to be*

$$\lambda \alpha L . \forall \beta . \forall p : \iota \to o . L\ \beta\ p \to \beta \in \alpha$$

of type $\iota \to (\iota \to (\iota \to o) \to o) \to o$ *identified by the following information:*

Pure Object Id:	*009fa745aa118f0d9b24ab4efbc4dc3a4560082e0d975fd74526e3c4d6d1511f*
Pure Object Address:	*TMYM1fGNqp8awnq5MVCDqBVSdy2y4KSQX4c*
Theory Object Id:	*a452a633e2734cc4531287f806932a19db788c9ebf515fd5c65361a126fcccda*
Theory Object Address:	*TMc7MFWWHbWct8gGwr9Jjykxysxajx7WxYR*

`PNo_lenbdd_strict_imv_extend0`

Theorem 21.43

$$\forall LR:\iota \to (\iota \to o) \to o.\forall \alpha.\mathsf{ordinal}\ \alpha \to \mathsf{PNo\_lenbdd}\ \alpha\ L \to \mathsf{PNo\_lenbdd}\ \alpha\ R \to \forall p:\iota \to o.\mathsf{PNo\_rel\_strict\_imv}\ L\ R\ \alpha\ p \to \mathsf{PNo\_rel\_strict\_imv}\ L\ R\ (\mathsf{ordsucc}\ \alpha)\ (\lambda \delta.p\ \delta \wedge \delta \neq \alpha).$$

The proposition is identified by the following information:

Pure Prop Id:	*17a11416cb52c3c428daf77fd88095938feda8129ca2851349863fa5a7f70fab*
Pure Prop Address:	*TMZ7dVvvzt9aBgJceuie2x9CTj7zzynBVKW*
Theory Prop Id:	*69327e8aa4dfac05b5218d04f861172313490347f560cc3142dc11e85c3d1de9*
Theory Prop Address:	*TMFuqsPkUmPBmPBLFudNnuZaej9uSQJFdHA*

`PNo_lenbdd_strict_imv_extend1`

Theorem 21.44

$$\forall LR:\iota \to (\iota \to o) \to o.\forall \alpha.\mathsf{ordinal}\ \alpha \to \mathsf{PNo\_lenbdd}\ \alpha\ L \to \mathsf{PNo\_lenbdd}\ \alpha\ R \to \forall p:\iota \to o.\mathsf{PNo\_rel\_strict\_imv}\ L\ R\ \alpha\ p \to \mathsf{PNo\_rel\_strict\_imv}\ L\ R\ (\mathsf{ordsucc}\ \alpha)\ (\lambda \delta.p\ \delta \vee \delta = \alpha).$$

The proposition is identified by the following information:

Pure Prop Id:	*ad69ec2daa15891f8d7b060e461a3f1b3e63589a17d9fe0b50275b428c54760d*
Pure Prop Address:	*TMHJrXcf4TaXkTgTPiRVvo9nkWreomZdRJn*
Theory Prop Id:	*5ff9936fa7da19e574c1e407d5dc4601e10f51aca435de66bb11cc9e7ada1df8*
Theory Prop Address:	*TMPvUJKpgzdwMRuCFNU2GRtAfonYYsF38vv*

`PNo_lenbdd_strict_imv_split`

Theorem 21.45

$$\forall LR:\iota \to (\iota \to o) \to o.\forall \alpha.\mathsf{ordinal}\ \alpha \to \mathsf{PNo\_lenbdd}\ \alpha\ L \to \mathsf{PNo\_lenbdd}\ \alpha\ R \to \forall p:\iota \to o.\mathsf{PNo\_rel\_strict\_imv}\ L\ R\ \alpha\ p \to \mathsf{PNo\_rel\_strict\_split\_imv}\ L\ R\ \alpha\ p.$$

The proposition is identified by the following information:

Pure Prop Id:	*76ae4f9f794260b3f0f83c999d55601052867 44cef38b1a0234287b13ad8db9e*
Pure Prop Address:	*TMaW9z99uWRswWnxdDU9cvmKc6HtRaCtVRU*
Theory Prop Id:	*f98998c738859a6ff5fbcf1701155ca36cbcb0caef56b13a9cff1b1c937b6ece*
Theory Prop Address:	*TMFZtcp8Jrq2c5Vbiocbri3d5UhPuowpAaw*

`PNo_rel_imv_bdd_ex`

Theorem 21.46

$$\forall LR : \iota \to (\iota \to o) \to o.\mathsf{PNoLt\_pwise}\ L\ R \to \forall \alpha.\mathsf{ordinal}\ \alpha \to \mathsf{PNo\_lenbdd}\ \alpha\ L \to \mathsf{PNo\_lenbdd}\ \alpha\ R \to \exists \beta \in \mathsf{ordsucc}\ \alpha.\exists p : \iota \to o.\mathsf{PNo\_rel\_strict\_split\_imv}\ L\ R\ \beta\ p.$$

The proposition is identified by the following information:

Pure Prop Id:	*9a98e4a421cb8eedc40a867b16cac1d2529c4dfabcc3ff3b451f9e220420f209*
Pure Prop Address:	*TMRSz1Na7BW752MS6vDye8Y6Vo8SfeVhV1j*
Theory Prop Id:	*25bcd2bc2f9d34e476f5e18c7b51c9075ed93e802655cc620f888c8e1fd16a22*
Theory Prop Address:	*TMQfEm1NLf1eHjLsYL7HyTLrAxiyrsEBsSQ*

Definition 21.14 *We define* PNo_strict_upperbd *to be*

$$\lambda L\alpha p.\forall \beta.\mathsf{ordinal}\ \beta \to \forall q : \iota \to o.L\ \beta\ q \to \mathsf{PNoLt}\ \beta\ q\ \alpha\ p$$

of type $(\iota \to (\iota \to o) \to o) \to \iota \to (\iota \to o) \to o$ *identified by the following information:*

Pure Object Id:	*6913bd802b6ead043221cf6994931f83418bb29a379dc6f65a313e7daa830dcc*
Pure Object Address:	*TMKnMYekteuHoeWstJHEmJ4JAdKPexnFA42*
Theory Object Id:	*3723abda991ef19534694f8f4cd239c37f64fc7c8d2f1cbae883cc9f9d0d69b3*
Theory Object Address:	*TMRkoJnHGKhQmpq21wXvTR2RBkLqLSagigX*

Definition 21.15 *We define* PNo_strict_lowerbd *to be*

$$\lambda R\alpha p.\forall \beta.\mathsf{ordinal}\ \beta \to \forall q : \iota \to o.R\ \beta\ q \to \mathsf{PNoLt}\ \alpha\ p\ \beta\ q$$

of type $(\iota \to (\iota \to o) \to o) \to \iota \to (\iota \to o) \to o$ *identified by the following information:*

Pure Object Id:	*87fac4547027c0dfeffbe22b750fcc4ce9638e0ae321aeaa99144ce5a5a6de3e*
Pure Object Address:	*TMZjYeWB7UQog19EjFL4PmaNhnokNCuMGJV*
Theory Object Id:	*3606d45e76891d3b7e28745aa1b61986d255603f7abeb8f63a847b1ccc62b95d*
Theory Object Address:	*TMXvc4Vsa77sgNvdnXny3CP9WCzsnoarmV1*

Definition 21.16 *We define* PNo_strict_imv *to be*

$$\lambda LR\alpha p.\mathsf{PNo\_strict\_upperbd}\ L\ \alpha\ p \land \mathsf{PNo\_strict\_lowerbd}\ R\ \alpha\ p$$

of type $(\iota \to (\iota \to o) \to o) \to (\iota \to (\iota \to o) \to o) \to \iota \to (\iota \to o) \to o$ *identified by the following information:*

Pure Object Id:	*ee97f6fc35369c0aa0ddf247ea2ee871ae4efd862b662820df5edcc2283ba7e3*
Pure Object Address:	*TMbEV7hkJGaPtHGWT7FsCyrBxQjHqPm9Ct9*
Theory Object Id:	*bc6c55650a1ceb99aefb5c3b39fed27e70fc45b811cc09f91f1fd23a0cfbe326*
Theory Object Address:	*TMHAeW8tAPaNuMKzRy2t7UZyujPe3oz8chT*

PNoEq_strict_upperbd

Theorem 21.47

$$\forall L : \iota \to (\iota \to o) \to o. \forall \alpha. \mathsf{ordinal}\ \alpha \to \forall p q : \iota \to o. \mathsf{PNoEq\_}\ \alpha\ p\ q \to \mathsf{PNo\_strict\_upperbd}\ L\ \alpha\ p \to \mathsf{PNo\_strict\_upperbd}\ L\ \alpha\ q.$$

The proposition is identified by the following information:

Pure Prop Id:	*9b7b8e2913af3ee84872e0fd4fca5bfeb65de5ae332f50e8005ce0b40c0390d1*
Pure Prop Address:	*TMMwWqqBxTemQcZaoYuQ5csnpPswRJuhRvB*
Theory Prop Id:	*f28b04874428203d54a6576e08aded8841fea8787b265c71cffb0a98a190b2b1*
Theory Prop Address:	*TMFSxkkKeWx8CjszfDWMQmeEXEdqcpXHZqL*

PNoEq_strict_lowerbd

Theorem 21.48

$$\forall R : \iota \to (\iota \to o) \to o. \forall \alpha. \mathsf{ordinal}\ \alpha \to \forall p q : \iota \to o. \mathsf{PNoEq\_}\ \alpha\ p\ q \to \mathsf{PNo\_strict\_lowerbd}\ R\ \alpha\ p \to \mathsf{PNo\_strict\_lowerbd}\ R\ \alpha\ q.$$

The proposition is identified by the following information:

Pure Prop Id:	*4db3f9364356674211f51fc9a183e838dff60a313c518ca7d65b13fd0698d5a1*
Pure Prop Address:	*TMG1CzmnW8AfKeL7zGEwMEtn4b1ApJy6q1g*
Theory Prop Id:	*67fa73a4227fa42a9ed6db8fd115f876749a15c619bbceedaf8114345d2a228b*
Theory Prop Address:	*TMFvqJhbAHTTaVpfbgHnnHh9Roc9bwF9YcE*

PNoEq_strict_imv

Theorem 21.49

$$\forall L R : \iota \to (\iota \to o) \to o. \forall \alpha. \mathsf{ordinal}\ \alpha \to \forall p q : \iota \to o. \mathsf{PNoEq\_}\ \alpha\ p\ q \to \mathsf{PNo\_strict\_imv}\ L\ R\ \alpha\ p \to \mathsf{PNo\_strict\_imv}\ L\ R\ \alpha\ q.$$

The proposition is identified by the following information:

Pure Prop Id:	*1ec3525cfde857873a05b1492e442895aa98e91b1a7a58db58116a8baa7863f3*
Pure Prop Address:	*TMWLyiXz4WsN9qiqmYUTwRb7RrR4sGpvULn*
Theory Prop Id:	*5f582f95ff13676a0730e043b393f09bf9b302f7c01959e268fdbaeaaba7bd23*
Theory Prop Address:	*TMZZL19r37MLXbo9tRES9F3kF1Cdd7rP1Vx*

PNo_strict_upperbd_imp_rel_strict_upperbd

Theorem 21.50

$$\forall L : \iota \to (\iota \to o) \to o. \forall \alpha. \mathsf{ordinal}\ \alpha \to \forall \beta \in \mathsf{ordsucc}\ \alpha. \forall p : \iota \to o. \mathsf{PNo\_strict\_upperbd}\ L\ \alpha\ p \to \mathsf{PNo\_rel\_strict\_upperbd}\ L\ \beta\ p.$$

The proposition is identified by the following information:

Pure Prop Id:	*2898f0821b6ee4c20a8875fb97191a1b12a50a1e963c11f9aa1a31c2bd46f8e2*
Pure Prop Address:	*TMXJEXJKF7JrSAhmPixqwmaahA97oWAoeLL*
Theory Prop Id:	*3cdd3b4f20ab67c1be5329318323deafeea009f34f28baa01090782c3bdfdd0e*
Theory Prop Address:	*TMbeJVU3GyejPAEuuCqdu9srTvCD2PVUAy8*

`PNo_strict_lowerbd_imp_rel_strict_lowerbd`

Theorem 21.51

$$\forall R : \iota \to (\iota \to o) \to o. \forall \alpha. \mathsf{ordinal}\ \alpha \to \forall \beta \in \mathsf{ordsucc}\ \alpha. \forall p : \iota \to o.$$
$$\mathsf{PNo\_strict\_lowerbd}\ R\ \alpha\ p \to \mathsf{PNo\_rel\_strict\_lowerbd}\ R\ \beta\ p.$$

The proposition is identified by the following information:

Pure Prop Id:	*442af59a4f76dc1ad25be2cc8abd5af2847bff7cd5f1b08ec480d479fe6f9148*
Pure Prop Address:	*TMMPUoKVYbXX3DBLQHFe5h9b2gv8eiAFaAb*
Theory Prop Id:	*04acc2c7664cb2baa2edfe5f073538b0cb855556d54d07337c5d7f3ccfcf5ab0*
Theory Prop Address:	*TMPPvnYPwEv5KBauhbm93s9dRpjMraJ2RMs*

`PNo_strict_imv_imp_rel_strict_imv`

Theorem 21.52

$$\forall L R : \iota \to (\iota \to o) \to o. \forall \alpha. \mathsf{ordinal}\ \alpha \to \forall \beta \in \mathsf{ordsucc}\ \alpha. \forall p : \iota \to o.$$
$$\mathsf{PNo\_strict\_imv}\ L\ R\ \alpha\ p \to \mathsf{PNo\_rel\_strict\_imv}\ L\ R\ \beta\ p.$$

The proposition is identified by the following information:

Pure Prop Id:	*f4fc71c90b443abe5f08ec2e1cbad2b8e178220ae7a28146d2debaa27c8dca9e*
Pure Prop Address:	*TMUZW37y1v2yvXxB4qDRF8ycQ12jacn99PZ*
Theory Prop Id:	*46a6aacddbb0224f249308bdbd8f4155dd2574ab91755a3faeebd246dd1ac8da*
Theory Prop Address:	*TMaWoAQeNZDif3qPS3T1yTknjm8pRRW8a4h*

`PNo_rel_split_imv_imp_strict_imv`

Theorem 21.53

$$\forall L R : \iota \to (\iota \to o) \to o. \forall \alpha. \mathsf{ordinal}\ \alpha \to \forall p : \iota \to o.$$
$$\mathsf{PNo\_rel\_strict\_split\_imv}\ L\ R\ \alpha\ p \to$$
$$\mathsf{PNo\_strict\_imv}\ L\ R\ \alpha\ p.$$

The proposition is identified by the following information:

Pure Prop Id:	*72be36124095b1c454cfc103ace8e578f6da1f3537bed3a63dff48a5ee9e67e3*
Pure Prop Address:	*TMGERrdQ6q2bYZah7DP6dYkxMqTrBkNs3qx*
Theory Prop Id:	*adfdb01bca46911cd3ac3a75bf022e1e4e0316883433f9218fddedd9a12aceeb*
Theory Prop Address:	*TMTEYb8yRqBikXo6Wt5MPUkSefJ4iSEG83i*

`ordinal_PNo_strict_imv`

Theorem 21.54

$$\forall LR : \iota \to (\iota \to o) \to o.\forall \alpha.\mathsf{ordinal}\ \alpha \to \forall p : \iota \to o.(\forall \beta \in \alpha.p\ \beta) \to (\forall \beta.\mathsf{ordinal}\ \beta \to \forall q : \iota \to o.L\ \beta\ q \to \beta \in \alpha) \to (\forall \beta \in \alpha.L\ \beta\ p) \to (\forall \beta.\mathsf{ordinal}\ \beta \to \forall q : \iota \to o.\neg R\ \beta\ q) \to \mathsf{PNo\_strict\_imv}\ L\ R\ \alpha\ p.$$

The proposition is identified by the following information:

Pure Prop Id:	*6ac14c06740c6ba05ec4cd77fc370b4915f44952ab2e3e843377f3ae0f4ef372*
Pure Prop Address:	*TMPF3ri47F9YTagy21jCuzhVZiWgwJhBagV*
Theory Prop Id:	*d6a73528e480611cb4cc96b7f8a095233cd0b5ce7a4c96d973c70a42ac001c03*
Theory Prop Address:	*TMTeL2CwHrU7yAsCK9SoepcoGiqPerjGfAP*

`PNo_lenbdd_strict_imv_ex`

Theorem 21.55

$$\forall LR : \iota \to (\iota \to o) \to o.\mathsf{PNoLt\_pwise}\ L\ R \to \forall \alpha.\mathsf{ordinal}\ \alpha \to \mathsf{PNo\_lenbdd}\ \alpha\ L \to \mathsf{PNo\_lenbdd}\ \alpha\ R \to \exists \beta \in \mathsf{ordsucc}\ \alpha.\exists p : \iota \to o.\mathsf{PNo\_strict\_imv}\ L\ R\ \beta\ p.$$

The proposition is identified by the following information:

Pure Prop Id:	*9485d658371746bd448a504cec578379db903809a34d0f3d7395e9ad92f52bc5*
Pure Prop Address:	*TMboc8u3wjuBmQSkALzv47nZjKdu9Ln6G1i*
Theory Prop Id:	*a864a57867c4b8fa4046e9c687fd4f648d42ccbee4cef69cf3b1c28bb983c0eb*
Theory Prop Address:	*TMXJMwoh8T8TYKwFJYwWGnhaeosMEU67QtG*

Definition 21.17 *We define* $\mathsf{PNo\_least\_rep}$ *to be*

$$\lambda LR\beta p.\mathsf{ordinal}\ \beta \wedge \mathsf{PNo\_strict\_imv}\ L\ R\ \beta\ p \wedge \forall \gamma \in \beta.\forall q : \iota \to o.\neg \mathsf{PNo\_strict\_imv}\ L\ R\ \gamma\ q$$

of type $(\iota \to (\iota \to o) \to o) \to (\iota \to (\iota \to o) \to o) \to \iota \to (\iota \to o) \to o$ *identified by the following information:*

Pure Object Id:	*0711e2a692a3c2bdc384654882d1195242a019a3fc5d1a73dba563c4291fc08b*
Pure Object Address:	*TMMrUbZHNUQojuJFRTNTGugMnkz3yaWyJjG*
Theory Object Id:	*e4c707e63a9d11d1b61d0db26f0cf3155ea6db0eca2d2f3dbf812d36a70e6c70*
Theory Object Address:	*TMdTiKpkh6t2DH3NuLyGfeGX79J5YUPNSBs*

`PNo_lenbdd_least_rep_ex`

Theorem 21.56

$$\forall LR : \iota \to (\iota \to o) \to o.\mathsf{PNoLt\_pwise}\ L\ R \to \forall \alpha.\mathsf{ordinal}\ \alpha \to \mathsf{PNo\_lenbdd}\ \alpha\ L \to \mathsf{PNo\_lenbdd}\ \alpha\ R \to \exists \beta.\exists p : \iota \to o.\mathsf{PNo\_least\_rep}\ L\ R\ \beta\ p.$$

The proposition is identified by the following information:

Pure Prop Id:	4541c94e58f35924a0da5fd80f4d2327c22d662287cf6c55c9962fcacd32b6c0
Pure Prop Address:	TMV8v9NRJQArdep3t94cASip2uDW46XdwHJ
Theory Prop Id:	fc95a5ba7826e45c22086496d12170955e43c43c454f7c838a38598d83edd933
Theory Prop Address:	TMQu9hsS7YcjLuJMR9UBzRz4gDzHhbZw56N

Definition 21.18 *We define* PNo_least_rep2 *to be*

$$\lambda LR\beta p.\mathsf{PNo\_least\_rep}\ L\ R\ \beta\ p \wedge \forall x.x \notin \beta \to \neg p\ x$$

of type $(\iota \to (\iota \to o) \to o) \to (\iota \to (\iota \to o) \to o) \to \iota \to (\iota \to o) \to o$ *identified by the following information:*

Pure Object Id:	1f6dc5b1a612afebee904f9acd1772cd31ab0f61111d3f74cfbfb91c82cfc729
Pure Object Address:	TMTPNGLhRLywGV7b2VErKKRqsBtTRqSdE63
Theory Object Id:	09d320f50b226d6de787cb7f04c0ac3eedf43d3981566d46692b73e884662b9b
Theory Object Address:	TMRAk8jd2vN252xuwfLzStCvRTrSjBsrXz3

`PNo_strict_imv_pred_eq`

Theorem 21.57

$$\forall LR :\iota \to (\iota \to o) \to o.\mathsf{PNoLt\_pwise}\ L\ R\to\forall\alpha.\mathsf{ordinal}\ \alpha\to\forall pq :\iota \to o.$$
$$\mathsf{PNo\_least\_rep}\ L\ R\ \alpha\ p\to\mathsf{PNo\_strict\_imv}\ L\ R\ \alpha\ q\to\forall\beta \in\alpha.$$
$$p\ \beta \Leftrightarrow q\ \beta.$$

The proposition is identified by the following information:

Pure Prop Id:	5e7822daeca655e2242d474ec86dd9579effe366031b4f9b73ec5ca4a659a545
Pure Prop Address:	TMZq9y1Uq2aENtjsxC2H2kmR5Sts8dSVeAn
Theory Prop Id:	72e14d37cd81d2a456f9878f785f26768d709f7848a4cf627865ba0c3662b1b3
Theory Prop Address:	TMP8YWfHwKekxFLMQmDee1eb36HPVWfDCkn

`PNo_lenbdd_least_rep2_exuniq2`

Theorem 21.58

$$\forall LR :\iota \to (\iota \to o) \to o.\mathsf{PNoLt\_pwise}\ L\ R\to\forall\alpha.\mathsf{ordinal}\ \alpha\to$$
$$\mathsf{PNo\_lenbdd}\ \alpha\ L\to\mathsf{PNo\_lenbdd}\ \alpha\ R\to$$
$$\exists\beta.(\exists p : \iota \to o.\mathsf{PNo\_least\_rep2}\ L\ R\ \beta\ p)\wedge$$
$$(\forall pq : \iota \to o.\mathsf{PNo\_least\_rep2}\ L\ R\ \beta\ p \to$$
$$\mathsf{PNo\_least\_rep2}\ L\ R\ \beta\ q \to p = q).$$

The proposition is identified by the following information:

Pure Prop Id:	5060a91293273a3d3416ab99070c6d1e15f0cbef0c24aa15399ee37c868f691c
Pure Prop Address:	TMT7sUcy4ndNzr9t94F3xrCG638UAh62av3
Theory Prop Id:	8b62c56d475417d4d361eaf3c3c3680d801387320141 22d77d426f4860f69081
Theory Prop Address:	TMNmdxAHFQ1AVyvBStP6JJjmC4WocYjvTV2

Definition 21.19 PNo_bd *is the opaque object of type*

$$(\iota \to (\iota \to o) \to o) \to (\iota \to (\iota \to o) \to o) \to \iota$$

identified by the following information:

Pure Object Id: *ed76e76de9b58e621daa601cca73b4159a437ba0e73114924cb92ec8044f2aa2*
Pure Object Address: *TMNBrUd5KSYmRWHN49oFdMFsUPTiSKvsa9Q*
Theory Object Id: *aa75efbc66bb94911a3c3b156ea65bbe1775870ce7a06e24c056f8d5cce3660d*
Theory Object Address: *TMV6kaag6YpB5pyMDkL4wftUcq2Diskvi7a*

Definition 21.20 PNo_pred *is the opaque object of type*

$$(\iota \to (\iota \to o) \to o) \to (\iota \to (\iota \to o) \to o) \to \iota \to o$$

identified by the following information:

Pure Object Id: *b2d51dcfccb9527e9551b0d0c47d891c9031a1d4ee87bba5a9ae5215025d107a*
Pure Object Address: *TMU5awsv39apDkXc2r3HzJRP37vuRruKsnf*
Theory Object Id: *2efd18fb25499ea37fc2af1b7a0ffdd36e6abc401c42458854e4f4288f7ca530*
Theory Object Address: *TMStxoqH3zLresqVCQdXwQ55q36aVygMDeL*

`PNo_bd_pred_lem`

Theorem 21.59

$$\forall L R : \iota \to (\iota \to o) \to o. \mathsf{PNoLt\_pwise}\ L\ R \to \forall \alpha. \mathsf{ordinal}\ \alpha \to \mathsf{PNo\_lenbdd}\ \alpha\ L \to \mathsf{PNo\_lenbdd}\ \alpha\ R \to \mathsf{PNo\_least\_rep2}\ L\ R\ (\mathsf{PNo\_bd}\ L\ R)\ (\mathsf{PNo\_pred}\ L\ R).$$

The proposition is identified by the following information:

Pure Prop Id: *749b8f4e860ae9a289350b300230645c0611ffff60798a2d0cdfd7fe0476bd2a*
Pure Prop Address: *TMYJBHCdV3Ct6gBy3MMb8STjafZb3SviQ9v*
Theory Prop Id: *5de4a33591f164add23849f5545d9322fa8885bd09e7ba1894ce774b2c9cab1a*
Theory Prop Address: *TMP6BAwWzoNKb47BSVYq4GJmL3WoFMQGbqf*

`PNo_bd_pred`

Theorem 21.60

$$\forall L R : \iota \to (\iota \to o) \to o. \mathsf{PNoLt\_pwise}\ L\ R \to \forall \alpha. \mathsf{ordinal}\ \alpha \to \mathsf{PNo\_lenbdd}\ \alpha\ L \to \mathsf{PNo\_lenbdd}\ \alpha\ R \to \mathsf{PNo\_least\_rep}\ L\ R\ (\mathsf{PNo\_bd}\ L\ R)\ (\mathsf{PNo\_pred}\ L\ R).$$

The proposition is identified by the following information:

Pure Prop Id:	*42224b3374ae5d0ed32e9420c84f18f8ea84d9eb8daae7337d3905a179b65d65*
Pure Prop Address:	*TMR9wZusZqBwrBJNUbqyESMWattt5sL41Jf*
Theory Prop Id:	*3309f6a3ddcb2319d01432fd7a6a7d3c05f3b9836222f520fb3ec000b2b7299f*
Theory Prop Address:	*TMH4SyZ8pRU9WGJb2eb6yLYAQnwpF555Lo5*

PNo_bd_ord

Theorem 21.61

$$\forall LR : \iota \to (\iota \to o) \to o.\mathsf{PNoLt\_pwise}\ L\ R \to \forall \alpha.\mathsf{ordinal}\ \alpha \to \mathsf{PNo\_lenbdd}\ \alpha\ L \to \mathsf{PNo\_lenbdd}\ \alpha\ R \to \mathsf{ordinal}\ (\mathsf{PNo\_bd}\ L\ R).$$

The proposition is identified by the following information:

Pure Prop Id:	*b40a91e75a7dcc94d55b6763376f5f366ab4a140e5e8b0380656e21f2b419fb8*
Pure Prop Address:	*TMZxHtMby9p5j448ESrb1iGeSp3QjaX8APB*
Theory Prop Id:	*bc6e649a31db5eaff908ac0cc857d64018672dfb788fc1bfef50fbd2a45f597c*
Theory Prop Address:	*TMYrVH5VdsuG5TzNCVy7afWTZ3eHsgNAjH3*

PNo_bd_pred_strict_imv

Theorem 21.62

$$\forall LR : \iota \to (\iota \to o) \to o.\mathsf{PNoLt\_pwise}\ L\ R \to \forall \alpha.\mathsf{ordinal}\ \alpha \to \mathsf{PNo\_lenbdd}\ \alpha\ L \to \mathsf{PNo\_lenbdd}\ \alpha\ R \to \mathsf{PNo\_strict\_imv}\ L\ R\ (\mathsf{PNo\_bd}\ L\ R)\ (\mathsf{PNo\_pred}\ L\ R).$$

The proposition is identified by the following information:

Pure Prop Id:	*b6398cfc0d268e1265cb766e4a3c763db5b5aba648cb2602837dfed49a6a1373*
Pure Prop Address:	*TMVx2aGZb6qmcJNfMJMoPgY2mzk5RmNZstN*
Theory Prop Id:	*ee0997fab4c7c13a66f2eda4601d185456ed98ab1a2c0269f8b7a47879f0a300*
Theory Prop Address:	*TMMZTkgpVapPLqBQTVn8NSVkQRWNow8gRiS*

PNo_bd_least_imv

Theorem 21.63

$$\forall LR : \iota \to (\iota \to o) \to o.\mathsf{PNoLt\_pwise}\ L\ R \to \forall \alpha.\mathsf{ordinal}\ \alpha \to \mathsf{PNo\_lenbdd}\ \alpha\ L \to \mathsf{PNo\_lenbdd}\ \alpha\ R \to \forall \gamma \in \mathsf{PNo\_bd}\ L\ R.\forall q : \iota \to o.\ \neg \mathsf{PNo\_strict\_imv}\ L\ R\ \gamma\ q.$$

The proposition is identified by the following information:

Pure Prop Id:	*8756915afa703c80eca22a6825041ae9983fe1efe0425560dd4d0e65970d8a91*
Pure Prop Address:	*TMEsGe3wG8yXWS94JJYUXQFxvwvV78FGr2C*
Theory Prop Id:	*266277c31581e02700387ec68a784741f97e44197639a61abe33b4bce3c8eccb*
Theory Prop Address:	*TMR3RGTyTYH8yzKLkmxhSTiEdgD4ZTKaapR*

`PNo_bd_In`

Theorem 21.64

$$\forall LR : \iota \to (\iota \to o) \to o.\mathsf{PNoLt\_pwise}\ L\ R \to \forall \alpha.\mathsf{ordinal}\ \alpha \to \mathsf{PNo\_lenbdd}\ \alpha\ L \to \mathsf{PNo\_lenbdd}\ \alpha\ R \to \mathsf{PNo\_bd}\ L\ R \in \mathsf{ordsucc}\ \alpha.$$

The proposition is identified by the following information:

Pure Prop Id:	*ad857b445d1196a6cfc19a14ac9d45dff7f3b25ed55d9764dcbb9d4d5efdea8d*
Pure Prop Address:	*TMasNPNdz1vv1jt29FCFyQpkupqkrcwKcUy*
Theory Prop Id:	*4719e12b28711d9d989a9ed9afa168f501d090c55ff5f971c2a9c4b951a3a4c8*
Theory Prop Address:	*TMZ1d5WRxZPZR9eT7BLzoFprMn6mNkfxFcZ*

Definition 21.21 *We define* PNoCutL *to be* $\lambda\alpha p\beta q.\beta \in \alpha \wedge \mathsf{PNoLt}\ \beta\ q\ \alpha\ p$ *of type* $\iota \to (\iota \to o) \to \iota \to (\iota \to o) \to o$ *identified by the following information:*

Pure Object Id:	*d272c756dc639d0c36828bd42543cc2a0d93082c0cbbe41ca2064f1cff38c130*
Pure Object Address:	*TMW89BeZ4LwBAWUSvcHTTffq4NjA9eURfe5*
Theory Object Id:	*5c2353f1a9132e4b0bef01f9d3b48e3dd377089926d3ec56a9c1b5f62d18564e*
Theory Object Address:	*TMXddSCNaHGduK9qNYrg5YeV7BsECCYxzJL*

Definition 21.22 *We define* PNoCutR *to be* $\lambda\alpha p\beta q.\beta \in \alpha \wedge \mathsf{PNoLt}\ \alpha\ p\ \beta\ q$ *of type* $\iota \to (\iota \to o) \to \iota \to (\iota \to o) \to o$ *identified by the following information:*

Pure Object Id:	*ff28d9a140c8b282282ead76a2a40763c4c4800ebff15a4502f32af6282a3d2e*
Pure Object Address:	*TMQ43uDGqp316dHj1xW6txbDkdt4BerVFRg*
Theory Object Id:	*6be9d2eca9c688e6504e0251a1344ee158dafeb178c09a30d02481f120ec9fdd*
Theory Object Address:	*TMJvHmHmkfm3PZnD7HiJxv464ZKQNcn3xK4*

`PNoCutL_lenbdd`

Theorem 21.65 $\forall \alpha.\forall p : \iota \to o.\mathsf{PNo\_lenbdd}\ \alpha\ (\mathsf{PNoCutL}\ \alpha\ p)$. *The proposition is identified by the following information:*

Pure Prop Id:	*2ce3cac589e0288fcb056a351d7b7f52ba86fb73ca736e136679ff9ef0ea1ac3*
Pure Prop Address:	*TMVhtYXkotNZng4p4VbWeUpqAzgiFcEJAs2*
Theory Prop Id:	*8d7ea3b990fcb5a72b452b48eff84626931337892eb12d3a2f14176e6d33964d*
Theory Prop Address:	*TMbipKXeU4Z2DY2jqE2CDGdVBX2xtErK7wW*

`PNoCutR_lenbdd`

Theorem 21.66 $\forall \alpha.\forall p : \iota \to o.\mathsf{PNo\_lenbdd}\ \alpha\ (\mathsf{PNoCutR}\ \alpha\ p)$. *The proposition is identified by the following information:*

Pure Prop Id:	*abe446408190f6d442a1f2eab7727c53458665db08e199262714d89da47c7000*
Pure Prop Address:	*TMNwdvruagyifCoqN3GpPqcvsoJxB98XesL*
Theory Prop Id:	*bba4ed960136adb5c77796704f207374e63fcf341cd9a438a50561365d6f14a0*
Theory Prop Address:	*TMcNY1d7D2m6KMq1LqqgqprtHuNXxok7xt6*

PNoCut_pwise

Theorem 21.67

$$\forall \alpha.\mathsf{ordinal}\ \alpha \rightarrow \forall p : \iota \rightarrow o.$$
$$\mathsf{PNoLt\_pwise}\ (\mathsf{PNoCutL}\ \alpha\ p)\ (\mathsf{PNoCutR}\ \alpha\ p).$$

The proposition is identified by the following information:

Pure Prop Id:	*8bb0692a31e5ad0bfd5013045a2fbf8c7889e948699261b72291eae629aa15fe*
Pure Prop Address:	*TMJo1MdhfFXdWBivM8BPiej6KZWhKLTQmBM*
Theory Prop Id:	*e9740986f55e09faff6e0e851bdf5251ed23276fc5747ca8a67afcdf70970cbb*
Theory Prop Address:	*TMZSEknJujgUGg2MoFZkCMMNLvQFRFD7SYC*

PNoCut_strict_imv

Theorem 21.68

$$\forall \alpha.\mathsf{ordinal}\ \alpha \rightarrow \forall p : \iota \rightarrow o.$$
$$\mathsf{PNo\_strict\_imv}\ (\mathsf{PNoCutL}\ \alpha\ p)\ (\mathsf{PNoCutR}\ \alpha\ p)\ \alpha\ p.$$

The proposition is identified by the following information:

Pure Prop Id:	*dd8b71e50df3e9a494df5a5572ce41f85b622aa24c1f3e1ff457aa4ba08718be*
Pure Prop Address:	*TMKFURgHEo6iHpwwz6JnZNJdRMvj8XUTA7D*
Theory Prop Id:	*6e3ce431e17b20e770f06809ce5182eb248cf10acb5c38a200119fe0520c9057*
Theory Prop Address:	*TMPNQHfdoNywBhUdkPA9hJADKARLduKe5aY*

PNoCut_bd_eq

Theorem 21.69

$$\forall \alpha.\mathsf{ordinal}\ \alpha \rightarrow \forall p : \iota \rightarrow o.$$
$$\mathsf{PNo\_bd}\ (\mathsf{PNoCutL}\ \alpha\ p)\ (\mathsf{PNoCutR}\ \alpha\ p) = \alpha.$$

The proposition is identified by the following information:

Pure Prop Id:	*9dda7c4906426d138c81234f3d50f5bc4402ad268025cc5da24c9b6fb4397ad3*
Pure Prop Address:	*TMVsjT6ktkVWYJDvH6eTEYnV2zKwFHA9xag*
Theory Prop Id:	*8b0d28708c0a512260761ccd7dc5373cc4fbff3ff56d623c12cc98c1db4e3747*
Theory Prop Address:	*TMR6CRmoDWEZoHBXWxwGBZgx6D5qz3GMeMa*

PNoCut_pred_eq

Theorem 21.70

$$\forall \alpha.\mathsf{ordinal}\ \alpha \rightarrow \forall p : \iota \rightarrow o.$$
$$\mathsf{PNoEq\_}\ \alpha\ p\ (\mathsf{PNo\_pred}\ (\mathsf{PNoCutL}\ \alpha\ p)\ (\mathsf{PNoCutR}\ \alpha\ p)).$$

The proposition is identified by the following information:

Pure Prop Id:	*3c5499b31499c700a1fb0b68f063a4b755a2658fb92632c82d2f048d7170c0f5*
Pure Prop Address:	*TMawNU4WyhEJede1jj4Sjss5i363KEbGi1F*
Theory Prop Id:	*95a2d638ee2907592549cba3eecd680371133a20f157bf39a1ec7d00b4682410*
Theory Prop Address:	*TMRmNdzKhB7GZ9dgSsWj9tM5jWyTG3QtGrZ*

21.2 Surreal Numbers as Sets

21.3 TaggedSets

Let *tag* : $\iota \to \iota$ be $\lambda\alpha.\mathsf{SetAdjoin}\ \alpha\ \{1\}$. **Notation.** We use $-'$ as a postfix operator corresponding to applying term *tag*.

`not_TransSet_Sing1`

Theorem 21.71 $\neg\mathsf{TransSet}\ \{1\}$. *The proposition is identified by the following information:*

Pure Prop Id:	*8f2d673650ebb57c98d6d8d2a5fc915f7f2d811cbaf7858a359e1b97bb6bde17*
Pure Prop Address:	*TMJBscGcLCjPWBHB8QVUutxyACNMVYf5z2Q*
Theory Prop Id:	*be9a70c02282395d672ab7c5e5f2551733683ad6270f097ab4c85edf46ad1a63*
Theory Prop Address:	*TMWGSzNuxQmGiDUtn5MjVnUHh75ihtU4J6W*

`not_ordinal_Sing1`

Theorem 21.72 $\neg\mathsf{ordinal}\ \{1\}$. *The proposition is identified by the following information:*

Pure Prop Id:	*99a3f6ae0ef1daee4a28fe3fcfd481e5a6573dfa220b080f34987386122d9c11*
Pure Prop Address:	*TMUKBsrmU2hKQg3EmpkpHjq5Vz3jGSJqAGL*
Theory Prop Id:	*db797718fa26a8f2ad7c62c261b77ebfd7b9de2b25f962911447dc5838d54b9c*
Theory Prop Address:	*TMKxsifEH9Ph4ovj8YaJwts3LDF7brCPFZZ*

`tagged_not_ordinal`

Theorem 21.73 $\forall y.\neg\mathsf{ordinal}\ (y')$. *The proposition is identified by the following information:*

Pure Prop Id:	*f9488c7d361c3059b61e3d76aaa5bb7df6cb13c210dcceb3c43c7b2388c0afea*
Pure Prop Address:	*TMSUbK9326n5Ft376LsWWx4xfGaDcP52PD5*
Theory Prop Id:	*aaf6e467102a87c8e9f73204645cc7de8b8c4dcd478bf528601bbcb36cf3a97d*
Theory Prop Address:	*TMVBkLkYkcbcmG1H2dngcHEkcuQeJt8yAkf*

`tagged_notin_ordinal`

Theorem 21.74 $\forall \alpha y.\mathsf{ordinal}\ \alpha \to (y') \notin \alpha$. *The proposition is identified by the following information:*

Pure Prop Id:	*b603ba84d8e3122663fa360bfa699dad0ab4e1f633f670e4cda297f9ba5b0208*
Pure Prop Address:	*TMFQrda9FfP9kMHatsRA4mRhspKL6dCBanm*
Theory Prop Id:	*c0e2c350bcd1226986c6e7fdbfa0f2c19884df170a172b85b757f9de370e72a5*
Theory Prop Address:	*TMVojt27k6SvjqAbUtQu8cJoDPMYpTdsKum*

tagged_eqE_Subq

Theorem 21.75 $\forall \alpha \beta.\mathsf{ordinal}\ \alpha \to \alpha\prime = \beta\prime \to \alpha \subseteq \beta$. *The proposition is identified by the following information:*

Pure Prop Id:	cf032f4043866a354e9e5c9a481f6041aefa5320fd3e30ce2a8a672c5cbdc19e
Pure Prop Address:	TMTWKSZoPeTMh9eB7LoRwuk94Bhfw2wU5zb
Theory Prop Id:	ef37f58abd6ded2b344094fa0f103848b972602d46465aa33548d6c5eeffd6ed
Theory Prop Address:	TMdBj6QvMekfbBeqoyfr1PZdrzpAAeNC5Ek

tagged_eqE_eq

Theorem 21.76 $\forall \alpha \beta.\mathsf{ordinal}\ \alpha \to \mathsf{ordinal}\ \beta \to \alpha\prime = \beta\prime \to \alpha = \beta$. *The proposition is identified by the following information:*

Pure Prop Id:	1f7dc940e194a7544299b8233619e1f491b2dde8acde374d90cb863cd307b0b7
Pure Prop Address:	TMKi9eMo4knWpH5ySxts7dqcS5g7CzxpzRQ
Theory Prop Id:	ec1608fa42f468b541a15ac8a164117d2d5256fb753cb09aa41258ec590a5e00
Theory Prop Address:	TMYDsoSZjgQSPetekXkj4s6GU5ub3XVJaWh

tagged_ReplE

Theorem 21.77

$$\forall \alpha \beta.\mathsf{ordinal}\ \alpha \to \mathsf{ordinal}\ \beta \to \beta\prime \in \{\gamma\prime | \gamma \in \alpha\} \to \beta \in \alpha.$$

The proposition is identified by the following information:

Pure Prop Id:	a1d43c66e0e7a187f286a4910c3a8578502c854c09b9ed79fd73f53b4d089435
Pure Prop Address:	TMNf9aFM1UsRXJkDzFmAMwGHYvakMgcytbx
Theory Prop Id:	70c25038dc2d795e7cd8a963a601a4e91c9ec37c890e90874b9e2ffe3758f817
Theory Prop Address:	TMTYBnzfsAwJny6EQ3AaAXwwpjV8sXaQQm4

ordinal_notin_tagged_Repl

Theorem 21.78 $\forall \alpha Y.\mathsf{ordinal}\ \alpha \to \alpha \notin \{y\prime | y \in Y\}$. *The proposition is identified by the following information:*

Pure Prop Id:	7f35b530bae383bd34f04f1aeb8e7f87cad1d17284bd4ad34bd07b9a9c0888ad
Pure Prop Address:	TMEiGdaBPeGmtidArg3tTV7mQKNB6Kjsv29
Theory Prop Id:	e7ff3b50c4b94c722012bd9c9578bf3faa3faed0b7e0a6728349051f41a649fb
Theory Prop Address:	TMUFgGXyhtzJfcJ5jwFoArpbHCNJMjtGFEC

Definition 21.23 *We define* $\mathsf{SNoElts\_}$ *to be* $\lambda \alpha.\alpha \cup \{\beta\prime | \beta \in \alpha\}$ *of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	c0ec73850ee5ffe522788630e90a685ec9dc80b04347c892d62880c5e108ba10
Pure Object Address:	TMchcPtmntTsG9a2jnVbo3rvhd4YRFGdXd1
Theory Object Id:	6f0c364e1773eaa440ffcfc12f522a8c0532bdf6abfa9b8337d5e824777d1834
Theory Object Address:	TMX8tC9smhsjcxum9MHG47kxXC2911bY8fh

SNoElts_mon

Theorem 21.79 $\forall \alpha \beta . \alpha \subseteq \beta \to \mathsf{SNoElts\_}\ \alpha \subseteq \mathsf{SNoElts\_}\ \beta$. *The proposition is identified by the following information:*

Pure Prop Id:	*49a08e4bc4f84602c91d1206ee5c208423e9081b680beb0711763d769896fcfb*
Pure Prop Address:	*TMcnJXLfgf6uG9ovz9CcN8AxtyhCcG7uj6R*
Theory Prop Id:	*85961174f4e845853804252a1ed8497fdf7987feab645886015281982943 5ac1*
Theory Prop Address:	*TMYbW9UULsAfXGSQRuG31aqjM693gccqxqg*

Definition 21.24 *We define* SNo_ *to be*

$$\lambda \alpha x . x \subseteq \mathsf{SNoElts\_}\ \alpha \wedge \forall \beta \in \alpha . \mathsf{exactly1of2}\ (\beta' \in x)\ (\beta \in x)$$

of type $\iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*4ab7e4afd8b51df80f04ef3dd42ae08c530697f7926635e26c92eb55ae427224*
Pure Object Address:	*TMWTZaSAkv4fbU4uw785MTxgHkscdn5nC4o*
Theory Object Id:	*c16a736947afa02fd21f8d698f4b28435c6d1a029515699f6dd4c54682d31685*
Theory Object Address:	*TMQpDMmMJpF7k2EcsWrxASKz43UULkewFnB*

Definition 21.25 *We define* PSNo *to be* $\lambda \alpha p . \{\beta \in \alpha | p\ \beta\} \cup \{\beta' | \beta \in \alpha, \neg p\ \beta\}$ *of type* $\iota \to (\iota \to o) \to \iota$ *identified by the following information:*

Pure Object Id:	*3657ae18f6f8c496bec0bc584e1a5f49a6ce4e6db74d7a924f85da1c10e2722d*
Pure Object Address:	*TMYCsiv2Uki9NzUmKkGH2kHd2zXGFWsYi5p*
Theory Object Id:	*0a835174a0cf9cb97b20e29b98340a75a9c5f19db25004ae0688b25995ff83b3*
Theory Object Address:	*TMTsRL2Kcwjhfick7fiiXD612CpaX6QFJea*

PNoEq_PSNo

Theorem 21.80

$$\forall \alpha . \mathsf{ordinal}\ \alpha \to \forall p : \iota \to o . \mathsf{PNoEq\_}\ \alpha\ (\lambda \beta . \beta \in \mathsf{PSNo}\ \alpha\ p)\ p.$$

The proposition is identified by the following information:

Pure Prop Id:	*c12e023a1bba6d9a9ec78087054b4057985c72eed826a9b312ba4f9091a65a61*
Pure Prop Address:	*TMZtDRwJxRs7oFkrbsWnLiWufDfjc6F5M8R*
Theory Prop Id:	*688518e3201b270ffc6ae7f6eaea9a64dca9d99cd02f4c980cb094834f164104*
Theory Prop Address:	*TMWKeiYteNuQ4ETYrWy5zjJ7ErjwUTNfcGU*

SNo_PSNo

Theorem 21.81

$$\forall \alpha . \mathsf{ordinal}\ \alpha \to \forall p : \iota \to o . \mathsf{SNo\_}\ \alpha\ (\mathsf{PSNo}\ \alpha\ p).$$

The proposition is identified by the following information:

Pure Prop Id:	*96e4e2ec36857754b409849e1c10332c78d89197c28a888ac4de3c1e39dee14e*
Pure Prop Address:	*TMHEHzxt38dr1Sf22Q4XwK3oTJfyqBauuXx*
Theory Prop Id:	*f03db005e0c97b51d3b6d91f5cc5af548be05d0a792345091ee88d22daa20473*
Theory Prop Address:	*TMUdwihfVPMkko4XmqQdHcyTjgzoGYZRGjh*

SNo_PSNo_eta_

Theorem 21.82

$$\forall \alpha x.\mathsf{ordinal}\ \alpha \to \mathsf{SNo\_}\ \alpha\ x \to x = \mathsf{PSNo}\ \alpha\ (\lambda \beta.\beta \in x).$$

The proposition is identified by the following information:

Pure Prop Id:	*1642af79554e4cf7b9318de374b2c7edd186172454054d947f607438269c9496*
Pure Prop Address:	*TMcuWpQkUmH1LMTde4kSNzc7YBDbA8sokbU*
Theory Prop Id:	*630119c550dd8b5c2e0bd53e2ad889d4d08f72bc50e97a63bd942e63a6413115*
Theory Prop Address:	*TMdU6L38rj7HbtERSXfWaxWZ7ZTCBb4pt8p*

Definition 21.26 SNo *is the opaque object of type* $\iota \to o$ *identified by the following information:*

Pure Object Id:	*11faa7a742daf8e4f9aaf08e90b175467e22d0e6ad3ed089af1be90cfc17314b*
Pure Object Address:	*TMN8jh3ustcw3ZSbqUeiv8YSjeH79sfuWBi*
Theory Object Id:	*116b7641893e1ad3998a7adf6c1fab6d1343fb60d499aef43ca12a4f847ef6b9*
Theory Object Address:	*TMcKkMrp1PUTPmiJaXVAxaYrixRdqgEjZje*

SNo_SNo

Theorem 21.83 $\forall \alpha.\mathsf{ordinal}\ \alpha \to \forall z.\mathsf{SNo\_}\ \alpha\ z \to \mathsf{SNo}\ z.$ *The proposition is identified by the following information:*

Pure Prop Id:	*929b7ac3599669289a870d939e634944e175e96faf30cb744f95d10af1ed5d72*
Pure Prop Address:	*TMMDuEgEyuT5E1BibeHE8pfR45YK1o1Bthr*
Theory Prop Id:	*a59341cc6c21bd7a5fd6bbf9d96c7141e1e9a5f9d341f307e50b304d6ed01584*
Theory Prop Address:	*TMUJvhsT86hcmvoRoqef5N9GPieDsaTYJah*

Definition 21.27 SNoLev *is the opaque object of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*293b77d05dab711767d698fb4484aab2a8843042567765be0733e6bd5348119e8*
Pure Object Address:	*TMLgHPbDhEczHhzhujz1Y6647YGuRwkP7uG*
Theory Object Id:	*83dce3cb89857056a3ed5cb650ae31e37721c49405fcf77a3b272f46d3ab6b80*
Theory Object Address:	*TMUZTu2xUcdtVkLHYv2sJFX8Yj2S6Jd5jnk*

SNoLev_uniq_Subq

Theorem 21.84

$$\forall x \alpha \beta.\mathsf{ordinal}\ \alpha \to \mathsf{ordinal}\ \beta \to \mathsf{SNo\_}\ \alpha\ x \to \mathsf{SNo\_}\ \beta\ x \to \alpha \subseteq \beta.$$

The proposition is identified by the following information:

Pure Prop Id:	*2ca114eb623f9aa0591aadc7c354900445384cf0262c752fa0430e62b230d79a*
Pure Prop Address:	*TMJjyz93md1dpAbsBqGeb8qeQUo8pB87vCg*
Theory Prop Id:	*65843d7b872576b70cff22d1e596ee268a921435cd21f9de56a360ecd581bb9a*
Theory Prop Address:	*TMUMCHaN9FLoTQWs3Q4fEncBkHicwqi4FfL*

SNoLev_uniq

Theorem 21.85

$$\forall x \alpha \beta . \mathsf{ordinal}\ \alpha \to \mathsf{ordinal}\ \beta \to \mathsf{SNo\_}\ \alpha\ x \to \mathsf{SNo\_}\ \beta\ x \to \alpha = \beta.$$

The proposition is identified by the following information:

Pure Prop Id:	*68f71dfd2001b27983e3c169b043d92f238faadc82a072f2d495cdc9761d2189*
Pure Prop Address:	*TMaDQTErBA2zHWoN1hNMatRbHaH7eAJwMWF*
Theory Prop Id:	*5d96d48026e74cd817ee90319b23cc2b42e20eba8d81f83f0edfcc2159f6ea5e*
Theory Prop Address:	*TMdvMVpBuYy5UApCwAKZdNQE9dn3JHxmPy6*

SNoLev_prop

Theorem 21.86

$$\forall x . \mathsf{SNo}\ x \to \mathsf{ordinal}\ (\mathsf{SNoLev}\ x) \wedge \mathsf{SNo\_}\ (\mathsf{SNoLev}\ x)\ x.$$

The proposition is identified by the following information:

Pure Prop Id:	*a8afccaac35cd67cae38ea5c7f8db5efcc00f46db4396c04d7693f267b847aa8*
Pure Prop Address:	*TMEuZs9WhvFjxtstDDHiS5zYgaamp73ip3m*
Theory Prop Id:	*21761c1d62638b04252039c756c82b2f54681ae14cef2e53b054c7eed1be1251*
Theory Prop Address:	*TMTeCDaoMd1YLP8PGYRivc2WeuktskbT8SH*

SNoLev_ordinal

Theorem 21.87 $\forall x . \mathsf{SNo}\ x \to \mathsf{ordinal}\ (\mathsf{SNoLev}\ x)$. *The proposition is identified by the following information:*

Pure Prop Id:	*7cacc5f1e879f627f484fd25c7b6c53a51432ebb794812d1bf85218be858870a*
Pure Prop Address:	*TMYpWCPBF4fDYL83uuTLHroYN9eaJFa1gjQ*
Theory Prop Id:	*f0e9f7da1d1eea44bc8152e3a3c9944f3397f3099eb97e82a027f58d2c354304*
Theory Prop Address:	*TMNg3i1WfBeNE9GZ3VagfsKKYV6q3v2MqxJ*

SNoLev_

Theorem 21.88 $\forall x . \mathsf{SNo}\ x \to \mathsf{SNo\_}\ (\mathsf{SNoLev}\ x)\ x$. *The proposition is identified by the following information:*

Pure Prop Id:	*4a10339549425e6a1bcdd71c2ab61157802ad96e57ac87b6d61abeeb14663bc2*
Pure Prop Address:	*TMF7HEEExZuLEM6qBULBQ44p5UsmzSwGq3s*
Theory Prop Id:	*49cd5a80c0a8afed4364ca62e2e89621ac660e834b9bc67ddf46be5aeb6d8009*
Theory Prop Address:	*TMHMibDhQbDodJRNsKTwFaztQh19uChMD8h*

SNo_PSNo_eta

Theorem 21.89 $\forall x.\mathsf{SNo}\ x \to x = \mathsf{PSNo}\ (\mathsf{SNoLev}\ x)\ (\lambda\beta.\beta \in x)$. *The proposition is identified by the following information:*

Pure Prop Id:	*6ffefe7aa4ffb2d1b5711998134756f9e5e8326d515f3a9f165fa2d847ae71c8*
Pure Prop Address:	*TMTaGgEBt5cPzRbi7CBcmpXvUfY5LgAtNjV*
Theory Prop Id:	*8c78704c4a4036ae8fdbd50b6aa2091682a9b6cc08d150bb5d09f8689293ae7d*
Theory Prop Address:	*TMPZ6V2HX31zAKKRowAqxj1W7MhW2vtEkh3*

SNoLev_PSNo

Theorem 21.90

$$\forall\alpha.\mathsf{ordinal}\ \alpha \to \forall p :\iota \to o.\mathsf{SNoLev}\ (\mathsf{PSNo}\ \alpha\ p) = \alpha.$$

The proposition is identified by the following information:

Pure Prop Id:	*07f1eb48a9b85d0d91b1644c1d205b7b5a1b4598605dbc92bcc702a5f6548159*
Pure Prop Address:	*TMS4B9MsCny6DcgEj6Q5jyZtqGjyadqVrXb*
Theory Prop Id:	*9c4890c1c16e11041619677ff7a9cab66327b15dc07aa9724197a3baf15864e4*
Theory Prop Address:	*TMH875f6xnwhLVBj8FhdaBY27tPcA7tc93w*

SNo_Subq

Theorem 21.91

$$\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNoLev}\ x \subseteq \mathsf{SNoLev}\ y \to (\forall\alpha \in \mathsf{SNoLev}\ x.\alpha \in x \Leftrightarrow \alpha \in y) \to x \subseteq y.$$

The proposition is identified by the following information:

Pure Prop Id:	*a244d79cdb92b7d27cf9364b054d3ab11f8fe1a80867c8aee4bfdbab426dc71b*
Pure Prop Address:	*TMZCRMtpxoXjbsEUAwQVnA3o3i8r9w9chR7*
Theory Prop Id:	*a428aff2b59424a8834bd98dcf9cccb10f9ccd50a36aac8cdb05bdd1f8b0dc7b*
Theory Prop Address:	*TMVu9eEtBLYvjJeiHT4owg9NLx7ADETn241*

Definition 21.28 *We define* SNoEq_ *to be*

$$\lambda\alpha xy.\mathsf{PNoEq\_}\ \alpha\ (\lambda\beta.\beta \in x)\ (\lambda\beta.\beta \in y)$$

of type $\iota \to \iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*5f11e279df04942220c983366e2a199b437dc705dac74495e76bc3123778dd86*
Pure Object Address:	*TMHgy5CsvqeCa98Aq7nixfsuoKHSwuUheGS*
Theory Object Id:	*12b5f61704c15020e2c024d9a3c4b44f386d2447a7bfd3a7ba2d9be7e534d79f*
Theory Object Address:	*TMPsS6FU8RDY8bj6bhgs9hNWme6t8mxkcHv*

SNoEq_I

Theorem 21.92 $\forall \alpha x y.(\forall \beta \in \alpha.\beta \in x \Leftrightarrow \beta \in y) \rightarrow \mathsf{SNoEq\_}\ \alpha\ x\ y$. *The proposition is identified by the following information:*

Pure Prop Id:	*a718b1b74fe6e25f89cd15b443e1050c4627de6e63f7fc13aed6604a5dff174d*
Pure Prop Address:	*TMaoxpA3u5BRdKqSyxRArRdQKxBCCb4Ynfd*
Theory Prop Id:	*fdcb780aed4882ebdf83c46eeec00af5f3c06f91828ac6f34b8cd53807a3f3f0*
Theory Prop Address:	*TMQV9XrhBpFPAX5ZadNCNzSub81tr3rR5pd*

SNoEq_E

Theorem 21.93 $\forall \alpha x y.\mathsf{SNoEq\_}\ \alpha\ x\ y \rightarrow \forall \beta \in \alpha.\beta \in x \Leftrightarrow \beta \in y$. *The proposition is identified by the following information:*

Pure Prop Id:	*322284f9b12d82137cb4547584023ca4089ff501f8b46d31017d6644359f67bc*
Pure Prop Address:	*TMUx62argfoVk7QVYzx9FBiQWTGGzPgSkMr*
Theory Prop Id:	*76c08c80b9ae86c43f88454225e5c2d37189dab1f7a4aaaab54885b6d1d584d5*
Theory Prop Address:	*TMcZ243SUoVmTGmz8GBWnNRSGSoQLcSekNM*

SNoEq_E1

Theorem 21.94 $\forall \alpha x y.\mathsf{SNoEq\_}\ \alpha\ x\ y \rightarrow \forall \beta \in \alpha.\beta \in x \rightarrow \beta \in y$. *The proposition is identified by the following information:*

Pure Prop Id:	*bc569f4fe8f8babfb01e30d1ce5330c3cd167c3a34a4d7f6865bbcb63ff0c246*
Pure Prop Address:	*TMcpXWEGwJyBfkywuB1xBLvouHwBSaFs2Lu*
Theory Prop Id:	*c702b67d3382793ff6831ae5d9948c42fcdf9959f67e4f34b70f105a4fa627c0*
Theory Prop Address:	*TMKUKLNUuNym8woqSCRiDcMxAfsCb1eUBk1*

SNoEq_E2

Theorem 21.95 $\forall \alpha x y.\mathsf{SNoEq\_}\ \alpha\ x\ y \rightarrow \forall \beta \in \alpha.\beta \in y \rightarrow \beta \in x$. *The proposition is identified by the following information:*

Pure Prop Id:	*04c55ef7be9110a6c332fe11ca098564ded45c1ae7f131199dc8a18eff3f728a*
Pure Prop Address:	*TMa2w3rdW3KJYQkLRhjwBtZCzeLY2CsSzUt*
Theory Prop Id:	*06351353e81c1d6cb1d4bd1b3275703327 81fae5e00a0975bce060a995406d00*
Theory Prop Address:	*TMTis8K7byF4GTjz4MpYG9BkyoJCzkUAJyW*

SNoEq_antimon_

Theorem 21.96

$$\forall \alpha.\mathsf{ordinal}\ \alpha \rightarrow \forall \beta \in \alpha.\forall x y.\mathsf{SNoEq\_}\ \alpha\ x\ y \rightarrow \mathsf{SNoEq\_}\ \beta\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*8d4936ed8e712c1dc231fb4b91a23637000f913b108f46cd2edc43f5e8726ba6*
Pure Prop Address:	*TMFyyANM4SzwuZGwLMdhVdAWuiVUHstHQug*
Theory Prop Id:	*95a70a9933c8b7e36b75adaa97b483d7f470c88d64871a9b077468393a7cee44*
Theory Prop Address:	*TMG63bj4zRBfeAkNC86WKZ7gtUpSwCSprJR*

`SNo_eq`

Theorem 21.97

$$\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNoLev}\ x = \mathsf{SNoLev}\ y \to \mathsf{SNoEq\_}\ (\mathsf{SNoLev}\ x)\ x\ y \to x = y.$$

The proposition is identified by the following information:

Pure Prop Id:	*b08bdc8e0533c6d335bfaefe372cf3eae5a6596181f4279fbee1bbbb96bd58d2*
Pure Prop Address:	*TMMawfQkhohvkcRg13ZBDTRwn1DgFdvbZN8*
Theory Prop Id:	*bf4ac3765cc4e77e69dfed6b06b79b2a135de381ed468aff0717b55ecd997219*
Theory Prop Address:	*TMXZ1YaK7uq7CL6xjMR9UpcJae85o8Qg6an*

Let $ctag : \iota \to \iota$ be $\lambda\alpha.\mathsf{SetAdjoin}\ \alpha\ \{2\}$. **Notation.** We use $-''$ as a postfix operator corresponding to applying term $ctag$.

`ctagged_not_ordinal`

Theorem 21.98 $\forall y.\neg\mathsf{ordinal}\ (y'')$. *The proposition is identified by the following information:*

Pure Prop Id:	*06ba78058810b2a66de61b42a989e96b9faf1830fe88331433cb796ec5e9efba*
Pure Prop Address:	*TMSJ67Y6zbYZXw4KipNfSDFXS7nCowqPsfv*
Theory Prop Id:	*762f4db9b4131082faa31f45fdf77ff2ecd9cada9a2b8be09a7c87338296ffaa*
Theory Prop Address:	*TMUUZwa9YUDXipGjVpau4QY3LpPAUNonrus*

`ctagged_notin_ordinal`

Theorem 21.99 $\forall\alpha y.\mathsf{ordinal}\ \alpha \to (y'') \notin \alpha$. *The proposition is identified by the following information:*

Pure Prop Id:	*a6d7ced029db2bf647e1bcf629a266dc8aa5350b597ac826ee1499c9488ef153*
Pure Prop Address:	*TMJtxbUm4vTnZsFYwmfoJXkdyU9Pa5iDTMG*
Theory Prop Id:	*33eb91696d423e27f32912360405bcff4635722555c6e964c127195a36bc3b46*
Theory Prop Address:	*TMU4pDKct3EFb68TQMdmzrbHxh4jXHXLAx4*

`Sing2_notin_SingSing1`

Theorem 21.100 $\{2\} \notin \{\{1\}\}$. *The proposition is identified by the following information:*

Pure Prop Id: *f44be8cc1b2eabcd18be5c1af7cc7c118087af9d39e568bff28319aa056768c9*
Pure Prop Address: *TMYYY1TCNHPhoU7ioo1yEjqPen8SD381J2H*
Theory Prop Id: *4504e136e5be3235264222f1b4fc92b8c82895870a0d5af8fb0ca383d0aa0b93*
Theory Prop Address: *TMXydjLtD7zAbYMVdsbyZnBWtZSoHs13VUk*

ctagged_notin_SNo

Theorem 21.101 $\forall xy.\mathsf{SNo}\ x \rightarrow (y\prime\prime) \notin x$. *The proposition is identified by the following information:*

Pure Prop Id: *e87bab002c13fd4c8f45d1fab441c351590059c0546ef7cb97b2f419eded12bb*
Pure Prop Address: *TMLi9ZahvNViXr94ACKuVosVxgffZSXBaQh*
Theory Prop Id: *fb05c7f6bcb781bf45159e3631adfc011ed3dcf028713a0a5e65ffa8442e7de6*
Theory Prop Address: *TMdvxEZ5qYtgLAENGC1mYi7PPenMSpHbbbG*

ctagged_eqE_eq

Theorem 21.102 $\forall xy.\mathsf{SNo}\ x \rightarrow \mathsf{SNo}\ y \rightarrow \forall u \in x.\forall v \in y.u\prime\prime = v\prime\prime \rightarrow u = v$. *The proposition is identified by the following information:*

Pure Prop Id: *9635c2246bd5ce564e4af804699383bd45289a4ce4fef5acfb843ea16be935ac*
Pure Prop Address: *TMWuHzjRejZ3vCngDLVxyx22KfXifdUFgrX*
Theory Prop Id: *7a7dec8b0cb27354ad3358e329e40af18501227e17ce9a3c573fb2ced3933325*
Theory Prop Address: *TMFudCkTrZ8qDZwsaKBW9Qsuj6AjLeAqYtM*

Definition 21.29 *We define* SNo_pair *to be* $\lambda xy.x \cup \{u\prime\prime | u \in y\}$ *of type* $\iota \rightarrow \iota \rightarrow \iota$ *identified by the following information:*

Pure Object Id: *0c801be26da5c0527e04f5b1962551a552dab2d2643327b86105925953cf3b06*
Pure Object Address: *TMQqNXnBZnkKSe8hed4os6KPYmLwpS71Jw7*
Theory Object Id: *88bcd797eb0ac50799968a11f7c23c8ed718c981b7a3bb422ca738771da70d49*
Theory Object Address: *TMS1SBjNkN4Fv979vSCovhJ3Hm5LmNC8vSP*

SNo_pair_prop_1

Theorem 21.103

$$\forall x1y1x2y2.\mathsf{SNo}\ x1 \rightarrow \mathsf{SNo}\ x2 \rightarrow \mathsf{SNo\_pair}\ x1\ y1 = \mathsf{SNo\_pair}\ x2\ y2 \rightarrow x1 = x2.$$

The proposition is identified by the following information:

Pure Prop Id: *55e11f2068e4bac205e7f81aa76c24ddba355ab4507585cc4d8d2d2f4087857f*
Pure Prop Address: *TMY6PgwrmwyiN6arDm1Yhg5JrTC3jXy8Z7g*
Theory Prop Id: *f4236401490970531 18cefcbb2f2260329493536ad3ae58e47efbdc5cf96a62f*
Theory Prop Address: *TMabWw9KKwUpjo1s2ey6gDkv7hUyHidq12D*

SNo_pair_prop_2

Theorem 21.104

$$\forall x1 y1 x2 y2.\mathsf{SNo}\ x1 \to \mathsf{SNo}\ y1 \to \mathsf{SNo}\ x2 \to \mathsf{SNo}\ y2 \to \mathsf{SNo\_pair}\ x1\ y1 = \mathsf{SNo\_pair}\ x2\ y2 \to y1 = y2.$$

The proposition is identified by the following information:

Pure Prop Id:	*0bfff99d8cefbebcd6db7207b1609c8e7c6338fc2381f6c937e65ef66435fa02*
Pure Prop Address:	*TMZQx8qdiMuWPMgQr92RnVZJPMwYU6eRoDU*
Theory Prop Id:	*cc5febed625436e972c38a7e71012d4d0459f0651c13dba74988508db075847c*
Theory Prop Address:	*TMboEtYmU1rhLndeftmE1xvAXa12uenk52T*

SNo_pair_prop

Theorem 21.105

$$\forall x1 y1 x2 y2.\mathsf{SNo}\ x1 \to \mathsf{SNo}\ y1 \to \mathsf{SNo}\ x2 \to \mathsf{SNo}\ y2 \to \mathsf{SNo\_pair}\ x1\ y1 = \mathsf{SNo\_pair}\ x2\ y2 \to x1 = x2 \wedge y1 = y2.$$

The proposition is identified by the following information:

Pure Prop Id:	*0c1fab5088198fbd78c0308f1d0a7d3e7a9067d7ed242c17489a18a01a610489*
Pure Prop Address:	*TMdh56g5Z6tVNavL7VJHdhuafzTH1Pmfs82*
Theory Prop Id:	*9f8e820a171395b7e1ea5696b5ba61536ab0a1efd23d479c1ad3f329425325a3*
Theory Prop Address:	*TMYoD7ZXdXJGFLDcspavQztKyvhuDPUyktz*

SNo_pair_0

Theorem 21.106 $\forall x.\mathsf{SNo\_pair}\ x\ 0 = x$. *The proposition is identified by the following information:*

Pure Prop Id:	*a24e9ca908c70bad12f23288cfeadb94cf8d4a42e36b819d9a269d2083a09ee7*
Pure Prop Address:	*TMLieswQhd4ZG8isZSgFpxUfMX6vYzwXue7*
Theory Prop Id:	*fe382ec7ec6028476f54901b6bab3e279d994b93eb8d072db22da7f5d6c039d6*
Theory Prop Address:	*TMZuZFbfyTNy3vBnkV22aEUTbRJMeouwCtc*

21.4 Surreal Numbers as Sets II

Definition 21.30 *We define* SNoLt *to be*

$$\lambda xy.\mathsf{PNoLt}\ (\mathsf{SNoLev}\ x)\ (\lambda\beta.\beta \in x)\ (\mathsf{SNoLev}\ y)\ (\lambda\beta.\beta \in y)$$

of type $\iota \to \iota \to o$ identified by the following information:

Pure Object Id:	*46e7ed0ee512360f08f5e5f9fc40a934ff27cfd0c79d3c2384e6fb16d461bd95*
Pure Object Address:	*TMQeBApbcRbbcKWZqVjr7EnqQ7iX7wkV5cx*
Theory Object Id:	*8c8aa9c7cd39e813a521dcca067bdc27aa59721372e00112d3a56e0af7f1c820*
Theory Object Address:	*TMKc4rFp1uKxLq8fj5pXFZziozMvupbPz34*

Notation. We use $<$ as an infix operator corresponding to applying term SNoLt.

Definition 21.31 *We define* SNoLe *to be*

$$\lambda xy.\mathsf{PNoLe}\ (\mathsf{SNoLev}\ x)\ (\lambda\beta.\beta \in x)\ (\mathsf{SNoLev}\ y)\ (\lambda\beta.\beta \in y)$$

of type $\iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*ddf7d378c4df6fcdf73e416f8d4c08965e38e50abe1463a0312048d3dd7ac7e9*
Pure Object Address:	*TMKo92SvXPoMheM1GrcBcnHBZDewaXgqJwX*
Theory Object Id:	*54f405ff1c52a47311b5896cb2435cbca5240391e95a4473bb33d4f389ef4b27*
Theory Object Address:	*TMcgcuHLpN7pQTJ2oMAMaweT9JaE3NrXnSu*

Notation. We use $\leq$ as an infix operator corresponding to applying term SNoLe.

`SNoLtLe`

Theorem 21.107 $\forall xy.x < y \to x \leq y$. *The proposition is identified by the following information:*

Pure Prop Id:	*c0fef1fff2b54737a2955837a95cd2fa4a51eaeab8bdcd197730a7dc32fdaa00*
Pure Prop Address:	*TMcpCGuF3ij7f7Q9otaxmn58tuxgzTAEELE*
Theory Prop Id:	*f09833f8c9ddb44132bc696213b112e392eb8293700f8a6493ef998a11ef5bf7*
Theory Prop Address:	*TMWx22uWdLVXSdBruTx9wkh1hYKRqrGaxYL*

`SNoLeE`

Theorem 21.108 $\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to x \leq y \to x < y \vee x = y$. *The proposition is identified by the following information:*

Pure Prop Id:	*1c9c62734eefe76ad0f740a1f3c250414dfb8d30fa252d1ca61a69f245fe533c*
Pure Prop Address:	*TMKZjxSSN6F5jwo8R4FwwTKANQJT8wVY6Zq*
Theory Prop Id:	*23cc476eb8b886081aab3ea46dc0b4ab0cbfb0a43df47487eade402bb40d21d1*
Theory Prop Address:	*TMR8soSdPW2WyGNYANuCH2tdDGwy3XK3ZZX*

`SNoEq_ref_`

Theorem 21.109 $\forall \alpha x.\mathsf{SNoEq\_}\ \alpha\ x\ x$. *The proposition is identified by the following information:*

Pure Prop Id:	*259cc3291296e3db90ffb96d963d0421fb6016e7a41218d81b92cd846193aa78*
Pure Prop Address:	*TMGJ1mcTbnMvajkoJpACBxXXWZPaQ116hPq*
Theory Prop Id:	*92ef044c4eddbbb417dc88765739163aa0b710f5b5646ffae3c372cffa7a575a*
Theory Prop Address:	*TMWvCZMB6S5XhCvHow2Zc5MwWv32acftDJc*

`SNoEq_sym_`

Theorem 21.110 $\forall \alpha x y.\mathsf{SNoEq\_}\ \alpha\ x\ y \to \mathsf{SNoEq\_}\ \alpha\ y\ x$. *The proposition is identified by the following information:*

Pure Prop Id:	*f8c61d4617dba1594bda7ef1b25d222315df97512407b3dc4d77c1e117f5ffae*
Pure Prop Address:	*TMYtRm1GvHqLhau3uVcMnTfMYn5u2YmwqHi*
Theory Prop Id:	*f756d24db9f6575a09cab35daef6bf28fa9b1b0101f4d013fedfef1e8c0f7741*
Theory Prop Address:	*TMZPd22JbRT6sSfR5cHZF68img19UYG8kuM*

`SNoEq_tra_`

Theorem 21.111

$$\forall \alpha x y z.\mathsf{SNoEq\_}\ \alpha\ x\ y \to \mathsf{SNoEq\_}\ \alpha\ y\ z \to \mathsf{SNoEq\_}\ \alpha\ x\ z.$$

The proposition is identified by the following information:

Pure Prop Id:	*0fff96e9fa087a01b4e627c5a43d8f712211ef77acbd62778d0d5fbdc9a2c11e*
Pure Prop Address:	*TMGUGWpuTgNVZaVHrjqqx3B3dvveuU3xb1w*
Theory Prop Id:	*246d040775eff6f1c7748648c0c49f13cbd822ffecf62018af28e16107c8db95*
Theory Prop Address:	*TMKhJm34Nbo1ihoXZ31EHQfG87mFoxCD2xF*

`SNoLtE`

Theorem 21.112

$$\begin{array}{c}\forall x y.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to x < y \to \forall p : o.\\ (\forall z.\mathsf{SNo}\ z \to \mathsf{SNoLev}\ z \in \mathsf{SNoLev}\ x \cap \mathsf{SNoLev}\ y \to\\ \mathsf{SNoEq\_}\ (\mathsf{SNoLev}\ z)\ z\ x \to \mathsf{SNoEq\_}\ (\mathsf{SNoLev}\ z)\ z\ y \to x < z \to\\ z < y \to \mathsf{SNoLev}\ z \notin x \to \mathsf{SNoLev}\ z \in y \to p)\\ \to\\ (\mathsf{SNoLev}\ x \in \mathsf{SNoLev}\ y \to \mathsf{SNoEq\_}\ (\mathsf{SNoLev}\ x)\ x\ y \to \mathsf{SNoLev}\ x \in y \to p)\\ \to\\ (\mathsf{SNoLev}\ y \in \mathsf{SNoLev}\ x \to \mathsf{SNoEq\_}\ (\mathsf{SNoLev}\ y)\ x\ y \to \mathsf{SNoLev}\ y \notin x \to p)\\ \to p.\end{array}$$

The proposition is identified by the following information:

Pure Prop Id:	*958711d28833ece2b40c3f3b22c5429d43fda518512b7e6cd75b687901aa585c*
Pure Prop Address:	*TMQaPacdRra3VaMe2NYNa6hUY8TFNJ1no3E*
Theory Prop Id:	*d12fc61e3017695d6fd60f1d88de8697d1f3b34fae6748b05bc36f7361185ea5*
Theory Prop Address:	*TMScxkqv8rKobWueAxfgzCcwhkfhaGV9qHT*

`SNoLtI2`

Theorem 21.113

$$\forall xy.\mathsf{SNoLev}\ x \in \mathsf{SNoLev}\ y \to \mathsf{SNoEq\_}\ (\mathsf{SNoLev}\ x)\ x\ y \to \mathsf{SNoLev}\ x \in y \to x < y.$$

The proposition is identified by the following information:

Pure Prop Id:	*282b0114a7969ac0a9f2f7d7736f7948b656946dfe0bd61474b62b3399f08148*
Pure Prop Address:	*TMSSV4UzJfz3EizcWCjQwJdMaAvo7dQx3od*
Theory Prop Id:	*82b82caa129216de39cbf73134cd56db7ca51386bfea22d4c8ddd6c051496ed1*
Theory Prop Address:	*TMTsvbaNTUL6yQqdnrt1ngGJ6yz18T2WJx7*

SNoLtI3

Theorem 21.114

$$\forall xy.\mathsf{SNoLev}\ y \in \mathsf{SNoLev}\ x \to \mathsf{SNoEq\_}\ (\mathsf{SNoLev}\ y)\ x\ y \to \mathsf{SNoLev}\ y \notin x \to x < y.$$

The proposition is identified by the following information:

Pure Prop Id:	*006afc7cf0f802618dbec43d665276d0ee81d35772edbf373b3bd651342a9d37*
Pure Prop Address:	*TMJ4VDCxrxGVAF5fK26vap4HDavQoHRMJ4m*
Theory Prop Id:	*938f67cc578e6abe82c5585824484d30503f3bcc5e3689cb6de279568d894afc*
Theory Prop Address:	*TMX95fsRv9YXCLYg3BLQgUEsosPv95mRaVf*

SNoLt_irref

Theorem 21.115 $\forall x.\neg\mathsf{SNoLt}\ x\ x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*d5ecf97e33d758999475a13ee895246c6ae7bc234feff76c2354f56f7b624826*
Pure Prop Address:	*TMbVeK3yqN3QTUGSqR22RZ7ufquPqAg75TV*
Theory Prop Id:	*06fac5ec34e79c463fc32f8c6ed17874daafb68995b87ca166ab687f4266c6e8*
Theory Prop Address:	*TMFNgRdNx48U2iQGe5ump9qwyA5gisWDyYk*

SNoLt_trichotomy_or

Theorem 21.116 $\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to x < y \lor x = y \lor y < x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*35a4c8a73fc7c1c7cbab4f63f7e92de7f75eec2e52fca95b9fd47f0ab2de7ae7*
Pure Prop Address:	*TMdfGBU7nCaoeRDa8DYFzE35sW9rF2BaYJT*
Theory Prop Id:	*8ad056deb63dd86dc6b1c668c0e389f14827e04273dcb3e1a8d1c1e74bad1877*
Theory Prop Address:	*TMdykwY9bimsgzdU8rQy52x82fcLDNP8gqm*

SNoLt_trichotomy_or_impred

Theorem 21.117

$$\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \forall p:o.(x < y \to p) \to (x = y \to p) \to (y < x \to p) \to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*ba04ef155f5e47607fdb0216fb6a06340f5cd9616eb6c817260c9f387b62270a*
Pure Prop Address:	*TMV8mPnyLgKgikuN7NsNsQJao2G8A2ya29W*
Theory Prop Id:	*7588e3b81e8c26b5bdfa83b3a50190f9f22e3e7c278731520d46b07ea230bf45*
Theory Prop Address:	*TMYEfgRm1btK3489ffptw9arr8NJ1tSuHYr*

SNoLt_tra

Theorem 21.118 $\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to x < y \to y < z \to x < z.$ *The proposition is identified by the following information:*

Pure Prop Id:	*ac0e241054ec1b90968f95ad1b556eec114498a58651d565ae4cac1b019867c7*
Pure Prop Address:	*TMddkASjPFXDxWgZwPsWeWLvopbdDModThm*
Theory Prop Id:	*ccfdf7a1275e7ae74a5f6ecc7d8b2def5778c8e9002abdb744c67fd331be61e1*
Theory Prop Address:	*TML6qNhmMdQ9xqiz61XAszuyuyC6VGsunnR*

SNoLe_ref

Theorem 21.119 $\forall x.\mathsf{SNoLe}\ x\ x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*c940c82ce0e40faca616524909eeb5303ed083f5290e54d8054cfbae1cf0ec2f*
Pure Prop Address:	*TMWbEJ22hQKnoK2eVaoyNrJEUXEkgbcRBJf*
Theory Prop Id:	*83bd2d49035b19f0998a95f22664723ce6d3c960022e6125f8249aa45f2b1292*
Theory Prop Address:	*TMKJtJGg7gcLDxHPUbGWPcUfzbs1mmF3Wki*

SNoLe_antisym

Theorem 21.120 $\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to x \leq y \to y \leq x \to x = y.$ *The proposition is identified by the following information:*

Pure Prop Id:	*f9f7571d97142cd27d147c350951113898e84e19eedaf980f36ed322766e0514*
Pure Prop Address:	*TMdEqGkU3vjpdRYetepi2YSrbnZ5VjCxYuZ*
Theory Prop Id:	*3abee4eacee94652f2f428c3dd42de2ad807637c433d8c2c868a6509d7a6427a*
Theory Prop Address:	*TMMewh6x1J5khDBzMVayDe8pgeEGnJt2fPL*

SNoLtLe_tra

Theorem 21.121 $\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to x < y \to y \leq z \to x < z.$ *The proposition is identified by the following information:*

Pure Prop Id:	*3bc66187a3c8c6c503fb4e0c28121c6f7b0e9fdb129f4989a4e877bda7ea2331*
Pure Prop Address:	*TMFeQ529o5NY2zuHeApHyghETBCcdzFrerN*
Theory Prop Id:	*ba8545b12dea14cf235ae9007eb85c28adfad81e6abd4a3c0afdc1263e529e5d*
Theory Prop Address:	*TMWXyUVVCVjMxZQeS5PSqxmnt47eXXrnZjj*

SNoLeLt_tra

Theorem 21.122 $\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to x \leq y \to y < z \to x < z.$ *The proposition is identified by the following information:*

Pure Prop Id:	*b2178f178198787e74b3d9f71600d68ffeef7e217a11d962036dfd3e08b75293*
Pure Prop Address:	*TMLec6uaS4DS4VwQAcpcPM3YPb74q9SMcDB*
Theory Prop Id:	*4a77a2a749de106eaac82a7c1e466ed95de5fc489f1820e8b49646e6b03c8d35*
Theory Prop Address:	*TMK9rwT18dhoTbiBXjrvgGExGkmYmRnFyZK*

SNoLe_tra

Theorem 21.123 $\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to x \leq y \to y \leq z \to x \leq z.$ *The proposition is identified by the following information:*

Pure Prop Id:	*a0c5c60c2ceffb86861009ffeca522474ef473926e20770226f32c90efdc18d0*
Pure Prop Address:	*TMKoicgHRM8yShdwoBNSAh9MEvz7q8PtkM9*
Theory Prop Id:	*0319892cfab3f54bb914dbb9f14645402420d106668b7793a5f85e69ecca72fc*
Theory Prop Address:	*TMXd83ANWfT4zya3nqhmvzCfE2ez1NuqTf3*

SNoLtLe_or

Theorem 21.124 $\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to x < y \lor y \leq x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*f595ac154b5f6c53eca4ea196af34004c4273a448817a2233791468da205e94a*
Pure Prop Address:	*TMQDjtGFE58mqHck4JTyF12SPbWSzrFAjzw*
Theory Prop Id:	*8965d2d632b616ea19510993aaf5d59d9c47af2a7d76be6d00d0b436c1309dc4*
Theory Prop Address:	*TMXGgukiEQFsFuC6uSDXeRYCUnqF5inCRTa*

SNoLt_PSNo_PNoLt

Theorem 21.125

$$\forall \alpha\beta.\forall pq : \iota \to o.\mathsf{ordinal}\ \alpha \to \mathsf{ordinal}\ \beta \to \mathsf{PSNo}\ \alpha\ p < \mathsf{PSNo}\ \beta\ q \to \mathsf{PNoLt}\ \alpha\ p\ \beta\ q.$$

The proposition is identified by the following information:

Pure Prop Id:	*6e122de5852708a5ca608843957b6c9ff2dae5ef5222ddf994aad35720219c8f*
Pure Prop Address:	*TMG8BGEKrgDCqPJMuPr9e9EppTdzunca7fq*
Theory Prop Id:	*535161c1205ef5061ddcd8bc9186161d3eacd8b5c97edac1dfa870877bb532ba*
Theory Prop Address:	*TMNDvRwj9viGBk1WZotrsPW535EA9L5CVb6*

PNoLt_SNoLt_PSNo

Theorem 21.126

$$\forall \alpha \beta . \forall p q : \iota \to o . \mathsf{ordinal}\ \alpha \to \mathsf{ordinal}\ \beta \to \mathsf{PNoLt}\ \alpha\ p\ \beta\ q \to \mathsf{PSNo}\ \alpha\ p < \mathsf{PSNo}\ \beta\ q.$$

The proposition is identified by the following information:

Pure Prop Id:	*11b6ed978d93a259a246a64548a3100d22b113d8b2de70d4392bb571d4078f73*
Pure Prop Address:	*TMXkNqDx5R69XDH3RVS3UbfdKQwDetvxzFa*
Theory Prop Id:	*6196d926339459b6e115949379593dc6e6a3f5922f1b60dd11abdd0193525e46*
Theory Prop Address:	*TMSzfRKTbTeWkajRMYPfF6mjUqxx7M2QY5i*

Definition 21.32 *We define* SNoCut *to be*

$$\lambda L R . \mathsf{PSNo}\ (\mathsf{PNo\_bd}\ (\lambda \alpha p . \mathsf{ordinal}\ \alpha \wedge \mathsf{PSNo}\ \alpha\ p \in L)\ (\lambda \alpha p . \mathsf{ordinal}\ \alpha \wedge \mathsf{PSNo}\ \alpha\ p \in R))\ (\mathsf{PNo\_pred}\ (\lambda \alpha p . \mathsf{ordinal}\ \alpha \wedge \mathsf{PSNo}\ \alpha\ p \in L)\ (\lambda \alpha p . \mathsf{ordinal}\ \alpha \wedge \mathsf{PSNo}\ \alpha\ p \in R))$$

of type $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*ec849efeaf83b2fcdbc828ebb9af38820604ea420adf2ef07bb54a73d0fcb75b*
Pure Object Address:	*TMRuDcB8ezdcHgdFjNy68Wt7HG9e2X1bxLT*
Theory Object Id:	*295bc4535cc47135a06fa73d8ca9d4df8cc9e2d68a2a449df25a754b7e9dc1a9*
Theory Object Address:	*TMP7d7RzZhj9qAvayw3UXZSiTmHLbd188MH*

Definition 21.33 *We define* SNoCutP *to be*

$$\lambda L R . (\forall x \in L . \mathsf{SNo}\ x) \wedge (\forall y \in R . \mathsf{SNo}\ y) \wedge (\forall x \in L . \forall y \in R . x < y)$$

of type $\iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*c083d829a4633f1bc9aeab80859255a8d126d7c6824bf5fd520d6daabfbbe976*
Pure Object Address:	*TMdYBKYErUYWZiKgnTvgUffZxEVBUVDaLvh*
Theory Object Id:	*775127720c53377db4bdf359fa2ddfb72fd332c9614eaf97b7db372fc3b6837a*
Theory Object Address:	*TMF5e4Sx4fER7UgD1xGTzVfv1apiMjDYdoK*

`SNoCutP_SNoCut`

Theorem 21.127

$$\forall L R . \mathsf{SNoCutP}\ L\ R \to \mathsf{SNo}\ (\mathsf{SNoCut}\ L\ R) \wedge \mathsf{SNoLev}\ (\mathsf{SNoCut}\ L\ R) \in \mathsf{ordsucc}\ ((\bigcup_{x \in L} \mathsf{ordsucc}\ (\mathsf{SNoLev}\ x)) \cup (\bigcup_{y \in R} \mathsf{ordsucc}\ (\mathsf{SNoLev}\ y))) \wedge (\forall x \in L . x < \mathsf{SNoCut}\ L\ R) \wedge (\forall y \in R . \mathsf{SNoCut}\ L\ R < y) \wedge (\forall z . \mathsf{SNo}\ z \to (\forall x \in L . x < z) \to (\forall y \in R . z < y) \to \mathsf{SNoLev}\ (\mathsf{SNoCut}\ L\ R) \subseteq \mathsf{SNoLev}\ z \wedge \mathsf{SNoEq\_}\ (\mathsf{SNoLev}\ (\mathsf{SNoCut}\ L\ R))\ (\mathsf{SNoCut}\ L\ R)\ z).$$

The proposition is identified by the following information:

Pure Prop Id:	*9398b7aeb19cac8a975ba6fbd95f5b59201e44209108095554 31c85cad9a916b*
Pure Prop Address:	*TMU2SS6hDznnxkCmvyEBsebcgVDL1S1H2x2*
Theory Prop Id:	*66456cd00f96a66ea2d8766a84096f51528bb65bd260c0bd0ee0a6d609286810*
Theory Prop Address:	*TMMddRAHYAMxfbYKpGLowF2aZGsCS8EmE5Q*

SNoCutP_SNoCut_impred

Theorem 21.128

$$\begin{array}{c}\forall LR.\mathsf{SNoCutP}\ L\ R\to\forall p:o.\\ (\mathsf{SNo}\ (\mathsf{SNoCut}\ L\ R)\to\\ \mathsf{SNoLev}\ (\mathsf{SNoCut}\ L\ R)\in\\ \mathsf{ordsucc}\ ((\bigcup_{x\in L}\mathsf{ordsucc}\ (\mathsf{SNoLev}\ x))\cup(\bigcup_{y\in R}\mathsf{ordsucc}\ (\mathsf{SNoLev}\ y)))\to\\ (\forall x\in L.x<\mathsf{SNoCut}\ L\ R)\to\\ (\forall y\in R.\mathsf{SNoCut}\ L\ R<y)\to\\ (\forall z.\mathsf{SNo}\ z\to(\forall x\in L.x<z)\to(\forall y\in R.z<y)\to\\ \mathsf{SNoLev}\ (\mathsf{SNoCut}\ L\ R)\subseteq\mathsf{SNoLev}\ z\wedge\\ \mathsf{SNoEq\_}\ (\mathsf{SNoLev}\ (\mathsf{SNoCut}\ L\ R))\ (\mathsf{SNoCut}\ L\ R)\ z)\\ \to p)\\ \to p.\end{array}$$

The proposition is identified by the following information:

Pure Prop Id:	*02c4ae1ea5a415b6ae779362a18ac7cc3070a6c3c3929db683bbe6328a699c64*
Pure Prop Address:	*TMZsxeiR2ByApNxNBoTaPP9YjGAmivj7Pn1*
Theory Prop Id:	*5bcc8fbe0029984d17a2e449797a375c7e0a555e9ac3699de6cb6126199cfb8e*
Theory Prop Address:	*TMLavVhWLmAEpRZmDjMboZyS2VSTuS99y2H*

SNoCutP_L_0

Theorem 21.129 $\forall L.(\forall x\in L.\mathsf{SNo}\ x)\to\mathsf{SNoCutP}\ L\ 0.$ *The proposition is identified by the following information:*

Pure Prop Id:	*47329ccaff17b8de852f941e3476b6e2a6155b82363ef200acdb240a53b6fda3*
Pure Prop Address:	*TMTjdd2ksvbLqB5FsqRdHpicLo8hBbx6oUB*
Theory Prop Id:	*ba61e1cdd15d766dc08200012803abfdba4ca254c084a1b0a956a1cce0e38bec*
Theory Prop Address:	*TMc8bJkCqfKyT4pUBfyifSdGoAoYMJU3VNT*

SNoCutP_0_R

Theorem 21.130 $\forall R.(\forall x\in R.\mathsf{SNo}\ x)\to\mathsf{SNoCutP}\ 0\ R.$ *The proposition is identified by the following information:*

Pure Prop Id:	*54eb826f8bd3be24b1ac7e770fe0c82339a00e86e3aa6ad66d6c724dfb7ef61d*
Pure Prop Address:	*TMYBbiGfDG8mCyX4fru38L5jAWs9jdk1E22*
Theory Prop Id:	*a06bcdb281c727dba6fa27a17f7f8cf4ed415ef56fdd112760f4ab9f181bdd15*
Theory Prop Address:	*TMVtzjAnWmMCfmCH1umdVeisYihbHtUcePx*

SNoCutP_0_0

Theorem 21.131 SNoCutP 0 0. *The proposition is identified by the following information:*

Pure Prop Id:	*3b144eeddb1a1e1da010e706f7ae7f7d3118085be6517aa02ae9e9baaabdbf9e*
Pure Prop Address:	*TMFvHEjaKbqPcs4FFJwfso2a1BUbAJ1s8cW*
Theory Prop Id:	*a5144d076fe9899c5c9e19256598fb91665b7f4ccdb1496e795e8e54b2d4dbd1*
Theory Prop Address:	*TMMq2uhqdU4HWokrwXJLACzASFLaAXQaemQ*

SNoCut_0_0

Theorem 21.132 SNoCut $0\ 0 = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*6a1f81c2c7f937ee3a0d8974a08070c8b8027c1f662f906b43877504c8d43386*
Pure Prop Address:	*TMXsrRAvtrifruw52jjYBXPcZSqXpXBYS6i*
Theory Prop Id:	*abbffb386e047e8707315aa435470d2a1b93eb4b65743a2e3d1e70c9bfac9822*
Theory Prop Address:	*TMNDz4jiqD3F4kCxcNG6hy3DkR89EsDykLk*

ordinal_SNoLt_In

Theorem 21.133 $\forall \alpha \beta.$ordinal $\alpha \to$ ordinal $\beta \to \alpha < \beta \to \alpha \in \beta$. *The proposition is identified by the following information:*

Pure Prop Id:	*d6f066102af8887431439634f1b57a9b0617aba84ba72851e224f021d81ee1fa*
Pure Prop Address:	*TMboKANLD7F64pb6YHEZed6jmzXMjejG8Hn*
Theory Prop Id:	*de775bfed53c50ad2f8415b8603d1aa62bc9e72e6af350f83a6b199c85e37f40*
Theory Prop Address:	*TMY34unv2doasrWYaYXyGVaCNDYHke47nRH*

ordinal_SNoLe_Subq

Theorem 21.134 $\forall \alpha \beta.$ordinal $\alpha \to$ ordinal $\beta \to \alpha \leq \beta \to \alpha \subseteq \beta$. *The proposition is identified by the following information:*

Pure Prop Id:	*c7c906306e5db042974931e626682e6d001f1926ae5d425fb53e074ecbc4a4b6*
Pure Prop Address:	*TMUky22qrn52gwZasuEjEB8QeZ16vSaUQda*
Theory Prop Id:	*7447913a7365f17ac2bd36550d83a144dba2081fdb9554f75855b60e9ab366d6*
Theory Prop Address:	*TMVmvfhvMNFxdUKaz49Zx12oefEK5cmZnmc*

Definition 21.34 *We define* SNoS_ *to be*

$$\lambda \alpha.\{x \in \text{Power } (\text{SNoElts\_ } \alpha) | \exists \beta \in \alpha.\text{SNo\_ } \beta\ x\}$$

of type $\iota \to \iota$ identified by the following information:

Pure Object Id:	*d5069aa583583f8fbe5d4de1ba560cc89ea048cae55ea56270ec3dea15e52ca0*
Pure Object Address:	*TMYeiHkopAzUFpJHiX6DyQTk3GQ6WaqpPTK*
Theory Object Id:	*4680ceebcb1ec94f9ea6976bf96fcfc7658bb190d9a29c5379cc475cc08324a0*
Theory Object Address:	*TMS7Rrpy3RhKZhHAm1BhXAbgLBRoK6QSvXq*

SNoS_E

Theorem 21.135

$$\forall\alpha.\mathsf{ordinal}\ \alpha\to\forall x\in\mathsf{SNoS\_}\ \alpha.\exists\beta\in\alpha.\mathsf{SNo\_}\ \beta\ x.$$

The proposition is identified by the following information:

Pure Prop Id:	*970ce204c4c603af34a26281cf79119bce7edf82baa409d90d59caebd267c095*
Pure Prop Address:	*TMRdGNRDtsqoEYn5nNq2tqy2tRb7GnvaRGr*
Theory Prop Id:	*bb731c8dff4704c5eeb508b1721d9396d9976279c8d5f0cd55142b3de33acf35*
Theory Prop Address:	*TMTy8u2pBq8VtTTWH867d9E9g248LG9VRPU*

21.5 TaggedSets2

Let *tag* : $\iota\to\iota$ be $\lambda\alpha.\mathsf{SetAdjoin}\ \alpha\ \{1\}$. **Notation.** We use $-'$ as a postfix operator corresponding to applying term *tag*.

SNoS_I

Theorem 21.136

$$\forall\alpha.\mathsf{ordinal}\ \alpha\to\forall x.\forall\beta\in\alpha.\mathsf{SNo\_}\ \beta\ x\to x\in\mathsf{SNoS\_}\ \alpha.$$

The proposition is identified by the following information:

Pure Prop Id:	*f5227dbc4fc6188f3716bce1eb047d891fe917014fdf489301acb45e2c964b31*
Pure Prop Address:	*TMMNh5GJRjfU6R9jYR9oSB51a82UR8JuQEx*
Theory Prop Id:	*04f24bbca0962b615517b9ad07d87ff686ee791137b3963827817df34a43adf4*
Theory Prop Address:	*TMKCtpebj2a31twyQ52NMFQ1XPqjxuuqVSL*

SNoS_I2

Theorem 21.137

$$\forall xy.\mathsf{SNo}\ x\to\mathsf{SNo}\ y\to\mathsf{SNoLev}\ x\in\mathsf{SNoLev}\ y\to x\in\mathsf{SNoS\_}\ (\mathsf{SNoLev}\ y).$$

The proposition is identified by the following information:

Pure Prop Id:	*7a14c7d2ef8827bceb18bb81afdc45412f5d9fea9a904ca562bfb0f81d89d276*
Pure Prop Address:	*TMJ7HAvGksViV5vVRG5CZDkJu7DwFRhbuix*
Theory Prop Id:	*b4042d0ea055be8fc3da99d768f853e3498c984596760bcc2ea38e088f32c744*
Theory Prop Address:	*TMKRKaYnYhhkyGaZmUUzgdvAQzZwXTCobPz*

SNoS_Subq

Theorem 21.138

$$\forall \alpha \beta.\mathsf{ordinal}\ \alpha \rightarrow \mathsf{ordinal}\ \beta \rightarrow \alpha \subseteq \beta \rightarrow \mathsf{SNoS\_}\ \alpha \subseteq \mathsf{SNoS\_}\ \beta.$$

The proposition is identified by the following information:

Pure Prop Id:	*6828e7536cfb2fa9917d83ea810ef035d006db4740719d8a39bb37385dbd855d*
Pure Prop Address:	*TMUi6zuABJzGjt2kXYnMDgkM42sqaRJMMSb*
Theory Prop Id:	*d928b8db0b481798073f21b6dda22ac8c152f53325fda84e77c9b8ec06d1d6e3*
Theory Prop Address:	*TMcJgHjSXytYqLvRxz9hc3gMG8529moaiYm*

SNoLev_uniq2

Theorem 21.139 $\forall \alpha.\mathsf{ordinal}\ \alpha \rightarrow \forall x.\mathsf{SNo\_}\ \alpha\ x \rightarrow \mathsf{SNoLev}\ x = \alpha.$ *The proposition is identified by the following information:*

Pure Prop Id:	*fbb46148e53cbe3acaacc8b0b0d7126ebd90194aeb82198e6da28510ce8036d1*
Pure Prop Address:	*TMPvNq7g9BwshVJwKXtCkaoFEnCVHUogHas*
Theory Prop Id:	*f48d47fd7fa9af8472cc826916e9dcead2eaad0845f212108c42b4b8599893ef*
Theory Prop Address:	*TMUESvsETvw3i1PT8zcwLiWdQvbBBUe1Xw5*

SNoS_E2

Theorem 21.140

$$\forall \alpha.\mathsf{ordinal}\ \alpha \rightarrow \forall x \in \mathsf{SNoS\_}\ \alpha.\forall p : o.$$
$$(\mathsf{SNoLev}\ x \in \alpha \rightarrow \mathsf{ordinal}\ (\mathsf{SNoLev}\ x) \rightarrow \mathsf{SNo}\ x \rightarrow \mathsf{SNo\_}\ (\mathsf{SNoLev}\ x)\ x \rightarrow p)$$
$$\rightarrow p.$$

The proposition is identified by the following information:

Pure Prop Id:	*deb03168c1f55ab1d7d74277ebc4f8f33c87b28b16265d557f144f3a25032de8*
Pure Prop Address:	*TMRabQVdMnYRsChGTYCN42TFfnEWrGyAXz2*
Theory Prop Id:	*62b490ef867368eaf329040a83310276c2ad2d046f5274d8c507fe860ea3918f*
Theory Prop Address:	*TMb5fbntT2N1CnACiMUeKM3JYGV5og2W5ZC*

SNoS_In_neq

Theorem 21.141 $\forall w.\mathsf{SNo}\ w \rightarrow \forall x \in \mathsf{SNoS\_}\ (\mathsf{SNoLev}\ w).x \neq w.$ *The proposition is identified by the following information:*

Pure Prop Id:	*ab85fd9a0fdd5f7f96ca2f75d5fd279bf5e0232396c952796f3fc30b522d427d*
Pure Prop Address:	*TMFyX2RhEpYBCTYG6ZNQWUrYk2pQxTkZujK*
Theory Prop Id:	*d5dbd7ce116b1b3b16cbebfad9387d99e477865cfc62e88ffd6e229c2248d86f*
Theory Prop Address:	*TMZeUEJeQ6MejSErRQj2VVEk9N7K2zaE9uV*

SNoS_SNoLev

Theorem 21.142

$$\forall z.\mathsf{SNo}\ z \to z \in \mathsf{SNoS\_}\ (\mathsf{ordsucc}\ (\mathsf{SNoLev}\ z)).$$

The proposition is identified by the following information:

Pure Prop Id:	*a9ba1fc0964d1c60416e8e5bf12e8514139434ea6fe325257b505b527b64eee1*
Pure Prop Address:	*TMW8wR8JRK9MYLigu7413p9VCMwfzGT34AQ*
Theory Prop Id:	*8efb88111c118260d0d7ef1b5cdc5079c9b4a3be002c1cb750eb87a510583725*
Theory Prop Address:	*TMbmaGPAkU5fqbmKhk6PvZkArv7GqJuJRmY*

Definition 21.35 *We define* SNoL *to be* $\lambda z.\{x \in \mathsf{SNoS\_}\ (\mathsf{SNoLev}\ z) | x < z\}$ *of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*8cd812b65c6c466abea8433d21a39d35b8d8427ee973f9bb93567a1402705384*
Pure Object Address:	*TMUvMXBQ19xuZkaN1Qr1oL9x2RgVQBhPiEP*
Theory Object Id:	*48bcf841807be9a2a8a7fdaa4062b7f700ed225f5219ffd13f26782d239d11ae*
Theory Object Address:	*TMGV9huV8KZCMkFQWMGc7fqW2LnTPhHVw9y*

Definition 21.36 *We define* SNoR *to be* $\lambda z.\{y \in \mathsf{SNoS\_}\ (\mathsf{SNoLev}\ z) | z < y\}$ *of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*73b2b912e42098857a1a0d2fc878581a3c1dcc1ca3769935edb92613cf441876*
Pure Object Address:	*TMSvKHoWGSuy2M3PmTt5s6iktHABF7qYPcr*
Theory Object Id:	*0cfe870d225f8cba537d2fe6552d285b3601e6ba81a7b2c47d955f4d490e3aba*
Theory Object Address:	*TMH8PQYnqVae2kcUxoh7NLmWhd5qkNUZU8E*

`SNoCutP_SNoL_SNoR`

Theorem 21.143 $\forall z.\mathsf{SNo}\ z \to \mathsf{SNoCutP}\ (\mathsf{SNoL}\ z)\ (\mathsf{SNoR}\ z)$. *The proposition is identified by the following information:*

Pure Prop Id:	*52486657c66c30f3a313a97245051168bef3ab087bda6a6d0afdc7a3a1f8843e*
Pure Prop Address:	*TMQrzjSnfMHhEaKZxuYJx3zZLL9WfuFojym*
Theory Prop Id:	*7aba571de6f479a36c61b1efbc3a3061f907bdbc5585aca815e7b788f51bd0a2*
Theory Prop Address:	*TMVyVwNCUniGutAsBRT2zh5MKBDjX715Vgi*

`SNoL_E`

Theorem 21.144

$$\forall x.\mathsf{SNo}\ x \to \forall w \in \mathsf{SNoL}\ x.\forall p : o.$$
$$(\mathsf{SNo}\ w \to \mathsf{SNoLev}\ w \in \mathsf{SNoLev}\ x \to w < x \to p)$$
$$\to p.$$

The proposition is identified by the following information:

Pure Prop Id: *cead4f50e23f6816dc9703016f0ed79b8bdbc32a7c09a0f9af78552aeebba9c0*
Pure Prop Address: *TMZmXPiB4BB3YvdBwZTD97bGggD7rSLF6eD*
Theory Prop Id: *7ccd8b9dc0186ef905a8d93999fdc69eb492737ff71320a53317c135b5d63fb4*
Theory Prop Address: *TMYKcvufFZ2FLebZaXDWknwUKjkcdzGsQTZ*

SNoR_E

Theorem 21.145

$$\forall x.\mathsf{SNo}\ x \to \forall z \in \mathsf{SNoR}\ x.\forall p : o.$$
$$(\mathsf{SNo}\ z \to \mathsf{SNoLev}\ z \in \mathsf{SNoLev}\ x \to x < z \to p)$$
$$\to p.$$

The proposition is identified by the following information:

Pure Prop Id: *efca05b2e45ac1d5303c49c52a3636406f36f5a77d8812e0200eaefc9712df2c*
Pure Prop Address: *TMbsBxRcWxxu6mkgykGDi4yRnEUFXwJJe4Z*
Theory Prop Id: *7dca045bf869829b43204d36be8d7632a5061acf12e145a99d1cd8d9be1edfda*
Theory Prop Address: *TMUsPcK3wqDZwDie4ZMQqUBLgLnuT57PPnY*

SNoL_SNoS

Theorem 21.146

$$\forall x.\mathsf{SNo}\ x \to \forall w \in \mathsf{SNoL}\ x.w \in \mathsf{SNoS\_}\ (\mathsf{SNoLev}\ x).$$

The proposition is identified by the following information:

Pure Prop Id: *6e3a2292c3997c29e78fcb520858907ddc605dae52383648872fd6c7ce45f0c1*
Pure Prop Address: *TMLcRAnSRqhUGWvLLTk89wzM2sZ4wvddz98*
Theory Prop Id: *ded78aa4dc105de48885dd5e055230505548286bce7f04f3f5050316c2c04cd0*
Theory Prop Address: *TMPHFqjx1BZx94gsexg3cyERVChRcfu9aMh*

SNoR_SNoS

Theorem 21.147

$$\forall x.\mathsf{SNo}\ x \to \forall z \in \mathsf{SNoR}\ x.z \in \mathsf{SNoS\_}\ (\mathsf{SNoLev}\ x).$$

The proposition is identified by the following information:

Pure Prop Id: *98051c05991e35d56daa447cc7976259612d6048fa82f9a44da11c916b906848*
Pure Prop Address: *TMJ2p8noSCp43KaJq9tZFaAKHrVfWm26tto*
Theory Prop Id: *e13f497f5dab3650206c4362046a1ec477ed8e40d771a06657c3db32f3b247b0*
Theory Prop Address: *TMJz37dSJBFjqARSMYVzEuc7L1KYXuTUBFc*

SNoL_SNoS_

Theorem 21.148 $\forall z.$SNoL $z \subseteq$ SNoS_ (SNoLev z). *The proposition is identified by the following information:*

Pure Prop Id:	*73f5b2cc52ef1daf3b3493d4b3a1b56e0f6a7fedf019873cb56ed5165fc129f6*
Pure Prop Address:	*TMVShTkL3TuhDkq8iCTfj9xfEecNaq6hZTr*
Theory Prop Id:	*409713becd3f086d3979ea36c2db06311233c54602ab8b2acc36bc0e856661f0*
Theory Prop Address:	*TMacyLTfdfpQiEh9iWRH45thaEffs457QES*

SNoR_SNoS_

Theorem 21.149 $\forall z.$SNoR $z \subseteq$ SNoS_ (SNoLev z). *The proposition is identified by the following information:*

Pure Prop Id:	*16bdfc8b4f768a6b37a41115958b385c5a1ec67f13570e242fd1933e158b499f*
Pure Prop Address:	*TMSz3ALjKBot9ZKASXvvTfDJErg7bfASa2J*
Theory Prop Id:	*2fe1c08fc3923f123fffe6c7d8be4b5955d977da8e53d13d4d6c6f092bdcc16d*
Theory Prop Address:	*TMbbztsQAvihxVwkJ9kvqAoNEe7VRXcgYMR*

SNoL_I

Theorem 21.150

$$\forall z.\mathsf{SNo}\ z \to \forall x.\mathsf{SNo}\ x \to \mathsf{SNoLev}\ x \in \mathsf{SNoLev}\ z \to x < z \to x \in \mathsf{SNoL}\ z.$$

The proposition is identified by the following information:

Pure Prop Id:	*ddd91ba46f8cb49763a2e555070ff9cf0d25b89336d5efd26582ff2019e6e033*
Pure Prop Address:	*TMFWNR9kjhSxmC1PpWAxELFPMCQrQDgS3ii*
Theory Prop Id:	*4f3d049c65e44bceaa7abebe992ac83d5e90855d0e811e33cecee9e4c04e2717*
Theory Prop Address:	*TMV173uXJoeNM1as5esbiKuaj6zEgDhUJka*

SNoR_I

Theorem 21.151

$$\forall z.\mathsf{SNo}\ z \to \forall y.\mathsf{SNo}\ y \to \mathsf{SNoLev}\ y \in \mathsf{SNoLev}\ z \to z < y \to y \in \mathsf{SNoR}\ z.$$

The proposition is identified by the following information:

Pure Prop Id:	*97693c3a27a8f00a3262b80f853884f0e0d68e482cbff49cb642b3f14d256b71*
Pure Prop Address:	*TMTPFvahbCyh9yFHZ4zMiRoweJ7ZXMnfLx8*
Theory Prop Id:	*383e88e28a8ea102937169b00da3f18efb2523746b17c0fc524867476a449c5a*
Theory Prop Address:	*TMSrHC17ffyA2enfcx5o1B7iUJn1JV3jwSc*

SNo_eta

Theorem 21.152 $\forall z.$SNo $z \to z =$ SNoCut (SNoL z) (SNoR z). *The proposition is identified by the following information:*

Pure Prop Id:	*3cc369ecf11e43c8a7a6cd9176f7edf2e2d6f77128e0bb8b46dd7e9adb3afd18*
Pure Prop Address:	*TMMbZPXPQWq2cEQg1eagYj6NwweZYGCHjiJ*
Theory Prop Id:	*dc9273d2d30d56ac9a4eff2b975bce32ce2e1e109d620f9318846e4176dda05b*
Theory Prop Address:	*TMZQ7w25dF2ftfDkjDshscxccT898iSijZ7*

SNoCutP_SNo_SNoCut

Theorem 21.153 $\forall LR.\mathsf{SNoCutP}\ L\ R \to \mathsf{SNo}\ (\mathsf{SNoCut}\ L\ R)$. *The proposition is identified by the following information:*

Pure Prop Id:	*9621aa2a6018b3eb1183066003e6daeb26c90761edfa02dd7e872bf831ccb4f8*
Pure Prop Address:	*TMNKaoM4RaxsmapryqxboP9xADcYUVy5VVL*
Theory Prop Id:	*ae46e89c23c7cd0b1fbccb0a1e97f383308cec395b5c73ec3d890f6d1641f001*
Theory Prop Address:	*TMMkfWp7u1CPUEtc9sUMWAX6ZkHiFKL3Tsx*

SNoCutP_SNoCut_L

Theorem 21.154 $\forall LR.\mathsf{SNoCutP}\ L\ R \to \forall x \in L.x < \mathsf{SNoCut}\ L\ R$. *The proposition is identified by the following information:*

Pure Prop Id:	*984a30c42556263d781f855feaa7c03ffd5961f0b9c95159ea8302b3ca16467c*
Pure Prop Address:	*TMKfLyuDzXLNY7PtnVTeZe7eLvSaxAynvFi*
Theory Prop Id:	*4bca14807188033563b691a30256c970867594d77736618ddaeed7f6ead01b56*
Theory Prop Address:	*TMcuZs8Sau5dRwSYifuF8HCBdKnRRmSKZYt*

SNoCutP_SNoCut_R

Theorem 21.155 $\forall LR.\mathsf{SNoCutP}\ L\ R \to \forall y \in R.\mathsf{SNoCut}\ L\ R < y$. *The proposition is identified by the following information:*

Pure Prop Id:	*9161db8c6978c922fbaa8ef29650182e4dc42c4f7e4bbe22e34280baae014e7e*
Pure Prop Address:	*TMdihdbyAcDNSyUkD3TdUvmSMiU3VY6TxqA*
Theory Prop Id:	*7c3d12a89f726793a0a60bd44097c71eaee25e353ad706ea323bc543dbf26c62*
Theory Prop Address:	*TMPBd6BZmniMTRdKnM7fbpYBkR7PiBbKsPm*

SNoCutP_SNoCut_fst

Theorem 21.156

$$\forall LR.\mathsf{SNoCutP}\ L\ R \to \forall z.\mathsf{SNo}\ z \to (\forall x \in L.x < z) \to (\forall y \in R.z < y) \to \mathsf{SNoLev}\ (\mathsf{SNoCut}\ L\ R) \subseteq \mathsf{SNoLev}\ z \wedge \mathsf{SNoEq\_}\ (\mathsf{SNoLev}\ (\mathsf{SNoCut}\ L\ R))\ (\mathsf{SNoCut}\ L\ R)\ z.$$

The proposition is identified by the following information:

Pure Prop Id:	*b52eb75a2c789c7e4e20b3f5065952940fdfe39af85380d08e4788785181a8fa*
Pure Prop Address:	*TMGpKvy9aLQxgW1tTVWtNezptYGvyfjG5Lj*
Theory Prop Id:	*8362f8acd554e760437ebf6fcb6ddf58a0b184474b06a457ee1b26d03f2fb41a*
Theory Prop Address:	*TMKQruVQCojmwaMqYHkmFCxgxgAAJu5hamZ*

SNoCut_Le

Theorem 21.157

$$\forall L1 R1 L2 R2.\mathsf{SNoCutP}\ L1\ R1 \to \mathsf{SNoCutP}\ L2\ R2 \to (\forall w \in L1.w < \mathsf{SNoCut}\ L2\ R2) \to (\forall z \in R2.\mathsf{SNoCut}\ L1\ R1 < z) \to \mathsf{SNoCut}\ L1\ R1 \leq \mathsf{SNoCut}\ L2\ R2.$$

The proposition is identified by the following information:

Pure Prop Id:	*bb8cdcd2d5fd2f00037512dfe3c9dc90186e99a11943fa34839c21c6d21327f9*
Pure Prop Address:	*TMNRFtNxDRZszFXoDioSFyUkctJH577Wgtm*
Theory Prop Id:	*4237313994e58f38bf2a6019231c10978a504fc59d522337bd78a558d606fc9d*
Theory Prop Address:	*TMU9RWn6464F25Pra7jPLPqiPfwa8i21YQ3*

SNoCut_ext

Theorem 21.158

$$\forall L1 R1 L2 R2.\mathsf{SNoCutP}\ L1\ R1 \to \mathsf{SNoCutP}\ L2\ R2 \to (\forall w \in L1.w < \mathsf{SNoCut}\ L2\ R2) \to (\forall z \in R1.\mathsf{SNoCut}\ L2\ R2 < z) \to (\forall w \in L2.w < \mathsf{SNoCut}\ L1\ R1) \to (\forall z \in R2.\mathsf{SNoCut}\ L1\ R1 < z) \to \mathsf{SNoCut}\ L1\ R1 = \mathsf{SNoCut}\ L2\ R2.$$

The proposition is identified by the following information:

Pure Prop Id:	*c1078eaf6d9d767331c2adfc239cf6d058aa0386b0e05639ec15f00d6bc8651b*
Pure Prop Address:	*TMQEDzfjypV741w6WAoTJECPB5vgdYS7NXj*
Theory Prop Id:	*f66618e40670b959c354add07597e52634943b95783e7550a46245c83744c351*
Theory Prop Address:	*TMQc8BkWk3YDRQ3ZDJXw13dYU48b624zt8h*

SNoLt_SNoL_or_SNoR_impred

Theorem 21.159

$$\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to x < y \to \forall p : o.(\forall z \in \mathsf{SNoL}\ y.z \in \mathsf{SNoR}\ x \to p) \to (x \in \mathsf{SNoL}\ y \to p) \to (y \in \mathsf{SNoR}\ x \to p) \to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*f3b15a8dbd76f9e501e20410a95606c6e4d8a53c81914a627374d46d3d907b2f*
Pure Prop Address:	*TMRDwmSiYvVpLRjvbAK1MWfXAVsfvotprEu*
Theory Prop Id:	*ffc0bf982d5e8f4c56f088c7cc742ee4ab02e0c490f0c7638cbc614dc1677aca*
Theory Prop Address:	*TMcckGGW8ynGG29S7CCB8qwfJasDqZZoUqS*

SNoL_or_SNoR_impred

Theorem 21.160

$$\forall xy.\mathsf{SNo}\ x\to\mathsf{SNo}\ y\to\forall p:o.(x=y\to p)\to(\forall z\in\mathsf{SNoL}\ y.z\in\mathsf{SNoR}\ x\to p)\to(x\in\mathsf{SNoL}\ y\to p)\to(y\in\mathsf{SNoR}\ x\to p)\to(\forall z\in\mathsf{SNoR}\ y.z\in\mathsf{SNoL}\ x\to p)\to(x\in\mathsf{SNoR}\ y\to p)\to(y\in\mathsf{SNoL}\ x\to p)\to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*f8a5e25fc885d4cd85a70c63657ca7547fbf75b02f27b82464c042547cc677f6*
Pure Prop Address:	*TMKsCmKUpLx2BhcK6jEDUspKVKvE5k5QiRa*
Theory Prop Id:	*68ba9809e987e498558a641b25900231b82622d983653583a201a59a4d1c0b55*
Theory Prop Address:	*TMKWiFqjuwfNEiTTVN1gfvvjWVrJhQj1gqg*

`ordinal_SNo_`

Theorem 21.161 $\forall\alpha.\mathsf{ordinal}\ \alpha\to\mathsf{SNo\_}\ \alpha\ \alpha$. *The proposition is identified by the following information:*

Pure Prop Id:	*4731fd4a3d0693e855e835e22a21f5e5977b26765eba190ec58fe9daa4d2afe8*
Pure Prop Address:	*TMWHKTo9dcgkBtGz7uaXaYTj4kgHyCDDyAP*
Theory Prop Id:	*7754e54755e12ec8adcd2c120c5d5474ec249b91da929ca1e7981dd9d443b092*
Theory Prop Address:	*TMbYC7hUX1cj11jwA8NrmcFtWzGyRNNcP3L*

`ordinal_SNoL`

Theorem 21.162 $\forall\alpha.\mathsf{ordinal}\ \alpha\to\mathsf{SNoL}\ \alpha=\mathsf{SNoS\_}\ \alpha$. *The proposition is identified by the following information:*

Pure Prop Id:	*fef60879d9dcebd791ff77a26af98cae1b05821d9fe106e7d892b781e23e857d*
Pure Prop Address:	*TMKCDKt4HyT8bYK3wPMhnwhhHEbQ5xacdeK*
Theory Prop Id:	*95823e3d326f1172e3884b6aa394d6e9d59d2ddf7578bca2e2f2aa935524fd70*
Theory Prop Address:	*TMd2zwjPUXZL6Vy46VFW4gwpc2zig21PKRe*

`ordinal_SNoR`

Theorem 21.163 $\forall\alpha.\mathsf{ordinal}\ \alpha\to\mathsf{SNoR}\ \alpha=\mathsf{Empty}$. *The proposition is identified by the following information:*

Pure Prop Id:	*07c69afcba017b5ee2b34caa8d7b084b72822e15942c9a589e012fc3354dadc3*
Pure Prop Address:	*TMEszERJsS849AqzpPGdTFGPRhenRZSY14a*
Theory Prop Id:	*f853d9f1e9d0ea7f8c0d0f6837e18502503c2b8c0a0f017d5f07a8be9f70fda3*
Theory Prop Address:	*TMRXLgDuMtvBV5siWdFYrySgsWpvnxN9925*

`ordinal_SNoCutP`

Theorem 21.164

$$\forall\alpha.\mathsf{ordinal}\ \alpha\to\mathsf{SNoCutP}\ (\mathsf{SNoS\_}\ \alpha)\ \mathsf{Empty}.$$

The proposition is identified by the following information:

Pure Prop Id:	*1e4d0d44a7461a68881734e0e91b185e38404b2e73f6e59f99b11fe839407b3d*
Pure Prop Address:	*TMW1UvmeHxUQLv7BE98ap334SUfCm8Fu73Y*
Theory Prop Id:	*0cfe8be75650c6e5a146099344f74fb66757e93b2cd2edac1ae4deefc984dc64*
Theory Prop Address:	*TMRkZRJr4waasoDGCAVbUFmg6pCVbwcgmgu*

`ordinal_SNoCut_eta`

Theorem 21.165

$$\forall \alpha.\mathsf{ordinal}\ \alpha \to \alpha = \mathsf{SNoCut}\ (\mathsf{SNoS\_}\ \alpha)\ \mathsf{Empty}.$$

The proposition is identified by the following information:

Pure Prop Id:	*a115354a2357cc66a492e6c27636f873c89ca659eeae89203adba376b623519f*
Pure Prop Address:	*TMSVcnu3qbhNj765CNa5JVutvQs8ZAUZbac*
Theory Prop Id:	*1f113a2b452bb89a7755a221d966a72de174752e7ff3e1d939ad96691e455817*
Theory Prop Address:	*TMFKp4mHaoLW3mEp9qD9XgwpSzYRMvq34F2*

`SNo_0`

Theorem 21.166 $\mathsf{SNo}\ 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*c8204b8b1882d371baace062a2960d93b16000698c7d6a09a142db659b3f1fca*
Pure Prop Address:	*TMN5B5ciEPfW7oQwW9YQcm3vLvkEtUZSZnt*
Theory Prop Id:	*94a310917df9f628979dfd6d1313a09153628ffb779d20e4011addf6dc4f2d0d*
Theory Prop Address:	*TMFnaFB9Pt5pY6PgwroCRcRZDgrkfSYBuSa*

`SNoLev_0`

Theorem 21.167 $\mathsf{SNoLev}\ 0 = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*45a0f7a396d9f9d63598766e2614400055684c37fc87764bd32b3d694a4f878e*
Pure Prop Address:	*TMJQ1Ke47ELvhoXkPaZRPfgMk7d8RGCakfG*
Theory Prop Id:	*3527a4181b9135af63b4982294e24684d20c27740e7c8367f29e25ad12624fa4*
Theory Prop Address:	*TMLKkLmPfzCPj82cSv1tmqe99GdjnMnFnvv*

`SNoL_0`

Theorem 21.168 $\mathsf{SNoL}\ 0 = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*c01c2ba737a470d613bec196a861574fc97eac3285aae68253f8ce0a1199ed06*
Pure Prop Address:	*TMPzAPKfbgig2tHnBj3tDTW5gQHRQr85Y6m*
Theory Prop Id:	*fdaa694c1dad3b51990c33828083df7857c7ce848cddbb249b15569df8cf4b8c*
Theory Prop Address:	*TMVPonV6zmxAnnnAJAxaJ4wWH75boedRK8V*

`SNoR_0`

Theorem 21.169 SNoR $0 = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*7329c7a0141d5faa6d6c735a4606f98929d053ab782ccfee0d527914d8d0bb82*
Pure Prop Address:	*TMHH23zks5tKNiFan71Zdnpm67D36aSbUoZ*
Theory Prop Id:	*51075148dc5c48d42db4d40072bca52ac7fb82c5a788b39d6f6b7e7c69cd1ecc*
Theory Prop Address:	*TMajtxBFtVd1rrqznKQdB7sqrYQ1JGnSYS3*

SNoL_1

Theorem 21.170 SNoL $1 = 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*84d688b29c78bb8b93ed259d6f9f639fde3ee97c6f1196e2a383bf07c901a937*
Pure Prop Address:	*TMSoMwfGW9Mu39jcotcU6thi6J4LWYEx7bn*
Theory Prop Id:	*15aa793de666a844e874c1d586a19989e272dfba20c5493bdc382a6d38d3b874*
Theory Prop Address:	*TMTBX6BysYj8xm3zeyBXfyKjENW9x7yzMib*

SNoR_1

Theorem 21.171 SNoR $1 = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*e6a83a40947575bc277f1b8c0862189045f237a6889acb55f060da536e5d0e57*
Pure Prop Address:	*TMPLYyxJkuB1KiJuFytWk8RBxXzyup6DPfW*
Theory Prop Id:	*8da427f5c15d8ec84a8b68f3c128839fd941641f566c3d76a8fc06701af24cd2*
Theory Prop Address:	*TMVQas5fNZ38HF4hftusV6FZz3JLDZMUN9n*

SNo_max_SNoLev

Theorem 21.172

$$\forall x.\mathsf{SNo}\ x \to (\forall y \in \mathsf{SNoS\_}\ (\mathsf{SNoLev}\ x).y < x) \to \mathsf{SNoLev}\ x = x.$$

The proposition is identified by the following information:

Pure Prop Id:	*ff019e872fd598ef8a2a3be672f6bcc7cc7017530d8e62908bad15bf571b16b0*
Pure Prop Address:	*TMTDt7E8q54KkcysEpcH7qcMKm5WwHpsKqT*
Theory Prop Id:	*3be24662ae052242ba894f8ccca93140cca84c843c9c37beb7a1560477952006*
Theory Prop Address:	*TMaz8UPXxX1bFVUXeXSuQwyrGjmQjY46y6C*

SNo_max_ordinal

Theorem 21.173

$$\forall x.\mathsf{SNo}\ x \to (\forall y \in \mathsf{SNoS\_}\ (\mathsf{SNoLev}\ x).y < x) \to \mathsf{ordinal}\ x.$$

The proposition is identified by the following information:

Pure Prop Id: *0b230ced38578b81f36ac519fa85acee92a243a563feccc69ef13c26cc93ac1*
Pure Prop Address: *TMJBDQRmWeDvJspje8G3NTmF5iRA5QFjvTX*
Theory Prop Id: *f28b9a03523768b82a0c58f1bbd347f928da765c1382c04c5d8307d059157531*
Theory Prop Address: *TMTYDjsbqqY8KJTxNg5X66HtVW1bn6sTmVP*

Definition 21.37 *We define* SNo_extend0 *to be*

$$\lambda x.\mathsf{PSNo}\ (\mathsf{ordsucc}\ (\mathsf{SNoLev}\ x))\ (\lambda\delta.\delta \in x \wedge \delta \neq \mathsf{SNoLev}\ x)$$

of type $\iota \to \iota$ identified by the following information:

Pure Object Id: *997d9126455d5de46368f27620eca2e5ad3b0f0ecdffdc7703aa4aafb77eafb6*
Pure Object Address: *TMQChUQBGYBvAJNKavV49dHDgfu4qGjxHQV*
Theory Object Id: *ff4161d4bf42181c2a7fa1c1b810f9d4878846fc9af335cc282ffd21f440f9f4*
Theory Object Address: *TMbS5DXTFrYci5baCfTAfaQPuyL4WdRc1rC*

Definition 21.38 *We define* SNo_extend1 *to be*

$$\lambda x.\mathsf{PSNo}\ (\mathsf{ordsucc}\ (\mathsf{SNoLev}\ x))\ (\lambda\delta.\delta \in x \vee \delta = \mathsf{SNoLev}\ x)$$

of type $\iota \to \iota$ identified by the following information:

Pure Object Id: *464e47790f44cd38285279f563a5a918d73224c78a9ef17ae1a46e62a95b2a41*
Pure Object Address: *TMKMddAcJS6FL2G9uiJereyb1tjokjtKa8Q*
Theory Object Id: *8c551b76c450b237e1236b32a569a43ca6b32a062d1d2a013da3dc303569fa68*
Theory Object Address: *TMQVotiw6nRDP1Q877RTRTfvRtH76B8baK8*

`SNo_extend0_SNo_`

Theorem 21.174

$$\forall x.\mathsf{SNo}\ x \to \mathsf{SNo\_}\ \ (\mathsf{ordsucc}\ (\mathsf{SNoLev}\ x))\ (\mathsf{SNo\_extend0}\ x).$$

The proposition is identified by the following information:

Pure Prop Id: *d41d3ef86775d62e2fc9b3c9a272033d4ea2944b3c8af763707dfdef923c968f*
Pure Prop Address: *TMQd3GQMR6hK1kRRtEeaAkFX4ThA8PYbx4s*
Theory Prop Id: *c8f8ea39e4cdce2253d47eb0646052513914897eaed4338727a5cd163df2b423*
Theory Prop Address: *TMH2yeAxqLFK5UHLb9b8dQ6DAmWA6XAxYHL*

`SNo_extend1_SNo_`

Theorem 21.175

$$\forall x.\mathsf{SNo}\ x \to \mathsf{SNo\_}\ \ (\mathsf{ordsucc}\ (\mathsf{SNoLev}\ x))\ (\mathsf{SNo\_extend1}\ x).$$

The proposition is identified by the following information:

Pure Prop Id:	*b7b8669f74b82f6386406947ed40fe7ac42ab269bbd6ea1fa28e5c6678ff4ef4*
Pure Prop Address:	*TMZZkw7fkUGWbN4TRGNYZKePU3uacXEAcNy*
Theory Prop Id:	*73bbf256bdb458b2821e16079f5ac4e447c94d365812f86f5c93f8bc91c0cf93*
Theory Prop Address:	*TMXbECAsrsXbVHqfqLcWLMGLrA5ADZz1k4Y*

SNo_extend0_SNo

Theorem 21.176 $\forall x.$SNo $x \to$ SNo (SNo_extend0 x). *The proposition is identified by the following information:*

Pure Prop Id:	*b353a2b441224520a178acfb9f0f817ca34603e17848371ce6ed746729e2d8a2*
Pure Prop Address:	*TMWGxYp3JwTkkqSrsYrfogqPN1vb2Xorvh1*
Theory Prop Id:	*76562ca12828cc8acac6c03cb0db1ff686b05c1fa8f57098d309a8fb90aee94c*
Theory Prop Address:	*TMQtywDaDPXseniJkxkpxbNAdKd38UE2dQ8*

SNo_extend1_SNo

Theorem 21.177 $\forall x.$SNo $x \to$ SNo (SNo_extend1 x). *The proposition is identified by the following information:*

Pure Prop Id:	*fa323a2025df47e7721db069b72ed5eee27d1ee3571a8a8af9723ad120927c88*
Pure Prop Address:	*TMSdMxvikAWSF7zWCcgb3FfvRNkDDipwxVq*
Theory Prop Id:	*5345a767fd64365f567ddd0c36e71e4585b5269ee70acf1e49fbec29ea6b20f8*
Theory Prop Address:	*TMY25eBqvG7PbCF867BfH8wXMFyJ3vZVa3z*

SNo_extend0_SNoLev

Theorem 21.178

$$\forall x.\text{SNo } x \to \text{SNoLev } (\text{SNo\_extend0 } x) = \text{ordsucc } (\text{SNoLev } x).$$

The proposition is identified by the following information:

Pure Prop Id:	*26efcd107d12caee8e8ee10adebe189fb528ee43376423b43537d0d980b96057*
Pure Prop Address:	*TMLE3zBjSiXJKVByLuNppcpDaWc5KxbJsij*
Theory Prop Id:	*5aa0afa406447583a2ffcc856c3c2270249851120d173ff811fd161bbb5476f1*
Theory Prop Address:	*TMFDXAViZJBzqyvBjZTxryG6TwYcAsVpsez*

SNo_extend1_SNoLev

Theorem 21.179

$$\forall x.\text{SNo } x \to \text{SNoLev } (\text{SNo\_extend1 } x) = \text{ordsucc } (\text{SNoLev } x).$$

The proposition is identified by the following information:

Pure Prop Id:	*33cf6044be79e5a605f13a1596393129b0521a4bbea91a8e8df65c3029e399ba*
Pure Prop Address:	*TMQdtthxiaHskccMdbmaNfQoxB2zGD8wrG3*
Theory Prop Id:	*5293470b79054a91a67c775285d231ecc230d1826978e4d088daf8390581d242*
Theory Prop Address:	*TMKdFGjuMvwpN2qMjfXbBzHx5ZvwNJUoX9Y*

`SNo_extend0_nIn`

Theorem 21.180 $\forall x.\mathsf{SNo}\ x \to \mathsf{SNoLev}\ x \notin \mathsf{SNo\_extend0}\ x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*217cffe317f555501a713a30362da2e786958f5eb0cb0d728c124c2cd996b872*
Pure Prop Address:	*TMda5fYn6yDo4M7LQx8adXmDoCPdX1T8aKm*
Theory Prop Id:	*0dfd17ffb30a9a53795a0c34f15c21b135e90759a30b52930e461dc926cfeab8*
Theory Prop Address:	*TMa2tRaEiFrQbXQteXN4nGzCFeoa2D3wqNJ*

`SNo_extend1_In`

Theorem 21.181 $\forall x.\mathsf{SNo}\ x \to \mathsf{SNoLev}\ x \in \mathsf{SNo\_extend1}\ x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*ac9570cd8593fef418bb2538fbb3c4c48095739f9ed56bba38b9f759f88d20c5*
Pure Prop Address:	*TMUeMNcoZvQFFpyFh5fDtrPSnekbcwi9gVi*
Theory Prop Id:	*8ca372c127a4d684d18db02fb51193ea2fd70537694faea2430652d07ea29128*
Theory Prop Address:	*TMRnfDntKJYsu4fKMmT6ZDjzF5aDKBGdaar*

`SNo_extend0_SNoEq`

Theorem 21.182

$$\forall x.\mathsf{SNo}\ x \to \mathsf{SNoEq\_}\ (\mathsf{SNoLev}\ x)\ (\mathsf{SNo\_extend0}\ x)\ x.$$

The proposition is identified by the following information:

Pure Prop Id:	*bd4a270b5c6aeb7814b4a5a42ab775e0d391605bb207e9a7d4771b5f610877a7*
Pure Prop Address:	*TMcVbrAftnCJg4KkPuCtBWErWfBTh4ncZk4*
Theory Prop Id:	*1299864bcabf2b68aae890ec79e8dac76d30b36a0b72f71d066855a7c6727939*
Theory Prop Address:	*TMJYn96t53oDn1QogAPqAu66JqLUXXXMPDh*

`SNo_extend1_SNoEq`

Theorem 21.183

$$\forall x.\mathsf{SNo}\ x \to \mathsf{SNoEq\_}\ (\mathsf{SNoLev}\ x)\ (\mathsf{SNo\_extend1}\ x)\ x.$$

The proposition is identified by the following information:

Pure Prop Id:	*ba974f3d548b12b8a092b7b2c74ff719f439552818912e00e7423c8b93a1bd73*
Pure Prop Address:	*TMF7TV22yna3gp5cUwPCUxbJ6xgGF2dzCxP*
Theory Prop Id:	*d6f8bb86911f71c00536c62a9ff4f7dcb502510ff47cbe6fa6e321ad985965b8*
Theory Prop Address:	*TMGZEXiMNjzk35x2Tqe624m5WkQJ1kfATtd*

`SNo_extend0_Lt`

Theorem 21.184 $\forall x.\mathsf{SNo}\ x \to \mathsf{SNo\_extend0}\ x < x$. *The proposition is identified by the following information:*

Pure Prop Id:	*bf669a947f56543b6f408b3b0d257dd39de5956a56aa8d486869eea444c8ce02*
Pure Prop Address:	*TMLC64iAmyHaHX6nQwFVRm3kwcL6vvU4P2u*
Theory Prop Id:	*ea5fd1a237ed14511ef2281ac67e33e3f2c0c0bf15a8e4ea87ffd5dd2258d376*
Theory Prop Address:	*TMdzN9ZkvjnY1AoibSuChkYwsx6anuPKYr4*

`SNo_extend1_Gt`

Theorem 21.185 $\forall x.\mathsf{SNo}\ x \to x < \mathsf{SNo\_extend1}\ x$. *The proposition is identified by the following information:*

Pure Prop Id:	*ca0a2d175740921eb82773f6f29e8a9d9ce71765975b1a2abc1810e62321e34d*
Pure Prop Address:	*TMPGx4oU5xXH99g4hKGMXgJyQojSJAw44zk*
Theory Prop Id:	*1c8d55e5a1c7bc27e06b6ece03d9153ad522999d088945e7671e790cafc9153f*
Theory Prop Address:	*TMSXKAxcPxJwDFbDGV7GdQ6dzrXb8t8Gtfp*

Definition 21.39 *We define* $\mathsf{eps\_}$ *to be* $\lambda n.\{0\} \cup \{(\mathsf{ordsucc}\ m)' | m \in n\}$ *of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*5e992a3fe0c82d03dd3d5c5fbd76ae4e09c16d4ce8a8f2bbff757c1617d1cb0b*
Pure Object Address:	*TMPREfrMv4dVM6bYC6yGeUixtCX8s5WtcZH*
Theory Object Id:	*ffee2b2b0138d17826ad706a96fc5a6a6ba8aae8fe09f0db5d0285ae918123b2*
Theory Object Address:	*TMWgmEV2PgET7GQLdd5raFjcRBfsyaTXCuK*

`eps_ordinal_In_eq_0`

Theorem 21.186 $\forall n \alpha.\mathsf{ordinal}\ \alpha \to \alpha \in \mathsf{eps\_}\ n \to \alpha = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*3046fd78a51853d1bb9786f97dc51dba530f419189bc9aeb3da38539fa9b461a*
Pure Prop Address:	*TMWe1qmzHkSSwwJFgrwPJoCfonbTQ4efjAN*
Theory Prop Id:	*1190f379f72b87aef60bf145308c2529adc42bd0db7ec4e1673c896c2a1c59a7*
Theory Prop Address:	*TMbpDBk3MTcVsKLqZPBPoYrpujcgPfiY5Wv*

`eps_0_1`

Theorem 21.187 $\mathsf{eps\_}\ 0 = 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*5bbce1fa3c1be09ebf6d116df2665c36b7b8f577503b6c121bb25e865d3b7238*
Pure Prop Address:	*TMR59LWQ8moWJ5QPeadff1HFh2ASfoVG2gN*
Theory Prop Id:	*d073f03f40b0eb3c8b0bbe33e8eba6928631935514fda8a0b3beeaf04316d10c*
Theory Prop Address:	*TMQ9LV548ZAa1dSucp5vhC4hNL2DmRGxCUz*

`SNo__eps_`

Theorem 21.188

$$\forall n \in \text{omega}.\mathsf{SNo\_}\ (\mathsf{ordsucc}\ n)\ (\mathsf{eps\_}\ n).$$

The proposition is identified by the following information:

Pure Prop Id:	*e9c4d9b771bd18106ae6b62b078902538b29d64f81f15926efc4103f8c00c9ba*
Pure Prop Address:	*TMHkLsYZsJdew3vMRGoV4eMkozdKinHgebF*
Theory Prop Id:	*1c1428769c5baa199f9145d7fd4da4efa99062ddab3a11093d0b91eaddea75ef*
Theory Prop Address:	*TMZLbVSbviYtp4AZzF9JbpEFvFr85s8f34z*

`SNo_eps_`

Theorem 21.189 $\forall n \in \text{omega}.\mathsf{SNo}\ (\mathsf{eps\_}\ n)$. *The proposition is identified by the following information:*

Pure Prop Id:	*9fb70039757911dbbd9302330aa8493957a0f839a3c637e3161bffc475cc017a*
Pure Prop Address:	*TMJo1r2k8j9rELnHY45xjWz1j6aXZKmg9xf*
Theory Prop Id:	*006b0fb6e9891e00a73714a2337f71ea81cd68e1d83e1ffbe6ff6ac2e2ebf23c*
Theory Prop Address:	*TMJdzF6hF6rddoE61FfB1UwP9FVejWKk4Nf*

`SNoLev_eps_`

Theorem 21.190 $\forall n \in \text{omega}.\mathsf{SNoLev}\ (\mathsf{eps\_}\ n) = \mathsf{ordsucc}\ n$. *The proposition is identified by the following information:*

Pure Prop Id:	*6891e35f6b0e6661135cf838e9a5fb81c80f47ec3798a2a896ac13c50244ab3b*
Pure Prop Address:	*TMdgBWSxkt84FLUVcqjHhWJ7ceSWs9iyw1B*
Theory Prop Id:	*75efd151c44e95bbe9b4c51e56bc5088f7629f12f537ea75601bc7ea9eaf088d*
Theory Prop Address:	*TMQsUJFDt5htkcABdDejoFJAbjZMEUCEAeH*

`SNo_eps_SNoS_omega`

Theorem 21.191 $\forall n \in \text{omega}.\mathsf{eps\_}\ n \in \mathsf{SNoS\_}\ \text{omega}$. *The proposition is identified by the following information:*

Pure Prop Id:	*736d2a5d04c2733a6003f64baced09360d5b4e3fca0641afbbf2823c83ba944a*
Pure Prop Address:	*TMbuXJoMG16dBYE5bnEUW2STmFeuKyFfvUX*
Theory Prop Id:	*52da1852246ddad0d39ed6113b9f3532b2d250e38f24dfcfeb5bb5b3d673d7d3*
Theory Prop Address:	*TMWwLLNiXwTEkwv5MiJv3H87ChU7x8J9ACC*

`SNo_eps_decr`

Theorem 21.192 $\forall n \in \text{omega}.\forall m \in n.\mathsf{eps\_}\ n < \mathsf{eps\_}\ m$. *The proposition is identified by the following information:*

Pure Prop Id:	*d6a46a372ecfabb245c4193957c4f0508d4998eb45deefc85c435af5ad5b3b94*
Pure Prop Address:	*TMPjtSvp9n2PhtKoxct2FCPEyUEF8mWwigi*
Theory Prop Id:	*67c76a0faead3dccb86543d7cb1d37298cd531be3091b54b04b83935dc85398b*
Theory Prop Address:	*TMFVwJv6i1e3XzsQSuzx3ek4mzSFTbbAQN6*

SNo_eps_pos

Theorem 21.193 $\forall n \in \mathsf{omega}. 0 < \mathsf{eps\_}\ n$. *The proposition is identified by the following information:*

Pure Prop Id:	*8b75de3a77baa9b92adae44b583582eb3b82463db2c14f404402a2bde097e88e*
Pure Prop Address:	*TMYSgyTwMAyFvd1rAZg1g3St39umQ4DAf3o*
Theory Prop Id:	*760b4aa07773231fa328d11d9a374bf04e0c43853a3127c676e77c310efc55af*
Theory Prop Address:	*TMPXpny51o2j5XrPqyJcP3FHvVjLPiqhf29*

SNo_pos_eps_Lt

Theorem 21.194

$$\forall n. \mathsf{nat\_p}\ n \to \forall x \in \mathsf{SNoS\_}\ n. 0 < x \to \mathsf{eps\_}\ n < x.$$

The proposition is identified by the following information:

Pure Prop Id:	*575b837b657e6e7a9b06a3cf27feee72addd6cc960da74d3e2bbceddc997f016*
Pure Prop Address:	*TMXiEk8xFJoDwT18FPGUHmWfruXdJYK5VR8*
Theory Prop Id:	*d115f0869d3b5e92d6a510264e9d3861cd7160f6be7616adc5574ddeffb04570*
Theory Prop Address:	*TMMiCk9Z6f7NptPRQzK4m6JDrmCyFfYUM7L*

21.6 Ordinals are Surreal Numbers

ordinal_SNo

Theorem 21.195 $\forall \alpha. \mathsf{ordinal}\ \alpha \to \mathsf{SNo}\ \alpha$. *The proposition is identified by the following information:*

Pure Prop Id:	*ac39bc678da7d4e002759c32fe07577995548274fcde56c0639a1d25743d1fce3*
Pure Prop Address:	*TMV1VGX8kBqSXJHN8BCPYuKwoW4HYVoSf7C*
Theory Prop Id:	*1eea33bee9fd0b26c50050d87b178445fa7eadba2d0a37fed8f158dba3c91ff0*
Theory Prop Address:	*TMe1KCf7jS8uH1DVEWLEFPafjhTXSfbJGKi*

ordinal_SNoLev

Theorem 21.196 $\forall \alpha. \mathsf{ordinal}\ \alpha \to \mathsf{SNoLev}\ \alpha = \alpha$. *The proposition is identified by the following information:*

Pure Prop Id:	*56f8b04f51175b2d3ad3ec438b868e145d9fb2c1d306f23dedc2aac5944efcc6*
Pure Prop Address:	*TMZ8sa3eLx4fHfcaC1ehbCESTMyVoJwHZet*
Theory Prop Id:	*fea5fc7aa601d4080a114440a967755767fa47253a68cef3729d30cee0317500*
Theory Prop Address:	*TMd5j8WdJGFuG8mnG6FCTaEsK9H3VxS3rpz*

ordinal_SNoLev_max

Theorem 21.197 $\forall \alpha.\mathsf{ordinal}\ \alpha \to \forall z.\mathsf{SNo}\ z \to \mathsf{SNoLev}\ z \in \alpha \to z < \alpha$. *The proposition is identified by the following information:*

Pure Prop Id:	*945794b25c51118cbb7056a200fc29b3beac4a0954ff35c7d46b8b10d9b490eb*
Pure Prop Address:	*TMW5wuDBjEPgptrHgu3NBPzWzTbiLpoMHFG*
Theory Prop Id:	*51ee47d6504231b0399ea5f21fb49d3219caefd43f8b7ed5b1fa6612bbd4856e*
Theory Prop Address:	*TMMaysrC6GCnCvik13kVigRPVk8Z3bWgCDj*

`ordinal_In_SNoLt`

Theorem 21.198 $\forall \alpha.\mathsf{ordinal}\ \alpha \to \forall \beta \in \alpha.\beta < \alpha$. *The proposition is identified by the following information:*

Pure Prop Id:	*e37d00b84165adfad6c68f140a3928d30ae38bcb8bc0d296cd08e27ee667c03c*
Pure Prop Address:	*TMKf8Fadm6Fho8o69tQZDnNutgimZmGKfvS*
Theory Prop Id:	*ced78e7d1eec9d129092ce29e5de303b7466ca2b9aed7d07aa7b3a24c939f5c3*
Theory Prop Address:	*TMX6tHgRu4ztsUaXjsy7UkVCDdBhk7fa6MQ*

`ordinal_SNoLev_max_2`

Theorem 21.199

$$\forall \alpha.\mathsf{ordinal}\ \alpha \to \forall z.\mathsf{SNo}\ z \to \mathsf{SNoLev}\ z \in \mathsf{ordsucc}\ \alpha \to z \leq \alpha.$$

The proposition is identified by the following information:

Pure Prop Id:	*72c5eb9b297f67eda46b4ae46e71bb272a5437c076df002405f197e27453a158*
Pure Prop Address:	*TMdSZyUHH9SYckxVb8GyE6S3rr2iCAbjF1T*
Theory Prop Id:	*c9a7312092068758770432c6a1ad0f802c09d7c8bd89c0d980c3a4006485a0bc*
Theory Prop Address:	*TMLopX51i5ivHHzxeprUDyP4DpoUWZeRmxK*

`ordinal_Subq_SNoLe`

Theorem 21.200 $\forall \alpha\beta.\mathsf{ordinal}\ \alpha \to \mathsf{ordinal}\ \beta \to \alpha \subseteq \beta \to \alpha \leq \beta$. *The proposition is identified by the following information:*

Pure Prop Id:	*7558ba76794eacd9296839fd46833c13e08c174ebbbd233a9e7e99572119dc6f*
Pure Prop Address:	*TMYpoMN7TNv38QhXrPvAmv9XN6Jwf9myViH*
Theory Prop Id:	*15c58afcaec3772f6849ca921cc76f9d5a42f0a409f34f90856359657725bb35*
Theory Prop Address:	*TMYJsKJKFxZReCyT6P9ehV99SK24YQvYPAK*

`SNo_etaE`

Theorem 21.201

$$\forall z.\mathsf{SNo}\ z \to \forall p : o. (\forall LR.\mathsf{SNoCutP}\ L\ R \to (\forall x \in L.\mathsf{SNoLev}\ x \in \mathsf{SNoLev}\ z) \to (\forall y \in R.\mathsf{SNoLev}\ y \in \mathsf{SNoLev}\ z) \to \to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*c2447381b74da897eaa58c3af9b1d3dde69eab20d337d33fba3e80ec78a2b254*
Pure Prop Address:	*TMMaNjK5BsDv7w7K5kZPoSC8eHU2ZpBXnb3*
Theory Prop Id:	*109f1d5e9cb018c3887e73bfa9bd86c12f6e0da2cf252e4591c3433b7220949e*
Theory Prop Address:	*TMZNw3WejhC9Mtc8ZSCkmGmtc6te5te6DBB*

`SNo_ind`

Theorem 21.202

$$\forall P : \iota \to o. (\forall L R. \mathsf{SNoCutP}\ L\ R \to (\forall x \in L. P\ x) \to (\forall y \in R. P\ y) \to P\ (\mathsf{SNoCut}\ L\ R)) \to \forall z. \mathsf{SNo}\ z \to P\ z.$$

The proposition is identified by the following information:

Pure Prop Id:	*5b4c39e8a208eba4acd367f82c3e27cebe26cfdc469fca07db4499164e4a2ce9*
Pure Prop Address:	*TMbQGwDAo4c4Qz7pDSP1Qrz23DzxEXC3zuv*
Theory Prop Id:	*94cbe61705cabfd1ca5e0c74b060f6cbf1387230afda6d17336d57953552fc1a*
Theory Prop Address:	*TMdM7tGeFyUKwEjhWyhtce31kgdVFHS9DFT*

21.7 SurrealRecI

Let $F : \iota \to (\iota \to \iota) \to \iota$ be given.

Definition 21.40 SNo_rec_i *is the opaque object of type $\iota \to \iota$ identified by the following information:*

Pure Object Id:	*be45dfaed6c479503a967f3834400c4fd18a8a567c8887787251ed89579f7be3*
Pure Object Address:	*TMbX65FfYCsMQpKH5CQAuQZ7t7Xzon4LVUj*
Theory Object Id:	*230ba80e6902fe44eb4a2d0e953e841148a6b8917a639f0a1a7683bf1567f6ed*
Theory Object Address:	*TMFnUMDQ6KoAKr1mcWtFHKKkSNoPUN4jaDz*

Assume the following.

$$\forall z. \mathsf{SNo}\ z \to \forall g h : \iota \to \iota. (\forall w \in \mathsf{SNoS\_}\ (\mathsf{SNoLev}\ z). g\ w = h\ w) \to F\ z\ g = F\ z\ h \tag{21.1}$$

`SNo_rec_i_eq`

Theorem 21.203

$$\forall z. \mathsf{SNo}\ z \to \mathsf{SNo\_rec\_i}\ z = F\ z\ \mathsf{SNo\_rec\_i}.$$

The proposition is identified by the following information:

Pure Prop Id:	*d5ffa87a5e29f9bd7db6e5f48c1339a209c1528efb466143479de577f4c925cf*
Pure Prop Address:	*TMFehK7LHhf71LYnf7ptxSG8U7UPEUDMtLB*
Theory Prop Id:	*f547c8e0e249495edd7c8c51106c6527c237807df09c30ce4aae1861b7fd8a48*
Theory Prop Address:	*TMWhuu65bzKbkqnknDmtEdbETx8pBAhrZni*

21.8 SurrealRecII

Let $F : \iota \to (\iota \to (\iota \to \iota)) \to (\iota \to \iota)$ be given.

Definition 21.41 SNo_rec_ii *is the opaque object of type* $\iota \to (\iota \to \iota)$ *identified by the following information:*

Pure Object Id: *e148e62186e718374accb69cda703e3440725cca8832aed55c0b32731f7401ab*
Pure Object Address: *TMQRggiyUu4JmM1iVoMTTJfKCnuvGKk9Coq*
Theory Object Id: *c3f21860c6b33b173235815c3bb495555ca57af947caf72a4ecda2bb7534873b*
Theory Object Address: *TMSBXU8YNhPPW34DsxuTsKMzbSfmRAeMJrj*

Assume the following.

$$\forall z.\mathsf{SNo}\ z \to \forall gh : \iota \to (\iota \to \iota).(\forall w \in \mathsf{SNoS\_}\ (\mathsf{SNoLev}\ z).g\ w = h\ w) \to F\ z\ g = F\ z\ h \tag{21.2}$$

`SNo_rec_ii_eq`

Theorem 21.204

$$\forall z.\mathsf{SNo}\ z \to \mathsf{SNo\_rec\_ii}\ z = F\ z\ \mathsf{SNo\_rec\_ii}.$$

The proposition is identified by the following information:

Pure Prop Id: *39e866e1054ce982b862417ffa3743bc0e8c324d6dd60d1efde2ee400235eb8c*
Pure Prop Address: *TMNXn2h6mjf3y4hBzx8Mu2JUFAE9hznp2cd*
Theory Prop Id: *1c5373d70e7017dcc19aae17ae9fa209e30fb1e64e38a8cf591d94eb9a8e2f3d*
Theory Prop Address: *TMHAFMpf4o9ixm66ZLoJTKWUsAPeGaCat6j*

21.9 SurrealRec2

Let $F : \iota \to \iota \to (\iota \to \iota \to \iota) \to \iota$ be given.

Definition 21.42 SNo_rec2 *is the opaque object of type* $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id: *7d10ab58310ebefb7f8bf63883310aa10fc2535f53bb48db513a868bc02ec028*
Pure Object Address: *TMJMVhuN9vEa6QPsu5PJo6CBxM3Wg6MgxXd*
Theory Object Id: *9511104c67c657c2ef57d259b59e1c5cb547d2196f59208de8f81071fa6bc63e*
Theory Object Address: *TMaJ9EqUKsqh8zZDNMETchCY8mQJMz7evJt*

Assume the following.

$$\begin{array}{c}\forall w.\mathsf{SNo}\ w \to \forall z.\mathsf{SNo}\ z \to \\ \forall gh : \iota \to \iota \to \iota.(\forall x \in \mathsf{SNoS\_}\ (\mathsf{SNoLev}\ w).\forall y.\mathsf{SNo}\ y \to g\ x\ y = h\ x\ y) \to \\ (\forall y \in \mathsf{SNoS\_}\ (\mathsf{SNoLev}\ z).g\ w\ y = h\ w\ y) \to \\ F\ w\ z\ g = F\ w\ z\ h\end{array} \tag{21.3}$$

SNo_rec2_eq

Theorem 21.205

$$\forall w.\mathsf{SNo}\ w \to \forall z.\mathsf{SNo}\ z \to \mathsf{SNo\_rec2}\ w\ z = F\ w\ z\ \mathsf{SNo\_rec2}.$$

The proposition is identified by the following information:

Pure Prop Id:	*e9ea41d7eb248cf4d9445fd415cfe25af80fb49c218714c2380e2789bcae97a8*
Pure Prop Address:	*TMRDgSHaazow3ek3rzLN6xJFhW5ZdzoekKh*
Theory Prop Id:	*ac9f2b2f651e85145eb6e3ce4dfeb2a9f0bd619baca63e2ceaa4a8bb59e326f5*
Theory Prop Address:	*TMJGL7o2tFChKYPUZdXzaYEMbNA7fM43fNy*

21.10 More Results about Surreal Numbers

SNo_ordinal_ind

Theorem 21.206

$$\forall P : \iota \to o.(\forall \alpha.\mathsf{ordinal}\ \alpha \to \forall x \in \mathsf{SNoS\_}\ \alpha.P\ x) \to (\forall x.\mathsf{SNo}\ x \to P\ x).$$

The proposition is identified by the following information:

Pure Prop Id:	*5b75fbe4f3fb968cae8d18aaf8e357e80492153f9fe23005c02bed666cc4dfac*
Pure Prop Address:	*TMZqP4gVUEbQmwPD8hDsBmcStaLjrwssf66*
Theory Prop Id:	*dc2c6d6f6bb1c43d5bbae8d635e29154247bd45fc51b459b16c9487e8936aec8*
Theory Prop Address:	*TMS5emF6ssGYWbwuHaDYXV6uwhf7edEugak*

SNo_ordinal_ind2

Theorem 21.207

$$\begin{array}{c}\forall P : \iota \to \iota \to o. \\ (\forall \alpha.\mathsf{ordinal}\ \alpha \to \forall \beta.\mathsf{ordinal}\ \beta \to \forall x \in \mathsf{SNoS\_}\ \alpha.\forall y \in \mathsf{SNoS\_}\ \beta.P\ x\ y) \\ \to (\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to P\ x\ y).\end{array}$$

The proposition is identified by the following information:

Pure Prop Id:	*ba3315e237ce1d4df16c16a8bf756e2bde89de0b18851b815b96450f882c55c7*
Pure Prop Address:	*TMJp9FY8VctR1fkJgGYxDdh1RLMeBqeS99x*
Theory Prop Id:	*4cd5acb49460e8a170f2661eb1292d8e3541c9317544fd5f2d82c4686e8763f0*
Theory Prop Address:	*TMN4ezAqsQi2C1fr3tcPbKS8LNDQoV38DBY*

`SNo_ordinal_ind3`

Theorem 21.208

$$\forall P : \iota \to \iota \to \iota \to o.$$
$$(\forall \alpha.\mathsf{ordinal}\ \alpha \to \forall \beta.\mathsf{ordinal}\ \beta \to \forall \gamma.\mathsf{ordinal}\ \gamma \to$$
$$\forall x \in \mathsf{SNoS\_}\ \alpha.\forall y \in \mathsf{SNoS\_}\ \beta.\forall z \in \mathsf{SNoS\_}\ \gamma.P\ x\ y\ z)$$
$$\to(\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to P\ x\ y\ z).$$

The proposition is identified by the following information:

Pure Prop Id:	*fccdb6672f955c6006797333286f8360f778fdb5570223fd20d9a063212269c4*
Pure Prop Address:	*TMHEmrGgS81dQJ2JGimKGXSByW4T5buGsKL*
Theory Prop Id:	*dece8cbbffb58ddf8cbf8dcf83ca0a96a6ef9a5448651a114c5355e9cdb4f7d8*
Theory Prop Address:	*TMSHTjXpVeYmg8pivxYP7m7ifgZn67S1TeJ*

`SNoLev_ind`

Theorem 21.209

$$\forall P : \iota \to o.(\forall x.\mathsf{SNo}\ x \to (\forall w \in \mathsf{SNoS\_}\ (\mathsf{SNoLev}\ x).P\ w) \to P\ x)\to$$
$$(\forall x.\mathsf{SNo}\ x \to P\ x).$$

The proposition is identified by the following information:

Pure Prop Id:	*8b2b421134af1c1c1d16aec0622c8efee4fd7ec43176635e591cc48df421e73b*
Pure Prop Address:	*TMdjvtesifW1vMYwLzwUFB8ubFQRxeDErX8*
Theory Prop Id:	*851a886ebf6c314dbc8a05caeee024ca4081f1783f55486fc9481e9aa423b8ed*
Theory Prop Address:	*TMXG4oEhSvYEBk3fcCYLLydMW8jZPQjHCwg*

`SNoLev_ind2`

Theorem 21.210

$$\forall P : \iota \to \iota \to o.$$
$$(\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to (\forall w \in \mathsf{SNoS\_}\ (\mathsf{SNoLev}\ x).P\ w\ y) \to$$
$$(\forall z \in \mathsf{SNoS\_}\ (\mathsf{SNoLev}\ y).P\ x\ z) \to$$
$$(\forall w \in \mathsf{SNoS\_}\ (\mathsf{SNoLev}\ x).\forall z \in \mathsf{SNoS\_}\ (\mathsf{SNoLev}\ y).P\ w\ z) \to P\ x\ y)$$
$$\to\forall xy.\mathsf{SNo}\ x\to\mathsf{SNo}\ y\to P\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id: *87eb9a4029d2214aeadffe89e3f00b768dc1f0ee672b65aa78beff4546554a8a*
Pure Prop Address: *TMPRSJVgigZNu3Ln5T8fN87EJE6rPBh6eyv*
Theory Prop Id: *800d4cef7892d2b9082248440b178c0db19d92fa16e49449e7934ce2a5cffeed*
Theory Prop Address: *TMdh3vKvQM9UhZmdEnwz2CChQjV8xXPGZtD*

SNoLev_ind3

Theorem 21.211

$$\begin{array}{c}\forall P:\iota\to\iota\to\iota\to o.\\ (\forall xyz.\mathsf{SNo}\ x\to\mathsf{SNo}\ y\to\mathsf{SNo}\ z\to\\ (\forall u\in\mathsf{SNoS\_}\ (\mathsf{SNoLev}\ x).P\ u\ y\ z)\to\\ (\forall v\in\mathsf{SNoS\_}\ (\mathsf{SNoLev}\ y).P\ x\ v\ z)\to\\ (\forall w\in\mathsf{SNoS\_}\ (\mathsf{SNoLev}\ z).P\ x\ y\ w)\to\\ (\forall u\in\mathsf{SNoS\_}\ (\mathsf{SNoLev}\ x).\forall v\in\mathsf{SNoS\_}\ (\mathsf{SNoLev}\ y).P\ u\ v\ z)\to\\ (\forall u\in\mathsf{SNoS\_}\ (\mathsf{SNoLev}\ x).\forall w\in\mathsf{SNoS\_}\ (\mathsf{SNoLev}\ z).P\ u\ y\ w)\to\\ (\forall v\in\mathsf{SNoS\_}\ (\mathsf{SNoLev}\ y).\forall w\in\mathsf{SNoS\_}\ (\mathsf{SNoLev}\ z).P\ x\ v\ w)\to\\ (\forall u\in\mathsf{SNoS\_}\ (\mathsf{SNoLev}\ x).\forall v\in\mathsf{SNoS\_}\ (\mathsf{SNoLev}\ y).\forall w\in\mathsf{SNoS\_}\ (\mathsf{SNoLev}\ z).P\ u\ v\ w)\to\\ P\ x\ y\ z)\\ \to\forall xyz.\mathsf{SNo}\ x\to\mathsf{SNo}\ y\to\mathsf{SNo}\ z\to P\ x\ y\ z.\end{array}$$

The proposition is identified by the following information:

Pure Prop Id: *f1d779fd469c0845a98e0f93f05143afba1386516af9d4603637aac985cd04d3*
Pure Prop Address: *TMSvcfsKu6jonEsCbEr8VWi4pk11xZ363HY*
Theory Prop Id: *ee16b90e02757ceeb6643978af1b3c0c4b10662feeea83cd645e3f428caa2f4e*
Theory Prop Address: *TMNAmAH2wAfd64peNfnqHxL8gBKfNxd1haR*

SNo_1

Theorem 21.212 $\mathsf{SNo}\ 1$. *The proposition is identified by the following information:*

Pure Prop Id: *ac50d79d994bfab93af900919749bb0b00bab77dd02496312cb99636c84de723*
Pure Prop Address: *TMLD7rYjwi3XCzS9wBFoZquU2fwKZu8hJfp*
Theory Prop Id: *9a733a1814c1f8aa807a0905095d4095b6b6c16bd5c1cbe8d42ff51d0476728e*
Theory Prop Address: *TMV4EbhEF9Fv43TrwcGbkmuL2KWwTbyEHzw*

SNo_2

Theorem 21.213 $\mathsf{SNo}\ 2$. *The proposition is identified by the following information:*

Pure Prop Id: *487591f799d85ccddbcd2b6f2e71c42ada69163cc874f37e171eabf04a2cf2c8*
Pure Prop Address: *TMLrvp9CHQ6VVsuUey7NKkjK9sekHgGeozC*
Theory Prop Id: *c524a421cfe10bd6c73ba8913472e276988f04c927f9d815d647884e770ef7e5*
Theory Prop Address: *TMR8t9mAk1WDjyns3E36LrbyZ4oDbwT7ZHW*

SNo_omega

Theorem 21.214 SNo omega. *The proposition is identified by the following information:*

Pure Prop Id:	*f6584889c3b559c0c57d54079ab6b3db3900f8646787147aba128aae330d55b6*
Pure Prop Address:	*TMNCVbGvT6eb68ZqscmNFNvebebAFmqdReB*
Theory Prop Id:	*db7b5f8d12a3f6e404156d15406fde7ccaacb30fcba4c04ca7993e10e5bc5231*
Theory Prop Address:	*TMGovFDv31hbwMQHyVzGRJD2AUNGQ55Jbau*

SNoLt_0_1

Theorem 21.215 $0 < 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*dd604718e43f44863c2707c951e0869b557b6fa3a70647ba47a17ec52b0029a2*
Pure Prop Address:	*TMdgV1jBe6EnZgcQwB7jjy19eTiHyJdqRgk*
Theory Prop Id:	*cb6392897bc04cb1365085f9f877dc153c53b66befbecc65b224bd4cc2f5ecd5*
Theory Prop Address:	*TMJwENHJCRTJn9GM5HVohBSxeqaDYfTPidF*

SNoLt_0_2

Theorem 21.216 $0 < 2$. *The proposition is identified by the following information:*

Pure Prop Id:	*7be36fc38585a8b130ffed59f4b86612f5a054fb66623f2e4759235722d406ef*
Pure Prop Address:	*TMaz8fiLqm9K7Mq8fWt23tudQikDLE3nVPq*
Theory Prop Id:	*f469347bc0b459c89c083e9dfd7070a8dfb29072eefa324603e21760ae54f647*
Theory Prop Address:	*TMHYveHQDjM9W8DaiRmfzZeA4rimU2E92Nx*

SNoLt_1_2

Theorem 21.217 $1 < 2$. *The proposition is identified by the following information:*

Pure Prop Id:	*cc44e8f0924d8a19e44142d916783f27b6cc875dba07d11a95a303ffd59e0426*
Pure Prop Address:	*TMUn8b7nVzm61CdAFH2Td6XCY5QtWtzLVFN*
Theory Prop Id:	*19d5e1b76af23ecf0247d684e971a9c96fb452c3323f62c2d67d14b22fa104d1*
Theory Prop Address:	*TMQQHPaJv8GbFZhqEM1fStxij59TT2UfDi7*

SNoLev_0_eq_0

Theorem 21.218 $\forall x.\mathsf{SNo}\ x \to \mathsf{SNoLev}\ x = 0 \to x = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*fe0bc4ac89543a28bb90f488d7033f084538a78425338f1c33f2d0f1095b8658*
Pure Prop Address:	*TMLtxEcKxjJ93DcQ4fddYfsYH8i24duAMQB*
Theory Prop Id:	*dfcf39481eba0eed8e498b098ea072eb9bf52e6e184da7a902a0d3cf7d28b799*
Theory Prop Address:	*TMQQUQVuJAU1SouCAxFzxgEkDK6uVzMBbqL*

`restr_SNo_`

Theorem 21.219

$$\forall x.\mathsf{SNo}\ x \to \forall \alpha \in \mathsf{SNoLev}\ x.\mathsf{SNo\_}\ \alpha\ (x \cap \mathsf{SNoElts\_}\ \alpha).$$

The proposition is identified by the following information:

Pure Prop Id:	*d784442182c6268660ac7e2ebd4ce1f49daf2eaccc2ca06df86cf30b983f10b3*
Pure Prop Address:	*TMKL6VTKgr5Lj5NeDnwUjkGyX4ocPdGDcir*
Theory Prop Id:	*e0eca8857b82373a353cb7cad078aeafe8a30a60da8a14c48961e659c679e16d*
Theory Prop Address:	*TMXSGP5nqZiFAfXN6HwbUUCGasSdVszy7Vr*

`restr_SNo`

Theorem 21.220

$$\forall x.\mathsf{SNo}\ x \to \forall \alpha \in \mathsf{SNoLev}\ x.\mathsf{SNo}\ (x \cap \mathsf{SNoElts\_}\ \alpha).$$

The proposition is identified by the following information:

Pure Prop Id:	*930b5633114b598786190b7f3e7477c4a66a906c5e9f9eba7449686faeef6a32*
Pure Prop Address:	*TMHnTUT8yBBnGwav9Bnqhxz5kqA5ggvvevz*
Theory Prop Id:	*d141ededdd7718b828cf689076cf982e403f154afe9f7dc65662f9704bddf669*
Theory Prop Address:	*TMSgjXpo7LeuFty8KqVjACKW1eSX1X9yniR*

`restr_SNoLev`

Theorem 21.221

$$\forall x.\mathsf{SNo}\ x \to \forall \alpha \in \mathsf{SNoLev}\ x.\mathsf{SNoLev}\ (x \cap \mathsf{SNoElts\_}\ \alpha) = \alpha.$$

The proposition is identified by the following information:

Pure Prop Id:	*cae8b53a142dc9bd4d328e356d5ea30fde7a5099b334cda244e99187209b8b99*
Pure Prop Address:	*TMS5WZkxehafSgkaqhUw2VJHacUTw9rwS1L*
Theory Prop Id:	*8b18538bf7d6bd1d9a848a9a9517a642deb5a218ec3d8a7caacd524d21dfd64a*
Theory Prop Address:	*TMJN6jAeeeX5njqn5SeaWsGAgEsFLV3XR4u*

`restr_SNoEq`

Theorem 21.222

$$\forall x.\mathsf{SNo}\ x \to \forall \alpha \in \mathsf{SNoLev}\ x.\mathsf{SNoEq\_}\ \alpha\ (x \cap \mathsf{SNoElts\_}\ \alpha)\ x.$$

The proposition is identified by the following information:

Pure Prop Id:	*829f8b1631e1372f4e65a822476f355e1c77a81a384b9cca1a4a8d552190fc4d*
Pure Prop Address:	*TMRgDXhz8Afh4H4FduKLm7tdh5mLBCuj1VX*
Theory Prop Id:	*704f5aec2f423db02021a4327ff03449e11de8691265c76b45d8784294ccc34c*
Theory Prop Address:	*TMZQtNrbPSFdXyaV4jvd6yZ7nzE191PFXRr*

restr_SNo_SNoCut

Theorem 21.223

$$\forall x.\mathsf{SNo}\ x \to \forall \alpha \in \mathsf{SNoLev}\ x.\forall p : o.$$
$$(\mathsf{SNoCutP}\ \{w \in \mathsf{SNoL}\ x | \mathsf{SNoLev}\ w \in \alpha\}\ \{z \in \mathsf{SNoR}\ x | \mathsf{SNoLev}\ z \in \alpha\} \to$$
$$x \cap \mathsf{SNoElts\_}\ \alpha = \mathsf{SNoCut}\ \{w \in \mathsf{SNoL}\ x | \mathsf{SNoLev}\ w \in \alpha\}\ \{z \in \mathsf{SNoR}\ x | \mathsf{SNoLev}\ z \in \alpha\}$$
$$\to p)$$
$$\to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*89a30eaccf26f7f6a96eaa958a6ba761b57a0f92bbecd3515a4233b677745527*
Pure Prop Address:	*TMEjpjmejvYuUCvTudHxbUChhzKXViX36A4*
Theory Prop Id:	*965ec873f3ef2f209dcc57482d884d08d8c7e0ccc428c255bb178794d09f6adc*
Theory Prop Address:	*TMJDSz4hS22ToFR3pnjsU5VSzcSsLte1yVF*

Definition 21.43 pack_e *is the opaque object of type* $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*dd8f2d332fef3b4d27898ab88fa5ddad0462180c8d64536ce201f5a9394f40dd*
Pure Object Address:	*TMZWdNrTTWaDgSjUhPMeWi1cahDuEhSHGWm*
Theory Object Id:	*02a4a758b68524d04d4d1b1347b10db0c244e6a85b8f75e8e031fe71887a51ea*
Theory Object Address:	*TMdVWrX6F4nw7pL1cx1YSLHmAWvAQPB5uyT*

pack_e_0_eq

Theorem 21.224 $\forall S X.\forall c : \iota.S = \mathsf{pack\_e}\ X\ c \to X = S\ 0.$ *The proposition is identified by the following information:*

Pure Prop Id:	*6ff73aba63bf83b93fdcd68f7f3b473ca425f7b7ecad18caa91ad3f3910771e8*
Pure Prop Address:	*TMF3p8wywFFn4tbwQTETRnnWJvQHuGheapA*
Theory Prop Id:	*00a7dba7aeb27089ddbf38d6a79dbefbcda54788a79f6438fdd968c1c97ac5b5*
Theory Prop Address:	*TMWi4vaXB8eNLtzU2oJt4rkxxuU1HmYd2Td*

pack_e_0_eq2

Theorem 21.225 $\forall X.\forall c : \iota.X = \mathsf{pack\_e}\ X\ c\ 0.$ *The proposition is identified by the following information:*

Pure Prop Id:	*e524616fe01439ccfbbdbe4f68ef616b835892e97fb13817028ebbf959c61f1f*
Pure Prop Address:	*TMNJYAKDBjmHn7HXSNC26jwp1s41xqC3inP*
Theory Prop Id:	*5904c567fc4acc3953bc24974e3fd7987245b906799d3f6bc8ba4f7f25608950*
Theory Prop Address:	*TMEr6GzQSymKEUjhZT6JqViiScEVyeyANgm*

`pack_e_1_eq`

Theorem 21.226 $\forall S X.\forall c : \iota.S = \mathsf{pack\_e}\ X\ c \to c = S\ 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*04f7f9a2df055819f0ee6e284dc3cf33cb7c8d5bca68b3910258bbbe11062c86*
Pure Prop Address:	*TMcNmrUzTXZFjuycPrfPwZEqsoyBeYPtDUL*
Theory Prop Id:	*a8d7da99aade4bcb64785eef8d10ca2392a20230bc7f92e5bde81d1b1470cddb*
Theory Prop Address:	*TMdRWR7pSiRTAZ3CjpuNGMk5aFmhMjmguaF*

`pack_e_1_eq2`

Theorem 21.227 $\forall X.\forall c : \iota.c = \mathsf{pack\_e}\ X\ c\ 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*9fdf3acdb0181694d27792d6b805cb1e854bc59fc0bff1362aa634222d642348*
Pure Prop Address:	*TMMriir6BTpwadY3sQTwcJ6UtGGpeNzruFP*
Theory Prop Id:	*571456f6d7bd181f2b902d4b966ebc6e588e0d89fa9a4ded6ad4bc4671514eaf*
Theory Prop Address:	*TMHqTypumR87KMZqxQykwqpxf2fLTDSKntV*

`pack_e_inj`

Theorem 21.228

$$\forall X X'.\forall c c'.\mathsf{pack\_e}\ X\ c = \mathsf{pack\_e}\ X'\ c' \to X = X' \wedge c = c'.$$

The proposition is identified by the following information:

Pure Prop Id:	*df5adc30165a60003f242f92d1c8fe389ba6bf0ef0a6d92e54e192638323f57f*
Pure Prop Address:	*TMbTR7tamHQov3r4vUKNrPZaM7iLEL5BaG9*
Theory Prop Id:	*0e751afc35195926201e850996d7021c60132bafee50cd3f7b50f532ad803b3f*
Theory Prop Address:	*TMXLaAMy8oAeTZ5Pr75ZnuM248oWLnFMqtq*

Definition 21.44 *We define* $\mathsf{struct\_e}$ *to be*

$$\lambda S.\forall q : \iota \to o.(\forall X : \iota.\forall c : \iota.c \in X \to q\ (\mathsf{pack\_e}\ X\ c)) \to q\ S$$

of type $\iota \to o$ *identified by the following information:*

Pure Object Id:	*d335d40e85580857cc0be3657b910d5bce202e0fab6875adec93e2fc5e5e57e4*
Pure Object Address:	*TMJWWWBc6dvdqjxZCTPUgLbN4aXWyUnq9bt*
Theory Object Id:	*59de6a6f7234e918742ecb64f84be2c8961660da62ee30ccf1365eab256ab3c0*
Theory Object Address:	*TMHJ9Le7KbYPQ93t9yavSzmxfaU78ZA7CZo*

`pack_struct_e_I`

Theorem 21.229 $\forall X.\forall c : \iota.c \in X \to \mathsf{struct\_e}\ (\mathsf{pack\_e}\ X\ c)$. *The proposition is identified by the following information:*

Pure Prop Id:	*91b51f2c281e33c4a047079e824292cc1811682fb17e3669dabee45608c97de2*
Pure Prop Address:	*TMakVN1dCuKrF2sSY6ifvjCSKdUn9w2h614*
Theory Prop Id:	*cc54f6f84b4c1ee3a02364985ad73f8f87488877bb3adf95720686a7b855b888*
Theory Prop Address:	*TMaSjR5JQ2VRSZZPHRxJNCLkwoNQtS9DD46*

pack_struct_e_E1

Theorem 21.230 $\forall X.\forall c:\iota.\mathsf{struct\_e}\ (\mathsf{pack\_e}\ X\ c) \to c \in X$. *The proposition is identified by the following information:*

Pure Prop Id:	*59653abe494ff81cf90bdae82d53b1f9a94dfc97f7ea3cb81712c80a9135ea4c*
Pure Prop Address:	*TMHRM45GC3aqyJTvtVy7hXGp2whYDye9QNF*
Theory Prop Id:	*550a75e54c585c80f080162a35f7e2478a8d8d392ae7b1a822044c132b01dd74*
Theory Prop Address:	*TMXc4Tt9w6ou7568GQpZHWHDita8JYAQUaJ*

struct_e_eta

Theorem 21.231 $\forall S.\mathsf{struct\_e}\ S \to S = \mathsf{pack\_e}\ (S\ 0)\ (S\ 1)$. *The proposition is identified by the following information:*

Pure Prop Id:	*fa919b3231deef7d4ef4a92653ceff69c800f9aa38bb03d41f4866f19342040d*
Pure Prop Address:	*TMUMVefsUP57yd7HoBLRFkwtdmzbuWgpQ5A*
Theory Prop Id:	*bdc4a9dce3edc9c64967e889aac3b081b436d30d2696fe495c04711a89f46239*
Theory Prop Address:	*TMbpmGPq3LT3TFSbSD5SYAp9JKZX2TbM5cp*

Definition 21.45 unpack_e_i *is the opaque object of type* $\iota \to (\iota \to \iota \to \iota) \to \iota$ *identified by the following information:*

Pure Object Id:	*91fff70be2f95d6e5e3fe9071cbd57d006bdee7d805b0049eefb19947909cc4e*
Pure Object Address:	*TMHRZBuSbhYF1JLr1Hs3TPKTCSk5Cr8Gnys*
Theory Object Id:	*fe41183ac024053e002b76da086e29818c4cac3a9cbc0104c1287b011dd6db98*
Theory Object Address:	*TMWRuKPw9UaH5vAfF8rtNPUaJrQyP5RPDCu*

unpack_e_i_eq

Theorem 21.232

$$\forall \Phi:\iota \to \iota \to \iota.\forall X.\forall c:\iota.\mathsf{unpack\_e\_i}\ (\mathsf{pack\_e}\ X\ c)\ \Phi = \Phi\ X\ c.$$

The proposition is identified by the following information:

Pure Prop Id:	*0a69b9763b0d02450063b2cb0f7352cf1dd65f6d2949e2ca0b1eec691efca645*
Pure Prop Address:	*TMXmwRkpi5UiyzYAcTTBq6WvNqKhaF2yCvj*
Theory Prop Id:	*98c7fcde5290487eba2c62e6af0c27ecf0bc59143da5bd14fca21628d4bed0ee*
Theory Prop Address:	*TMacpHNzxrWLsqUVSD652GArvNuF3vDpqCP*

Definition 21.46 unpack_e_o *is the opaque object of type* $\iota \to (\iota \to \iota \to o) \to o$ *identified by the following information:*

Pure Object Id:	*8e748578991012f665a61609fe4f951aa5e3791f69c71f5a551e29e39855416c*
Pure Object Address:	*TMPPKW3r2wphDdHkkrGqms7psPywvYzk8Zv*
Theory Object Id:	*c263bd473f626626582658a793421c02e2c9f3611ce4c4c4ca819f56cc256993*
Theory Object Address:	*TMUEDqJEoZmYehm2AX3Mv7koPW8daCwQirK*

unpack_e_o_eq

Theorem 21.233

$$\forall \Phi : \iota \to \iota \to o. \forall X. \forall c : \iota . \mathsf{unpack\_e\_o}\ (\mathsf{pack\_e}\ X\ c)\ \Phi = \Phi\ X\ c.$$

The proposition is identified by the following information:

Pure Prop Id:	*77684bc447f76184367525d0a412859ca7b0df8f1ddaa773da8fd0cf59b16bd3*
Pure Prop Address:	*TMJs57aPr7U7dVBfH8ahQ3aD8tBBNA1oDyb*
Theory Prop Id:	*b57b69660e83e8ccaf201f80e31bf02a3b6acbbac6d5fee57edfebc26325a9eb*
Theory Prop Address:	*TMcntFFtgaj5E7fiL6PwsfE3P6zHsyUTdwA*

Definition 21.47 pack_u *is the opaque object of type* $\iota \to (\iota \to \iota) \to \iota$ *identified by the following information:*

Pure Object Id:	*119d13725af3373dd508f147836c2eff5ff5acf92a1074ad6c114b181ea8a933*
Pure Object Address:	*TMZ2wxaB4wuhubUPHPk1gaizEbWbJzhsYbT*
Theory Object Id:	*3c7a5062deb09ce886a722a6a9cf08f879843995de990dfc0630269e3b8b19fa*
Theory Object Address:	*TMbVCB5AE2QsSb8QjhV8k61yAVcNXc6gybs*

pack_u_0_eq

Theorem 21.234 $\forall S X. \forall F : \iota \to \iota . S = \mathsf{pack\_u}\ X\ F \to X = S\ 0.$ *The proposition is identified by the following information:*

Pure Prop Id:	*e90120fb66c6e33b66df727e70deffca1cecf1c815f69ee141bf89989b70edd3*
Pure Prop Address:	*TMGVK5PJBrEie7g7JjvK6x8kgyoW4ZFtpwu*
Theory Prop Id:	*699268c2f39933c4fa055b801b3afb571c28045dae8bb5a6bbf0863f15e50660*
Theory Prop Address:	*TMUSVbwK29VTMzLPbxR5gXuNs6fXBzN4CnG*

pack_u_0_eq2

Theorem 21.235 $\forall X. \forall F : \iota \to \iota . X = \mathsf{pack\_u}\ X\ F\ 0.$ *The proposition is identified by the following information:*

Pure Prop Id:	*079fb39255923eff39682a3718d622c8a40d34674fdf95cc10f606920887dd81*
Pure Prop Address:	*TMKXKSi4pZddPxmhpCtcE6jh66s6fafGkNB*
Theory Prop Id:	*7eff940d0a6d0f8a81104b51146a0c8a93448c75c898a3382cc9544744401e18*
Theory Prop Address:	*TMLLQY93heT4wwbJsE7WPX6bYakekk5LE6a*

pack_u_1_eq

Theorem 21.236

$$\forall SX.\forall F:\iota\to\iota.S=\mathsf{pack\_u}\ X\ F\to\forall x\in X.F\ x=\mathsf{decode\_u}\ (S\ 1)\ x.$$

The proposition is identified by the following information:

Pure Prop Id:	*53d04c6be0df4b0a8a73f3e7962b6fd76f97d1b0e8aeb6753d8f2eceb28170d7*
Pure Prop Address:	*TMRRG8USJ8fkv5kE7Bjm2NXEz8mHWTTU7BF*
Theory Prop Id:	*9e80d0e685e0714450401020d0dda11302329314e7b5acad3049662aa8eb2796*
Theory Prop Address:	*TMGeytgzBv7rLRFtZPQUVZVT21LKHsmGiHi*

`pack_u_1_eq2`

Theorem 21.237

$$\forall X.\forall F:\iota\to\iota.\forall x\in X.F\ x=\mathsf{decode\_u}\ (\mathsf{pack\_u}\ X\ F\ 1)\ x.$$

The proposition is identified by the following information:

Pure Prop Id:	*7fca7f65973aa80d2bc3a1cb75163816f61e4d6d2dc4f5765d6e297e96abdae4*
Pure Prop Address:	*TMF37qTtWa8UwmQcuk4KkUAWcW9y1oiAjSP*
Theory Prop Id:	*c23d46052a337e70be0e9c4349faf01e97c334cd076ac5bb2026a88115fd2db1*
Theory Prop Address:	*TMdriRWWtDBheXF7Muhs1C7R3CgVDTDwMUM*

`pack_u_inj`

Theorem 21.238

$$\forall XX'.\forall FF':\iota\to\iota.\mathsf{pack\_u}\ X\ F=\mathsf{pack\_u}\ X'\ F'\to X=X'\wedge\forall x\in X.F\ x=F'\ x.$$

The proposition is identified by the following information:

Pure Prop Id:	*6b8b57fe8b77ff40918b3007f4711e989111aaafa13fccea532be661542391c6*
Pure Prop Address:	*TMF2hVeN68NYWiNVPfgquFhA14Cd2WfFA7h*
Theory Prop Id:	*6db3de9d9bc5bed83a7ab61a9d1445e0d1cca2c76182e965cc3d32fa08f1982c*
Theory Prop Address:	*TMP5sxvmCFkYyNJUa6v8DPY99xURLaEs34f*

`pack_u_ext`

Theorem 21.239

$$\forall X.\forall FF':\iota\to\iota.(\forall x\in X.F\ x=F'\ x)\to\mathsf{pack\_u}\ X\ F=\mathsf{pack\_u}\ X\ F'.$$

The proposition is identified by the following information:

Pure Prop Id:	*1aa49317bb1aafb72496316763 68bd94610841b4a1c159800bc86f2708d6bf6d*
Pure Prop Address:	*TMTX8evwr1dHzX2eGgpZhxbk3s1kai52w5w*
Theory Prop Id:	*981b39139ef57621b1bf8329aab2dea9fd20d3e11c2d121f340018c1b8ec9ada*
Theory Prop Address:	*TMVTvfHNDMHTzwo3D4fzu5ZXxnQc8m1VEZQ*

Definition 21.48 *We define* struct_u *to be*

$\lambda S.\forall q : \iota \to o.(\forall X.\forall F : \iota \to \iota.(\forall x \in X.F\ x \in X) \to q\ (\mathsf{pack\_u}\ X\ F)) \to q\ S$

of type $\iota \to o$ *identified by the following information:*

Pure Object Id:	*df14296f07f39c656a6467de607eb3ced299a130355447584c264061b5275b93*
Pure Object Address:	*TML4zt5QzeK1oosb9zeqYtcvb5j6UrWUQ83*
Theory Object Id:	*df0642555ff24c4e0093386810bb87b51992acb9f16b14c5ddacf37ee589c227*
Theory Object Address:	*TMcCXGRFwbxwpFTxpWqSgdFmjnxeAe1DkZn*

`pack_struct_u_I`

Theorem 21.240

$$\forall X.\forall F : \iota \to \iota.(\forall x \in X.F\ x \in X) \to \mathsf{struct\_u}\ (\mathsf{pack\_u}\ X\ F).$$

The proposition is identified by the following information:

Pure Prop Id:	*9ac1bfd471c8e0c07b3a8b8d8489ca22d34c772c104098df03fdc0f478c661f1*
Pure Prop Address:	*TMaiejfiKboUsvt1vPnNj7r4ZdJKsYHEBm9*
Theory Prop Id:	*9556db5aef8e77543f91bc07bf376517d9a6dee8137c27d45a1b83aa366667f6*
Theory Prop Address:	*TMKaRisyF4e8WBZv1N8ynvGUUuSkJVtA9ad*

`pack_struct_u_E1`

Theorem 21.241

$$\forall X.\forall F : \iota \to \iota.\mathsf{struct\_u}\ (\mathsf{pack\_u}\ X\ F) \to \forall x \in X.F\ x \in X.$$

The proposition is identified by the following information:

Pure Prop Id:	*98bace69f4799a59fdccc70ae5066e40837f3561cd06571f111486811be16df0*
Pure Prop Address:	*TMdoPRYZsVB9wjKk8Ri2fsV5T5ZW37aXDb2*
Theory Prop Id:	*7db00f022f515797b4fb0d821fe7489995399358fa823f779c3b601870d2a4d9*
Theory Prop Address:	*TMPmBmMcF9nNqTcrtPRKZjJNYvhvAYDxtgc*

`struct_u_eta`

Theorem 21.242

$$\forall S.\mathsf{struct\_u}\ S \to S = \mathsf{pack\_u}\ (S\ 0)\ (\mathsf{decode\_u}\ (S\ 1)).$$

The proposition is identified by the following information:

Pure Prop Id:	*0e0fc8f8e80ea783e6affd9347bd4f6d0900540ce0323735068c9fd37da332b6*
Pure Prop Address:	*TMbYJxMnDh5AP5uka6PSR5qdqZPbtHMnZT3*
Theory Prop Id:	*558efaeb63c323d6b2e00428a406b8ff6f02d42ae11b33350ebd4d63f3b15f18*
Theory Prop Address:	*TMZbzGyYczQAU2uwNtWM8TsVekdsMpRhw5A*

Definition 21.49 unpack_u_i *is the opaque object of type*

$$\iota \to (\iota \to (\iota \to \iota) \to \iota) \to \iota$$

identified by the following information:

Pure Object Id:	*111dc52d4fe7026b5a4da1edbfb7867d219362a0e5da0682d7a644a362d95997*
Pure Object Address:	*TMJ9S9HN4bipGWyfsvMApvMV12tYYa7YBTd*
Theory Object Id:	*9b53a2081c6da4b6da58cf2e2cd87b51c01cb3aad8d0198d435a1a60d5dae91a*
Theory Object Address:	*TMb9dRUCqQE14noMaQR9cGoqGSrgHMch1D6*

unpack_u_i_eq

Theorem 21.243

$$\forall \Phi : \iota \to (\iota \to \iota) \to \iota . \forall X . \forall F : \iota \to \iota .$$
$$(\forall F' : \iota \to \iota . (\forall x \in X . F\ x = F'\ x) \to \Phi\ X\ F' = \Phi\ X\ F) \to$$
$$\mathsf{unpack\_u\_i}\ (\mathsf{pack\_u}\ X\ F)\ \Phi = \Phi\ X\ F.$$

The proposition is identified by the following information:

Pure Prop Id:	*8381e9777cbe016dda8f9bcd84257b15371d3bdd4e26a59a561cc83844b28df2*
Pure Prop Address:	*TMGPuFBSKYSnC5Db2gcefmM9REhBKV3ciSg*
Theory Prop Id:	*133f42e59e5b3675fa1a64ca68c346da0148a3bcb8db86ae506fe34387ed80cc*
Theory Prop Address:	*TMMw9upsAnfEWn83orKSgX3D13dW8cXKsJ1*

Definition 21.50 unpack_u_o *is the opaque object of type*

$$\iota \to (\iota \to (\iota \to \iota) \to o) \to o$$

identified by the following information:

Pure Object Id:	*eb32c2161bb5020efad8203cd45aa4738a4908823619b55963cc2fd1f9830098*
Pure Object Address:	*TMHXEyss3bgpR5WZwP9nbSJUGVKgpx8hR7D*
Theory Object Id:	*8b490cedc3faf1e839d758df391095fb43853467123acd68380ad2bbd213e5d7*
Theory Object Address:	*TMbyJnusRWuvLh37RbmVLuA1HeDayMwKt84*

unpack_u_o_eq

Theorem 21.244

$$\forall \Phi : \iota \to (\iota \to \iota) \to o . \forall X . \forall F : \iota \to \iota .$$
$$(\forall F' : \iota \to \iota . (\forall x \in X . F\ x = F'\ x) \to \Phi\ X\ F' = \Phi\ X\ F) \to$$
$$\mathsf{unpack\_u\_o}\ (\mathsf{pack\_u}\ X\ F)\ \Phi = \Phi\ X\ F.$$

The proposition is identified by the following information:

Pure Prop Id:	*a862c11904e8a98362cb2ca4a94b3e16aea77824a968c54d67fdef41dbf14208*
Pure Prop Address:	*TMbGK4qZjEcqzu2hydjMJie8FfBg6EAAg2S*
Theory Prop Id:	*3daff9f51b582e2a164301f59f8428029a2d119064e12ffa327165a9612fbdfa*
Theory Prop Address:	*TMJJCNJVx3YKK8m4XGTUTRRXe5o7Hf4aW9V*

Definition 21.51 pack_b *is the opaque object of type* $\iota \to (\iota \to \iota \to \iota) \to \iota$ *identified by the following information:*

Pure Object Id:	*855387af88aad68b5f942a3a97029fcd79fe0a4e781cdf546d058297f8a483ce*
Pure Object Address:	*TMMVdjfJFDf8CbzqwNoQ3mUFc4V83Wt6E2j*
Theory Object Id:	*965d60e636461e17d0b088d4ef59f38d71f00297a7cae99d4a96cffeca6ece15*
Theory Object Address:	*TMYH1EdF9fP7KtMSudvx2S1z6Dg93V2gFPZ*

pack_b_0_eq

Theorem 21.245 $\forall SX.\forall F : \iota \to \iota \to \iota . S = \mathsf{pack\_b}\ X\ F \to X = S\ 0.$ *The proposition is identified by the following information:*

Pure Prop Id:	*88fe38e1e3d1bdd87d28927800e97536c017c5b945ed6ee12835147827096678*
Pure Prop Address:	*TMPXf5qRUESrQFHe3eEvNw8NfS5DMZgoyEd*
Theory Prop Id:	*71d9298b5c2a83f3a1c602ceb0f739f9dcf08025e28432169e400a5bcb826cf0*
Theory Prop Address:	*TMPunAXGyfUGF19jJ33E8oNYMXCDFxt7kMg*

pack_b_0_eq2

Theorem 21.246 $\forall X.\forall F : \iota \to \iota \to \iota . X = \mathsf{pack\_b}\ X\ F\ 0.$ *The proposition is identified by the following information:*

Pure Prop Id:	*149dd8442141d0c1c06226a4b7125eda7c7a5d109dbeccc3ea0c8d1839f3838d*
Pure Prop Address:	*TMcbsnvdHZJbA47etXzNDVwQbBnkePS3ghy*
Theory Prop Id:	*9bf81b992e14b803ddd19aefb763a8770611c9c5fa17470effe652aab3c9183e*
Theory Prop Address:	*TMUryUSP1YfaUUeoeisdwCd4DRwdF8qmqUb*

pack_b_1_eq

Theorem 21.247

$$\forall SX.\forall F : \iota \to \iota \to \iota . S = \mathsf{pack\_b}\ X\ F \to \forall xy \in X . F\ x\ y = \mathsf{decode\_b}\ (S\ 1)\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*d3a61aaa2c6fa85326d0d1fd27c8f573ecc708ec4d76ad1a1abe5e6bbbd6abd3*
Pure Prop Address:	*TMdzKNDCM8ocaiRdtLkRwCDFiqddCD9st6w*
Theory Prop Id:	*21712e8eea1a4e0f13bd9f8e8190b72c53104fcbeadbebef78accf2b3b33b851*
Theory Prop Address:	*TMS4NWDzzDuvWpiWKuvFVqLtaZ3ZcX8WtUc*

pack_b_1_eq2

Theorem 21.248

$$\forall X.\forall F : \iota \to \iota \to \iota. \forall xy \in X. F\ x\ y = \mathsf{decode\_b}\ (\mathsf{pack\_b}\ X\ F\ 1)\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*454eade5d6916e25226236256e11334f175b7f7cd644b120cdad3d7dd5c5e6e9*
Pure Prop Address:	*TMWdWKh8rp339UeC9sDaWqD8eAVpLX7Z5iS*
Theory Prop Id:	*d1a9a05acd0ffe92475f293616460f7d80da98dc26534cad8a38968507a1ae00*
Theory Prop Address:	*TMLZ8co6rk5aziSxSmJeTMBZPFjrDuvVx1w*

pack_b_inj

Theorem 21.249

$$\forall XX'.\forall FF' : \iota \to \iota \to \iota. \mathsf{pack\_b}\ X\ F = \mathsf{pack\_b}\ X'\ F' \to X = X' \land \forall xy \in X. F\ x\ y = F'\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*4ac18a20f5c60c34580cd550af2aff402c9f508b648a114d3e5e2ea05ef2375a*
Pure Prop Address:	*TMT3PAPfzsJZYdkFmXBPKRriTJ16RQbe8yt*
Theory Prop Id:	*e7c2d83ccf845c0d90123e0d147039f6eba9174a7529219510a7421b37427b1f*
Theory Prop Address:	*TMXz4wfFqKWJRm9mNE9tXUB4V3tWeTeWyYk*

pack_b_ext

Theorem 21.250

$$\forall X.\forall FF' : \iota \to \iota \to \iota. (\forall xy \in X. F\ x\ y = F'\ x\ y) \to \mathsf{pack\_b}\ X\ F = \mathsf{pack\_b}\ X\ F'.$$

The proposition is identified by the following information:

Pure Prop Id:	*6d6939a2b043243248ceb1ea23d9b40ec6bc8fb261736d1831d9e10f23783a31*
Pure Prop Address:	*TMaHYCJSjfZiY7qjR4iN2RGbjhwh6EVLJYN*
Theory Prop Id:	*3ac95dba16cba3dbbc90bc1fca2e869e6936625f308470e76d356473cba7627a*
Theory Prop Address:	*TMK2351yJNApHnAS7155Cuqed26ESErCWJa*

Definition 21.52 *We define* struct_b *to be*

$$\lambda S.\forall q : \iota \to o.(\forall X : \iota.\forall F : \iota \to \iota \to \iota.(\forall xy \in X.F\ x\ y \in X) \to q\ (\mathsf{pack\_b}\ X\ F)) \to q\ S$$

of type $\iota \to o$ *identified by the following information:*

Pure Object Id:	*54dbd3a1c5d7d09ab77e3b5f4c62ce5467ada0cf451d47ee29b7359cc44017cc*
Pure Object Address:	*TMEse5C6BhcvrdqN1kLCmCVchTMXrGpcjTH*
Theory Object Id:	*ec21dab5d2ad24a500e876b5370a93fef45587d8ab5c00a38135c1c00796c9eb*
Theory Object Address:	*TMdmavgqTvcPWn1s1jXYNzYjU9nxbzEFY2H*

pack_struct_b_I

Theorem 21.251

$$\forall X.\forall F:\iota\to\iota\to\iota.(\forall xy\in X.F\ x\ y\in X)\to\mathsf{struct\_b}\ (\mathsf{pack\_b}\ X\ F).$$

The proposition is identified by the following information:

Pure Prop Id:	*83f82c2d40c9edc96f854042ffdcba40bb79fc137957f7a157a7a521b94a3b29*
Pure Prop Address:	*TMcH7M63CKv6BPcLYnrSwWU8dgENp9N8WyC*
Theory Prop Id:	*0ca86fc209d99165c44f41c8c4242bd06e0bf4fe4ba50f5451a78c59d396264d*
Theory Prop Address:	*TMLzvnmLiuZxd4WpcgtmT4HzkYeu9oJmPW3*

pack_struct_b_E1

Theorem 21.252

$$\forall X.\forall F:\iota\to\iota\to\iota.\mathsf{struct\_b}\ (\mathsf{pack\_b}\ X\ F)\to\forall xy\in X.F\ x\ y\in X.$$

The proposition is identified by the following information:

Pure Prop Id:	*e76b711f4da2dc7324a0ca8bbaa3e9a0a1776119d6fc68ddc487c19f1b3b6ef8*
Pure Prop Address:	*TMUep5KWt973mJbiAFReu4N3fD7JzxVBrgG*
Theory Prop Id:	*a2f525ee0bd09ac467ed896b71a426bb93c2628f11efb722b69cb07e276d724b*
Theory Prop Address:	*TMHG4dTesLSYiYcwpuDT1tcpdD6a4fPzy5F*

struct_b_eta

Theorem 21.253

$$\forall S.\mathsf{struct\_b}\ S\to S=\mathsf{pack\_b}\ (S\ 0)\ (\mathsf{decode\_b}\ (S\ 1)).$$

The proposition is identified by the following information:

Pure Prop Id:	*536ed4571cfde24c5f0dc64b9e9ffd2b657f23afd3f82086d621030805fb5114*
Pure Prop Address:	*TMPEVs55RYKYcnco5aHg1EiBp9jRoaqAA6G*
Theory Prop Id:	*575bcffb4afcd6c37db437a9a89df09d1b9daa8bc2638184256049e29841eeb2*
Theory Prop Address:	*TMSYCVHcDX5UQYtD2RKHK5UWHY2zE6APwEV*

Definition 21.53 unpack_b_i *is the opaque object of type*

$$\iota\to(\iota\to(\iota\to\iota\to\iota)\to\iota)\to\iota$$

identified by the following information:

Pure Object Id:	*b3bb92bcc166500c6f6465bf41e67a407d3435b2ce2ac6763fb02fac8e30640e*
Pure Object Address:	*TMNZmh6j71sWmZmpeCQLCVGUVXxgvXyatB5*
Theory Object Id:	*b47c344128564d38b19aee94b230561e50981b9bc34aeabcd1027b52d36240d4*
Theory Object Address:	*TMTJ1tRG35mQbCDWChzhKgGESVZZrLHceGP*

`unpack_b_i_eq`

Theorem 21.254

$$\forall \Phi : \iota \to (\iota \to \iota \to \iota) \to \iota . \forall X . \forall F : \iota \to \iota \to \iota . \\ (\forall F' : \iota \to \iota \to \iota . (\forall xy \in X . F\ x\ y = F'\ x\ y) \to \Phi\ X\ F' = \Phi\ X\ F) \to \\ \mathsf{unpack\_b\_i}\ (\mathsf{pack\_b}\ X\ F)\ \Phi = \Phi\ X\ F .$$

The proposition is identified by the following information:

Pure Prop Id:	*0aa7882234fc6912b6b3e4f22d40592682f0ceb44c55fca9b6028e9957ed26ba*
Pure Prop Address:	*TMPfkFXN7gTRe1Y9msSvdvswaFPKLKJsqsF*
Theory Prop Id:	*659ca7a7505887c37bfdaad5fbda8fffb6e2e6f602ffd247af88305eb1f4a862*
Theory Prop Address:	*TMNHSFWaR67WngH7FgbwYKUvfmTNTF5riaM*

Definition 21.54 $\mathsf{unpack\_b\_o}$ *is the opaque object of type*

$$\iota \to (\iota \to (\iota \to \iota \to \iota) \to o) \to o$$

identified by the following information:

Pure Object Id:	*0601c1c35ff2c84ae36acdecfae98d553546d98a227f007481baf6b962cc1dc8*
Pure Object Address:	*TMFT42eJTKM7rrEafjCbY27VvguwAhgSpBS*
Theory Object Id:	*bea45508f7d246468c085f3f7525bacaaeae7831061f927ae066064ce4ebadd6*
Theory Object Address:	*TMcFFPrXTWYs7rZkur73nsizTsDbGiwvHnj*

`unpack_b_o_eq`

Theorem 21.255

$$\forall \Phi : \iota \to (\iota \to \iota \to \iota) \to o . \forall X . \forall F : \iota \to \iota \to \iota . \\ (\forall F' : \iota \to \iota \to \iota . (\forall xy \in X . F\ x\ y = F'\ x\ y) \to \Phi\ X\ F' = \Phi\ X\ F) \to \\ \mathsf{unpack\_b\_o}\ (\mathsf{pack\_b}\ X\ F)\ \Phi = \Phi\ X\ F .$$

The proposition is identified by the following information:

Pure Prop Id:	*36249217e9f4af2de97729e96889f2f46a4aee8abae10f4a7b6eeac7725e8565*
Pure Prop Address:	*TMVcr4vjHzNUUr7gkxQ7RQc49JkgGGy3YGA*
Theory Prop Id:	*d10e77888bbcf1e2f90eec0314e98757bb89a0d253b83086addf858363f79c3b*
Theory Prop Address:	*TMXR7UKpoi2iubM8VrTd9gbFCVWHtV2vx6V*

Definition 21.55 $\mathsf{pack\_p}$ *is the opaque object of type* $\iota \to (\iota \to o) \to \iota$ *identified by the following information:*

Pure Object Id:	*d5afae717eff5e7035dc846b590d96bd5a7727284f8ff94e161398c148ab897f*
Pure Object Address:	*TMZKq2rBy4uupXawQMQLzwHsbyNXwKbrHtK*
Theory Object Id:	*327f9f7a1cd5c793e02cabe2c78bd943eebf7e17640b5569cb69b7c6e867d288*
Theory Object Address:	*TMaXW2e5TX9atqycshV6aWebjBHBw3qBaDQ*

pack_p_0_eq

Theorem 21.256 $\forall SX.\forall P : \iota \to o.S = \mathsf{pack\_p}\ X\ P \to X = S\ 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*3e2bc1d01adbc1e08fb9a40b80d1078f5686a19087b5dc6a4a28f13ce1ed245d*
Pure Prop Address:	*TMcoBzsTRTrztFDNo8R9KfFyzrSSNLQi8J1*
Theory Prop Id:	*ab13889079587ea5409a49e8143a5bc1d13215ca88c52957afee0795cb787e1c*
Theory Prop Address:	*TMLxrx1gZmbDxCm2cf8kvHNCEjJYYTKG1cH*

pack_p_0_eq2

Theorem 21.257 $\forall X.\forall P : \iota \to o.X = \mathsf{pack\_p}\ X\ P\ 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*11696272b13bb134c1f814d0377d8f961c2444ec3276903b6197cb8b5c617a6e*
Pure Prop Address:	*TMRPLYPYL8Gsv7Hv3ycXTxChYbDMhuZEaAd*
Theory Prop Id:	*fac8f41e2043963665d5308100a90c3cd449fe7866184fd601f0987c8846c9b8*
Theory Prop Address:	*TMWKukhfMDzMZttRystq1Qgu6vLp3c8H2Bp*

pack_p_1_eq

Theorem 21.258

$$\forall SX.\forall P : \iota \to o.S = \mathsf{pack\_p}\ X\ P \to \forall x \in X.P\ x = \mathsf{decode\_p}\ (S\ 1)\ x.$$

The proposition is identified by the following information:

Pure Prop Id:	*e77d74a3d40ec36d1d39189b6f349ea6192ca304bb30166c7982e27ab5f4463d*
Pure Prop Address:	*TMZx1CjFBL7v9V4kJAPoMUFc6MQWeEFyAxE*
Theory Prop Id:	*8d778314b90e2c5d3ca7e90a948c37b0116287bf590445df492647685304f11c*
Theory Prop Address:	*TMSC2Fhr34MeQfa9SHEFBMjJpxN3mr5w7x6*

pack_p_1_eq2

Theorem 21.259

$$\forall X.\forall P : \iota \to o.\forall x \in X.P\ x = \mathsf{decode\_p}\ (\mathsf{pack\_p}\ X\ P\ 1)\ x.$$

The proposition is identified by the following information:

Pure Prop Id:	*edd77250b9082459c51c2404779121d28df39720c403c1ba615bdb98cea24155*
Pure Prop Address:	*TMZkKdo5noTu4SrYs3Y9ypPqsY1t9ZBbkQj*
Theory Prop Id:	*675cc21279443bb463c127734eaa6a938e37ac42884e7189535cb2f3f9554e3f*
Theory Prop Address:	*TMS3zfU1RcgvXFsbKoXKLfUi5bvRNheLzFV*

pack_p_inj

Theorem 21.260

$$\forall XX'.\forall PP' : \iota \to o.\mathsf{pack\_p}\ X\ P = \mathsf{pack\_p}\ X'\ P' \to X = X' \wedge \forall x \in X.P\ x = P'\ x.$$

The proposition is identified by the following information:

Pure Prop Id:	*447155bb83c20b64c9aa17c73431b30e27f1182f50ed0138f70ee0f5dd7546a7*
Pure Prop Address:	*TMdpN21GTqfTxNJNdjzawg4vyzWqRoBABY4*
Theory Prop Id:	*123b7446fa7f51381b466df69493c943b07297a406a23d2837d853425f1b81c5*
Theory Prop Address:	*TMZezSHn85gE3VL3kzSaUAnpeLhgzoznQxZ*

`pack_p_ext`

Theorem 21.261

$$\forall X.\forall PP' : \iota \to o.(\forall x \in X.P\ x \Leftrightarrow P'\ x) \to \mathsf{pack\_p}\ X\ P = \mathsf{pack\_p}\ X\ P'.$$

The proposition is identified by the following information:

Pure Prop Id:	*e20088be64022ef4177b25ae43ecbe098bcd4ec9889eafa67356e50f074d9b3c*
Pure Prop Address:	*TMQHBKHTocgt95FDHHgmVG6terquvvMXAX7*
Theory Prop Id:	*6327591dd89a183819d2315a107277c5ff966837b4658a1900a8927c37586a95*
Theory Prop Address:	*TMLRUuJpnLfZbAuashKkp6abZVYuyhErHms*

Definition 21.56 *We define* $\mathsf{struct\_p}$ *to be*

$$\lambda S.\forall q : \iota \to o.(\forall X : \iota.\forall P : \iota \to o.q\ (\mathsf{pack\_p}\ X\ P)) \to q\ S$$

of type $\iota \to o$ *identified by the following information:*

Pure Object Id:	*b927035076bdb3f2b856017733102a46ad6a0c52f1f4808b6c0cc6bde29328b6*
Pure Object Address:	*TMRQShbXQEMogWKiAvjj4S23VY1PBpfLExm*
Theory Object Id:	*5b2817559944fc0fdd6a8750018508b650f31cb6af63d767ded0c8472309f282*
Theory Object Address:	*TMHTvYZ6KkSUTR8CfNH2bmXGhb7SHNNrJWd*

`pack_struct_p_I`

Theorem 21.262 $\forall X.\forall P : \iota \to o.\mathsf{struct\_p}\ (\mathsf{pack\_p}\ X\ P)$. *The proposition is identified by the following information:*

Pure Prop Id:	*cb62efe4da65c84588ce56c8e842510106a18f82c93832ddb1e2a2acbd57adcf*
Pure Prop Address:	*TMUKcJ75mxMagfVFQFZmrB8tJdXNnpRBhMM*
Theory Prop Id:	*d2435fc55b93c69cbafe0cf9620c74888bf5617bb7d8f276f90ee91956e13a67*
Theory Prop Address:	*TMFUi9YSFkPJLd7uyyf4idZheyd2iSc8y4U*

`struct_p_eta`

Theorem 21.263

$$\forall S.\mathsf{struct\_p}\ S \to S = \mathsf{pack\_p}\ (S\ 0)\ (\mathsf{decode\_p}\ (S\ 1)).$$

The proposition is identified by the following information:

Pure Prop Id:	*28d313b94faf78be69428bf15c4bdc56ac3b3ede6ea32a5d46da14eb79f02556*
Pure Prop Address:	*TMGg8n9XFwykSXD4FQ2Q9USayRvoceEmnsL*
Theory Prop Id:	*b1c3ea7aa5b4e91e8d35f1c894d953beb735090d5a08ce1508df140c7731f22a*
Theory Prop Address:	*TMGh2mx2CkQeB1Xsb3gcjYRXiW8X2Wmjik9*

Definition 21.57 unpack_p_i *is the opaque object of type*

$$\iota \to (\iota \to (\iota \to o) \to \iota) \to \iota$$

identified by the following information:

Pure Object Id:	*dda44134499f98b412d13481678ca2262d520390676ab6b40e0cb0e0768412f6*
Pure Object Address:	*TMd8rM7X2eKWiHzb3iCj8EMQSAmUxWqsTAS*
Theory Object Id:	*02aa6a6b1d35a6d31321d714052e48ac29520943baf0867d4b7c81047c04b1df*
Theory Object Address:	*TMUMBD9eJTd7hghvMZ9kDCBBszGxHy9JpsM*

`unpack_p_i_eq`

Theorem 21.264

$$\begin{gathered}\forall \Phi :\iota \to (\iota \to o) \to \iota.\forall X.\forall P :\iota \to o.\\ (\forall P' : \iota \to o.(\forall x \in X.P\ x \Leftrightarrow P'\ x) \to \Phi\ X\ P' = \Phi\ X\ P)\to\\ \mathsf{unpack\_p\_i}\ (\mathsf{pack\_p}\ X\ P)\ \Phi = \Phi\ X\ P.\end{gathered}$$

The proposition is identified by the following information:

Pure Prop Id:	*320b78af7b08593888681ec40a36b6b9beffa499779c5c98d98c054e07741e79*
Pure Prop Address:	*TMLCGUfRCP9vQnp7GJB41cjD4Vrux4buaBB*
Theory Prop Id:	*2653932c0e7cc4c5374e965bdab745a870e265ec1529e1be48b856200430735c*
Theory Prop Address:	*TMUXbW94tZQYgwiXWFCbc4aLdmWdtpXkDaR*

Definition 21.58 unpack_p_o *is the opaque object of type*

$$\iota \to (\iota \to (\iota \to o) \to o) \to o$$

identified by the following information:

Pure Object Id:	*30f11fc88aca72af37fd2a4235aa22f28a552bbc44f1fb01f4edf7f2b7e376ac*
Pure Object Address:	*TMWT49Ai1jUqLkomFLu5DJZESHDY57rpkpp*
Theory Object Id:	*d42ffe55ca758d1a2808e2bf09432336b6012dd98176642489f5b352163c839b*
Theory Object Address:	*TML2ogMCitPSn4cnkwyxFywgE1nHMrQtk2b*

`unpack_p_o_eq`

Theorem 21.265

$$\forall \Phi : \iota \to (\iota \to o) \to o. \forall X. \forall P : \iota \to o.$$
$$(\forall P' : \iota \to o. (\forall x \in X. P\ x \Leftrightarrow P'\ x) \to \Phi\ X\ P' = \Phi\ X\ P) \to$$
$$\mathsf{unpack\_p\_o}\ (\mathsf{pack\_p}\ X\ P)\ \Phi = \Phi\ X\ P.$$

The proposition is identified by the following information:

Pure Prop Id:	*ff2e0a66eb757950b800c844729a119d800324b515bcf1715aa55d8b539d2bc7*
Pure Prop Address:	*TMLWdSrJohgGH4MnsHuPzU99KXhMztfBiB8*
Theory Prop Id:	*91b507a8afecc71b5b3acab9f63a3baddd24e6d5cfff2801c72bf2ba365cebf8*
Theory Prop Address:	*TMa2dnR7HWNn2drguqdGnbBgzCEo6PraPi3*

Definition 21.59 pack_r *is the opaque object of type* $\iota \to (\iota \to \iota \to o) \to \iota$ *identified by the following information:*

Pure Object Id:	*217a7f92acc207b15961c90039aa929f0bd5d9212f66ce5595c3af1dd8b9737e*
Pure Object Address:	*TMUctEexwaiGRFvuiMsx3Luij4T9hycQFR1*
Theory Object Id:	*653348825eafd6be52649491cbf9da0b90728ef636545f2fb600630bb8ede9d7*
Theory Object Address:	*TMN3CVcVBoGZd48wWhbqzV4YvsURA2XpsYf*

`pack_r_0_eq`

Theorem 21.266 $\forall S X. \forall R : \iota \to \iota \to o. S = \mathsf{pack\_r}\ X\ R \to X = S\ 0.$ *The proposition is identified by the following information:*

Pure Prop Id:	*1a61666ba8ed6054918adb678e03f4f0f4e52214f5fadb9c9f34e65f12236f35*
Pure Prop Address:	*TMPmyRQDsArPocUGo2q8pp2H9XidTrdzpim*
Theory Prop Id:	*aa16a193ecac4ece1fe16115e1798004924cd7546cb19979d6a048737c8c576c*
Theory Prop Address:	*TMNVChZMMuxdfe2iosJrMwXRxkaeUcZ4kvc*

`pack_r_0_eq2`

Theorem 21.267 $\forall X. \forall R : \iota \to \iota \to o. X = \mathsf{pack\_r}\ X\ R\ 0.$ *The proposition is identified by the following information:*

Pure Prop Id:	*e9f3791f72bbaf693f3addf117ed97ffe7d9c8870838c07f30d122aca64e046a*
Pure Prop Address:	*TMYru4HRQErL3eWLnb1NqojeVchvXn9Xjn6*
Theory Prop Id:	*09249bf17addc69ba6832be8e5f9b66a70f21059e88e762e548b6a234e305fba*
Theory Prop Address:	*TMWSrqYbCVb7RWvvsJJK5qwBoNom14geqRG*

`pack_r_1_eq`

Theorem 21.268

$$\forall S X. \forall R : \iota \to \iota \to o. S = \mathsf{pack\_r}\ X\ R \to \forall x y \in X. R\ x\ y = \mathsf{decode\_r}\ (S\ 1)\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*ae868a8d8478c46e95f6c4e452385c0ab754ed15d225e4264fb7670471ce9a56*
Pure Prop Address:	*TMPd9JAAJq5xDzmCczLyPedvrgGVdat5jz5*
Theory Prop Id:	*0bcee669141558ba17e2cbffb4dad25f53e895091c050e4ba640aade835538ec*
Theory Prop Address:	*TMP9bEFP2SXcQRXBRBqazJ4BAu3Ltzc4nFL*

`pack_r_1_eq2`

Theorem 21.269

$$\forall X.\forall R :\iota \to \iota \to o.\forall xy \in X.R\ x\ y = \mathsf{decode\_r}\ (\mathsf{pack\_r}\ X\ R\ 1)\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*673e8ce75e264e65efac5d4112b07bf2f956aba214e048e5976a0c7d4f26c86b*
Pure Prop Address:	*TMJsAy3ztkMuG5mtkuA7BGdKzRUaKs2tppt*
Theory Prop Id:	*bf564258305254769de120596e1680500b51275da445c8e6331abbea10f5cc33*
Theory Prop Address:	*TMKdqYogPTdzaPhkJNLJz4RJYuFFHgZ97g5*

`pack_r_inj`

Theorem 21.270

$$\forall XX'.\forall RR' :\iota \to \iota \to o.\mathsf{pack\_r}\ X\ R = \mathsf{pack\_r}\ X'\ R' \to X = X' \wedge \forall xy \in X.R\ x\ y = R'\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*7f2df97fb2f06e6818243ca4aa3a49505e174015422b4165e4c0e85797545d17*
Pure Prop Address:	*TMYtfZhkJBGMfEfGyEJ4nLXSF6xrqbv7xVn*
Theory Prop Id:	*3cd95991edbbb560c316012eca8622649a1b54474a30700935008c0e6f36b321*
Theory Prop Address:	*TMLB6JT8F1NtoYVen4ywitVmvi85JmSkvXH*

`pack_r_ext`

Theorem 21.271

$$\forall X.\forall RR' :\iota \to \iota \to o.(\forall xy \in X.R\ x\ y \Leftrightarrow R'\ x\ y) \to \mathsf{pack\_r}\ X\ R = \mathsf{pack\_r}\ X\ R'.$$

The proposition is identified by the following information:

Pure Prop Id:	*2f6a43369281b532971499e36bcc96d546582f450723f54cdd7373e9206c145e*
Pure Prop Address:	*TMFKB6PjAimGYtZ9yNw7JAPcySU1UJJ8zFa*
Theory Prop Id:	*94a5d9602cf57a3f5654fc82347cce52c34639293e929a5536fb236f16c4a584*
Theory Prop Address:	*TMNkz5vYZPjFcLYn1uGSKSZPVSM8RjwnAry*

Definition 21.60 *We define* struct_r *to be*

$$\lambda S.\forall q : \iota \to o.(\forall X : \iota.\forall R : \iota \to \iota \to o.q\ (\mathsf{pack\_r}\ X\ R)) \to q\ S$$

of type $\iota \to o$ *identified by the following information:*

Pure Object Id:	*2c39441e10ec56a1684beb291702e235e2a9e44db516e916d62ce426108cfd26*
Pure Object Address:	*TMcKwAb1Y239cNAKRVhgscrwaw8GNLJkYPp*
Theory Object Id:	*62b6df4a9733ff8dc3d14aedd832787d9847bd18b34fcf2c6bff876818a1847d*
Theory Object Address:	*TMYJ8WJhMWwP81uepXsK3pFf4gfjynNf86z*

`pack_struct_r_I`

Theorem 21.272 $\forall X.\forall R : \iota \to \iota \to o.\mathsf{struct\_r}\ (\mathsf{pack\_r}\ X\ R)$. *The proposition is identified by the following information:*

Pure Prop Id:	*b8a6bd24313c25cdf6f5d780f72c0aa8fc8d00573ca62ac10879aab64b058d71*
Pure Prop Address:	*TMLYEZeYcGNJ17buuXRfr7jgkECAGmexZCE*
Theory Prop Id:	*e305fee3cf88a27e70cab3757abe5760dbb2dfa53a2681cf0007eae77a1ac0a5*
Theory Prop Address:	*TMN1pwx29rgBN5sqW8xRekY99KzuFAfTFfC*

`struct_r_eta`

Theorem 21.273

$$\forall S.\mathsf{struct\_r}\ S \to S = \mathsf{pack\_r}\ (S\ 0)\ (\mathsf{decode\_r}\ (S\ 1)).$$

The proposition is identified by the following information:

Pure Prop Id:	*84273bb2eff84eb8f3c9054433916df932828a737f7c3ca3ab694990398c765d*
Pure Prop Address:	*TMV5JzEiDcbVXBiiQYZZiJfEwYjacKQ4MxV*
Theory Prop Id:	*a9afdff67a397303a2b3281aa9aec54d7e19dc89f19756ffe0b41bed021a4e74*
Theory Prop Address:	*TMMrznhQDCUW9B4yjAoMNMThp6vbuGof6jH*

Definition 21.61 unpack_r_i *is the opaque object of type*

$$\iota \to (\iota \to (\iota \to \iota \to o) \to \iota) \to \iota$$

identified by the following information:

Pure Object Id:	*3ace9e6e064ec2b6e839b94e81788f9c63b22e51ec9dec8ee535d50649401cf4*
Pure Object Address:	*TMN9skP6MgqmkRJ4cwifVmq6ZbEz41jqRV2*
Theory Object Id:	*076245d35f8b67c6c189c5aef9cfc61ae9e0e18184f6385bcb6a3dbdd97661c4*
Theory Object Address:	*TMJh6uB6VQPwpShTzHxgdReiKesNatUru1Z*

`unpack_r_i_eq`

Theorem 21.274

$$\begin{gathered}\forall \Phi : \iota \to (\iota \to \iota \to o) \to \iota.\forall X.\forall R : \iota \to \iota \to o. \\ (\forall R' : \iota \to \iota \to o.(\forall xy \in X.R\ x\ y \Leftrightarrow R'\ x\ y) \to \Phi\ X\ R' = \Phi\ X\ R) \to \\ \mathsf{unpack\_r\_i}\ (\mathsf{pack\_r}\ X\ R)\ \Phi = \Phi\ X\ R.\end{gathered}$$

The proposition is identified by the following information:

Pure Prop Id:	*16a31beed02eceef7aa5ad8abd3e417faefd8f5add89a3ffc21c18be48b02225*
Pure Prop Address:	*TMNMHd4hurTht88FezFD988n9Kwz38gbmMT*
Theory Prop Id:	*9d8a6e472c07d73be0ef8a14ffae4a0b986fb8834986991cc7f1aaabb3f3bce2*
Theory Prop Address:	*TMNFGGSBwtHNzfP26abSeRhNQbNe6qxGKxv*

Definition 21.62 unpack_r_o *is the opaque object of type*

$$\iota \to (\iota \to (\iota \to \iota \to o) \to o) \to o$$

identified by the following information:

Pure Object Id:	*e3e6af0984cda0a02912e4f3e09614b67fc0167c625954f0ead4110f52ede086*
Pure Object Address:	*TMZmDPx4PJagAzuKD7GKyixmxfa7zoZhWLS*
Theory Object Id:	*6bcb7cdbf5bd449df3c8deb49a0d9cdf76962583716fcb71c59d999f4bcc3fca*
Theory Object Address:	*TMR5CinBdVnunwxcK6781T5WjV1NMdFfYig*

`unpack_r_o_eq`

Theorem 21.275

$$\forall \Phi : \iota \to (\iota \to \iota \to o) \to o. \forall X. \forall R : \iota \to \iota \to o.$$
$$(\forall R' : \iota \to \iota \to o. (\forall xy \in X. R\ x\ y \Leftrightarrow R'\ x\ y) \to \Phi\ X\ R' = \Phi\ X\ R) \to$$
$$\mathsf{unpack\_r\_o}\ (\mathsf{pack\_r}\ X\ R)\ \Phi = \Phi\ X\ R.$$

The proposition is identified by the following information:

Pure Prop Id:	*2033874915fc7ab80e92006c39e273588d52da2c880d44c7a1709b538dcbf75e*
Pure Prop Address:	*TMbXxr57ewDhvFG8is2EuTAvHskooz91px9*
Theory Prop Id:	*52c8fc566349ffa2b8dd50a78e4a51cb1ce6221b8fc68c78bc7b7d1cfcec0507*
Theory Prop Address:	*TMHK3aoepzguxxe9uvkDTJLq9eEjx81iFMe*

Definition 21.63 pack_c *is the opaque object of type* $\iota \to ((\iota \to o) \to o) \to \iota$ *identified by the following information:*

Pure Object Id:	*cd75b74e4a07d881da0b0eda458a004806ed5c24b08fd3fef0f43e91f64c4633*
Pure Object Address:	*TMd1myShkR5W1i73HwjDPr8KSegvcYRGNkj*
Theory Object Id:	*385b4fc06e188b2cbb0ea30c0d01cf4136c17844fda6436fa4bc50f1ca06705a*
Theory Object Address:	*TMRLzP6f981VnW9r3th3udMRX2Ub6wT7if3*

`pack_c_0_eq`

Theorem 21.276 $\forall SX. \forall C : (\iota \to o) \to o. S = \mathsf{pack\_c}\ X\ C \to X = S\ 0.$ *The proposition is identified by the following information:*

Pure Prop Id:	*7a818cf8506cfd3b76666875ebb3fc61c82548b59fc9ee11d386b244c6479132*
Pure Prop Address:	*TMazxFUxbSMsNHSVTNyhQVMvZQVYMtn4rey*
Theory Prop Id:	*33b154a9ab4ecbefbf89524fafe42b917315c09c6052ab1c51e598aafe09f4d8*
Theory Prop Address:	*TMGfBTiq6Sx1weZYbUWpWq2HwMh165mnVTc*

pack_c_0_eq2

Theorem 21.277 $\forall X.\forall C : (\iota \to o) \to o.X = \mathsf{pack\_c}\ X\ C\ 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*e668a001a1c285f7e59f4c803795dd7d4779ae52441d54042fd5ffe717dddcce*
Pure Prop Address:	*TMU1E9mEhTwbdJ719haDZxuihHr9UCvVdNH*
Theory Prop Id:	*c88b55706909d090d96e8cc6cfa99496cd78e11afd38eec353935f17985d1464*
Theory Prop Address:	*TMTQUoVGyRyem2GnrMo5TZzGhHbcftwsL1J*

pack_c_1_eq

Theorem 21.278

$$\forall SX.\forall C :(\iota \to o) \to o.S = \mathsf{pack\_c}\ X\ C \to \forall U :\iota \to o.(\forall x.U\ x \to x \in X)\to C\ U = \mathsf{decode\_c}\ (S\ 1)\ U.$$

The proposition is identified by the following information:

Pure Prop Id:	*5c09442310e12d4810ed9f3e048015114950d37b0e8b72105b9b1bb99cf8e28a*
Pure Prop Address:	*TMQYfBviDQMhm9XsosJVREqZzaDe4FqcxjB*
Theory Prop Id:	*821fb2213762f86b1740f4279d2eab6d7fb93fd32c8ebe4a6391c58ea8623af8*
Theory Prop Address:	*TMdmyWGVAiB85Sa4EovKeQWGmrrH97ZjkPg*

pack_c_1_eq2

Theorem 21.279

$$\forall X.\forall C :(\iota \to o) \to o.\forall U :\iota \to o.(\forall x.U\ x \to x \in X)\to C\ U = \mathsf{decode\_c}\ (\mathsf{pack\_c}\ X\ C\ 1)\ U.$$

The proposition is identified by the following information:

Pure Prop Id:	*db75edb08a1a1b58545b4c2359316ff590916926eb8c93afcd90edcae6a56de8*
Pure Prop Address:	*TMLMrphf4gsuDFSivQWRvVEok1B1ijTDDi3*
Theory Prop Id:	*db8f0c9ec729f71005b0d39a0dd45dcc57e2e402d85bc7972239e4dcb3491445*
Theory Prop Address:	*TMHHqUa2BTRQWnruKb9Gi2QmNFrYUVn9izm*

pack_c_inj

Theorem 21.280

$$\forall XX'.\forall CC' :(\iota \to o) \to o.\mathsf{pack\_c}\ X\ C = \mathsf{pack\_c}\ X'\ C' \to X = X' \wedge \forall U : \iota \to o.(\forall x.U\ x \to x \in X) \to C\ U = C'\ U.$$

The proposition is identified by the following information:

Pure Prop Id:	*b5342db4d933f411377c97f1764dfc463670f61e9e5c7d2e559cd02374533b74*
Pure Prop Address:	*TMb2WjtZGnPxZepZzY1QdwFcCN59VzNjmQv*
Theory Prop Id:	*1616e034dd768bac9158e8d3698f121fc76f282512edd80730dffa3671b83dbb*
Theory Prop Address:	*TMN41sVNtmGNFNseJjz7CtVxUTGca3zf8Fx*

pack_c_ext

Theorem 21.281

$$\forall X.\forall C C' :(\iota \to o) \to o.(\forall U : \iota \to o.(\forall x.U\ x \to x \in X) \to (C\ U \Leftrightarrow C'\ U)) \to \mathsf{pack\_c}\ X\ C = \mathsf{pack\_c}\ X\ C'.$$

The proposition is identified by the following information:

Pure Prop Id:	*4c016262198de07b1a463845c2d49e035d3595ebda34bc6acc4687e728e1bf21*
Pure Prop Address:	*TMKA12T7rWqWi37qnNX1Sdao9MdhMjth6Cy*
Theory Prop Id:	*c56d9be727cc8fe44ba7dff7e4a52d3142c3847f950d436c7c3a985a836a38f3*
Theory Prop Address:	*TMMNarnCqG3uNHZiwphmNNhpz7GKTtcnVzm*

Definition 21.64 *We define* struct_c *to be*

$$\lambda S.\forall q : \iota \to o.(\forall X : \iota.\forall C : (\iota \to o) \to o.q\ (\mathsf{pack\_c}\ X\ C)) \to q\ S$$

of type $\iota \to o$ identified by the following information:

Pure Object Id:	*9e6b18b2a018f4fb700d2e18f2d465854 71c8c96cd648d742b7dbda08c349b17*
Pure Object Address:	*TMWcyWdA8JkYry5Pcnkkq5U1QPa36w6b1L5*
Theory Object Id:	*7d5f09aa0fe499776ea89c1aa772c888bce1fc0e89851f4347dec84efa0b4ea1*
Theory Object Address:	*TMW7M6E5Xmwk23LWFWBgLSDEoAq8oj8xvjy*

pack_struct_c_I

Theorem 21.282 $\forall X.\forall C : (\iota \to o) \to o.\mathsf{struct\_c}\ (\mathsf{pack\_c}\ X\ C)$. *The proposition is identified by the following information:*

Pure Prop Id:	*708ab1416bde59517467bf78b5c03e29243d5f7dcaab69b7f4984e6e5e44bd0c*
Pure Prop Address:	*TMb4doWh2x5zNeTDHTUgTiSD9NtGRrxK76r*
Theory Prop Id:	*9e3725e9cd266ee0013e48acce207c124782c57dbfda0f074f6a8c2e7f15fc39*
Theory Prop Address:	*TMacYJooBqqThT8EFbSspPBtC9gFg81RNZ5*

struct_c_eta

Theorem 21.283

$$\forall S.\mathsf{struct\_c}\ S \to S = \mathsf{pack\_c}\ (S\ 0)\ (\mathsf{decode\_c}\ (S\ 1)).$$

The proposition is identified by the following information:

Pure Prop Id:	*e9a19baf027baa0b457c0b4ad53e06f86965db35b095e21bbfdcc856488ce205*
Pure Prop Address:	*TMGzKMqTDCSUsm3E8XZBzrbDXqT5nLqjeH7*
Theory Prop Id:	*492cc612750919ec1d22e8bdc9fb6755693cce05b75a791a2964562629d0c741*
Theory Prop Address:	*TMGqxPLPiHT83m18jBgJDFekewkT2SUQYiZ*

Definition 21.65 unpack_c_i *is the opaque object of type*

$$\iota \to (\iota \to ((\iota \to o) \to o) \to \iota) \to \iota$$

identified by the following information:

Pure Object Id:	*a01360cac9e6ba0460797b632a50d83b7fadb5eadb897964c7102639062b23ba*
Pure Object Address:	*TMbQScLkWSLe1HYb5m6QUhTBUkH2XSZfNEv*
Theory Object Id:	*fa69d4911c2be7aef9c6749803f6c7d2e452fdb2d239cc2c8efdd2104077e48a*
Theory Object Address:	*TMZxmf46bwJxe9YDsDHKRKXQrDdkLoPAP4T*

`unpack_c_i_eq`

Theorem 21.284

$$\forall \Phi : \iota \to ((\iota \to o) \to o) \to \iota . \forall X . \forall C : (\iota \to o) \to o .$$
$$(\forall C' : (\iota \to o) \to o . (\forall U : \iota \to o . (\forall x . U\ x \to x \in X) \to (C\ U \Leftrightarrow C'\ U)) \to$$
$$\Phi\ X\ C' = \Phi\ X\ C)$$
$$\to \mathsf{unpack\_c\_i}\ (\mathsf{pack\_c}\ X\ C)\ \Phi = \Phi\ X\ C.$$

The proposition is identified by the following information:

Pure Prop Id:	*1c35a1ec49283779d9dad9c87ccddab3c155a1e81dfd7c30d2e5bf413a4903ca*
Pure Prop Address:	*TMJGk56zQ9ppwWePYzrdvyPXUCDe2sEqZ19*
Theory Prop Id:	*3a893b44e699dd6503cf16fc256ea51e13c996f8fdbfb14928fb14d405d95013*
Theory Prop Address:	*TMY29KidZk9kJp7G1gonGvtrEyAKfE9y3mh*

Definition 21.66 unpack_c_o *is the opaque object of type*

$$\iota \to (\iota \to ((\iota \to o) \to o) \to o) \to o$$

identified by the following information:

Pure Object Id:	*939baef108d0de16a824c79fc4e61d99b3a9047993a202a0f47c60d551b65834*
Pure Object Address:	*TMZ52NsRLMZey7UxdWrY5r5LQnLT6warXH5*
Theory Object Id:	*8b56a8588064b5d7d1c47e8fcf2acc9e2ce6c5f269ba5d05ed77c59568ec81b1*
Theory Object Address:	*TMZkMyksvwVPJJNYLovJrphjDV7ZP1519ja*

`unpack_c_o_eq`

Theorem 21.285

$$\forall \Phi : \iota \to ((\iota \to o) \to o) \to o . \forall X . \forall C : (\iota \to o) \to o .$$
$$(\forall C' : (\iota \to o) \to o . (\forall U : \iota \to o . (\forall x . U\ x \to x \in X) \to (C\ U \Leftrightarrow C'\ U)) \to \Phi\ X\ C' = \Phi\ X\ C)$$
$$\to \mathsf{unpack\_c\_o}\ (\mathsf{pack\_c}\ X\ C)\ \Phi = \Phi\ X\ C.$$

The proposition is identified by the following information:

Pure Prop Id:	*5145ca206371461257e3dbbda8cd67e11446e95f699a7643af128584410c440e*
Pure Prop Address:	*TMMcXtq6qovUwYmcRYmCXc121HsSojJs5fr*
Theory Prop Id:	*ba3042e75e86ab022bb94d2eb8a4e5e23f21d516fccd571acb857200dcc5ac3d*
Theory Prop Address:	*TMZ6bYNJXuLEh438fMAjBWK4p44Add2Frrj*

Definition 21.67 canonical_elt *is the opaque object of type* $(\iota \to \iota \to o) \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*24615c6078840ea77a483098ef7fecf7d2e10585bf1f31bd0c61fbaeced3ecb8*
Pure Object Address:	*TMTxaocPHR7KaoAb7prHbkajsp6MUfshdsg*
Theory Object Id:	*a18d95a2ab347610921018619 24b9134534850495d74fd75a2443135e945ea6f*
Theory Object Address:	*TMTEe99JMMf634KDuVmQ44i85VxLmTHvB9y*

canonical_elt_rel

Theorem 21.286

$$\forall R : \iota \to \iota \to o . \forall x : \iota . R\ x\ x \to R\ x\ (\mathsf{canonical\_elt}\ R\ x).$$

The proposition is identified by the following information:

Pure Prop Id:	*409f7f93609b8bbc78e9b27c91b8e8b15d16e673c0445c98fe5cbae6fe93e350*
Pure Prop Address:	*TMQQtrdvb5MuDAgqNkxaL8Yxdiut4fDtB6a*
Theory Prop Id:	*a82ff2dd9b739269814be3b2b90d4dcd5d7e5f49fd6fa941daaf8f2fac8b7e4b*
Theory Prop Address:	*TMahFABoX7qUN4TKbs8SxWDdavcsNyazvny*

canonical_elt_eq

Theorem 21.287

$$\forall R : \iota \to \iota \to o . \mathsf{per}\ R \to \forall x y : \iota . R\ x\ y \to \mathsf{canonical\_elt}\ R\ x = \mathsf{canonical\_elt}\ R\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*77a0bb710d7f6fd22c2038b8e8a7aa53cb38b8625399eda5a768a92f86f94529*
Pure Prop Address:	*TMNGDho8FciVfUN7CiWcH22sTikdZ3nEXxy*
Theory Prop Id:	*a3a6535b47fa636b2a37d0fd9d2cbc572ab41c638fc41180620163a965a862a9*
Theory Prop Address:	*TMPA9JRmixkhj8Ft97qJcZE9RMXboWNNtQA*

canonical_elt_idem

Theorem 21.288

$$\forall R : \iota \to \iota \to o . \mathsf{per}\ R \to \forall x : \iota . R\ x\ x \to \mathsf{canonical\_elt}\ R\ x = \mathsf{canonical\_elt}\ R\ (\mathsf{canonical\_elt}\ R\ x).$$

The proposition is identified by the following information:

Pure Prop Id:	*f360e8148073fe4e1c4b5a43fcb23f3a4230874ab3afbbbea65f8ac02b0385d5*
Pure Prop Address:	*TMGnsgqbHzSxjuur9NxxHHErkCtdk6tipsC*
Theory Prop Id:	*0881fbf40b7b603f1f612763b2238f0020aa675fdfbc4d16389c60ebcc4735ed*
Theory Prop Address:	*TMNx4jXjzYQJoqNMEri4pBsyV3LhSvqYYqt*

Definition 21.68 quotient *is the opaque object of type* $(\iota \to \iota \to o) \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*185d8f16b44939deb8995cbb9be7d1b78b78d5fc4049043a3b6ccc9ec7b33b41*
Pure Object Address:	*TMQgacYA6wSdteHEaGGZB5pfV8odVjnD3Ld*
Theory Object Id:	*eceb7632185ce5062420ae1f8b27cdcf062b7c243c6d9b7856e5e2a4e937e2af*
Theory Object Address:	*TMdVzy3y6H8DfNQEHsaYmphZTWHJpPCs1dv*

`quotient_prop1`

Theorem 21.289 $\forall R : \iota \to \iota \to o.\forall x : \iota.\mathsf{quotient}\ R\ x \to R\ x\ x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*c9cf06df33dd5d9e1508e9c920dbee7c4fbe3ac5ca1796506c238279d7024d76*
Pure Prop Address:	*TMEpRYaApsS9NBgYkPfxvKjtbKoMxckLJkg*
Theory Prop Id:	*8eea9432bbf60de4e1871846f2697db36eb0c3579081190809f73c887aa8932c*
Theory Prop Address:	*TMHSpwoRWkRrT199Pr4Zw6NBfBmFt5FaZDd*

`quotient_prop2`

Theorem 21.290

$$\forall R : \iota \to \iota \to o.\mathsf{per}\ R \to \forall xy : \iota.\mathsf{quotient}\ R\ x \to \mathsf{quotient}\ R\ y \to R\ x\ y \to x = y.$$

The proposition is identified by the following information:

Pure Prop Id:	*54b14f4b32c6d1e65fbf61514c95ad09fa776454efdf44259705e3654a8de198*
Pure Prop Address:	*TMWEzaUyoSBC2cv5ikpYBmh1Wxg9bwCWMaW*
Theory Prop Id:	*c5f241148436a9448eb8c5fe8b100b16191b8a22b99dbbc971e37dd9e6442a9c*
Theory Prop Address:	*TMSQAtmzR5PbKfWrMfYzL1xj4QFSrYQWPLq*

Definition 21.69 canonical_elt_def *is the opaque object of type*

$$(\iota \to \iota \to o) \to (\iota \to \iota) \to \iota \to \iota$$

identified by the following information:

Pure Object Id:	*cd4f601256fbe0285d49ded42c4f554df32a64182e0242462437212fe90b44cd*
Pure Object Address:	*TMXTWshnbJ4phtQfmjt8rT3VUcaRS5EkRgM*
Theory Object Id:	*c6ce994fa2a7c7d9b9936df4d487c851d3b5a14f9d5b842e6d7950167473ab6e*
Theory Object Address:	*TMZLpA4pBtPQmtYCGc2fcZVLLXtrZbwNRz2*

`canonical_elt_def_rel`

Theorem 21.291

$$\forall R{:}\iota \to \iota \to o.\forall d{:}\iota \to \iota.\forall x{:}\iota.R\ x\ x{\to}R\ x\ (\mathsf{canonical\_elt\_def}\ R\ d\ x).$$

The proposition is identified by the following information:

Pure Prop Id:	*427d7106df945e01d1e010f6b5f54619d5e305852962b29fd074411773f7a5fb*
Pure Prop Address:	*TMQseUYfkBpoXqbKv8vXzWb7zmRmZGRt6ho*
Theory Prop Id:	*ba911f1fb82909fa0343a89423fd8007a1eea7266f3a64d4f344fc4454b45e97*
Theory Prop Address:	*TMSAmmXFWnmPQSVvG6xXns5c29knAy7NoJL*

`canonical_elt_def_eq`

Theorem 21.292

$$\forall R{:}\iota \to \iota \to o.\mathsf{per}\ R{\to}\forall d{:}\iota \to \iota.(\forall xy : \iota.R\ x\ y \to d\ x = d\ y){\to}\forall xy{:}\iota.R\ x\ y{\to}\\ \mathsf{canonical\_elt\_def}\ R\ d\ x = \mathsf{canonical\_elt\_def}\ R\ d\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*08b2b96ed66c9cf40b455864a9805316a228566ea2f3dee98727b4ddf6fdee29*
Pure Prop Address:	*TMdbiHGV2yacZL1C6aVnpN4nqxmFxeFRfVh*
Theory Prop Id:	*8b36f1b656a7909d9a962189ecd03e0e7de3681872a48796808275153f9b010f*
Theory Prop Address:	*TMQkuYZZz1p5gpefQX7a59yAUSxmoaUZ2B1*

`canonical_elt_def_idem`

Theorem 21.293

$$\forall R{:}\iota \to \iota \to o.\mathsf{per}\ R{\to}\forall d{:}\iota \to \iota.(\forall xy : \iota.R\ x\ y \to d\ x = d\ y){\to}\forall x{:}\iota.R\ x\ x{\to}\\ \mathsf{canonical\_elt\_def}\ R\ d\ x = \mathsf{canonical\_elt\_def}\ R\ d\ (\mathsf{canonical\_elt\_def}\ R\ d\ x).$$

The proposition is identified by the following information:

Pure Prop Id:	*49efe82bbd3304683bf8464c2a34622dd4c9bffd857d2336f4501a2673d7d8f4*
Pure Prop Address:	*TMFnpUHvtfsuux3kkiYp7792SjHSqdtGr2C*
Theory Prop Id:	*c305bfd1e77d91b812fe3dc8843e58279f1bb98be2cbbed8b4fedf7a73f5d6ef*
Theory Prop Address:	*TMKtsdWDnNkcBLMACntcRypwVFCS39wBpEg*

Definition 21.70 quotient_def *is the opaque object of type*

$$(\iota \to \iota \to o) \to (\iota \to \iota) \to \iota \to o$$

identified by the following information:

Pure Object Id:	*612d4b4fd0d22dd5985c10cf0fed7eda4e18dce70710ebd2cd5e91acf3995937*
Pure Object Address:	*TMHzGWujGt1Siv1ANFttshwyzCe6VYRGi4K*
Theory Object Id:	*b18ffb4e6bef489f8b9eecd55fec81b0c0593f62e6a1386eb07f6c9250a2d493*
Theory Object Address:	*TMQgkTfPUbwLT8jGwoRckaDnvCcP3Vd9PPz*

quotient_def_prop0

Theorem 21.294

$$\forall R : \iota \to \iota \to o.\mathsf{per}\ R \to \forall d : \iota \to \iota. \forall x : \iota. R\ x\ (d\ x) \to x = d\ x \to \mathsf{quotient\_def}\ R\ d\ x.$$

The proposition is identified by the following information:

Pure Prop Id:	*b6730b9da18d165aefaece5098a92db470398156beb51a1a0c1278d4e36152dc*
Pure Prop Address:	*TMVu99E7LXXZ91pqY2zxxjzevyHiyRuDk2L*
Theory Prop Id:	*1e2c729d64e136e7b405ff57ee6ac780a1b626f5f145e0dc7b9f1d1c894c154b*
Theory Prop Address:	*TMUnYQqkYvFzvC8Rsud6AfF4JJ7DqXnWQxo*

quotient_def_prop1

Theorem 21.295

$$\forall R : \iota \to \iota \to o. \forall d : \iota \to \iota. \forall x : \iota. \mathsf{quotient\_def}\ R\ d\ x \to R\ x\ x.$$

The proposition is identified by the following information:

Pure Prop Id:	*96ac558305488884573f3e141434b062bf9a4c5864092d3e70136808df96b01ad*
Pure Prop Address:	*TMXLZPFcYNs9m56G1QCx9QktiHeUscd41Rw*
Theory Prop Id:	*0ba02cd229346ac664fd2fadc979d704d3e12575ce6ce56adbcbe00e2e55b9db*
Theory Prop Address:	*TMT4xAb2VRQxAVPG9ZG39hM8kN2w5vHWSNB*

quotient_def_prop2

Theorem 21.296

$$\forall R : \iota \to \iota \to o.\mathsf{per}\ R \to \forall d : \iota \to \iota. (\forall xy : \iota. R\ x\ y \to d\ x = d\ y) \to \forall xy : \iota. \mathsf{quotient\_def}\ R\ d\ x \to \mathsf{quotient\_def}\ R\ d\ y \to R\ x\ y \to x = y.$$

The proposition is identified by the following information:

Pure Prop Id:	*fd0707aa86d2b360392ba2fdfe2a63261a390d4e8aa5535320c6e57d0b01996e*
Pure Prop Address:	*TMU2h1hwqUi1KnjLzjZCzhLM9nK9DZmiDRP*
Theory Prop Id:	*0dc38ca2997b0761433b4fc94c031c565be7652963fd19f4257dd60e41928762*
Theory Prop Address:	*TMNeKehn9tffwmMaonuDau88ErbSbo4Wzam*

Chapter 22

Explicit Number Structures

22.1 explicit_Nats

Let $N : \iota$ be given. Let $base : \iota$ be given. Let $S : \iota \to \iota$ be given.

Definition 22.1 explicit_Nats *is the opaque object of type o identified by the following information:*

Pure Object Id:	*3fb62f75e778736947d086a36d35ebe45a5d60bf0a340a0761ba08a396d4123a*
Pure Object Address:	*TMVtLiLBPcSiAyimBNLrSsPPvNGxNnQTLgx*
Theory Object Id:	*499c1253e694386778e3a86aec4d8d86e3e968012315d84ff3d8771c7ca33ca6*
Theory Object Address:	*TMYM3xBCn91GR8gYvvjmLwhSW8eWQhQUR5V*

`explicit_Nats_I`

Theorem 22.1

$$(base \in N) \to (\forall m \in N.S\ m \in N) \to (\forall m \in N.S\ m \neq base) \to (\forall mn \in N.S\ m = S\ n \to m = n) \to (\forall p : \iota \to o.p\ base \to (\forall m.p\ m \to p\ (S\ m)) \to (\forall m \in N.p\ m)) \to \mathsf{explicit\_Nats}.$$

The proposition is identified by the following information:

Pure Prop Id:	*1e6abb88cfbf0a3803f631d2180983c78469316b36f44986957690ca50046636*
Pure Prop Address:	*TMS7xTLnefHd6AjUKGkQBpdVrdEKnFWi6GV*
Theory Prop Id:	*1c1fb7a66f9f17cd55170faa1827e232bbd7515b8fbb9a57a4ccf3c34eaa501a*
Theory Prop Address:	*TMZjQDjppyHzU9Sm626Nv7EXMsNVVAqkaJT*

`explicit_Nats_E`

Theorem 22.2

$$\forall q : o. \\ (\mathsf{explicit\_Nats} \to (base \in N) \to (\forall m \in N.S\ m \in N) \to \\ (\forall m \in N.S\ m \neq base) \to \\ (\forall mn \in N.S\ m = S\ n \to m = n) \to \\ (\forall p : \iota \to o.p\ base \to (\forall m.p\ m \to p\ (S\ m)) \to (\forall m \in N.p\ m)) \to q) \\ \to \mathsf{explicit\_Nats} \to q.$$

The proposition is identified by the following information:

Pure Prop Id:	*5de31c779c105b713f97c938846c5369a751bf11d4d1c91131667de452b85d52*
Pure Prop Address:	*TMN27YWUyKn3QBjt1t4WK1VcstkTSsWazb6*
Theory Prop Id:	*80e22881183427762 8e910ca46e86ac23fcdf98d3b95ab2ee85c1f1697c9842b*
Theory Prop Address:	*TMXQDL2aLuWPAFGh9xNGSkbkMuMSyVTuvG2*

`explicit_Nats_ind`

Theorem 22.3

$$\mathsf{explicit\_Nats} \to \forall p : \iota \to o.p\ base \to (\forall m \in N.p\ m \to p\ (S\ m)) \to \forall m \in N.p\ m.$$

The proposition is identified by the following information:

Pure Prop Id:	*72f6cf6a8db779d851cead9db4030c8cf14cdb66b3dba2256273ea5c82f6cf75*
Pure Prop Address:	*TMRCzQGX3LAr7ohapsy9xQiuobUsWPbHmfG*
Theory Prop Id:	*3e626e6f6e97b94d761465a195a81a506097757289b1852c22484904835f624b*
Theory Prop Address:	*TMLtcGPvoYCa4mMihX67t4Lm78GzLsmB2B6*

Definition 22.2 explicit_Nats_primrec *is the opaque object of type*

$$\iota \to (\iota \to \iota \to \iota) \to \iota \to \iota$$

identified by the following information:

Pure Object Id:	*a61e60c0704e01255992ecc776a771ad4ef672b2ed0f4edea9713442d02c0ba9*
Pure Object Address:	*TMY3C6hyjHYFvPNDaQK728xpbMFkgTGPbUQ*
Theory Object Id:	*02adae4d88dbb61570b0e9d5d5ea2d21abfb3e9a8408276329ddc4e3b1fc2a9a*
Theory Object Address:	*TMUhjTmqPCD758LckQJPJ6DHuAFTQeHLoqS*

`explicit_Nats_primrec_base`

Theorem 22.4

$$\forall a.\forall f : \iota \to \iota \to \iota.\mathsf{explicit\_Nats} \to \\ \mathsf{explicit\_Nats\_primrec}\ a\ f\ base = a.$$

The proposition is identified by the following information:

Pure Prop Id:	*ec9dac528f5b9d80aeb2b078f81657da0725083292683597b6ed0f6bcda0033c*
Pure Prop Address:	*TMJxuYA6EQJ2GxewDFKrBTK4gZX2qiFpNuM*
Theory Prop Id:	*18887ff3f12d4f358090a38d2eecb57a5531dabfe6535ad15a910933b602f16c*
Theory Prop Address:	*TMGkBbmvyBif6wxjhQzqdSRLtTP5TBd5e9z*

explicit_Nats_primrec_S

Theorem 22.5

$$\forall a.\forall f :\iota \to \iota \to \iota.\text{explicit\_Nats}{\to}\forall n \in N.$$
$$\text{explicit\_Nats\_primrec } a\ f\ (S\ n) = f\ n\ (\text{explicit\_Nats\_primrec } a\ f\ n).$$

The proposition is identified by the following information:

Pure Prop Id:	*c79ce9883e3c0c86ed88d57e78ed6269f94c26749772f51c6ad325234a554663*
Pure Prop Address:	*TMSv5jrFNLuwkARow9Y11SWFGFXBqcbUbLK*
Theory Prop Id:	*bf9939e2289e86288ccefbe010777a3faf16479bf896f3b41a2c5a5a73e561cb*
Theory Prop Address:	*TMUpUUqwaxvwAg3jgXfPz8RdqaXuW1kMAxx*

explicit_Nats_primrec_P

Theorem 22.6

$$\text{explicit\_Nats}{\to}\forall P :\iota \to o.\forall a.P\ a{\to}\forall f :\iota \to \iota \to \iota.$$
$$(\forall n \in N.\forall b.P\ b \to P\ (f\ n\ b)){\to}\forall n \in N.$$
$$P\ (\text{explicit\_Nats\_primrec } a\ f\ n).$$

The proposition is identified by the following information:

Pure Prop Id:	*7ffca5b3e4e2357fc59f35b26bcc159af8962fe6020bca14a2c9246bac7165e9*
Pure Prop Address:	*TMPVTJWhZ2JZ6V3t3BHXTUQ7k318dSnYpfW*
Theory Prop Id:	*a23b352d53e7ed810c1716d20ebfd8cf9fd43007e863a3265aa1009d6883d822*
Theory Prop Address:	*TMSyXZfNTKDx39oHHCRdmhL4WfJE3gghudc*

22.2 Omega gives Explicit Naturals

explicit_Nats_omega

Theorem 22.7 explicit_Nats omega 0 ordsucc. *The proposition is identified by the following information:*

Pure Prop Id:	*a3ba10e61c3b75d16024bffd1f732e7b904270375 33d34ee2e091771480c74a6*
Pure Prop Address:	*TMdGdgcJArCKDPYXVndpbm1SU3AEVwyfoDy*
Theory Prop Id:	*4d074b5527b3d4f670e267d1341927f6e1665b1b695a8fa1d7b296e3b59b616f*
Theory Prop Address:	*TMN5Yqfwk5yD9vYjacHHUZqkgRCThYxgE6u*

22.3 explicit_Nats_zero

Let $N : \iota$ be given. Let $zero : \iota$ be given. Let $S : \iota \to \iota$ be given.

Definition 22.3 explicit_Nats_zero_plus *is the opaque object of type* $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*9683ebbbd2610b6b9f8f9bb32a63d9d3cf8c376a919e6989444d6d995da2aceb*
Pure Object Address:	*TMEmJ2AUSSkSo3PgswMDx4FDuVNXWTFp2Xv*
Theory Object Id:	*a492e421e4fb131a6ece5ab66d32b1664ac5750c65f21c1ee016f0b281db4c68*
Theory Object Address:	*TMZHNQrttsjehBJHhEo49zc8mrT3L85obE5*

Notation. We use $+$ as a right associative infix operator corresponding to applying term explicit_Nats_zero_plus.

Definition 22.4 explicit_Nats_zero_mult *is the opaque object of type* $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*7cf43a3b8ce0af790f9fc86020fd06ab45e597b29a7ff1dbbe8921910d68638b*
Pure Object Address:	*TMWE5mZjYWyApPQfEURTERyzUQDmzqxvrRZ*
Theory Object Id:	*11a63fc4e4e266de185d7864c9ae0a5059a920dc1af5eb33480e4b6257a39712*
Theory Object Address:	*TMa5dmS6AU8GbHfa7BizTDCqP4sRj1UmdGg*

Notation. We use $*$ as a right associative infix operator corresponding to applying term explicit_Nats_zero_mult. Assume the following.

$$\text{explicit\_Nats } N\ zero\ S \tag{22.1}$$

`explicit_Nats_zero_plus_N`

Theorem 22.8 $\forall nm \in N.n + m \in N$. *The proposition is identified by the following information:*

Pure Prop Id:	*56523af04adfa34d579665edd443f3c8c991a55c2e1d59fe86eb7cb534626054*
Pure Prop Address:	*TMKvkMhHVgvBVAMZ58PR6NKqCWx4kMPZmfY*
Theory Prop Id:	*d3decda97e93dc588cf7d1be70d082fee93bb6edd68e6c451ca3e4e4685258a4*
Theory Prop Address:	*TMXZCAYSxCqnbikDjS9v4JyUbJQi41N3Beg*

`explicit_Nats_zero_plus_0L`

Theorem 22.9 $\forall m \in N.zero + m = m$. *The proposition is identified by the following information:*

Pure Prop Id:	*eda4ea107573c4741f17b9e4cfc4ad2c0fca35856cb39a9f9d0c7082eeef8edc*
Pure Prop Address:	*TMZFAJoCr87Twhz3ue2eaZWtjHZrDbriNS2*
Theory Prop Id:	*6cbaa15873e859402e00e7efd7313e6202c9580bb8dc07d3eaff7cb6dd528689*
Theory Prop Address:	*TMXGbC3fvTNoibPqk8dSmxUYfFe6A2jTjVA*

explicit_Nats_zero_plus_SL

Theorem 22.10 $\forall nm \in N.S\ n + m = S\ (n + m)$. *The proposition is identified by the following information:*

Pure Prop Id:	*4b64cc27697767078865ed93f819202d8095e44f34703cacb427d0ff277b7772*
Pure Prop Address:	*TMUB4sk69KXoqr9KeeXuQvkBtdzEy5uGSGY*
Theory Prop Id:	*67b5266ae2b89bd4f290f8483d37fed2bec3392b27b55c35022e5621958309a4*
Theory Prop Address:	*TMUxY6JJgbyA2wvfR5kWXPg76FJgumm8rc1*

explicit_Nats_zero_mult_N

Theorem 22.11 $\forall nm \in N.n * m \in N$. *The proposition is identified by the following information:*

Pure Prop Id:	*242c80020d9e0983d413a8726b0f8dd31a5d6ed1c4a614a9e372f46e71b811d7*
Pure Prop Address:	*TMPyVVnedkqiLXtyLzYCYMHVttNM5sfuXaF*
Theory Prop Id:	*4d99dbaf92926086a5db90b0b45da73422e998d71f9d9b53551ce90cc1bc2a97*
Theory Prop Address:	*TMUjvpTR8zMFez8UREoFWmF1mfB2Ck1G9qd*

explicit_Nats_zero_mult_0L

Theorem 22.12 $\forall m \in N.zero * m = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*b80ff108406a373e38422d03ee6a993a3300f355e3972522d8834e8f32e727eb*
Pure Prop Address:	*TMdPBQEb6r1jazTfaSsAkVi4HU5BuA9SgCr*
Theory Prop Id:	*8b905296c07181890992e4efb2371e7d4c2eacd4d79aa5f2ae5bee8d137d8e99*
Theory Prop Address:	*TMFy6HpCwNacy45vXp7VgoS5A62a6koR3ib*

explicit_Nats_zero_mult_SL

Theorem 22.13 $\forall nm \in N.S\ n * m = m + n * m$. *The proposition is identified by the following information:*

Pure Prop Id:	*ceea17fc60f9c29a157639cceb082002f336d7ada992c4bf24f10ec2107aaeef*
Pure Prop Address:	*TMN5mn9NySXen2SeWDjUyYFPcUTDr2zeAhC*
Theory Prop Id:	*58cb3351db2fc2413401ac6beecc0c5aa8cb6e94862e9b0888a08fab43ce10ae*
Theory Prop Address:	*TMYNaqKknxFESuZan8Ztx9TK21bLv9z72UQ*

22.4 explicit_Nats_one

Let $N : \iota$ be given. Let $one : \iota$ be given. Let $S : \iota \to \iota$ be given.

Definition 22.5 explicit_Nats_one_plus *is the opaque object of type $\iota \to \iota \to \iota$ identified by the following information:*

Pure Object Id:	*c14dd5291f7204df5484a3c38ca94107f29d636a3cdfbd67faf1196b3bf192d6*
Pure Object Address:	*TMVj1gkGc7wpNZ6S9o6JmxmKVGCvLP3HmTd*
Theory Object Id:	*a0dc24bec298fc49cdd175bfad0801c6dda7e88d2602021d5fc71cc01e269ee8*
Theory Object Address:	*TMW7qLS9YPVnzwPSYkBdmmRwk5ZbPbLdnpL*

Notation. We use $+$ as a right associative infix operator corresponding to applying term explicit_Nats_one_plus.

Definition 22.6 explicit_Nats_one_mult *is the opaque object of type* $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*ec4f9ffffa60d2015503172b35532a59cebea3390c398d0157fd3159e693eb97*
Pure Object Address:	*TMJBfR7vUcShqA32xB1n6LKgJDcAtK2PWgY*
Theory Object Id:	*7cbccf9874bf38cebb42cd5bbc730880796dbf9ee8b00e588914593a33f3616e*
Theory Object Address:	*TMdYg5kd19jjyivNeoE1Fq5Z77fpmPN4Ghy*

Notation. We use $*$ as a right associative infix operator corresponding to applying term explicit_Nats_one_mult.

Definition 22.7 explicit_Nats_one_exp *is the opaque object of type* $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*cbcee236e6cb4bea1cf64f58905668aa36807a85032ea58e6bb539f5721ff4c4*
Pure Object Address:	*TMGRgxwxS6aYefXQew7xTarwkrNkekEv1vL*
Theory Object Id:	*f451e86804defa9f4c49ef8b31319235986f06a57cc47e224e0e616e81349e41*
Theory Object Address:	*TMYMj3b1DyJ9Lbh2dkML476LtBBK6RhxAGa*

Notation. We use $\uparrow$ as a right associative infix operator corresponding to applying term explicit_Nats_one_exp. Assume the following.

$$\text{explicit\_Nats}\ N\ one\ S \tag{22.2}$$

`explicit_Nats_one_plus_N`

Theorem 22.14 $\forall nm \in N.n + m \in N$. *The proposition is identified by the following information:*

Pure Prop Id:	*b870373860de2f7ad912a4078539010f085139c691292d7e7d3c34db1f0cdb6a*
Pure Prop Address:	*TMMGxvc17zs14pSXUk5XHB547EgcLE5Qj1X*
Theory Prop Id:	*2031f14ef57b3e0a0f0bb3c5d4e8d7cc6c01908d84f8ae8a58422536a7ebd671*
Theory Prop Address:	*TMKsochxf4GKDVGFjEfDV7zBLDZ3NZmTqN8*

`explicit_Nats_one_plus_1L`

Theorem 22.15 $\forall m \in N.one + m = S\ m$. *The proposition is identified by the following information:*

Pure Prop Id:	*e0ca47b5062d5ffc6896d07746c59aaca4bc4acc06788ddcfa7d606c68f00a95*
Pure Prop Address:	*TMSjCGdV6bWn9b6ayvqYSEN5ezLiifHBvqP*
Theory Prop Id:	*b861add16dde10cbd273ebb51a08c3bde324096c7b892a0f54908292dbd67d14*
Theory Prop Address:	*TMRmkTUjpfWnjUnbtZRbTeUiDDjnxz6PKXn*

explicit_Nats_one_plus_SL

Theorem 22.16 $\forall nm \in N.S\ n + m = S\ (n + m)$. *The proposition is identified by the following information:*

Pure Prop Id:	*d527faf460c66eb2a605937d7bc284a505c5e71e11b5819b3ebe45aa5ae3df30*
Pure Prop Address:	*TMZrvHTgr8uHgFqVfwzAjaHWjcyWUe9ku88*
Theory Prop Id:	*272ea646fea8c08826956116e3b06f06b4f1a29035d7099f3e17dbff9743db74*
Theory Prop Address:	*TMdJyqAKqfUoMVXYdkRynaBe5Y4UCycBCSi*

explicit_Nats_one_mult_N

Theorem 22.17 $\forall nm \in N.n * m \in N$. *The proposition is identified by the following information:*

Pure Prop Id:	*71dc3009d99275dcb7507e7e7f7b53e10abcd721e2a9f4479dfb1d9626492798*
Pure Prop Address:	*TMUF1CggpPWJLEjwmb369yXyFWWhKTPPS6u*
Theory Prop Id:	*6cea9937d528c3ed3844899b07adc36352b53c024fca3a3fb796832b3cb817eb*
Theory Prop Address:	*TMLGNEexWqJMgReiwzYkhaf87Wi9rCre9U7*

explicit_Nats_one_mult_1L

Theorem 22.18 $\forall m \in N.one * m = m$. *The proposition is identified by the following information:*

Pure Prop Id:	*248ac634eb92e6d0725101234c0f2912ffe95ed5d81f7568c5ebace9788fe8c6*
Pure Prop Address:	*TMTS3c1HETRVj5fANAi3ixWKrd9G35hxXTk*
Theory Prop Id:	*cf9d37fbd63485076deb649fa124b9d2400952c5423fe11ea9f085d236188b80*
Theory Prop Address:	*TMTMa1DWHDPBUbN5Uh2GsHTp6jg2Ve8n4iF*

explicit_Nats_one_mult_SL

Theorem 22.19 $\forall nm \in N.S\ n * m = m + n * m$. *The proposition is identified by the following information:*

Pure Prop Id:	*f260dfa960c7ea2ad1c80083a06eaed1460025b578976944e9d439072b54a833*
Pure Prop Address:	*TMYCaevEfmMyUdnjxigfSRQFKL62bwDtseb*
Theory Prop Id:	*84048a4e0ac742533042895099975cda2d4ce7777ccfe9a38dfebad9d348452b*
Theory Prop Address:	*TMT3bNHPRqWLxd5k165RfwDrLuFDDnTU4GR*

explicit_Nats_one_exp_N

Theorem 22.20 $\forall nm \in N.n^m \in N$. *The proposition is identified by the following information:*

Pure Prop Id:	*e06153d7a791dd306b96ac5fa124b22a5e0e6f3c79dc892e8973612dbdbcf965*
Pure Prop Address:	*TMbPBpFHKw4tdRPzKEnzY36NJm6Wo5NA1vq*
Theory Prop Id:	*dc83c194406168e2322b693e0db9e9c2935f9b5890252230aac13dbba5dd04f4*
Theory Prop Address:	*TMXt3nAqYNYZHtj3LbChY5GKH6imEYDSJJA*

explicit_Nats_one_exp_1L

Theorem 22.21 $\forall n \in N.n^{one} = n$. *The proposition is identified by the following information:*

Pure Prop Id:	*3135e1cbd0675df0f6c911e6f05dd6ea73bc4c458266e5d736412045200aae78*
Pure Prop Address:	*TMd59g2sbanMoRmaQmddMbB1kEpMZkGsjHd*
Theory Prop Id:	*35371d46a876899103c3b0122a82b56515f75a0378f71012b5d765398f35d30b*
Theory Prop Address:	*TMKNGaufMJdNVayeapLn2aUCzuEgW63wrfh*

explicit_Nats_one_exp_SL

Theorem 22.22 $\forall nm \in N.n^{(S\ m)} = n * n^m$. *The proposition is identified by the following information:*

Pure Prop Id:	*7a9c28b365c7612ae00bcf5eae2460615c9a85f39e5734a2fe11b4129c6272d2*
Pure Prop Address:	*TMbzoYWnxTt9ZksjQ6HvVxXro6tokwCYzCB*
Theory Prop Id:	*61fbfd0911e934537afe54a0c8fd8369d881dbd3b94b5b74c40a418954a55d7d*
Theory Prop Address:	*TMJ3ASygyK5PDcYrS3MhKvqvt7Buzmpj1BC*

Definition 22.8 *We define* explicit_Nats_one_lt *to be*

$$\lambda mn.m \in N \wedge n \in N \wedge \exists k \in N.m + k = n$$

of type $\iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*a887fc6dd84a5c55d822e4ceb932c68ead74c9292ff8f29b32a732a2ee261b73*
Pure Object Address:	*TMaZtSJJReNwaSNEigaPZg3GCC7FWiry1eM*
Theory Object Id:	*d1d62283501ce0525b4885ac683b5f8872a63a79b85eaa1912e4259776dcde37*
Theory Object Address:	*TMQmi5rw5Vmk7auDTyrnNtW84cLiTH49cT4*

Definition 22.9 *We define* explicit_Nats_one_le *to be*

$$\lambda mn.m \in N \wedge n \in N \wedge (m = n \vee \exists k \in N.m + k = n)$$

of type $\iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*8d7cd4bd6b239e3dc559142aa4d11d731c7a888de9faa3b9eeec2e7d5651c3fb*
Pure Object Address:	*TMM32WFKaju25pJnuVLyA3borpF7YFkso2V*
Theory Object Id:	*0165a966c5de8b542de020a29a4531b3e01f1d9ab3a4aa7fba893a0bf28f0580*
Theory Object Address:	*TMMNCaFpJUV2w4YZrBepTz2BXoitsuYhDUE*

Notation. We use $<$ as an infix operator corresponding to applying term explicit_Nats_one_lt. **Notation.** We use $\leq$ as an infix operator corresponding to applying term explicit_Nats_one_le.

22.5 explicit_Nats_transfer

Let $N : \iota$ be given. Let $base : \iota$ be given. Let $S : \iota \to \iota$ be given. Let $N' : \iota$ be given. Let $base' : \iota$ be given. Let $S' : \iota \to \iota$ be given. Let $f : \iota \to \iota$ be given.

`explicit_Nats_transfer`

Theorem 22.23

$$\mathsf{explicit\_Nats}\ N\ base\ S \to \mathsf{bij}\ N\ N'\ f \to f\ base = base' \to (\forall n \in N. f\ (S\ n) = S'\ (f\ n)) \to \mathsf{explicit\_Nats}\ N'\ base'\ S'.$$

The proposition is identified by the following information:

Pure Prop Id:	*3a37116910f44ef4e3c8921b5dcaba3ab229f567fd494a0d9f6d3fd6ac9823f8*
Pure Prop Address:	*TMGVh4mHULAwdH24Rh3RJUPVxrmkg1EWVue*
Theory Prop Id:	*6c4761bc5fcc2f65480e8a6ac8bd7faf5d89a85926881bb7e4724dd397f0746d*
Theory Prop Address:	*TMbddV6bagEzzT8wfXKJuLdPeN4XyXPbp2U*

Chapter 23

Groups

23.1 AssocComm

Let $R : \iota$ be given. Let $plus : \iota \to \iota \to \iota$ be given. **Notation.** We use $+$ as a right associative infix operator corresponding to applying term $plus$.

`AssocComm_identities`

Theorem 23.1

$$(\forall xy \in R.x + y \in R) \to (\forall xyz \in R.x + (y + z) = (x + y) + z) \to (\forall xy \in R.x + y = y + x) \to \forall p : o.$$
$$((\forall xyz \in R.x + y + z = y + x + z) \to$$
$$(\forall xyz \in R.x + y + z = z + x + y) \to$$
$$(\forall xyzw \in R.(x + y) + (z + w) = (x + z) + (y + w)) \to$$
$$(\forall xyzw \in R.x + y + z + w = w + x + y + z) \to$$
$$(\forall xyzw \in R.x + y + z + w = z + w + x + y) \to p)$$
$$\to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*c45653e3c80cd999509820d81f22ae3c47b5cd44421738f32eda320ba570c87e*
Pure Prop Address:	*TMGvdvL6fepmXewTz9sJwhigZicrLoSXLZZ*
Theory Prop Id:	*40e443867ab4b8a14aafd8ab67f7ac4ae69383ab44ea2a4da6420b281d0d0628*
Theory Prop Address:	*TMQfH9YcUAFRqr7yeAgZh2h8mk1nc3GYCto*

23.2 Group1

Let $G : \iota$ be given.

23.2.1 Group1Explicit

Let $op : \iota \to \iota \to \iota$ be given. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term op.

Definition 23.1 *We define* explicit_Group *to be*

$$(\forall ab \in G.a * b \in G) \wedge (\forall abc \in G.a * (b * c) = (a * b) * c)\wedge$$
$$\exists e \in G.(\forall a \in G.e * a = a \wedge a * e = a)\wedge$$
$$(\forall a \in G.\exists b \in G.a * b = e \wedge b * a = e)$$

of type o identified by the following information:

Pure Object Id:	*0edee4fe5294a35d7bf6f4b196df80681be9c05fa6bb1037edb8959fae208cea*
Pure Object Address:	*TMKHXb8sJYXkkV8waqUHeCymoBGxrmp2eJ7*
Theory Object Id:	*a0d4c8bd9f788ef787589b54b2745f33864b9386aad8d7b99e0c747d5cd502b8*
Theory Object Address:	*TMNeD6sRqkAPQrbpVxeWHbWkWuomaavLf35*

explicit_Group_identity_unique

Theorem 23.2 $\forall ee' \in G.(\forall a \in G.e * a = a) \to (\forall a \in G.a * e' = a) \to e = e'$.
The proposition is identified by the following information:

Pure Prop Id:	*fe12d74789aa4b30c5b553f9b583cc7dac65508fc7b435345c330c9c50b7bda4*
Pure Prop Address:	*TMX43i28gCdiXowFyXeB4bgH5QZTmM6g1nh*
Theory Prop Id:	*4b4d44f80512969adc0813a098b3d4bc6f42912746d2cd4de30c9b4dc1fd8c1d*
Theory Prop Address:	*TMcvB6UWPmzCrw9QT1edWJq9i5gU4ihPnYh*

Assume the following.

$$\text{explicit\_Group} \tag{23.1}$$

Definition 23.2 *We define* explicit_Group_identity *to be*

$$\text{Eps\_i}\ (\lambda e.e \in G \wedge ((\forall a \in G.e * a = a \wedge a * e = a) \wedge \forall a \in G.\exists b \in G.a * b = e \wedge b * a = e))$$

of type ι identified by the following information:

Pure Object Id:	*8a818c489559985bbe78176e9e201d0d162b3dd17ee57c97b44a0af6b009d53b*
Pure Object Address:	*TMMh2NL7Q85JJXCFBeNsaMqMcMPaFhqKbfP*
Theory Object Id:	*84a58742704bbb5cd7767ccb82e978851abf4fb9bc5ab7c27851fdc99d2cd664*
Theory Object Address:	*TMdVb1nKZjH4P19ib11PFY6Cka8NUiF79Re*

Let e be explicit_Group_identity.

Definition 23.3 *We define* explicit_Group_inverse *to be*

$$\lambda a.\mathsf{Eps\_i}\ (\lambda b.b \in G \wedge (a * b = e \wedge b * a = e))$$

of type $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*e8cf29495c4bd3a540c463797fd27db8e2dfcc7e9bd1302dad04b40c11ce4bab*
Pure Object Address:	*TMHFMiwrvo9LoSX8Wi5EZvS4ccFWXeWxrgx*
Theory Object Id:	*a89687cfcbd45caa52b5cdf460d31291544f843439c0fb0cac9bd218223d0a98*
Theory Object Address:	*TMJDvZFoNydRtF58jVXVmTRo5GVfkFbJTHb*

Notation. We use $-^{-}$ as a postfix operator corresponding to applying term explicit_Group_inverse.

`explicit_Group_identity_prop`

Theorem 23.3

$$e \in G \wedge ((\forall a \in G.e * a = a \wedge a * e = a) \wedge \forall a \in G.\exists b \in G.a * b = e \wedge b * a = e).$$

The proposition is identified by the following information:

Pure Prop Id:	*068ffab150dc76c7de09d187943d2888faceaba1fbdc6449b322c8d4ef63ac4c*
Pure Prop Address:	*TMM9Eh1qhaiVeZj5Wgcxuvug9QQBNRfsEoz*
Theory Prop Id:	*c36841621d7b57d11c84af6cd0ed9c8fba75205040141ec30f15c1792884c83e*
Theory Prop Address:	*TMWKjEwmnVGHJzPZq7tPzu7AEs2mtZfK8zJ*

`explicit_Group_identity_in`

Theorem 23.4 $e \in G$. *The proposition is identified by the following information:*

Pure Prop Id:	*696ec40c6c7d83676b8638c42bd7832dd823bca9c973ae6f9c4f06cfa23bfc08*
Pure Prop Address:	*TMSoutuNF8eeWkkhpwkABkzngwXxdhPEeii*
Theory Prop Id:	*8cf78c6448b0a5a1eafb0e5c957793ae6ea2c4988a86e53cde584dd0291d61f3*
Theory Prop Address:	*TMHmwXi5SHMeK3Us86CEgisvDGA9xNi341B*

`explicit_Group_identity_lid`

Theorem 23.5 $\forall a \in G.e * a = a$. *The proposition is identified by the following information:*

Pure Prop Id:	*0a8b96c11b2713c863850c5c7d4857de37aa4fca73f7c304457b56e2f95e16e3*
Pure Prop Address:	*TMbD19oggdaE7cVLwcWS7ssWyd1V2HcUnDW*
Theory Prop Id:	*c412492ac795885c6900db2fe22e1dd552814f2557d6d79041f6191b3689beed*
Theory Prop Address:	*TMPTJy8uG71biAxCkf8AvzGecNsAuAUaKnK*

`explicit_Group_identity_rid`

Theorem 23.6 $\forall a \in G.a * e = a$. *The proposition is identified by the following information:*

Pure Prop Id:	*f6f5808dcc0c330452a517b4ebf8456cd5238d7b46e8111ab177813e331aa8b8*
Pure Prop Address:	*TMVgRDZiCVC1sCRnAp2nQ2asZP7Bjg3oe5e*
Theory Prop Id:	*ac30373bfbf244191de0c19f090210f9903310858129ab85808112737072b614*
Theory Prop Address:	*TMLhbf6oyewQ17oxmbzXFC3ejnSF86C5A6t*

explicit_Group_identity_invex

Theorem 23.7 $\forall a \in G.\exists b \in G.a * b = e \wedge b * a = e$. *The proposition is identified by the following information:*

Pure Prop Id:	*a48e2c7de86dcd94f3d0b7870a3a456284feaaf5d5c20e80a45b93114c6d50d5*
Pure Prop Address:	*TMW4eSZUkr3refSrUsK582eNHJhJwavEYV6*
Theory Prop Id:	*aee17204445b1eac60e7b66a3cdd8608a0182bd2a434fc82de1a92678591d2a3*
Theory Prop Address:	*TMTWCgk93fRtERh1PSJNhibGca6EnEgcnfe*

explicit_Group_inverse_prop

Theorem 23.8 $\forall a \in G.a^- \in G \wedge (a * a^- = e \wedge a^- * a = e)$. *The proposition is identified by the following information:*

Pure Prop Id:	*0f17df9988cfc01139158b40656a874d8f2118364e3b47b12b829509448c3c11*
Pure Prop Address:	*TMNhTk43Kf2YzVaFqj6KragMTmudKkw3kGf*
Theory Prop Id:	*6781d5952539e6702dfb38b38d848b596c60f3f0e775d5ca957f11e8938fcfe3*
Theory Prop Address:	*TMMJ2fwNJJe1W18rM6iSWAfyEcKkyVumryi*

explicit_Group_inverse_in

Theorem 23.9 $\forall a \in G.a^- \in G$. *The proposition is identified by the following information:*

Pure Prop Id:	*0f9bac9fe458ed7636280b89adbff7d9c00215f5f4364d4666a09f2fed583992*
Pure Prop Address:	*TMJm8VUj9hoafxu2Z8k99ys52dNjtstV9hY*
Theory Prop Id:	*c688e8369c999b2423047a981d3b5c198fb153c1d0f7e6dbee92f27ff38da254*
Theory Prop Address:	*TMbVkHwsuxJFdmgosfxN8dJkV4edBZBQ8Xf*

explicit_Group_inverse_rinv

Theorem 23.10 $\forall a \in G.a * a^- = e$. *The proposition is identified by the following information:*

Pure Prop Id:	*5faedae00b2ff3112d18fc67a44cc32e8836e3aefdb6f82dd4e11df8648381dc*
Pure Prop Address:	*TMNsYoMt2v2YNvxeJ1MzMMweHHpoJi62UCS*
Theory Prop Id:	*6522bcdcc934d3748a0dfcbd204413a2501816629e1eddf53c5b4d871f78ca57*
Theory Prop Address:	*TMc4sqjSkyuS9R2hVFzNDY16nePbzE5g4h8*

explicit_Group_inverse_linv

Theorem 23.11 $\forall a \in G.a^{-} * a = e$. *The proposition is identified by the following information:*

Pure Prop Id:	*434e096992c877eb38298cea7cc6fcf3ec01d9afbbcbd3b0783a632c67f5deff*
Pure Prop Address:	*TMTD5RfGMs4wShS2nexxcTshjmbsydNaghn*
Theory Prop Id:	*6b2fca7f46e1532922eaca0735e71402b9a2629f54eb3f3571f4ca378487902d*
Theory Prop Address:	*TMExrQ2X7HUbLBQZmQuMUtKf8RDh4mQWHWu*

`explicit_Group_lcancel`

Theorem 23.12 $\forall abc \in G.a * b = a * c \to b = c$. *The proposition is identified by the following information:*

Pure Prop Id:	*33b1b149cdeee23d3bb33fc0a6188a68ffa7bd8e41332d8be527ae438267f3d4*
Pure Prop Address:	*TMX6h7qhm3NenK6T4aPYQn1M9426HgYTMLw*
Theory Prop Id:	*826b2e6618fa8fe5b8cff0da58e5a8f4627dcb7b4323cd9e55e91ce57f05280e*
Theory Prop Address:	*TMSLWhCZYGhVwR8odE7nnVNZCRLJR4iruFv*

`explicit_Group_rcancel`

Theorem 23.13 $\forall abc \in G.a * c = b * c \to a = b$. *The proposition is identified by the following information:*

Pure Prop Id:	*ef05cefa790fecef7769c37f99ae58c2563aaa45db53a8bee86258c6b886132a*
Pure Prop Address:	*TMWrcUsJHruC5nYp5yAKHRzmwgkFPmSofTL*
Theory Prop Id:	*953b77b05a77af06ccb9cc940e52712be361bd38ef21fd159599be906700fe65*
Theory Prop Address:	*TMGiX5S1yoJfx8UWN5xD5FJqKeuhVYzgttD*

`explicit_Group_rinv_rev`

Theorem 23.14 $\forall ab \in G.a * b = e \to b = a^{-}$. *The proposition is identified by the following information:*

Pure Prop Id:	*891f7754b53564fca6f0343cad2dc494efc6e05e34c918bed88fcbddc533265f*
Pure Prop Address:	*TMGNPsYapqknYHxQsZ2F5XoqR6d9prQyeeR*
Theory Prop Id:	*a04aee87622e0b7dc5811e087ae8a1b75632125b99c1aa5cb6c6c3c9beceb1fc*
Theory Prop Address:	*TMSE4wKdLGWD9NZm2jRusJ5H4TwxUYDkdzK*

`explicit_Group_inv_com`

Theorem 23.15 $\forall ab \in G.a * b = e \to b * a = e$. *The proposition is identified by the following information:*

Pure Prop Id:	*663c8f6ce8987057ef1a016fc2d2eb826895e0f18480b99833b0f0990206672c*
Pure Prop Address:	*TMaSbuU5FwcNTsVwQ6PtZ6jLP8LYTgUxsUb*
Theory Prop Id:	*f1098fa01f9673e741bc27d240745fb3bfa00795281c4048f9003c7cc6b2f23d*
Theory Prop Address:	*TMYsR6YLMhEDxsZDgKWug5MYnBqnSoiNUZg*

`explicit_Group_inv_rev2`

Theorem 23.16 $\forall ab \in G.(a * b) * (a * b) = e \to (b * a) * (b * a) = e$. *The proposition is identified by the following information:*

Pure Prop Id:	*68dbad203c9e4e0b7716e921ed000127a9cf37e288fd5c525a1cdd3a75b5471f*
Pure Prop Address:	*TML3EFwc6aWd8wgzpvyuEJ5Ygy5et6QXvtZ*
Theory Prop Id:	*c5aeaedf7b6e943800ba4479c3f4fbb8514a2e20caea2699de720d8d40aec27b*
Theory Prop Address:	*TMRZWZ4JLKtefjP9iqT3BqVemyLiAo6X7VF*

Definition 23.4 *We define* explicit_abelian *to be* $\forall ab \in G.a * b = b * a$ *of type* o *identified by the following information:*

Pure Object Id:	*05d54a9f11f12daa0639e1df1157577f57d8b0c6ede098282f5eca14026a360f*
Pure Object Address:	*TMSbRNehAadQ85Kiirs6CeWkovxtJVvZdLQ*
Theory Object Id:	*afb9d62b583f9c34b2932e79c800886506cf83a6d1957f0f38ea4b6120525bf5*
Theory Object Address:	*TMWoaUJJFHkS4LvMBAUyKZxmsGTDk35A5mR*

23.2.2 Group1Explicit2

Let $op : \iota \to \iota \to \iota$ be given. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term op.

Group1Explicit2RepIndep

Let $op' : \iota \to \iota \to \iota$ be given. **Notation.** We use $\times$ as a right associative infix operator corresponding to applying term op'. Assume the following.

$$\forall ab \in G.a * b = a \times b \tag{23.2}$$

`explicit_Group_repindep_imp`

Theorem 23.17

$$\text{explicit\_Group } op \to \text{explicit\_Group } op'.$$

The proposition is identified by the following information:

Pure Prop Id:	*f5012ed8643556d71d97efacc96365a3e64c4075b67a87d5e41d6946ea117e15*
Pure Prop Address:	*TMK2g6qJebDnBquMej7EVWnZiqivTV1697r*
Theory Prop Id:	*9d8852ec071692482dd632725271f66464e62a8187bc25f252ba7aa90d3a2c38*
Theory Prop Address:	*TMUCkaJVFQzcD3xM53V7UGDtKmLnoAfRCF8*

Let e be explicit_Group_identity op. Let e' be explicit_Group_identity op'.

`explicit_Group_identity_repindep`

Theorem 23.18 explicit_Group $op \to e = e'$. *The proposition is identified by the following information:*

Pure Prop Id:	*598ff1a23c1fe4126915975f010569b525215d6ff2b98a5ab546dd324cbccaf2*
Pure Prop Address:	*TMcA9q6i8Vw98yFDFEvLQqJQQv6mxymW78w*
Theory Prop Id:	*bf4147f42cb6909b3123e55bbdd9abcd5e4500bf4c228aba6d90e85f3863fc07*
Theory Prop Address:	*TMGGsZcTZdyUMrW4cjQ2TioN1uRYTKdww5G*

Let inv be explicit_Group_inverse op. Let inv' be explicit_Group_inverse op'.

`explicit_Group_inverse_repindep`

Theorem 23.19 explicit_Group $op \to \forall a \in G.$inv $a = inv'\ a$. *The proposition is identified by the following information:*

Pure Prop Id:	*1b3e083dd9cf397ee3bc561cfa6db02990a3cf16bf8021b5b480a16f23aeb4b6*
Pure Prop Address:	*TMGDnT1Aqj6HwvAs6czLDGM8YE7vWNHc8xc*
Theory Prop Id:	*bce8beb409ddbfeedc408e86bebac1a0f9ce7b545243ee799f091e40547b0865*
Theory Prop Address:	*TMcrPmRNvDAbfSmF1GFzJbteMNNZLQJGYpX*

`explicit_abelian_repindep_imp`

Theorem 23.20 explicit_abelian $op \to$explicit_abelian op'. *The proposition is identified by the following information:*

Pure Prop Id:	*ef5af4d7ebcdc54f16dcbd6735446df711e99129ddf89eacd2c278c216ee4783*
Pure Prop Address:	*TMPnVDeP4gfRfuDBEtT7nJNFcLgKCXaRHWg*
Theory Prop Id:	*6dc553047a706e22ba94296abf5b0db1652898d70d735cf338004f26e3e4c8b4*
Theory Prop Address:	*TMQeZC612yR6Xczxn9AVL8mF3Ukm9V2F2bL*

23.2.3 Group1Explicit3RepIndep

Let $op : \iota \to \iota \to \iota$ be given. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term op. Let $op' : \iota \to \iota \to \iota$ be given. **Notation.** We use $\times$ as a right associative infix operator corresponding to applying term op'. Assume the following.

$$\forall ab \in G.a * b = a \times b \tag{23.3}$$

`explicit_Group_repindep`

Theorem 23.21 explicit_Group $op \Leftrightarrow$ explicit_Group op'. *The proposition is identified by the following information:*

Pure Prop Id:	*4c880a72b808ad924e81f56a1ed90dd384b42d8c05b5a34106946de015681f8b*
Pure Prop Address:	*TMYoUUzVknsDHuBdb4CTMFGTQJ8RavVXrEb*
Theory Prop Id:	*f0f9a2f129898373c5727edb2324561e41b4d2ee82534892365f8605e104fa10*
Theory Prop Address:	*TMdA43jg23cA8VmWUWe6knaNXVhz835Psjq*

`explicit_abelian_repindep`

Theorem 23.22 explicit_abelian $op \Leftrightarrow$ explicit_abelian op'. *The proposition is identified by the following information:*

Pure Prop Id:	*4bf05bb4c1ce5486164c944aa718fe7c76541eaa979f90733925e0df5da80b31*
Pure Prop Address:	*TMK9RV3nDAmdTWLAZyJDHS6BozLokJ5gF8H*
Theory Prop Id:	*9c96138929f1974912de1c755238da49ddca4b64627b220e7717c3142c9bffbb*
Theory Prop Address:	*TMJUpZdPq3Ps6CrXDg4pSuMXobz6F6aTgdA*

23.3 Groups Encoded as a Set

Definition 23.5 *We define* Group *to be*

$$\lambda G.\mathsf{struct\_b}\ G \wedge \mathsf{unpack\_b\_o}\ G\ \mathsf{explicit\_Group}$$

of type $\iota \to o$ identified by the following information:

Pure Object Id:	*3bcfdf72871668bce2faf4af19b82f05b1ff30e94a64bbc83215c8462bc294ca*
Pure Object Address:	*TMPgSXfSPyvihsFWLDa9H7w9DDaHAAqfKWh*
Theory Object Id:	*48ad758ca1725528973592bef2eca92dd3b4cc57fcc1cec0478a8f8e97376e86*
Theory Object Address:	*TMVKTH5owDCXYfd1wFd2cm4yxGHnuLN5CXz*

Definition 23.6 *We define* abelian_Group *to be*

$$\lambda G.\mathsf{Group}\ G \wedge \mathsf{unpack\_b\_o}\ G\ \mathsf{explicit\_abelian}$$

of type $\iota \to o$ identified by the following information:

Pure Object Id:	*c9c27ca28ffd069e766ce309b49a17df75d70111ce5b2b8e9d836356fb30b20a*
Pure Object Address:	*TMZYJxov52ZDnfLrrgP436fnEtrVohjgYF3*
Theory Object Id:	*82b65a8b4d905ce66d83d96cd51c72493e71be4e889671636b43a03ef4993b11*
Theory Object Address:	*TMKxwHVb8mydLee3wEkwZAbNhhw23ui2AP7*

`Group_unpack_eq`

Theorem 23.23

$$\forall G.\forall op :\iota \to \iota \to \iota.$$
$$\mathsf{unpack\_b\_o}\ (\mathsf{pack\_b}\ G\ op)\ \mathsf{explicit\_Group} = \mathsf{explicit\_Group}\ G\ op.$$

The proposition is identified by the following information:

Pure Prop Id:	*aceab134c9adba80930143837f535ea11b55ad2ca8eaf42a51a35901b0783c35*
Pure Prop Address:	*TMd7GQcSo21uLBQ7A49xYT78iKSY3PhWQFB*
Theory Prop Id:	*f34cbdd88f4d553a25bb8ac2222863ca028a51086ae3e0f2d11a1fe78e716303*
Theory Prop Address:	*TMZ6K2cAEkL2uXg3fBxTcxoJSRUaJHicJmH*

`GroupI`

Theorem 23.24

$$\forall G.\forall op :\iota \to \iota \to \iota.\mathsf{explicit\_Group}\ G\ op \to \mathsf{Group}\ (\mathsf{pack\_b}\ G\ op).$$

The proposition is identified by the following information:

Pure Prop Id:	*f59584e74c871bfb0a1036e424d097143949b1923d9616b3320eb90f399a91ed*
Pure Prop Address:	*TMQUiQQxmEfgMnDao2t2u2KdoezR3zraGZY*
Theory Prop Id:	*7a80f33492e5b3d03f1841fd7ccbd5fb77c2faa6b30b5c242db4c0059fc6cdb9*
Theory Prop Address:	*TMTYbJ6XVymYaZkodwF8XHTGRBpsABB8dxk*

`GroupE`

Theorem 23.25

$$\forall G.\forall op :\iota \to \iota \to \iota.\mathsf{Group}\ (\mathsf{pack\_b}\ G\ op) \to \mathsf{explicit\_Group}\ G\ op.$$

The proposition is identified by the following information:

Pure Prop Id:	*f3f463245e1c5c9686fda5a34894e9f282a8f86a019ada8b14ef0e11b0976d07*
Pure Prop Address:	*TMHNfqMd8twRy3pLuxLepizuzSei8dgrR2i*
Theory Prop Id:	*2239355bbc80224799ae9be8d5818f67132ab3353e50b12be897026966f41a8f*
Theory Prop Address:	*TMUKjYUwFEKr2Brho6Mkc5ukMBKuMH2PP4M*

`abelian_Group_unpack_eq`

Theorem 23.26

$$\forall G.\forall op :\iota \to \iota \to \iota.$$
$$\mathsf{unpack\_b\_o}\ (\mathsf{pack\_b}\ G\ op)\ \mathsf{explicit\_abelian} = \mathsf{explicit\_abelian}\ G\ op.$$

The proposition is identified by the following information:

Pure Prop Id:	*2346855e53255b79e76009eda280acf374dad7dcc5e72ab74d2f42201d79140a*
Pure Prop Address:	*TMMHnLwWhH3nJZkk281hptKVfm4dKceyAwW*
Theory Prop Id:	*6dfe13367c39b0c86ac8656ad108a540b7a3f5c3364369c3dea90e33b297a4a2*
Theory Prop Address:	*TMWfTBZCZjhKbb8QKapfLpSa2pyTTfAE71m*

`abelian_Group_E`

Theorem 23.27

$$\forall G.\forall op :\iota \to \iota \to \iota.\mathsf{abelian\_Group}\ (\mathsf{pack\_b}\ G\ op) \to$$
$$\mathsf{Group}\ (\mathsf{pack\_b}\ G\ op) \wedge \mathsf{explicit\_abelian}\ G\ op.$$

The proposition is identified by the following information:

Pure Prop Id:	*0ddfdd60c899db663169e18602174fc26a195f4e543a49ea6ff9d85c51d1db1d*
Pure Prop Address:	*TMFf47YDuvj4E1VPha1jb6n3UhV7xC9VtyU*
Theory Prop Id:	*6ec272a62e16860ef6fdf6e1827bf60d5f1da411baf1baa0a7b2319ae3c3300a*
Theory Prop Address:	*TMQz71B8ZwyWMJ8Y7PhfRsqJmZ5rqBNeqbW*

23.4 Group2

Let $G : \iota$ be given. Let $op : \iota \to \iota \to \iota$ be given. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term op. **Notation.** We use $-^{-}$ as a postfix operator corresponding to applying term explicit_Group_inverse $G\ op$. Let $H : \iota$ be given.

Definition 23.7 *We define* explicit_subgroup *to be* Group (pack_b $H\ op$) $\wedge H \subseteq G$ *of type o identified by the following information:*

Pure Object Id:	*4f273e5c6c57dff24b2d1ca4088c333f4bbd2647ab77e6a03beedd08f8f17eb9*
Pure Object Address:	*TMNFW3sUiRGF4QZDyZs4U1ZFwLXYNZndEUA*
Theory Object Id:	*a5189d9d38cb9e37388edfc6167f21fbbac2dae96c0d30a42796f7f45363ba49*
Theory Object Address:	*TMUKd1esxsYP6brvBoQj3SzvUxCbvF2x7bC*

Definition 23.8 *We define* explicit_normal *to be* $\forall x \in G.\{x * a * x^{-} | a \in H\} \subseteq H$ *of type o identified by the following information:*

Pure Object Id:	*1cb8f85f222724bb5ee5926c5c1c0da8acd0066ba3fa91c035fe2fb8d511a035*
Pure Object Address:	*TMUp97JhNka3EMtq317xqSLU7QwwU9Rv3di*
Theory Object Id:	*d7a9568bb9deb69b03d47d288813c29f08fcc8800ee11aed9df4f6a44df35147*
Theory Object Address:	*TMdaRbf7U1umHdo2j1hDVsveTbdoBQj6QLo*

Assume the following.

$$\text{Group (pack\_b } G\ op) \tag{23.4}$$

Let e be explicit_Group_identity $G\ op$.

`explicit_subgroup_test`

Theorem 23.28

$$H \subseteq G \to e \in H \to (\forall a \in H.a^{-} \in H) \to (\forall ab \in H.a * b \in H) \to \text{explicit\_subgroup}.$$

The proposition is identified by the following information:

Pure Prop Id:	*a0764d61788853004917f998bf172f98ea252833c7ffffbd4cc8f9fc6d366640*
Pure Prop Address:	*TMF72nFcvhbTi7RSPrKiVdQH7Zno6gqKNzX*
Theory Prop Id:	*febaefb669b389c2e7af77327b7c126d3c2296175d831b5152819092c60df749*
Theory Prop Address:	*TMSe5XgyjkAZsLZRAWWesUNSkSR5BgeN71j*

Assume the following.

$$\text{explicit\_subgroup} \tag{23.5}$$

Let e' be explicit_Group_identity $H\ op$.

`explicit_subgroup_identity_eq`

Theorem 23.29 $e = e'$. *The proposition is identified by the following information:*

Pure Prop Id:	*d608de6b443a17bb92b076f03900648de80c1688b0a9b18c0f6f2a87d48e9494*
Pure Prop Address:	*TMMi9FM87WTYChZcDCYSiR1mUgcxriaPpTH*
Theory Prop Id:	*29bcb214853a5b2b4c2ba9cd9171fe4cd7583e3c7526e1f14f5c9d35785657f7*
Theory Prop Address:	*TMMErcjnubQrVeuBCzarecXxmRfNpUwDVWL*

`explicit_subgroup_inv_eq`

Theorem 23.30

$$\forall a \in H.\text{explicit\_Group\_inverse}\ G\ op\ a = \text{explicit\_Group\_inverse}\ H\ op\ a.$$

The proposition is identified by the following information:

Pure Prop Id:	*23d0fd40782cc5d113a6f8abbffe684eabb63f361450d6781bee3a513762cc1c*
Pure Prop Address:	*TMSzo58obcQmEn5FMDAcyyCsxm1mW4emXNT*
Theory Prop Id:	*c0bd13173f3bd1ccdf37652d679abb58404d427bcf1cb4bb3209c92633bb1cb0*
Theory Prop Address:	*TMMes5AgQed1v3SDwaTYoyxExk9LqUvpPHw*

`explicit_abelian_normal`

Theorem 23.31 $\text{explicit\_abelian}\ G\ op \rightarrow \text{explicit\_normal}$. *The proposition is identified by the following information:*

Pure Prop Id:	*07d5b2fc245834d75ec2a313ddb687a0a4ac1c4858adfb47fb56a2b4cdadfcbb*
Pure Prop Address:	*TMWykJghtnE3YAmhYszqL4L2WUWyWs8bESa*
Theory Prop Id:	*142e7aa60e5da0bf6b3fb1a0a03445c8098ce5c7613ce5f3f698f2493d8311bb*
Theory Prop Address:	*TMPfm754BwmkjxHRPzWnL1tdpxUusE7tyXy*

23.5 Group3

Let $HG : \iota$ be given. Let $op, op' : \iota \to \iota \to \iota$ be given. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term op. **Notation.** We use $-^{-}$ as a postfix operator corresponding to applying term $\text{explicit\_Group\_inverse}\ G\ op$. **Notation.** We use $\times$ as a right associative infix operator corresponding to applying term op'. **Notation.** We use $-^{-}$ as a postfix operator corresponding to applying term $\text{explicit\_Group\_inverse}\ G\ op'$. Assume the following.

$$\text{explicit\_Group}\ G\ op \tag{23.6}$$

Assume the following.

$$H \subseteq G \tag{23.7}$$

Assume the following.

$$\forall ab \in G.a * b = a \times b \tag{23.8}$$

explicit_normal_repindep_imp

Theorem 23.32 explicit_normal $G\ op\ H \to$ explicit_normal $G\ op'\ H$. *The proposition is identified by the following information:*

Pure Prop Id:	*3085191447907ed870314c89a8d28cb2745242ad5fc8d8fac5e605f123406e41*
Pure Prop Address:	*TMF9MV6sqG4RPotXsm4dDP2WeBrREMZMpDz*
Theory Prop Id:	*9e1540196415c21f2eff94a4d69154a46a0a7f369764af1fe8c67431af6bab59*
Theory Prop Address:	*TMFwHf4QsoVfQnoVRPxuR1DDrZAUtc9aPLc*

23.6 Subgroup as a Relation on Encoded Groups

Definition 23.9 *We define* subgroup *to be*

$$\lambda HG.\mathsf{struct\_b}\ G \wedge \mathsf{struct\_b}\ H \wedge \\ \mathsf{unpack\_b\_o}\ G \\ (\lambda G' op.\mathsf{unpack\_b\_o}\ H \\ (\lambda H'\_.H = \mathsf{pack\_b}\ H'\ op \wedge \mathsf{Group}\ (\mathsf{pack\_b}\ H'\ op) \wedge H' \subseteq G'))$$

of type $\iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*994c6976ade70b6acb23366d7ac2e5d6cf47e35ce09a6d71154e38fd46406ad1*
Pure Object Address:	*TMLRkyR7sy5G519JM6FKX7WCKVJc2Xnbd7n*
Theory Object Id:	*cb09cb23c96a2afb0a031b5795b5906505c2d764fd807be41d9c9ed8115f7b0e*
Theory Object Address:	*TMGMFfq46mwEhdYGyNc2EdYoj7nzTA8Jomq*

Notation. We use $\leq$ as an infix operator corresponding to applying term subgroup.

Definition 23.10 *We define* subgroup_index *to be*

$$\lambda HG.\mathsf{unpack\_b\_i}\ G \\ (\lambda G' op.\{n \in \mathsf{omega} | \exists f \in G'^{\mathsf{ordsucc}\ n}.\forall ij \in \mathsf{ordsucc}\ n.i \neq j \to \forall ab \in H\ 0.op\ (f\ i)\ a \neq op\ (f\ j)$$

of type $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*1dd206bf7d1aea060662f06e19ec29564e628c990cb17f043a496e180cf094e8*
Pure Object Address:	*TMPcqQwKgtAXH5j1TdcLe1pH8wNiDMvYyEf*
Theory Object Id:	*7ebea62699ca1f7386cd7aa4aca981fd2c9635dc208c7d879649411943005d9d*
Theory Object Address:	*TMHQwMsB4gE9yRryzdXDQJVWgJ17RM6aMp4*

Definition 23.11 *We define* normal_subgroup *to be*

$$\lambda HG.H \leq G \wedge \\ \mathsf{unpack\_b\_o}\ G \\ (\lambda G' op.\mathsf{unpack\_b\_o}\ H\ (\lambda H'\_.\mathsf{explicit\_normal}\ G'\ op\ H'))$$

of type $\iota \to \iota \to o$ *identified by the following information:*

Pure Object Id: *071c71e291b8c50d10f9d7f5544da801035d99ff3a68c9e5386b4087d22b5db2*
Pure Object Address: *TMd7xWz8FWuB8j5DzNNXPoPYjwwcw9iGFfh*
Theory Object Id: *e2b2818e909567261bb44f9241b012f127d9ff02c3dd0d03ad5c7d708a41ee32*
Theory Object Address: *TMUQxZTG9uZr52Krep3t5yfZUjXQwpCSMDD*

`pack_b_subgroup_E`

Theorem 23.33

$$\forall HG:\iota.\forall opHop:\iota\to\iota\to\iota.\mathsf{pack\_b}\ H\ opH \le \mathsf{pack\_b}\ G\ op\to \mathsf{pack\_b}\ H\ opH = \mathsf{pack\_b}\ H\ op \wedge \mathsf{explicit\_subgroup}\ G\ op\ H.$$

The proposition is identified by the following information:

Pure Prop Id: *7125ffccb47b1cd6d83c70896cd0d77ebc0a5c375a659546c75d43ff0d142474*
Pure Prop Address: *TMUbUKT8Uymhwq1sqJm6JGztSHZZZywozop*
Theory Prop Id: *071cdd18db9355616c94b43f39efdd84fb6821f364314b2cb4f40e9bd1675033*
Theory Prop Address: *TMaQ7YHCEZFXsZgr3AL8pKTCzucZbFxr6jW*

`subgroup_E`

Theorem 23.34

$$\forall HG.H\le G\to\forall q:\iota\to\iota\to o.(\forall HG.\forall op:\iota\to\iota\to\iota.(\forall ab\in G.op\ a\ b\in G)\to \mathsf{Group}\ (\mathsf{pack\_b}\ H\ op)\to H\subseteq G\to q\ (\mathsf{pack\_b}\ H\ op)\ (\mathsf{pack\_b}\ G\ op)) \to q\ H\ G.$$

The proposition is identified by the following information:

Pure Prop Id: *8d2e3a1f4dd63f83ee62c1339fd126e053185c4e48fc3cc95c24bf2ccac05098*
Pure Prop Address: *TMYiapjALEQg8YorewqjMBBDNZSfdKWuMUd*
Theory Prop Id: *664898a15a2a44476b34fcd20187cb6b36a7be22f8e5a6208da45360a43d39bf*
Theory Prop Address: *TMR3yhY1JLi3dUUZzQR9GUXSZroHRWT43WE*

`abelian_group_normal_subgroup`

Theorem 23.35

$$\forall G.\mathsf{abelian\_Group}\ G\to\forall H.H\le G\to\mathsf{normal\_subgroup}\ H\ G.$$

The proposition is identified by the following information:

Pure Prop Id: *4382dad503528fbf8173cf9e47b154aca06b57bf02b22eb6830a50e23b94f865*
Pure Prop Address: *TMTKNLNxcPh9EVds4cHdQmAhf5w3KjMpXZk*
Theory Prop Id: *31f5b8608189f9d346c09a8ca7beb6a0955a20f1ee768bbad1c3a8d8c9eff610*
Theory Prop Address: *TMRsrDZNbLqAcJgVD7iy9CvAaGQiKeavic1*

`subgroup_transitive`

Theorem 23.36 $\forall KHG.K\le H\to H\le G\to K\le G.$ *The proposition is identified by the following information:*

Pure Prop Id: *c84fb0f00761163c826038c45b5a080d551a4af82e13ecdbec4190f7d264c8c6*
Pure Prop Address: *TMRVsYwzXoyQoBSWcrLFgzK5m4LjCaiomjX*
Theory Prop Id: *026e9055ebb50e4182d43a6f9cdfa349ac1e29a7844be46cdc9b70e9aa507fce*
Theory Prop Address: *TMQpoQCnMNzGQ5c655fPoRUpabq3NF9LuZs*

23.7 Group4

Let $A : \iota$ be given. Let G be $\{f \in A^A | \mathsf{bij}\ A\ A\ (\lambda x.f\ x)\}$. Let op be $\lambda f g : \iota.\lambda x \in A.g\ (f\ x)$.
Notation. We use $*$ as a right associative infix operator corresponding to applying term op. Let id be $\lambda x \in A.x$.

explicit_Group_symgroup

Theorem 23.37 $\mathsf{explicit\_Group}\ G\ op$. *The proposition is identified by the following information:*

Pure Prop Id:	75533e8a735e606e28b8daa28ccee75f6b02e88687f5737d356a9e287a3636e0
Pure Prop Address:	TMKpf7yzXWNWoB7pTFZ9CSXQc1WgFWKDum8
Theory Prop Id:	2004dca00796c3260ddfc226ff73cd52a9ab5d9464afb109a5e9626139500ed8
Theory Prop Address:	TMUjucvp4GP3FkV7CeMpFR5TdJdrvtmChGd

explicit_Group_symgroup_id_eq

Theorem 23.38 $\mathsf{explicit\_Group\_identity}\ G\ op = id$. *The proposition is identified by the following information:*

Pure Prop Id:	21cf5cbe5a190e3996df417982f48894c7fcd25e942984729664b5b8bc6e0914
Pure Prop Address:	TMZFox5vkuvToBeKHtjvAXUWRxd5zPHKZEF
Theory Prop Id:	fce19ddc04b4e28eca8656cf372366102a641140d6d39cdbbd0e47fd939dd647
Theory Prop Address:	TMXkzvHQA9cxQakivoT7RYrtHpaUzWEVmaX

explicit_Group_symgroup_inv_eq

Theorem 23.39

$$\forall f \in G.\mathsf{explicit\_Group\_inverse}\ G\ op\ f = (\lambda x \in A.\mathsf{inv}\ A\ (\lambda x.f\ x)\ x).$$

The proposition is identified by the following information:

Pure Prop Id:	569ca8a98ff593e668e81c3c75a51082eb69d41932f6f1b3eb2d602a460fcb77
Pure Prop Address:	TMSEXyQTYWdXvSSZeGpr2AiQNQYfM8psrG6
Theory Prop Id:	9b2215cf55968e6869adb6aba2a45d58065e1806de069a6c1f8eba372d33e15e
Theory Prop Address:	TMKp3LXskT1sHdd7hevEZu5XUkBG1q5YCR9

Let $B : \iota$ be given. Let H be $\{f \in A^A | \mathsf{bij}\ A\ A\ (\lambda x.f\ x) \wedge \forall x \in B.f\ x = x\}$.

explicit_subgroup_symgroup_fixing

Theorem 23.40 $B \subseteq A \to \mathsf{explicit\_subgroup}\ G\ op\ H$. *The proposition is identified by the following information:*

Pure Prop Id:	bbacee9bc55d4bb9d8a095e058cf6c04648223d4da35a401d88ff34482c4bfa6
Pure Prop Address:	TMWhKgrPUBPXmguhqdDXCXSxNBeVZqTWqt7
Theory Prop Id:	9bbf1dff7d86d24b11b8fc750a9c1358c88d26f302226f1a0f7af5c4b779e747
Theory Prop Address:	TMQPazgEs2ZAxG7K58ru8aJWKhEasAJanUc

23.8 Groups of Permutations

Definition 23.12 *We define* symgroup *to be*

$$\lambda A.\mathsf{pack\_b}\ \{f \in A^A | \mathsf{bij}\ A\ A\ (\lambda x.f\ x)\}\ (\lambda f g.\lambda x \in A.g\ (f\ x))$$

of type $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*b5e0e5eb78d27c7fd8102e3c6741350d4444905a165833aa81250cef01a1acea*
Pure Object Address:	*TMZELiVF4MBqaUvQtfAy3PdnvoJi11ixgrf*
Theory Object Id:	*5a86a668a5aa4ec3f8070179712ef77612eb82d194a9d09eba866f04a67f1bb9*
Theory Object Address:	*TMLr56c3xH1KtYDpD7smvX5TNYc9XxLWkp7*

Definition 23.13 *We define* symgroup_fixing *to be* $\lambda AB.\mathsf{pack\_b}\ \{f \in A^A | \mathsf{bij}\ A\ A\ (\lambda x.f\ x) \wedge \forall x \in B.f$
of type $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*683becb457be56391cd0ea1cefb5016e2185efebd92b7afd41cd7b3a4769ac57*
Pure Object Address:	*TMVDj8UpCzz8NDMvJKFnUV46Ac7avCYjHaA*
Theory Object Id:	*3ec65f6d528b1474770fc9b11779c3256c697e2ffcb88e012e04a72be7cb29e1*
Theory Object Address:	*TMFNyn7aj1wSxx41MtoZKZ7b1VE1wtZVNQz*

`Group_symgroup`

Theorem 23.41 $\forall A.$Group (symgroup A). *The proposition is identified by the following information:*

Pure Prop Id:	*990875f1186a5c690222b6df93ca46ef542493573c3ec402b83ecaa74a396595*
Pure Prop Address:	*TMUqQYsqDvdQsVAxhFJHuJf8qqVjQaX4xft*
Theory Prop Id:	*aa2f4e58437f0a2d91605cef84771180bffe4c75c9f5c4c510c1123991d31284d*
Theory Prop Address:	*TMQ1kebtip2R8A29YT8cryCxBqcSdnZRkFP*

`Group_symgroup_fixing`

Theorem 23.42 $\forall AB.B \subseteq A \to$ Group (symgroup_fixing A B). *The proposition is identified by the following information:*

Pure Prop Id:	*4c494d5b78b3e2f30e56cfecea27ca083d44f00a781f32acffbe38a3616a3bd0*
Pure Prop Address:	*TMVs3oeD3WthGX2etWEzZB5jM5vfjmrzdv8*
Theory Prop Id:	*79320ab5a30ce81496e65d23083aa7bf3b2669673cf4d3f1da6f4cd56754af32*
Theory Prop Address:	*TMJxYyCcFvZLw4yZZSr8PNBQfMkiujqpSn7*

`subgroup_symgroup_fixing`

Theorem 23.43 $\forall AB.B \subseteq A \to$symgroup_fixing A $B \leq$ symgroup A. *The proposition is identified by the following information:*

Pure Prop Id:	*ea4ea2cb60a4490fa7dd3a8d1121a24afe0c4f02b1e69057fa3ac1cbaa488ce1*
Pure Prop Address:	*TMUgyamfautEr7neHo5s168PJsHpp3YAAxn*
Theory Prop Id:	*e7086b5067a4adfaef6ac76b57d03ca28dd628c134faaba9dd82bdbe075035ba*
Theory Prop Address:	*TMNMREeoQruvS2q4Z9DbBPztZroZJLfFJtw*

`subgroup_symgroup_fixing2`

Theorem 23.44

$$\forall ABC.C \subseteq B \to B \subseteq A \to \mathsf{symgroup\_fixing}\ A\ B \leq \mathsf{symgroup\_fixing}\ A\ C.$$

The proposition is identified by the following information:

Pure Prop Id:	*5b06faf92f0eabd16f89b9270b775c6a00de6b3dbc7910aba9398f6587c745ba*
Pure Prop Address:	*TMFXcztsEWX26DZ48NuWwuvxCKRxMqxgYEw*
Theory Prop Id:	*93e5e57f39b365e446a7903858b9899067c88e034d90efdc581b4175c2a760c5*
Theory Prop Address:	*TMVocWTn2DD7Q6zJVrVcTrKtjJ566LnziCr*

`nonnormal_subgroup`

Theorem 23.45 $\exists HG.\mathsf{Group}\ G \wedge H \leq G \wedge \neg \mathsf{normal\_subgroup}\ H\ G.$ *The proposition is identified by the following information:*

Pure Prop Id:	*4af1eff8f561099d5165fbd9166ec526c420d387fa5843d448689ae34c64a1fd*
Pure Prop Address:	*TMaJXbhnzpZH2dG2PZNn1vcD2Zrzy99NNUq*
Theory Prop Id:	*f4475069cf93e4f1ed33d4f2b63efb7114719af9f10d271d932e038b09e2f797*
Theory Prop Address:	*TMK3fNtE5vExNjZ8gT49kLhgbW7s8Mujkhm*

Definition 23.14 *We define* normal_subgroup_equiv *to be*

$$\lambda GNab.\mathsf{unpack\_b\_o}\ G\ (\lambda Gop.a \in G \wedge b \in G \wedge op\ a\ (\mathsf{explicit\_Group\_inverse}\ G\ op\ b) \in N\ 0)$$

of type $\iota \to \iota \to \iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*8d6a0ebb123230b9386afac5f3888ea2c3009a7caabad13e57153f9a737c8d0b*
Pure Object Address:	*TMRASNP9xsTmfEHZVz86EScPbtm9LE7RB3s*
Theory Object Id:	*6de522f05b2ef6abc7f410d16bdebf811c59bb4ef7f25e5a80615298e85ecef2*
Theory Object Address:	*TMHp4buRe3CwdShZ32fRxcFLnxds4BLiMTy*

Definition 23.15 *We define* quotient_Group *to be*

$$\lambda GN.\mathsf{unpack\_b\_i}\ G$$
$$(\lambda G'op.\mathsf{pack\_b}\ \{a \in G' | \mathsf{quotient}\ (\mathsf{normal\_subgroup\_equiv}\ G\ N)\ a\}$$
$$(\lambda ab.\mathsf{canonical\_elt}\ (\mathsf{normal\_subgroup\_equiv}\ G\ N)\ (op\ a\ b)))$$

of type $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*971902aba30554ada3e97388da3b25c677413e2b4df617edba4112889c123612*
Pure Object Address:	*TMRMVZTtqZEm7ZeJHfgYoXZDgFGDJRPPgV8*
Theory Object Id:	*ef67eb20b92ec20f2b10d37a0da1f0c16b334205004d4b4bdc4b498bcd06a015*
Theory Object Address:	*TMJprm3Y7UHzW78sFefvKVSjQLMnLJqyagm*

Definition 23.16 *We define* trivial_Group_p *to be* $\lambda G.\mathsf{Group}\ G \wedge \forall xy \in G\ 0.x = y$ *of type* $\iota \to o$ *identified by the following information:*

Pure Object Id: *759fd51778137326dda8cf75ce644e3ec8608dfa7e59c54b364069dd6cf6dd0d*
Pure Object Address: *TMXgRAihjx6p6LVB2gjGrWcrkQxxPLzMt3u*
Theory Object Id: *4a843e3be3a632d671082290bcb4335685122c8f5b5168ab2de4fffb849942c4*
Theory Object Address: *TMMyfsrQ9kVyrT23cGp37GGgZap9HT6THXK*

Definition 23.17 *We define* solvable_Group_p *to be*

$$\lambda G.\exists n \in \text{omega}.\exists Gseq.(\forall i \in \text{ordsucc } n.\text{Group } (Gseq\ i))\wedge$$
$$(\forall i \in n.\text{normal\_subgroup } (Gseq\ (\text{ordsucc } i))\ (Gseq\ i))\wedge$$
$$(\forall i \in n.\text{abelian\_Group } (\text{quotient\_Group } (Gseq\ i)\ (Gseq\ (\text{ordsucc } i))))\wedge$$
$$G = Gseq\ 0 \wedge \text{trivial\_Group\_p } (Gseq\ n)$$

of type $\iota \to o$ *identified by the following information:*

Pure Object Id: *49803500fa75d5030dddbb3d7a18d03eeebfdd720d57630629384cec6d8b3afc*
Pure Object Address: *TMX5XpRdUwhyCYB1UE83KHG1aehAuoESN6w*
Theory Object Id: *da0c1bb253cd8664d898fe71430bbdd8202b6943c0866c96597b014889d43008*
Theory Object Address: *TMNb2P5ANiNoTvE4HeSJnPNGJDNyZCjuTD5*

Definition 23.18 *We define* Group_carrier *to be* $\lambda Gs.Gs\ 0$ *of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id: *f0bb69e74123475b6ecce64430b539b64548b0b148267ea6c512e65545a391a1*
Pure Object Address: *TMY8sbcPiVs9VL311KPHpbm67U6sLM9JvF7*
Theory Object Id: *9211e41eaf81bb8c152618cbe35230be60d736fcd5684c0b7bdc5ccc1ff70181*
Theory Object Address: *TMZQE6tPrFvJMaBcLrcMn85ZgscSeFbm4pT*

Definition 23.19 *We define* Group_op *to be* $\lambda Gs.\text{decode\_b } (Gs\ 1)$ *of type* $\iota \to \iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id: *6968cd9ba47369f21e6e4d576648c6317ed74b303c8a2ac9e3d5e8bc91a68d9d*
Pure Object Address: *TMYTM1QQGqgVyRM9unR8CLL2scwRHYvV3c3*
Theory Object Id: *ed06aa34b4112f8507370b46c1c5c7de7f2f9af4f24b831d45893ad3066e994e*
Theory Object Address: *TMRiY7KU6BpU28KD19iwgBDLsQ5XyfpcbYL*

23.9 Group2

Let $Gs : \iota$ be given. Let $Gs' : \iota$ be given. Let $G : \iota$ be Group_carrier Gs. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term Group_op Gs. Let $G' : \iota$ be Group_carrier Gs'. **Notation.** We use $\times$ as a right associative infix operator corresponding to applying term Group_op Gs'.

Definition 23.20 *We define* Group_Hom *to be*

$$\lambda g.\text{Group } Gs \wedge \text{Group } Gs' \wedge g \in {G'}^{G} \wedge \forall ab \in G.g\ (a * b) = g\ a \times g\ b$$

of type $\iota \to o$ *identified by the following information:*

Pure Object Id:	*163d1cc9e169332d8257b89d93d961798c6847300b6756e818c4906f0e37c37f*
Pure Object Address:	*TMG33bGq8oXGhbwW9bdnHoQwrAnZGRnqbDo*
Theory Object Id:	*04a4493700bf4496fc9785ab649cc965ed6f6509b5bedeb92b3cfd0295c6ab1d*
Theory Object Address:	*TMbt3jCCTS2GpiirQWPe7YD1PGZoafrvFeD*

Definition 23.21 *We define* Group_Iso *to be* $\lambda g.$Group_Hom $g \wedge$ bij $G\ G'\ (\lambda x.g\ x)$ *of type* $\iota \to o$ *identified by the following information:*

Pure Object Id:	*3bc46030cc63aa36608ba06b3b00b696e1a1eb1de5346ff216ca33108220fbda*
Pure Object Address:	*TMNoSwhDXv7bcUbtfNVM1cTexdvsoLbdLJQ*
Theory Object Id:	*3578401f9ab8d370c4c75e61d9c1049d50026478ddeaef7cb2f37a857cf8c8cd*
Theory Object Address:	*TMLoxwKVttJ32tGGRhbrdMTf4SuCUFyYCkX*

Definition 23.22 *We define* Group_Isomorphic *to be* $\exists g.$Group_Iso g *of type* o *identified by the following information:*

Pure Object Id:	*727fd1e7d5e38e7145387c0a329b4de2b25f2de5d67989f11e8921bc96f4b6bd*
Pure Object Address:	*TMPtRTKnaVTXR2dtDWrcvKMrZogxkw6v58N*
Theory Object Id:	*8b3f77534efc12910b019fd5efaa573bd03c8fbe3aaf9677848757f8741adaf1*
Theory Object Address:	*TMKSQinvAqsVekHU9Jcu1GLLV1RU4ESajgh*

Chapter 24

Rings

24.1 explicit_Ring

Let $R : \iota$ be given. Let $zero : \iota$ be given. Let $plus, mult : \iota \to \iota \to \iota$ be given. **Notation.** We use $+$ as a right associative infix operator corresponding to applying term $plus$. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term $mult$.

Definition 24.1 explicit_Ring *is the opaque object of type o identified by the following information:*

Pure Object Id:	*aa9e02604aeaede16041c30137af87a14a6dd9733da1e227cc7d6b966907c706*
Pure Object Address:	*TMKWfmiqi9982enMMW61SmX2e5KJYZZRvW1*
Theory Object Id:	*9c479e9dd34e64c77a730debb88d122fdd5adfceda11ca779159e45aed2bc6dc*
Theory Object Address:	*TMJC4zWDzgdV3bp53NDJM4PjrbuESuq67yR*

`explicit_Ring_I`

Theorem 24.1

$$(\forall xy \in R.x + y \in R) \to (\forall xyz \in R.x + (y + z) = (x + y) + z) \to (\forall xy \in R.x + y = y + x) \to zero \in R \to (\forall x \in R.zero + x = x) \to (\forall x \in R.\exists y \in R.x + y = zero) \to (\forall xy \in R.x * y \in R) \to (\forall xyz \in R.x * (y * z) = (x * y) * z) \to (\forall xyz \in R.x * (y + z) = x * y + x * z) \to (\forall xyz \in R.(x + y) * z = x * z + y * z) \to \mathsf{explicit\_Ring}.$$

The proposition is identified by the following information:

Pure Prop Id:	*22c5848914731e1acb3f2bdd3b5bea1e0b842aaaa63773bdad8403ec9062cf0b*
Pure Prop Address:	*TMbhVQoNomR8RucZMffmjCGsSENbsaFffzF*
Theory Prop Id:	*c833f7cffb3a99d6a671445f80dbc49660a3082bc66d24ff01289ad8615ffb3b*
Theory Prop Address:	*TMWeeT9b2ScxzB9LDbXgaQkiZTzmNTSjyAf*

`explicit_Ring_E`

Theorem 24.2

$$\begin{gathered}\forall q : o.\\ (\mathsf{explicit\_Ring} \to (\forall xy \in R. x + y \in R) \to\\ (\forall xyz \in R. x + (y + z) = (x + y) + z) \to (\forall xy \in R. x + y = y + x) \to\\ (zero \in R) \to (\forall x \in R. zero + x = x) \to\\ (\forall x \in R. \exists y \in R. x + y = zero) \to\\ (\forall xy \in R. x * y \in R) \to\\ (\forall xyz \in R. x * (y * z) = (x * y) * z) \to\\ (\forall xyz \in R. x * (y + z) = x * y + x * z) \to\\ (\forall xyz \in R. (x + y) * z = x * z + y * z) \to q)\\ \to \mathsf{explicit\_Ring} \to q.\end{gathered}$$

The proposition is identified by the following information:

Pure Prop Id:	*950aaf3139b76e70ba05068465d32a78e3f118d81f4d1e94b22cced92de5bfd0*
Pure Prop Address:	*TMQfB6fMj33EVzRKYCyD87MzvedosSwVhJD*
Theory Prop Id:	*4b8365d9c8e0b8b788552aff606ae36d533041b7da02b2cec22a70afcfbd1597*
Theory Prop Address:	*TMJpTMc6Ws4ojK4kibHze5L8RFuTnDjA4Dk*

Definition 24.2 explicit_Ring_minus *is the opaque object of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*2f43ee814823893eab4673fb76636de742c4d49e63bd645d79baa4d423489f20*
Pure Object Address:	*TMaKRKGs3HSLGHhE6tiRaiCAnrUSperUAZx*
Theory Object Id:	*39f82b17ca020bfc8f422ff1f573e4b4d16cb568809ce43d0dd8180c479bac80*
Theory Object Address:	*TMHJ1R7iBuvhhAf2k1Cz8ffCtedaaEHuKX8*

Notation. We use $--$ as a prefix operator corresponding to applying term explicit_Ring_minus.

`explicit_Ring_minus_prop`

Theorem 24.3 explicit_Ring $\to \forall x \in R. -x \in R \wedge x + -x = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*1505204a877b431f2568dc7ac6d12545244a55ed333849833f1c46b8183d336e*
Pure Prop Address:	*TMRsPoCLMDVwLK3rcKy5p43qryZKDLTiRyB*
Theory Prop Id:	*611ba38f9c0f91f22d522aa1088c3a0d2e970bdac08517c046f57e9723dea677*
Theory Prop Address:	*TMEt9CzbdZzeQX9Zc7KR9eWrwfT3fpbr79B*

`explicit_Ring_minus_clos`

Theorem 24.4 explicit_Ring $\to \forall x \in R. -x \in R$. *The proposition is identified by the following information:*

Pure Prop Id:	*f7fd70d2da1e992a18a6daffc8e53b6d72df3587a6d38db3c311d3d07eadeb34*
Pure Prop Address:	*TMMXdMN1Ek9HzsMkM2k8YJ2yoC8vLSVSxaF*
Theory Prop Id:	*407c5021d140befb145ee35be8f062958adbf49ed32321837f9bdfd8cbf8bbf3*
Theory Prop Address:	*TMFPcMzjC3fh84y93frtUFrQHpvWkhBH7dU*

explicit_Ring_minus_R

Theorem 24.5 explicit_Ring $\to \forall x \in R. x + -x = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*50d8d21d95cbe4b19e35eceb1a27541f47149bceed25c6463ea708464904f821*
Pure Prop Address:	*TMX1UUFqGBXBNndnGRpkvSGovMpgc9gU2DF*
Theory Prop Id:	*f36bcb516e35dbe4a659c52f22434f2db5232803762843e14d255d39ff7cfe6f*
Theory Prop Address:	*TMZLsVX3DiMMGuzxPY5SdVeZeJrSUrUPMu1*

explicit_Ring_minus_L

Theorem 24.6 explicit_Ring $\to \forall x \in R. -x + x = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*0f88f84d640e937f6b72601b5dab987366579483fa4d481bcdda23f4a4ccf24c*
Pure Prop Address:	*TMKnaY97FsvKqr4oNHqfkqDUzfWR5EDriF8*
Theory Prop Id:	*265b233b823a9b5403a7808c8d9fe40e67443a240aa436ac39880031c4b97c6b*
Theory Prop Address:	*TMJDyjNAiaJZm8otfnLTZeaKs3qBbk4a5En*

explicit_Ring_plus_cancelL

Theorem 24.7 explicit_Ring $\to \forall xyz \in R. x + y = x + z \to y = z$. *The proposition is identified by the following information:*

Pure Prop Id:	*0d0971cc93610ddafc2ae881eca4b9cc1f517b4d2883cf319421aee337430896*
Pure Prop Address:	*TMZdhjuSoUtSbh7ZariasMQr3GnYBuCj2DP*
Theory Prop Id:	*29aba412161a15a184e51ae63d56496c3de978192ab699d167c460212c724412*
Theory Prop Address:	*TMF4XmuUfy2u5jbYgngzAWNZqQjoXEqyx7T*

explicit_Ring_plus_cancelR

Theorem 24.8 explicit_Ring $\to \forall xyz \in R. x + z = y + z \to x = y$. *The proposition is identified by the following information:*

Pure Prop Id:	*faa841b125d630a9a1f491a07ed3a5d60de8dd20716db81e18b6f781bdc4684b*
Pure Prop Address:	*TMcgiwGBhdYyV5TGZfhy58toP2rsnQBF51d*
Theory Prop Id:	*db44c76a5e03e020f6aecd4b489349da20eea01ed3ccb5c1e7aba14d3b807824*
Theory Prop Address:	*TMPsdV2xq6oy1c4yFNQy1vMJWTw1hohLykV*

explicit_Ring_minus_invol

Theorem 24.9 explicit_Ring $\to \forall x \in R. --x = x$. *The proposition is identified by the following information:*

Pure Prop Id:	*d53c5c6286ff24824884d7e268b83ab80bc56b72e3ac4a5d7f66f254d781235c*
Pure Prop Address:	*TMRSgzKJYUXvGzJtkNSKyvXy2ejXYwUhA9g*
Theory Prop Id:	*ae91428e6764c3c11715f8090decf3cb92e118e24893f18575481d2e6e141b7f*
Theory Prop Address:	*TMb8n5HKH6hot5PBfq3FJxG7Uy6tzs4YLHa*

24.2 explicit_Ring_with_id

Let $R : \iota$ be given. Let $zero, one : \iota$ be given. Let $plus, mult : \iota \to \iota \to \iota$ be given. **Notation.** We use $+$ as a right associative infix operator corresponding to applying term $plus$. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term $mult$.

Definition 24.3 explicit_Ring_with_id *is the opaque object of type o identified by the following information:*

Pure Object Id:	*51b1b6cf343b691168d1f26c39212233b95f9ae7efa3be71d53540eaa3c849ab*
Pure Object Address:	*TMdaa5S4BKNjpqS1g4ZsN2vwScQyufyRHC2*
Theory Object Id:	*3383bfe28df5b99c25f43d596c0ef0217f346f7b4b9482bb0cbae34091149924*
Theory Object Address:	*TMPc9cZFPkXaxhQPst97YbsFqjZe58N9wQs*

`explicit_Ring_with_id_I`

Theorem 24.10

$$(\forall xy \in R.x + y \in R) \to (\forall xyz \in R.x + (y + z) = (x + y) + z) \to (\forall xy \in R.x + y = y + x) \to zero \in R \to (\forall x \in R.zero + x = x) \to (\forall x \in R.\exists y \in R.x + y = zero) \to (\forall xy \in R.x * y \in R) \to (\forall xyz \in R.x * (y * z) = (x * y) * z) \to (one \in R) \to (one \neq zero) \to (\forall x \in R.one * x = x) \to (\forall x \in R.x * one = x) \to (\forall xyz \in R.x * (y + z) = x * y + x * z) \to (\forall xyz \in R.(x + y) * z = x * z + y * z) \to \mathsf{explicit\_Ring\_with\_id}.$$

The proposition is identified by the following information:

Pure Prop Id:	*2811338562140c0cd2270de2c5b8b90a2220baae01b626fc5a2384dc56f6ee59*
Pure Prop Address:	*TMJooYZ4CrgUzVo3Bzu48i5pHmBC9ifXNhY*
Theory Prop Id:	*0109ea8d1cdf78b4c4198b9dc0579e38153af90331137e279dca9bb43be8a24b*
Theory Prop Address:	*TMZPzA5o83etUX2nE84DdwyKQ7dHD7hyBLA*

`explicit_Ring_with_id_E`

Theorem 24.11

$$\begin{array}{c}\forall q{:}o.\\ (\mathsf{explicit\_Ring\_with\_id} \to\\ (\forall xy \in R.x+y \in R) \to\\ (\forall xyz \in R.x+(y+z) = (x+y)+z) \to\\ (\forall xy \in R.x+y = y+x) \to\\ (zero \in R) \to (\forall x \in R.zero + x = x) \to\\ (\forall x \in R.\exists y \in R.x+y = zero) \to\\ (\forall xy \in R.x*y \in R) \to\\ (\forall xyz \in R.x*(y*z) = (x*y)*z) \to\\ (one \in R) \to (one \neq zero) \to\\ (\forall x \in R.one * x = x) \to\\ (\forall x \in R.x * one = x) \to\\ (\forall xyz \in R.x*(y+z) = x*y + x*z) \to\\ (\forall xyz \in R.(x+y)*z = x*z + y*z) \to q)\\ \to \mathsf{explicit\_Ring\_with\_id} \to q.\end{array}$$

The proposition is identified by the following information:

Pure Prop Id:	*8ccd42001003ed1d73a68a98715061082c13cf9b4045c5ad0972d6d2a4abc865*
Pure Prop Address:	*TMRhQLAyL1ePgWjSAjbaKVJM3aPA2Kad84X*
Theory Prop Id:	*94e4966c9127d4ec26511ff21d9016555b1c64c09b5a25b9a0ff7b14ca157b51*
Theory Prop Address:	*TML8qDN4dhpVpcJvnzRzd6pGv3x7VGoQ4ZE*

`explicit_Ring_with_id_Ring`

Theorem 24.12 explicit_Ring_with_id→explicit_Ring $R\ zero\ plus\ mult$. *The proposition is identified by the following information:*

Pure Prop Id:	*de7598000bb3e8d9d0c079d8c393c0647add3611485f554beb972cd63b0c3e18*
Pure Prop Address:	*TMN4a8Jp21Suwa418T4HpRTPuYS9i9K2pG9*
Theory Prop Id:	*6333af3f823898e2e5ef5587ca605674e78e08fc142b015e8dde0012a1de2ec4*
Theory Prop Address:	*TMc6f6BJAGYUkRp6vVLE5Qq4oekR9n7gJ85*

Notation. We use $--$ as a prefix operator corresponding to applying term explicit_Ring_minus $R\ zero\ plus\ mult$.

`explicit_Ring_with_id_minus_clos`

Theorem 24.13 explicit_Ring_with_id $\to \forall x \in R. -x \in R$. *The proposition is identified by the following information:*

Pure Prop Id:	*5a4877220e6d8a5f449750b45b954c43d7bb295e3e890da80fd7c2235434f467*
Pure Prop Address:	*TMPQuSyrrDPcLZfuACKu4c53aSTWev3qgN5*
Theory Prop Id:	*3cfc7850da9c3580b1c5ea83a321bd1140e03b38529cd3d45e38826d00c9ef51*
Theory Prop Address:	*TMWoZMxoSrCdixGNZ9VBKwUMqr5CAaYD96g*

`explicit_Ring_with_id_minus_R`

Theorem 24.14

$$\mathsf{explicit\_Ring\_with\_id}{\rightarrow}\forall x \in R. x + -x = zero.$$

The proposition is identified by the following information:

Pure Prop Id:	*f46952af2450086f002e85d2d6e0556f7f24acc2118735482f95a69d45fa52ab*
Pure Prop Address:	*TMHCSKcKw6w715v3zWTNQjH8owToUBSXdea*
Theory Prop Id:	*8841ef802d29f2a91dcb38ac2f2ee8e696c5b4536b7328b1511b09500d2d0012*
Theory Prop Address:	*TMWtKvbayU8SfroSFkejQVC6W6nbnfrvb1B*

`explicit_Ring_with_id_minus_L`

Theorem 24.15

$$\mathsf{explicit\_Ring\_with\_id}{\rightarrow}\forall x \in R. -x + x = zero.$$

The proposition is identified by the following information:

Pure Prop Id:	*e8d98642ee9931149e776bcdddbfcdf98466a43b2db827516266be45bd30d348*
Pure Prop Address:	*TMb7CRWhEaMLL4W5WF5t5amnUDdRobFSVb9*
Theory Prop Id:	*165cdf21cd0f400b4f6c03be7116abc9566fabac1d816fc347de2a525694e3b9*
Theory Prop Address:	*TMX35b4kkmEtvQXdySEQugbu9V37WDaikHr*

`explicit_Ring_with_id_plus_cancelL`

Theorem 24.16

$$\mathsf{explicit\_Ring\_with\_id}{\rightarrow}\forall xyz \in R. x + y = x + z{\rightarrow}y = z.$$

The proposition is identified by the following information:

Pure Prop Id:	*71bef6786a50fbc6a6a273f4a9eff4374f26d88994c8365b72e0daa0e23143e3*
Pure Prop Address:	*TMKJ2P9pdxXugWYcPhKAPqtjdx8qCYhWmMY*
Theory Prop Id:	*85138e19f6a6723c0ea841496568c64f40c18e1db10b2723c6d77f8596e268df*
Theory Prop Address:	*TMVGif5zhTjdUyXp5anvQUtBGG5rfbX7ddZ*

`explicit_Ring_with_id_plus_cancelR`

Theorem 24.17

$$\text{explicit\_Ring\_with\_id} \rightarrow \forall xyz \in R.x + z = y + z \rightarrow x = y.$$

The proposition is identified by the following information:

Pure Prop Id:	187848b39ef969f3fde507982eafd051de7ab1e0f840087b77e121e96cd2c1fc
Pure Prop Address:	TMYzuoTxJr6MhTadsdKwqXhkYK3ur8MvhKJ
Theory Prop Id:	302fc924e5436eb9d2e5f9b7edefcaf0847335356058020c631fbda9c01bc7b1
Theory Prop Address:	TMGbuMdJUtXoPXDJ5sUq7cTdniqN9CM7pRd

explicit_Ring_with_id_minus_invol

Theorem 24.18 explicit_Ring_with_id $\rightarrow \forall x \in R. - -x = x$. *The proposition is identified by the following information:*

Pure Prop Id:	7e2ce6a225ab10fc23adde10859d367f9dbabbf928990919a5bc29910205476e
Pure Prop Address:	TMQK76BB3wvByKH1DMd1mwhQ7mBUo3KzPwY
Theory Prop Id:	b433b46a9cbc815a0d2612acd26255f6df34bacc5c54530277b91926d351f631
Theory Prop Address:	TMTzGdyieMSypyUHY9Q64TiWWdbBynCRuoF

explicit_Ring_with_id_minus_one_In

Theorem 24.19 explicit_Ring_with_id $\rightarrow -one \in R$. *The proposition is identified by the following information:*

Pure Prop Id:	85b0236407eef117b01e63557a6f6a217a4948757c19b01f2fb5174081d84eed
Pure Prop Address:	TMXLf2xnGebu3EyRH1esAtDb4r35PFjLqhZ
Theory Prop Id:	aed03cc170861db941e2cd313b3ec44f90bf47a77311cec49b28c6e3e4d48540
Theory Prop Address:	TMGRtpEhwtc8eg1XWzU4Sk8czRAk24M35AH

explicit_Ring_with_id_zero_multR

Theorem 24.20

$$\text{explicit\_Ring\_with\_id} \rightarrow \forall x \in R.x * zero = zero.$$

The proposition is identified by the following information:

Pure Prop Id:	4a62e9fcb1d64a183b6295fd4ed4aba9eb09b3e0d5f275fcd8cc12c94aa442d0
Pure Prop Address:	TMcnjcPqg8wqwQFAz8NTDCL51KPVyypCAig
Theory Prop Id:	abe208295f59516f79837f5f471c2dda2bf8f33f4cf65bcfd158530eff7010c3
Theory Prop Address:	TMbdUb6eVFEUNrJSreGzs2bAZr6eyyrhWfd

explicit_Ring_with_id_zero_multL

Theorem 24.21

$$\text{explicit\_Ring\_with\_id} \rightarrow \forall x \in R. zero * x = zero.$$

The proposition is identified by the following information:

Pure Prop Id:	*46fb0e1fe2518ba1e903c00b5d02eb550a82f2839ac2bc021f0537f349aec4cb*
Pure Prop Address:	*TMLkHw8gKwxRm3iSphg4bG4HnxhbaKjTqE4*
Theory Prop Id:	*8352e717b959e3341c61f0f05eabef91e77b66df445e9d75d2b708e96dd469f5*
Theory Prop Address:	*TMW7Z4P7jE8uDrrpvsx73yL58TmGFqNpuwk*

`explicit_Ring_with_id_minus_mult`

Theorem 24.22

$$\text{explicit\_Ring\_with\_id} \rightarrow \forall x \in R. -x = (-one) * x.$$

The proposition is identified by the following information:

Pure Prop Id:	*9d127a8ccb192ce461b0057b3348e4222c87b1c14666f7af75a91213930b4bdd*
Pure Prop Address:	*TMdw33XLkkeMx1fCtBccHF6mmrg1MHhJCTk*
Theory Prop Id:	*a85fe0130867e7b63136e642b5d12bba6d678e84770dedb29cddcd2d54c237c1*
Theory Prop Address:	*TMXjFmdfwUTe58v3XycBBnWLN3ecVdoynv2*

`explicit_Ring_with_id_mult_minus`

Theorem 24.23

$$\text{explicit\_Ring\_with\_id} \rightarrow \forall x \in R. -x = x * (-one).$$

The proposition is identified by the following information:

Pure Prop Id:	*8837bf363dd2a389c6264ecdd6145862e06c0d8603070 51430ec19cdd534d488*
Pure Prop Address:	*TMakKZcgjKjCdtVgW3SZdCgaxwFVjQbR5Rr*
Theory Prop Id:	*c61eb6196305d87d5e1ff179d767abbaeb21b0e821b3a9aa65c5e08b2da13c70*
Theory Prop Address:	*TMWyT8XGK3tNij3sgsKDRvCMa7UVxEV34Sd*

`explicit_Ring_with_id_minus_one_square`

Theorem 24.24

$$\text{explicit\_Ring\_with\_id} \rightarrow (-one) * (-one) = one.$$

The proposition is identified by the following information:

Pure Prop Id:	*f6b3aca4e35bed72696bb58194e762fe71df22af5aa2ba4a20eb3cb185e29174*
Pure Prop Address:	*TMbKTvnVWYvy7JDi45TnZqMzLQeGTT488j7*
Theory Prop Id:	*f15095975e8203242ccee867846db7d1ff22f185cd0eb5d42934be01a000ec8a*
Theory Prop Address:	*TMXsC4FvKMruMUkYRjfKG4VMBJC1cbu9rU9*

explicit_Ring_with_id_minus_square

Theorem 24.25

$$\text{explicit\_Ring\_with\_id} \rightarrow \forall x \in R.(-x) * (-x) = x * x.$$

The proposition is identified by the following information:

Pure Prop Id:	*cf6988a1ad6d9a8623c0b2b14f24faadb38aef31eae52d0060f2892b40f1403e*
Pure Prop Address:	*TMctyY5gHQHkN8MBCzjY1dQYHmXfR3DRMpt*
Theory Prop Id:	*218d33fbe8087bda46c6ffc2373aac19b542cc5eca4aa7125ccf7b7701998f41*
Theory Prop Address:	*TMWeA74hJJNZgWcZWB1H6PzQVHCow6mJYFu*

Definition 24.4 *We define* explicit_Ring_with_id_exp_nat *to be*

$$\lambda xn.\text{nat\_primrec}\ one\ (\lambda\_r.x * r)\ n$$

of type $\iota \rightarrow \iota \rightarrow \iota$ *identified by the following information:*

Pure Object Id:	*da14f3f79323b9ad1fb102c951a6ba616694bc14e5602b46a53c3b0e4928a55e*
Pure Object Address:	*TMH1FdiM8AidR1Vbxis6wfjL483LJqfQGSF*
Theory Object Id:	*8058d9275d3dce79f0324a94ee62abfb6077060a9ed0a22dff3624c09ede793f*
Theory Object Address:	*TMPvyvYkA2Szt3fWkneXYns8V2R3guxHXSK*

Notation. We use $\uparrow$ as a right associative infix operator corresponding to applying term explicit_Ring_with_id_exp_nat.

Definition 24.5 *We define* explicit_Ring_with_id_eval_poly *to be*

$$\lambda ncsx.\text{nat\_primrec}\ zero\ (\lambda mr.cs\ m * x^m + r)\ n$$

of type $\iota \rightarrow \iota \rightarrow \iota \rightarrow \iota$ *identified by the following information:*

Pure Object Id:	*7e80cba90c488f36c81034790c30418111d63919ffb3a9b47a18fd1817ba628e*
Pure Object Address:	*TMJ6rFroSnnr3YyYokqB1fFwqTDMseMA9Pc*
Theory Object Id:	*a93ca6874dd0fc9d10e725ebdc680764de2c751fe2955a812a3550b87334ae51*
Theory Object Address:	*TMMeQp2kjHK2x54q5VmuFKELPoVkugf6jb6*

24.3 explicit_Ring_with_id_RepIndep2

Let $R : \iota$ be given. Let $zero, one : \iota$ be given. Let $plus, mult : \iota \rightarrow \iota \rightarrow \iota$ be given. **Notation.** We use $+$ as a right associative infix operator corresponding to applying term $plus$. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term $mult$. Let $plus', mult' : \iota \rightarrow \iota \rightarrow \iota$

be given. **Notation.** We use $\oplus$ as a right associative infix operator corresponding to applying term $plus'$. **Notation.** We use $\times$ as a right associative infix operator corresponding to applying term $mult'$. Assume the following.

$$\forall ab \in R.a + b = a \oplus b \tag{24.1}$$

Assume the following.

$$\forall ab \in R.a * b = a \times b \tag{24.2}$$

`explicit_Ring_with_id_repindep`

Theorem 24.26

$$\text{explicit\_Ring\_with\_id}\ R\ zero\ one\ plus\ mult$$
$$\Leftrightarrow \text{explicit\_Ring\_with\_id}\ R\ zero\ one\ plus'\ mult'.$$

The proposition is identified by the following information:

Pure Prop Id:	*a5dfcc96cb00f2205c3d5f2ac6a23140d6df64432d0b8b0a6e7cbe9701a3a2b6*
Pure Prop Address:	*TMM9oEeg2JCz3Bqz65W4NLj1nv1sKS8dasp*
Theory Prop Id:	*67e2ce901a576edc313e31090eebfa992581693662f54b580c5d20f054153db7*
Theory Prop Address:	*TMaNX5eBf7WxzhXPJWRFWthj8sKB2ruu2kf*

24.4 explicit_CRing_with_id

Let $R : \iota$ be given. Let $zero, one : \iota$ be given. Let $plus, mult : \iota \to \iota \to \iota$ be given. **Notation.** We use $+$ as a right associative infix operator corresponding to applying term $plus$. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term $mult$.

Definition 24.6 explicit_CRing_with_id *is the opaque object of type o identified by the following information:*

Pure Object Id:	*83e7eeb351d92427c0d3bb2bd24e83d0879191c3aa01e7be24fb68ffdbb9060c*
Pure Object Address:	*TMKfVuNP6dqFRwExvJ9WyXcYZ3NfhsZyNwK*
Theory Object Id:	*d75af9f3e353a0cff15b19ee5f3b63b1f8cb7ac51e67f0cc5138140b1f6e3fdd*
Theory Object Address:	*TMMxvjm7y4j3vySsRsh1Va6jygpFMDeRtdb*

`explicit_CRing_with_id_I`

Theorem 24.27

$$(\forall xy \in R.x + y \in R) \to (\forall xyz \in R.x + (y + z) = (x + y) + z) \to (\forall xy \in R.x + y = y + x) \to$$
$$zero \in R \to (\forall x \in R.zero + x = x) \to (\forall x \in R.\exists y \in R.x + y = zero) \to (\forall xy \in R.x * y \in R) \to$$
$$(\forall xyz \in R.x * (y * z) = (x * y) * z) \to (\forall xy \in R.x * y = y * x) \to (one \in R) \to (one \neq zero) \to$$
$$(\forall x \in R.one * x = x) \to (\forall xyz \in R.x * (y + z) = x * y + x * z) \to$$
$$\text{explicit\_CRing\_with\_id}.$$

The proposition is identified by the following information:

Pure Prop Id: *889096929a3004fb09e1dfb86cc3b031dce78b5ce5b2ab064f27905045936a27*
Pure Prop Address: *TMRNEU81o8JKA2Eh2gUrS2b8pFv5FoeCm3s*
Theory Prop Id: *679bfe2ab11592d4d80ec52820c46e5c80300e609e19aa4dc28ae56e77854229*
Theory Prop Address: *TMKhUWhmjdKqdspZLxVwpVyWxcqC5gtPAPX*

explicit_CRing_with_id_E

Theorem 24.28

$$\begin{gathered}\forall q{:}o.\\ (\mathsf{explicit\_CRing\_with\_id} \to\\ (\forall xy \in R.x + y \in R) \to\\ (\forall xyz \in R.x + (y + z) = (x + y) + z) \to\\ (\forall xy \in R.x + y = y + x) \to\\ (zero \in R) \to (\forall x \in R.zero + x = x) \to\\ (\forall x \in R.\exists y \in R.x + y = zero) \to\\ (\forall xy \in R.x * y \in R) \to\\ (\forall xyz \in R.x * (y * z) = (x * y) * z) \to\\ (\forall xy \in R.x * y = y * x) \to\\ (one \in R) \to (one \neq zero) \to\\ (\forall x \in R.one * x = x) \to\\ (\forall xyz \in R.x * (y + z) = x * y + x * z) \to q)\\ \to \mathsf{explicit\_CRing\_with\_id} \to q.\end{gathered}$$

The proposition is identified by the following information:

Pure Prop Id: *faabb6a45219704de2f0e3fef26389bffec0af08019c3749078473ced2cb3d11*
Pure Prop Address: *TMMxE19BawJennoso8H8tRbuhZucCddTgHK*
Theory Prop Id: *efd3a525fb8c1361391f2c1f17911c282c72ed050492dba2ca64c199693f7866*
Theory Prop Address: *TMJJgUD4bReLi7AP1TsY9TnoFikcsgyuy3G*

explicit_CRing_with_id_Ring_with_id

Theorem 24.29

$$\begin{gathered}\mathsf{explicit\_CRing\_with\_id} \to\\ \mathsf{explicit\_Ring\_with\_id}\ R\ zero\ one\ plus\ mult.\end{gathered}$$

The proposition is identified by the following information:

Pure Prop Id: *fbe43396cb1d43136aff0d303b13abd85b1ea630689e55ec6a2b988778f24e48*
Pure Prop Address: *TMQFMJBWxHz51Y8fYrL48pNMGk7r3p4736r*
Theory Prop Id: *9be399985767fb38ca0d7d6e6d6b473480388de775deabbb47204f82989a81c7*
Theory Prop Address: *TMSCnJQ4PTpCxDB9v4DeaGQZuAAtWyEj5cy*

explicit_CRing_with_id_Ring

Theorem 24.30

$$\text{explicit\_CRing\_with\_id} \rightarrow \text{explicit\_Ring}\ R\ zero\ plus\ mult.$$

The proposition is identified by the following information:

Pure Prop Id:	*0a1028f53dfffdb66a5bc8450b30b06f47584d5a8ebb7659caccc2431eab79cd*
Pure Prop Address:	*TMWR4HYXqHQbNAtCPpCW4WyX1g6sEgeqM81*
Theory Prop Id:	*d4833d0cb5524bc30358a3bb532e4e71813c9a5770dc5a75251e671d49156b0c*
Theory Prop Address:	*TMW54V1xY2GACHQtTgGxeQnSjcgYE13QF5h*

Notation. We use $--$ as a prefix operator corresponding to applying term explicit_Ring_minus R *zero plus mult.*

`explicit_CRing_with_id_minus_clos`

Theorem 24.31 explicit_CRing_with_id $\rightarrow \forall x \in R. -x \in R.$ *The proposition is identified by the following information:*

Pure Prop Id:	*3ae85ff26a5f71ad4869ca5b097a67259c6de609119efeb4bdc60ed927184f6f*
Pure Prop Address:	*TMdwWexqPB1YB6dqTefAgb1hYBMdY5om5oU*
Theory Prop Id:	*0765b182f4deb4a94a40f4675d75a50065d365c9670a0ab0f49e558d0f85993d*
Theory Prop Address:	*TMJ4pj8CmckG9Wt5bG8Q52MwrXF25dyk3kb*

`explicit_CRing_with_id_minus_R`

Theorem 24.32

$$\text{explicit\_CRing\_with\_id} \rightarrow \forall x \in R. x + -x = zero.$$

The proposition is identified by the following information:

Pure Prop Id:	*bb27a98e4f753c9ce2de93efd3a6c3a66fcfa71c58a26ed9feff5958784d9524*
Pure Prop Address:	*TMLxMqEk88QyHzY7bcXNWKPPvM2GmbJkpT8*
Theory Prop Id:	*60d4131fe989a7c001f81787ae3ed44b78153d1f852ce54a3d4141b279c5c457*
Theory Prop Address:	*TML4H1hcc9CreDDS8GT4dGMNCmQRxHBJUkG*

`explicit_CRing_with_id_minus_L`

Theorem 24.33

$$\text{explicit\_CRing\_with\_id} \rightarrow \forall x \in R. -x + x = zero.$$

The proposition is identified by the following information:

Pure Prop Id:	*50baa96d414935b78c8004fe8327833fdc95dbb21e677a91ab1cf2c6b0fe04ca*
Pure Prop Address:	*TMaiKT6dJBUA6GYD8CEipgDz31FuNhAsVPZ*
Theory Prop Id:	*f84540b0cdb980f22b39fce9eca7602a4d61e86a70a72452220292d78080cb9f*
Theory Prop Address:	*TMWLryjTxXP9PfTDUBQTJVrMoHtCqMf2Toq*

explicit_CRing_with_id_plus_cancelL

Theorem 24.34

$$\text{explicit\_CRing\_with\_id} \rightarrow \forall xyz \in R. x + y = x + z \rightarrow y = z.$$

The proposition is identified by the following information:

Pure Prop Id:	6f6e462ce78421c4a368b9489c238c0a56b788c33381051bc30b2344cc6fc9dd
Pure Prop Address:	TMYtj8Hh526XhqURvJhj6jZ4hduf9SfkVLZ
Theory Prop Id:	1efdbbbd0d4626f6ed562dd6cf586bde2ad4047d77bf071a55596278332fd212
Theory Prop Address:	TMHKn5Uz9yj1C4oNmL951CdC7QTuio1h7e7

explicit_CRing_with_id_plus_cancelR

Theorem 24.35

$$\text{explicit\_CRing\_with\_id} \rightarrow \forall xyz \in R. x + z = y + z \rightarrow x = y.$$

The proposition is identified by the following information:

Pure Prop Id:	f7bef7cef51fefac0a8a968cd48339390194bc43db609d321297622f290f854b
Pure Prop Address:	TMXr6FaGNVmrtogzEG7TRA9wchey2iGQPw1
Theory Prop Id:	251f9b25df85d0dc24f0e3a92c389f8e0d2abf089dabb0942376c7f836fada37
Theory Prop Address:	TMNaQYvw47cG7Q58RrnEKj8rrfammDMrR62

explicit_CRing_with_id_minus_invol

Theorem 24.36 explicit_CRing_with_id $\rightarrow \forall x \in R. --x = x$. *The proposition is identified by the following information:*

Pure Prop Id:	ee28f648bb60af4ab802e7550db486a9b5f33435a2ddcfad83e57e7aeb38b90c
Pure Prop Address:	TMPixMacvRwLbJEEYF3Tu3BXhGhgayHXYiq
Theory Prop Id:	01d19d582f4cd5e2997e85901f8234fd2844833c0ea422c25b30fb4cb7dcfba6
Theory Prop Address:	TMaM5mPayy7LFWdA8ncy5w63W2sh9AEjLdV

explicit_CRing_with_id_minus_one_In

Theorem 24.37 explicit_CRing_with_id $\rightarrow -one \in R$. *The proposition is identified by the following information:*

Pure Prop Id:	7608d9188365499939cb1fe551eecd223870593a73f2f473c50fdf0812b607fe
Pure Prop Address:	TMbawiEWXmjkJn3cNvBUYBL28Cpc9LDQEGD
Theory Prop Id:	954fe6bb57ae5b479c5d7182e7aa2ca52a9d14d7226f7296a1bc495ed1c16908
Theory Prop Address:	TMXjU4EzqoWzujVJA3GSZFqn43jFuMhi3Sf

explicit_CRing_with_id_zero_multR

Theorem 24.38

$$\text{explicit\_CRing\_with\_id} \to \forall x \in R. x * zero = zero.$$

The proposition is identified by the following information:

Pure Prop Id:	*9f49c5a1afdc52b7b5433d4d3367501e3b8caf0412846c4cf5f182b8c277de85*
Pure Prop Address:	*TMG7cDqVVyiPU92qFsQzXosDNLq1NTSreqP*
Theory Prop Id:	*0c4690feb0a694a7e1bb0c027da7c70d01be7f914b759b801e5e9fdb4cf100c9*
Theory Prop Address:	*TMSRjqxP1jRu2t7GgGMpBgEXn8ivZQWM9um*

`explicit_CRing_with_id_zero_multL`

Theorem 24.39

$$\text{explicit\_CRing\_with\_id} \to \forall x \in R. zero * x = zero.$$

The proposition is identified by the following information:

Pure Prop Id:	*0a092c2a6dd656ce3cf2cbab1ff88bd05ddbfb27fe53be361696f86c4312822c*
Pure Prop Address:	*TMMgnbsVdPLJ3iwrHZPnZiErXzyVeTyZRU2*
Theory Prop Id:	*924526e9dfa6060f77cadce4784 65ee75ea541ecc409f563b2ffe02f103abfd9*
Theory Prop Address:	*TMMFp4RVmjUKJA9f9cUetYE3usofxBj3uuC*

`explicit_CRing_with_id_minus_mult`

Theorem 24.40

$$\text{explicit\_CRing\_with\_id} \to \forall x \in R. -x = (-one) * x.$$

The proposition is identified by the following information:

Pure Prop Id:	*2f95e375de8d8d7d33d150a3a832a3f28d3f4736facce19ec50f7bf1278b5de7*
Pure Prop Address:	*TMc3H8URNXwHnJda77991VAXe6HmNqtnjWB*
Theory Prop Id:	*fd44c89e251ce67e81ce86520a199d00039c96c2d1f0bcfba1e2da4ebecd8add4*
Theory Prop Address:	*TMU9zhbrpi1cxTikjTTd5CSMd1G6Kst4j5w*

`explicit_CRing_with_id_mult_minus`

Theorem 24.41

$$\text{explicit\_CRing\_with\_id} \to \forall x \in R. -x = x * (-one).$$

The proposition is identified by the following information:

Pure Prop Id:	*960b3084b8e49beeea6746780f075296d84ba89bfe0c23064365d8255960dfa6*
Pure Prop Address:	*TMT8SWemm7sAFAGRv6i8SpVrYKufCRcTNkj*
Theory Prop Id:	*58dd34b96bd3da4dda59ac8f027f0144f9aa188ef626da02bf86692dc0f5a89c*
Theory Prop Address:	*TMXMyqEZX2K4g8LgCYyD3LRKSt3urgcv9Gt*

`explicit_CRing_with_id_minus_one_square`

Theorem 24.42

$$\text{explicit\_CRing\_with\_id} \rightarrow (-one) * (-one) = one.$$

The proposition is identified by the following information:

Pure Prop Id:	*9027680ad43bbdc05b9ac2585988bf9bca1f56435309fcf8330934d3a4e55242*
Pure Prop Address:	*TMKHMtgM4RegJL7J6H3aDWWmZYyuG6dGt4F*
Theory Prop Id:	*27e1d09ca5cd60ad129f6b679f7062bed626f17e6e37987c960df24c3b5ef58b*
Theory Prop Address:	*TMJQU82As7FBLwuBdVYR5vt1kuAL1jNF9cr*

`explicit_CRing_with_id_minus_square`

Theorem 24.43

$$\text{explicit\_CRing\_with\_id} \rightarrow \forall x \in R.(-x) * (-x) = x * x.$$

The proposition is identified by the following information:

Pure Prop Id:	*cc64ff002d9d1fee374c40859d2a8b13e6a2e01c0daf12ebb732af37740d2369*
Pure Prop Address:	*TMNzA9VqA3PfzJZ7FjCXuhvJBqkuXeLycnt*
Theory Prop Id:	*cba06f4f773cf52216bcff465b0e35eeda22efd81812f03664505820cd4d217b*
Theory Prop Address:	*TMaX7AFX3UeZnZX4SBaoUAegDF4y8TjJD2i*

24.5 explicit_CRing_with_id_RepIndep2

Let $R : \iota$ be given. Let $zero, one : \iota$ be given. Let $plus, mult : \iota \to \iota \to \iota$ be given. **Notation.** We use $+$ as a right associative infix operator corresponding to applying term $plus$. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term $mult$. Let $plus', mult' : \iota \to \iota \to \iota$ be given. **Notation.** We use $\oplus$ as a right associative infix operator corresponding to applying term $plus'$. **Notation.** We use $\times$ as a right associative infix operator corresponding to applying term $mult'$. Assume the following.

$$\forall ab \in R.a + b = a \oplus b \tag{24.3}$$

Assume the following.

$$\forall ab \in R.a * b = a \times b \tag{24.4}$$

`explicit_CRing_with_id_repindep`

Theorem 24.44

$$\text{explicit\_CRing\_with\_id}\ R\ zero\ one\ plus\ mult$$
$$\Leftrightarrow \text{explicit\_CRing\_with\_id}\ R\ zero\ one\ plus'\ mult'.$$

The proposition is identified by the following information:

Pure Prop Id:	*ac2d59ca206daa22783237146a060cbb078337457a0a1298597e6fcae4da5f00*
Pure Prop Address:	*TMJtWqjqttZHff5TE3g13Vjiu7BhtNnvzfB*
Theory Prop Id:	*6531240d433090d437bf90709c4ea3cb3ed4e135a6c52863192d1c5d24238853*
Theory Prop Address:	*TMYe8NzBzkkswszXrNbVN74ksiBLPimnNkq*

24.6 Packing two Binary Operations and a Constant into a Set

Definition 24.7 pack_b_b_e *is the opaque object of type*

$$\iota \to (\iota \to \iota \to \iota) \to (\iota \to \iota \to \iota) \to \iota \to \iota$$

identified by the following information:

Pure Object Id:	*c9ca029e75ae9f47e0821539f84775cc07258db662e0b5ccf4a423c45a480766*
Pure Object Address:	*TMcBhLLE6SjWU8YK3zBnZTiJ441rzSVQZ4f*
Theory Object Id:	*21d4d088757e7386d904218652e603fec7ce704c78740c732cc6f2bc8d0f15b9*
Theory Object Address:	*TMT1KQaS6hSV4qnwVuKuKkuNrS6Em96tqXN*

Definition 24.8 struct_b_b_e *is the opaque object of type* $\iota \to o$ *identified by the following information:*

Pure Object Id:	*755e4e2e4854b160b8c7e63bfc53bb4f4d968d82ce993ef8c5b5c176c4d14b33*
Pure Object Address:	*TMVRzggDuRW6mHLPpsN6eZtf5G2DQ1nm6ug*
Theory Object Id:	*3be2a81c29bbdf8205c87f99ef9048632fba2bced44a1e8c51e6c55f41781fad*
Theory Object Address:	*TMYHoLuYL7HNg4NL1sS21yHunHjH3Z1FY6X*

Definition 24.9 unpack_b_b_e_i *is the opaque object of type*

$$\iota \to (\iota \to (\iota \to \iota \to \iota) \to (\iota \to \iota \to \iota) \to \iota \to \iota) \to \iota$$

identified by the following information:

Pure Object Id:	*7004278ccd490dc5f231c346523a94ad983051f8775b8942916378630dd40008*
Pure Object Address:	*TMQ74XmyoP47CnXDEBHKEFtMzk18jjqu6dt*
Theory Object Id:	*52a106befe8ad370345a8e6ab30f464040cc96b3bd3f78b38dc7599fc5e7de99*
Theory Object Address:	*TMPRW7TWzKwNSZnPUoDa8Aj3P5cvnhJZi47*

Definition 24.10 unpack_b_b_e_o *is the opaque object of type*

$$\iota \to (\iota \to (\iota \to \iota \to \iota) \to (\iota \to \iota \to \iota) \to \iota \to o) \to o$$

identified by the following information:

Pure Object Id:	*b94d6880c1961cc8323e2d6df4491604a11b5f2bf723511a06e0ab22f677364d*
Pure Object Address:	*TMVcrr5WR2rjHynX8x5DLY3reHfggeDReXu*
Theory Object Id:	*7b0871aa00521156ece9021408011d93cc22a3a3d8be2ffab32e46cde2c7017a*
Theory Object Address:	*TMRmp4Vwef25JwbPLckTqR8B8PxPtoyHXZ1*

Definition 24.11 *We define* Ring *to be*

$$\lambda R.\mathsf{struct\_b\_b\_e}\ R \wedge$$
$$\mathsf{unpack\_b\_b\_e\_o}\ R\ (\lambda R\ plus\ mult\ zero.\mathsf{explicit\_Ring}\ R\ zero\ plus\ mult)$$

of type $\iota \to o$ *identified by the following information:*

Pure Object Id:	*61bbe03b7930b5642e013fa138d7c8e2353bab0324349fce28b7cbce8edce56a*
Pure Object Address:	*TMVG5WKysVhbYYd6hg9dUY2aCKLj1h9qkSs*
Theory Object Id:	*ffaf640f43cef8fec01ec428ab080a16b7b8c9fe29fc01c79b9a62fc32f9e1ab*
Theory Object Address:	*TMVey7JB9XreF71vPMXWv4MfPYVqXDJNqi5*

Definition 24.12 *We define* Ring_minus *to be*

$$\lambda Rx.\mathsf{unpack\_b\_b\_e\_i}\ R\ (\lambda R\ plus\ mult\ zero.\mathsf{explicit\_Ring\_minus}\ R\ zero\ plus\ mult\ x)$$

of type $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*e5405bc7309f2aa971bcab97aadae821ba1e89825a425c7bf10000998c5266c9*
Pure Object Address:	*TMGB3zNFrM8bRch4GNc7TY8X7Y5CDFkWQMu*
Theory Object Id:	*c725983606b8ac83202398408bea35fe4ce0b6c34d2d5411b8a4d54f4b73256f*
Theory Object Address:	*TMGzuX373w32Vo8i8c1MYDeruu5iffk14jV*

Definition 24.13 pack_b_b_e_e *is the opaque object of type*

$$\iota \to (\iota \to \iota \to \iota) \to (\iota \to \iota \to \iota) \to \iota \to \iota \to \iota$$

identified by the following information:

Pure Object Id:	*6859fb13a44929ca6d7c0e598ffc6a6f82969c8cfe5edda33f1c1d81187b9d31*
Pure Object Address:	*TMHt74X3xjgCgvDbTBeJDSgoSU6NBFqLo3t*
Theory Object Id:	*cf1c51f21701c7f90f031360baccab3d9f9cc6bcbb438ab90424d004540beecf*
Theory Object Address:	*TMY3Jeq6qFWZoiMV17oUBbd2wzL4nynScQo*

Definition 24.14 struct_b_b_e_e *is the opaque object of type* $\iota \to o$ *identified by the following information:*

Pure Object Id: *7685bdf4f6436a90f8002790ede7ec64e03b146138f7d85bc11be7d287d3652b*
Pure Object Address: *TMMq6LTJYr6FfwYvoFthxRCCWMJeJ3PPcbc*
Theory Object Id: *93fa62123840de71d70cd336cd76c191a66b2a255f74ff9d886c0ce7bd229764*
Theory Object Address: *TMKaRBHLAH7X2upT8ygEK87KwVQkCv88tVc*

Definition 24.15 unpack_b_b_e_e_i *is the opaque object of type*

$$\iota \to (\iota \to (\iota \to \iota \to \iota) \to (\iota \to \iota \to \iota) \to \iota \to \iota \to \iota) \to \iota$$

identified by the following information:

Pure Object Id: *0708055ba3473c2f52dbd9ebd0f0042913b2ba689b64244d92acea4341472a1d*
Pure Object Address: *TMPWkrRKKhARHw7yZVhmEDea2FrJHMb9vQ2*
Theory Object Id: *e7d717c6dc1d45afea7f0ffa393d0e0f9d8162bd1099d59986e7a90eae668a3d*
Theory Object Address: *TMPQgdNg6eUMGGtvqYwfi9ZggR6mb3tYxPU*

Definition 24.16 unpack_b_b_e_e_o *is the opaque object of type*

$$\iota \to (\iota \to (\iota \to \iota \to \iota) \to (\iota \to \iota \to \iota) \to \iota \to \iota \to o) \to o$$

identified by the following information:

Pure Object Id: *1bcc21b2f13824c926a364c5b001d252d630f39a0ebe1f8af790facbe0f63a11*
Pure Object Address: *TMLUiT5tyYDHruUZiDx7gvU1hj4TEKsQReX*
Theory Object Id: *3325ddb79a400302ecc0a153bdd9c21b9425489ac6ae7b8d16a00c9f9386f079*
Theory Object Address: *TMUV6wYXdEfCF2Zrhg4Q7de7PEc2arc5QiC*

Definition 24.17 *We define* Ring_with_id *to be*

$$\lambda R.\mathsf{struct\_b\_b\_e\_e}\ R \land \\ \mathsf{unpack\_b\_b\_e\_e\_o}\ R \\ (\lambda R\ plus\ mult\ zero\ one.\mathsf{explicit\_Ring\_with\_id}\ R\ zero\ one\ plus\ mult)$$

of type $\iota \to o$ *identified by the following information:*

Pure Object Id: *e87205d21afb0774d1a02a5d882285c7554f7b824ee3d6ff0a0c0be48dac31d6*
Pure Object Address: *TMMXH8sbGq7TCys16vmazU94M6z99RUZJ4e*
Theory Object Id: *5b1d1fc429c4ad1b59f175a48a3a442a9707b676450c6ff34ff21d9e21b47f80*
Theory Object Address: *TMHpeNjPXgpyDcsaGcgo2ZDP3VuziQZ8U4x*

`Ring_with_id_unpack_eq`

Theorem 24.45

$$\forall R.\forall plus\ mult : \iota \to \iota \to \iota.\forall zero\ one. \\ \mathsf{unpack\_b\_b\_e\_e\_o}\ (\mathsf{pack\_b\_b\_e\_e}\ R\ plus\ mult\ zero\ one) \\ (\lambda R\ plus\ mult\ zero\ one.\mathsf{explicit\_Ring\_with\_id}\ R\ zero\ one\ plus\ mult) \\ = \mathsf{explicit\_Ring\_with\_id}\ R\ zero\ one\ plus\ mult.$$

The proposition is identified by the following information:

Pure Prop Id:	*e125aaa25b8e15f0cb5142f069cd1158c31c0a22e2ba188ed03651f2b50f7f69*
Pure Prop Address:	*TMYhZgSfEnPXAcYrzYgAs5j3TvrmvSQUYwo*
Theory Prop Id:	*6ff04cdbf65065a1048fb4ed5c01d06bc600971c026c0e88f07528dee5a7753b*
Theory Prop Address:	*TMHgUEdaJ2656FZdQffXPyg9M1r2869W3gG*

Definition 24.18 *We define* CRing_with_id *to be*

$$\lambda R.\mathsf{struct\_b\_b\_e\_e}\ R \wedge$$
$$\mathsf{unpack\_b\_b\_e\_e\_o}\ R$$
$$(\lambda R\ plus\ mult\ zero\ one.\mathsf{explicit\_CRing\_with\_id}\ R\ zero\ one\ plus\ mult)$$

of type $\iota \to o$ *identified by the following information:*

Pure Object Id:	*38dd910342442210ba796f0e96ab9a7e6b36c17352f74ba8c9c2db4da2aebf0e*
Pure Object Address:	*TMQgd2PoDs8mxEHY4Lw1N9XM2MuPgtY8d9w*
Theory Object Id:	*a1021e16f50fc0f2bd0b04593e51a03b9457694823e7aa3bc57cf1efe615c1f7*
Theory Object Address:	*TMbRJRM2RQRfp7E2CEx4AJhw2m5Gxt5HthV*

`CRing_with_id_unpack_eq`

Theorem 24.46

$$\forall R.\forall plus\ mult :\iota \to \iota \to \iota.\forall zero\ one.$$
$$\mathsf{unpack\_b\_b\_e\_e\_o}\ (\mathsf{pack\_b\_b\_e\_e}\ R\ plus\ mult\ zero\ one)$$
$$(\lambda R\ plus\ mult\ zero\ one.\mathsf{explicit\_CRing\_with\_id}\ R\ zero\ one\ plus\ mult)$$
$$= \mathsf{explicit\_CRing\_with\_id}\ R\ zero\ one\ plus\ mult.$$

The proposition is identified by the following information:

Pure Prop Id:	*36ff594b81ff0f9289211e517dfadc9825f390bdb5273a1c1da1b64814838ac3*
Pure Prop Address:	*TMGSuWhhM3Pypq6dXURWZnpYUJPYcx7vkJ2*
Theory Prop Id:	*7707613ab3233c231329185e9013759e858082c5e03f957f33f5a266d7c92331*
Theory Prop Address:	*TMFpD2MTMvgWJJuwcM5sqPZKwuope6VFnQg*

Definition 24.19 *We define* Ring_of_Ring_with_id *to be*

$$\lambda R.\mathsf{unpack\_b\_b\_e\_e\_i}\ R\ (\lambda R\ plus\ mult\ zero\ one.\mathsf{pack\_b\_b\_e}\ R\ plus\ mult\ zero)$$

of type $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*9f7755a326e730a1c98395c05032d068f1d3a5762700ae73598c3c57a6bd51ea*
Pure Object Address:	*TMJ3Jckg9eHwYuoWNm91qRwiRDGjP1Wv1u3*
Theory Object Id:	*119ffb040b7ae9e440cde0cba75f90e17ecab1fe0ecf8a8ace0582b41a80ab5e*
Theory Object Address:	*TMSZtFuoH5AXFQXg4j9tyqGVoKFDW7ouQ8H*

`CRing_with_id_is_Ring_with_id`

Theorem 24.47 $\forall R.\mathsf{CRing\_with\_id}\ R \to \mathsf{Ring\_with\_id}\ R.$ *The proposition is identified by the following information:*

Pure Prop Id:	*815b955355c5b9a3a4bba1901f861f2e42a8aba0b2e264541eb5237d2b7defed*
Pure Prop Address:	*TMUZNUomruVanp8ziSGJjGo6mnjbL8FcKLU*
Theory Prop Id:	*e157b3b577d0c147bf7e94da6f8ae23dc7ce8e6040bdb6473b6afe038837f0b1*
Theory Prop Address:	*TMPJAdDaLrQhhspajeStvBhg4f1PCFGYJTp*

Chapter 25

Explicit Reals

25.1 explicit_Reals

Let $R : \iota$ be given. Let $zero, one : \iota$ be given. Let $plus, mult : \iota \to \iota \to \iota$ be given. **Notation.** We use $+$ as a right associative infix operator corresponding to applying term $plus$. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term $mult$.

Definition 25.1 explicit_Field *is the opaque object of type o identified by the following information:*

Pure Object Id:	*32dcc27d27b5003a86f8b13ba9ca16ffaec7168a11c5d9850940847c48148591*
Pure Object Address:	*TMbJPQHnQf1mY3onQrsrhqCR24yapErnf21*
Theory Object Id:	*0020dc091a770bc26640d405438dea5752e187e4f46e3ea423aa06fbf99169cd*
Theory Object Address:	*TMctGNriWgkHut2Cii7sum1tGGmeJ1TQf2z*

explicit_Field_I

Theorem 25.1

$$(\forall xy \in R.x + y \in R)\to(\forall xyz \in R.x + (y + z) = (x + y) + z)\to(\forall xy \in R.x + y = y + x)\to zero \in R\to(\forall x \in R.zero + x = x)\to(\forall x \in R.\exists y \in R.x + y = zero)\to(\forall xy \in R.x * y \in R)\to(\forall xyz \in R.x * (y * z) = (x * y) * z)\to(\forall xy \in R.x * y = y * x)\to(one \in R)\to(one \neq zero)\to(\forall x \in R.one * x = x)\to(\forall x \in R.x \neq zero \to \exists y \in R.x * y = one)\to(\forall xyz \in R.x * (y + z) = x * y + x * z)\to\mathsf{explicit\_Field}.$$

The proposition is identified by the following information:

Pure Prop Id:	*3ccc525dac877738bde04411aa5b2802762d08d54c0801938fbdb890674a0002*
Pure Prop Address:	*TMdJ9fH9ZyRuWnaU1MAQEZ1tDGUfPtL2cec*
Theory Prop Id:	*51ce1195007755d317528633fca2947880002298355b66559cc3f47f888639540*
Theory Prop Address:	*TMKFU5TXhW1AbfR8o181j3pjJGyWs64AHai*

`explicit_Field_E`

Theorem 25.2

$$\begin{gathered}\forall q:o.\\ (\textsf{explicit\_Field} \to \\ (\forall xy \in R.x + y \in R) \to \\ (\forall xyz \in R.x + (y + z) = (x + y) + z) \to \\ (\forall xy \in R.x + y = y + x) \to \\ (zero \in R) \to (\forall x \in R.zero + x = x) \to \\ (\forall x \in R.\exists y \in R.x + y = zero) \to \\ (\forall xy \in R.x * y \in R) \to \\ (\forall xyz \in R.x * (y * z) = (x * y) * z) \to \\ (\forall xy \in R.x * y = y * x) \to \\ (one \in R) \to (one \neq zero) \to \\ (\forall x \in R.one * x = x) \to \\ (\forall x \in R.x \neq zero \to \exists y \in R.x * y = one) \to \\ (\forall xyz \in R.x * (y + z) = x * y + x * z) \to q \\ \to \textsf{explicit\_Field} \to q.\end{gathered}$$

The proposition is identified by the following information:

Pure Prop Id:	*bb3db9eded830bf9d92132c0016be1d1a259929bceda5bdf57b4f64fcba43184*
Pure Prop Address:	*TMXJPRqy8mBTRo6EPJpYzNS1rE86xJ2ABjo*
Theory Prop Id:	*6c4aa1700c811949523aab7956f68644700f9d543e3915471880f5956881b35e*
Theory Prop Address:	*TMcxwBH2nbJB8F5LxJwz4A2as2v9Ejx9zoD*

Definition 25.2 explicit_Field_minus *is the opaque object of type $\iota \to \iota$ identified by the following information:*

Pure Object Id:	*5be570b4bcbe7fefd36c2057491ddcc7b11903d8d98ca54d9b37db60d1bf0f7e*
Pure Object Address:	*TMMPF8zir9fkfAtJFGxe63EcXTEPLHDtBA4*
Theory Object Id:	*7a10c2aa3843a7152cec6319a52f2f9ff01bb82b441aed667fd99a775dab9980*
Theory Object Address:	*TMYcEMbnauFAYpJxp2DpEB5nscfUnNcsbZy*

Notation. We use $--$ as a prefix operator corresponding to applying term explicit_Field_minus.

`explicit_Field_minus_prop`

Theorem 25.3 explicit_Field $\to \forall x \in R.-x \in R \land x + -x = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*dfdfb501a72ee10c72ad8933d6f1bd3d1d25a2009a655a5683cf351539aa70bc*
Pure Prop Address:	*TMRiffBv5SK7Dv3GZ8Qc3WgwrC1MxCSpHC7*
Theory Prop Id:	*be5356f76e63b2d1a7cc20f125e927198f6005afc05464565ca4304115a8e435*
Theory Prop Address:	*TMKQfrW9q8xeEbq3YsFtzRuMpXJcsRviQhq*

explicit_Field_minus_clos

Theorem 25.4 explicit_Field $\rightarrow \forall x \in R. -x \in R$. *The proposition is identified by the following information:*

Pure Prop Id:	*19192856a86a06ac1d56d571d172b3dd60e77b95edea0e5a195a7849ea9fdd1d*
Pure Prop Address:	*TMLDrc2xd6CY9VqXqvmt3SZypH79h73yzYt*
Theory Prop Id:	*853f414afb3b6db2964375b75fc2bdfa06eba2d8a9cf8c7993e79be8c16c241c*
Theory Prop Address:	*TMMd78EndbrgewWKg7cug4aFmuNLcdvLETi*

explicit_Field_minus_R

Theorem 25.5 explicit_Field $\rightarrow \forall x \in R. x + -x = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*16bd02b0d5c539b89ab6c3b290f99da455cb893382173ba554e3a9499850d228*
Pure Prop Address:	*TMWcY3CmCi3C7TchQ2ZWaU2w6UYvY5EyRBX*
Theory Prop Id:	*e00e5ccf74077747e0399d61c172ce48cfbd48eee9e6d1a7ccc3d55adb0814dd*
Theory Prop Address:	*TMJphf6VbWpCYsxdqnUTWXSj422gfKJmxEo*

explicit_Field_minus_L

Theorem 25.6 explicit_Field $\rightarrow \forall x \in R. -x + x = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*1e484c596156e799c8b1be02b2f184d891d04b570a626b0f353d58fb19b04a98*
Pure Prop Address:	*TMZbY42rURTo9ZUysPTCKb3htjwFZXbEw1T*
Theory Prop Id:	*3c1cf77f15777c38163c09cb6f75c87835fac72965b443566fab8a7a40e9665f*
Theory Prop Address:	*TMYDLj9BQtJLVtxTosQ1aMz4YRJvYdWceaV*

explicit_Field_plus_cancelL

Theorem 25.7 explicit_Field $\rightarrow \forall xyz \in R. x + y = x + z \rightarrow y = z$. *The proposition is identified by the following information:*

Pure Prop Id:	*b9cc3f21432ad1dcc45524e4ac02e5182053a3f7477ca552e87c5132962558cb*
Pure Prop Address:	*TMLW7dUp3ubrm2KQb1j1qqXX7SiHc2FMGYH*
Theory Prop Id:	*da9a6bc0e051dd48f33393f832d94859ae59087f843e5647b7d48be40b16752c*
Theory Prop Address:	*TMGDxEVrun2MVRqLiWkibA4xgq5mXZYRefM*

explicit_Field_plus_cancelR

Theorem 25.8 explicit_Field $\rightarrow \forall xyz \in R. x + z = y + z \rightarrow x = y$. *The proposition is identified by the following information:*

Pure Prop Id:	*8e7410c2408dd9fc6349ca4ae571c9cfc54c4cdbd647f030a422a0f138d33342*
Pure Prop Address:	*TMLaB8tvdGpW8DwHyHVQH6BkFA5qABz62Xw*
Theory Prop Id:	*148a14870a33e0b1322a50fde11a6ff30e6ed7d2cf59e7b846b0a4fece952554*
Theory Prop Address:	*TMdwAiT4yYz4w2xxTmzsG9vifx1CtnVpbbK*

explicit_Field_minus_invol

Theorem 25.9 explicit_Field $\to \forall x \in R. --x = x$. *The proposition is identified by the following information:*

Pure Prop Id:	*d9245973a06673bbaca0fe2244ae2b76cb719974a7a1a5be04c9786c8ccd521a*
Pure Prop Address:	*TMSFpQWK1zrTS89dhaG5MbKGZ3CzKuw2Hez*
Theory Prop Id:	*e4a26d2ce9a3a793e18ddeb34b9e5f5689c952c20c234bbeaccb1e36b6ed3934*
Theory Prop Address:	*TMSuivsorWRWJ8izeRUNUJJmCYigXx7CwUM*

explicit_Field_minus_one_In

Theorem 25.10 explicit_Field $\to -one \in R$. *The proposition is identified by the following information:*

Pure Prop Id:	*3dea9f1e7c156a2c2a714f0dcbe2bb48e13a8f0786132f71784621f4566ba3de*
Pure Prop Address:	*TMKembKWo9bsc7rvXjS3b9tPDernwnfN69C*
Theory Prop Id:	*c3141394b9d6650476539d3de9c24c2094fafd277233c838e05eece229179706*
Theory Prop Address:	*TMYYz1ipvpJRoYVLBTTHwW5inLAqHEZJNZj*

explicit_Field_zero_multR

Theorem 25.11 explicit_Field $\to \forall x \in R. x * zero = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*f6ea109454bd63c383487f79cde58c39e60809206cdeecfb0cf5c33ba7a39a78*
Pure Prop Address:	*TMc9K3rxpDhdcV4fiwudpCAvCbo4EgSX2FM*
Theory Prop Id:	*cddbcdbd4bc296b7c12e6e905456a622a658af38644ec81e091b58a94847904f*
Theory Prop Address:	*TMTpJuV9Dgs6cEeCYFcQ71NUT9zLc55yQ6G*

explicit_Field_zero_multL

Theorem 25.12 explicit_Field $\to \forall x \in R. zero * x = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*a6219e7c7ce408f1a4ed2d0bbb6897dd8a32479b32efbe9caa5735d0b3fe8fa2*
Pure Prop Address:	*TMbHRokQ9EFeZHPhTtnpAgJYqraYBTYVR9D*
Theory Prop Id:	*bf3f6a5eb75519377a3a0ab3681dee400b8bb378f33c7dc4dcd864f874a61fca*
Theory Prop Address:	*TMWNakNQV1Rea25NYbFjsGGZVZqZY7C1sHo*

explicit_Field_minus_mult

Theorem 25.13 explicit_Field $\to \forall x \in R. -x = (-one) * x$. *The proposition is identified by the following information:*

Pure Prop Id:	*b725ff0fba328afa96eaf8d8cee4c89ac94d1e750fd3006aba3b6f6f6dc8d6fa*
Pure Prop Address:	*TMTsU5wxazHBfmvDRU6dzpmJe6y8taJi1zr*
Theory Prop Id:	*6c32dff82b3955c923f8ad0af52f709a8d9750b5932c46a80584bf6fe86d4079*
Theory Prop Address:	*TMG8caBgEaUDhsJtbdypcFcgYXKH7VumZf2*

`explicit_Field_minus_one_square`

Theorem 25.14 explicit_Field $\rightarrow (-one) * (-one) = one$. *The proposition is identified by the following information:*

Pure Prop Id:	*33687daadedd9048924a66186e9b1d11d2849f1c4c403989aa1dde1a2c8d1839*
Pure Prop Address:	*TMRYgfX4tEirAsP8NiXBD231RunzNSeTGaQ*
Theory Prop Id:	*81b69b4f65a19abefefc96e03e44e28ccab7aa2d8fdd98ec035a25ceeb9f9c01*
Theory Prop Address:	*TMWNc4GkfcpRjYQn5ss6k1srmNCX8pS3RTW*

`explicit_Field_minus_square`

Theorem 25.15 explicit_Field $\rightarrow \forall x \in R.(-x) * (-x) = x * x$. *The proposition is identified by the following information:*

Pure Prop Id:	*db0127263baf3b745b4b535f56b1f7c123744b30c2db578852e18d3b417f004c*
Pure Prop Address:	*TMTV85Vo7eTyFBiu5YCvAEyGtkJGhZnmztE*
Theory Prop Id:	*cc20f4507a4b8ab01b79ded3da8b0945e39532ee96a70c6ef10bab5a6e176273*
Theory Prop Address:	*TMFLG2o5FZ3crQm2waGZhJKk41t1KZmfnsC*

`explicit_Field_minus_zero`

Theorem 25.16 explicit_Field $\rightarrow -zero = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*677fffd7ef5af74d4eaf3075a3db01ed4e7685052f86b5da66de86c40ac0029c*
Pure Prop Address:	*TMbHTa1NSQxgViQGA5LgQhajneJu1oTW2n7*
Theory Prop Id:	*d58ae486546dede085e27a56fa6854b755e9bfc2f0b3c78aa184624c4d4535e2*
Theory Prop Address:	*TMHxghZtCQFNpCShXEagGPGXBetd1ZxTcCB*

`explicit_Field_dist_R`

Theorem 25.17 explicit_Field $\rightarrow \forall xyz \in R.(x + y) * z = x * z + y * z$. *The proposition is identified by the following information:*

Pure Prop Id:	*de79567ee223c72fabfe843b7d7526a3a12428317b799a662ff4ce3ad76f9472*
Pure Prop Address:	*TMMz7KwL6xBwXeCscdhDKUbgZEeMqZXz2n1*
Theory Prop Id:	*2a896a6970c2a6f3d291e0b5a6330cabecc63ba17b6f9f6b4338f1a0e03a08a0*
Theory Prop Address:	*TMXM5285AmLUf7pvii4Ym8YsRizGaSngE2Z*

`explicit_Field_minus_plus_dist`

Theorem 25.18 explicit_Field $\rightarrow \forall xy \in R.-(x + y) = -x + -y$. *The proposition is identified by the following information:*

Pure Prop Id:	*7c28f73f41fb9362c802db194a80e809ab40b5fcbf86889187a241fb04c45f07*
Pure Prop Address:	*TMMbYxMRKezKTJauH5tFeT6YcdVqNqaoNyb*
Theory Prop Id:	*3477df803cb462a1c67676532b809a8085f5965e8eded20de22c081f8b380c1f*
Theory Prop Address:	*TMJEy6ntH8yniiQ1XpVapukWC9af7szg259*

explicit_Field_minus_mult_L

Theorem 25.19 explicit_Field $\rightarrow \forall xy \in R.(-x) * y = -(x * y)$. *The proposition is identified by the following information:*

Pure Prop Id:	*d4042ab05af8694ed9f50ea48d73fd8241fe9e096a57f64ca3767ec2ee8622b8*
Pure Prop Address:	*TMMDqKcMu6spUHkosKvvDfVjBChv657VpPo*
Theory Prop Id:	*2c1cad10f4a09ca8c4864cae7b040bf84788c98182553b04deca95d667a74739*
Theory Prop Address:	*TMb8pKtSFLQTYNxrQHoTXXs92fMEGh22svD*

explicit_Field_minus_mult_R

Theorem 25.20 explicit_Field $\rightarrow \forall xy \in R.x * (-y) = -(x * y)$. *The proposition is identified by the following information:*

Pure Prop Id:	*1c896cfcf91413c6c45f35892ce57731b0adad535619672d8857f0281523657a*
Pure Prop Address:	*TMHHkG7bnn6fLtxgsof8vd6CC3u39fp5sYn*
Theory Prop Id:	*46e01b365cf641485dae65460ad1681ecaab879cc9dd36e6f2dc58b3a94a1cd0*
Theory Prop Address:	*TMPM6UjftWZ8u7qznYUZoTw3xxFGQZUT3ce*

explicit_Field_square_zero_inv

Theorem 25.21 explicit_Field $\rightarrow \forall x \in R.x * x = zero \rightarrow x = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*18939b3ff00eabb11a4a28ae19cc0aac425c72704752d525860f80dfe5375212*
Pure Prop Address:	*TMNAKKNruWjcXkrAW9Rb6BxwgJg9jKX4Lbd*
Theory Prop Id:	*fe699ea22147eff4c7086ff7bf4954794c5e61d9d3e22c587a86681568f2423d*
Theory Prop Address:	*TMP55pVZRC8XUcxBEuENBHAqrd7cau5uiY4*

explicit_Field_mult_zero_inv

Theorem 25.22

$$\text{explicit\_Field} \rightarrow \forall xy \in R.x * y = zero \rightarrow x = zero \vee y = zero.$$

The proposition is identified by the following information:

Pure Prop Id:	*48f3d685f5c19ffb943e4e92bf1143cd300504313d7025cbcd85d11fa57e29aa*
Pure Prop Address:	*TMMoGSv87AzopLCYaS7tpk3DNioBcaw6e6Z*
Theory Prop Id:	*d90b744848de4a3c083b3ce57e25e069c0d9b625f137f9e544dde929638e2794*
Theory Prop Address:	*TManTc7e6UWwgQhhz3U1c3VVG946TvLTxR9*

Let $leq : \iota \rightarrow \iota \rightarrow o$ be given. **Notation.** We use $\leq$ as an infix operator corresponding to applying term leq.

Definition 25.3 explicit_OrderedField *is the opaque object of type o identified by the following information:*

Pure Object Id:	*a18f7bca989a67fb7dc6df8e6c094372c26fa2c334578447d3314616155afb72*
Pure Object Address:	*TMSMYTTginXTnkcVZtpjAKPAaL7BnqqgRds*
Theory Object Id:	*eea7fafdf682c37d08f8505d8389845b96987326a92b0823dac083b00eda754c*
Theory Object Address:	*TMUVEidBNjgBhKRvbJD1JJ2kD28DQi7APKy*

`explicit_OrderedField_I`

Theorem 25.23

$$\text{explicit\_Field}\to(\forall xyz \in R.x \leq y \to y \leq z \to x \leq z)\to \\ (\forall xy \in R.x \leq y \wedge y \leq x \Leftrightarrow x = y)\to \\ (\forall xy \in R.x \leq y \vee y \leq x)\to(\forall xyz \in R.x \leq y \to x + z \leq y + z)\to \\ (\forall xy \in R.zero \leq x \to zero \leq y \to zero \leq x * y)\to\text{explicit\_OrderedField}.$$

The proposition is identified by the following information:

Pure Prop Id:	*80eb1d0d6d1ae59e7716b600362a8edadf3a3d46606eefe17eb4379546b5b255*
Pure Prop Address:	*TMVbN3H3LfjoJc5JN3kFVvxfeyGkhWvjBwf*
Theory Prop Id:	*71586c9655f0d1781fbbb7e18105c89958843a5ccb8e7deeffe6b9e62b1b1e09*
Theory Prop Address:	*TMTVD1npwX9pr1azvDC9toE4RLXrqq7MsyC*

`explicit_OrderedField_E`

Theorem 25.24

$$\forall q :o. \\ (\text{explicit\_OrderedField} \to \text{explicit\_Field} \to \\ (\forall xyz \in R.x \leq y \to y \leq z \to x \leq z) \to \\ (\forall xy \in R.x \leq y \wedge y \leq x \Leftrightarrow x = y) \to \\ (\forall xy \in R.x \leq y \vee y \leq x) \to \\ (\forall xyz \in R.x \leq y \to x + z \leq y + z) \to \\ (\forall xy \in R.zero \leq x \to zero \leq y \to zero \leq x * y) \to q) \\ \to\text{explicit\_OrderedField}\to q.$$

The proposition is identified by the following information:

Pure Prop Id:	*1c269ba4e553561a1cc6de2a41b6545ec8ac75e48e3c6cb23b2b6a324a64952b*
Pure Prop Address:	*TMVNKTpEnn5zTRZtpuHVXwYgJDgVq64jhQb*
Theory Prop Id:	*5c835e8cb20f579d989cd542e60e75f77e9b09c2976f6070c05a17cfca4d407a*
Theory Prop Address:	*TMaCBRHe1Qvk5uX74Rit6toSqNPaYfiTK4D*

`explicit_OrderedField_minus_leq`

Theorem 25.25 explicit_OrderedField $\rightarrow \forall xy \in R.x \leq y \rightarrow -y \leq -x$. *The proposition is identified by the following information:*

Pure Prop Id:	*0d0f5cad7897b1fc9859fb0fb353f392e444d5b3b7e1db20a4f125e0e21ce6bf*
Pure Prop Address:	*TMMwfqJswL3g7Cij23YHpxZpvqT7KfRqNMe*
Theory Prop Id:	*ee304f2ce86cb10f1be68e56828aad0317279e773397f75e93647653c83be151*
Theory Prop Address:	*TMMzgVBuTQhmpTV2B9ybeJX6tiEkbUXhbe3*

explicit_OrderedField_square_nonneg

Theorem 25.26 explicit_OrderedField $\rightarrow \forall x \in R.zero \leq x * x$. *The proposition is identified by the following information:*

Pure Prop Id:	*ebf414f2c6ff9848d35121369f85472ab25e8af5cc2728af838e40787e67ac6f*
Pure Prop Address:	*TMV8VUHDk12iC7xpNnNDLjZD9BtuYf7ESyK*
Theory Prop Id:	*891e8662d8b95f38bd875c846330ed8d495c4375ade2fe5aac04a6d4c9daa3d3*
Theory Prop Address:	*TMJ9LeAui7MyNuXXvPgJDye1dzUzpzKZanv*

explicit_OrderedField_sum_squares_nonneg

Theorem 25.27 explicit_OrderedField$\rightarrow\forall xy \in R.zero \leq x * x + y * y$. *The proposition is identified by the following information:*

Pure Prop Id:	*7596b76c40f5977c5d12346563941ea1bd17dedf0fdc6d4677b21fd667046cd0*
Pure Prop Address:	*TMXYnd1YrHHrvjWcoZ6XYMkRe7mUtnUsHMB*
Theory Prop Id:	*d9447e2431709c9a2271ba5d7c8d2c42675d962980e8d0c6a945b47a84e3f7f2*
Theory Prop Address:	*TMR3ypJ2YAEpeYUYejDaWcqzkgc8VHN9BYU*

explicit_OrderedField_sum_nonneg_zero_inv

Theorem 25.28

$$\text{explicit\_OrderedField}\rightarrow\forall xy \in R.zero \leq x\rightarrow zero \leq y\rightarrow x + y = zero\rightarrow x = zero \wedge y = zero.$$

The proposition is identified by the following information:

Pure Prop Id:	*072f56b723bef4ade29f644b20f32711dd9f8872653a4603191255a02ed12aa5*
Pure Prop Address:	*TMbqPNKUAZHZZvbj8kXQXLfL56CpAozuS97*
Theory Prop Id:	*5f6c78eb0411a6fb156cf3b2848ebb0c759d7a6a6b653d97dec27052916e94e5*
Theory Prop Address:	*TMXUo7T7pnNdX24nuWLNaNSHmUgB2UFnGt4*

explicit_OrderedField_sum_squares_zero_inv

Theorem 25.29

$$\text{explicit\_OrderedField}\rightarrow\forall xy \in R.x * x + y * y = zero\rightarrow x = zero \wedge y = zero.$$

The proposition is identified by the following information:

Pure Prop Id:	*3475f408fb7b3dec70f825cb0332b28e033d25bc6e4cbd6a9b382bbd3d531484*
Pure Prop Address:	*TMGde3gx4w4Uh9Awwi1qwfiWQ4CBJfJAtS1*
Theory Prop Id:	*5736f60e1ddc12d7525825b06f528d3d62fe1e6481630bd0d8e030f123c54b60*
Theory Prop Address:	*TMG76tdh9rhRwpDqWUkbVYQDxYySTKTuYYE*

`explicit_OrderedField_leq_refl`

Theorem 25.30 explicit_OrderedField $\to \forall x \in R.x \leq x$. *The proposition is identified by the following information:*

Pure Prop Id:	*6d89bac73c97399406da5761a4e05ca142b2af3c9ea45f75c5674037832d0352*
Pure Prop Address:	*TMJPTfciQTSdnS3SFhTmePpEG61KkYyXyh8*
Theory Prop Id:	*915e8fb0a5a50db0b4874494c5e1fbe27f4a9e913998db7bdbe6335c2782cef8*
Theory Prop Address:	*TMUeqaV9ny8ooGx2mDKgZUyR36krjn7twu6*

`explicit_OrderedField_leq_antisym`

Theorem 25.31 explicit_OrderedField$\to\forall xy \in R.x \leq y \to y \leq x \to x = y$. *The proposition is identified by the following information:*

Pure Prop Id:	*92c2c21b46e137352ea2b13877fba3bccf89bb379a6f9cb05d766c8493fbc378*
Pure Prop Address:	*TMdF3uckvYNFsZLo2N2AUXRf961iqhpb4nY*
Theory Prop Id:	*2313aae88e7fee79d5231f7afbc5542b995b3af3ba7cc654b0066c73120152d4*
Theory Prop Address:	*TMNmtcpgnXCEWQdjZhJoxe1nhrEE87jGdwk*

`explicit_OrderedField_leq_tra`

Theorem 25.32 explicit_OrderedField$\to\forall xyz \in R.x \leq y \to y \leq z \to x \leq z$. *The proposition is identified by the following information:*

Pure Prop Id:	*2d02678a2209fd726235ddb09c2aa9290b48375519487773c818f9d57860c62b*
Pure Prop Address:	*TMcsuosBpLY4AFhGspmWx2rN1cH12b2XPqe*
Theory Prop Id:	*642b84d1691774bc188f593803d0d85f1012bda7c66ff4ef994e821e3570a030*
Theory Prop Address:	*TMNgAZiMJeVFoYAuFpPJez3pVME9zrth8Xd*

`explicit_OrderedField_leq_zero_one`

Theorem 25.33 explicit_OrderedField $\to zero \leq one$. *The proposition is identified by the following information:*

Pure Prop Id:	*4f14f3aee0dd811396902d2e346ed9447d586fecfa610cb7f80d8c018b78445c*
Pure Prop Address:	*TMcxPXzviwMyx5FiSvb9JnufFyGPAdVhiA2*
Theory Prop Id:	*a0dbd6bf9eb8bf6f8529771cbf53aa3be49e04449a97763b5e01ea7d7ea2a870*
Theory Prop Address:	*TMGr9TnXkfhuvkbv1dMj5JtTZ8RSfzoP95A*

Definition 25.4 *We define* lt *to be* $\lambda xy.x \leq y \wedge x \neq y$ *of type* $\iota \to \iota \to o$ *identified by the following information:*

Pure Object Id: *d326e5351e117c7374d055c5289934dc3a6e79d66ed68f1c6e23297feea0148e*
Pure Object Address: *TMGFyhEzMLotVozJF1yiunrkGLuFdfjWkng*
Theory Object Id: *f91dfd2380597226f5c8900812832b2a5a67ae6568fe2b2a605848bd8d22540a*
Theory Object Address: *TMcecisvAF6JJ66jafWS8ZR87w7SMBwYATJ*

Notation. We use $<$ as an infix operator corresponding to applying term lt.

Definition 25.5 natOfOrderedField_p *is the opaque object of type* $\iota \to o$ *identified by the following information:*

Pure Object Id: *f1c45e0e9f9df75c62335865582f186c3ec3df1a94bc94b16d41e73bf27899f9*
Pure Object Address: *TMUGM8F7ocg9jGYhPZVwzCfNfPzPmAanw4Z*
Theory Object Id: *f89652d7f0e1787e89129826561eeccb4f5558676c7910ea999306a32285e218*
Theory Object Address: *TMFAmRM27u8JLyZ7Jf1HAC8DeSELjytd8Ua*

Let N be $\{n \in R | \text{natOfOrderedField\_p}\ n\}$. Let $Npos$ be $\{n \in N | n \neq zero\}$.

`explicit_Nats_natOfOrderedField`

Theorem 25.34

$$\text{explicit\_OrderedField}{\to}\text{explicit\_Nats}\ N\ zero\ (\lambda m.m + one).$$

The proposition is identified by the following information:

Pure Prop Id: *9cb324808fc5cf72d2d653a2418ab30cb9b85476e00be35fcd9b57f1bb93ad72*
Pure Prop Address: *TMWcMhihbJqy1w5uYhuM3Es9nEdDsCM8SJn*
Theory Prop Id: *8d1a312e7d98b7df2205e4cfa93c99a5f6dfa4019921c333b9d177cc70aca004*
Theory Prop Address: *TMZXnwRs21hpHmTpXy9R1JnG8n2ykq4d1cS*

`explicit_PosNats_natOfOrderedField`

Theorem 25.35

$$\text{explicit\_OrderedField}{\to}\text{explicit\_Nats}\ Npos\ one\ (\lambda m.m + one).$$

The proposition is identified by the following information:

Pure Prop Id: *36987abea2f716243c8b427e45e8b2e1ae58f4dd9e1dd000b423f2e6f4bc48a0*
Pure Prop Address: *TMWy3odsbnhVAitSJ5hzGBv9T8fmV4ZP3S7*
Theory Prop Id: *7b42ab4ebcb5779370bac5766235443 25cab108670d64b30ebe19743a3fa9054*
Theory Prop Address: *TMZEMUgieWVVNR5ysQcJpnbXL1KpnZvBLMh*

Let Z be $\{n \in R | -n \in Npos \vee n = zero \vee n \in Npos\}$.

Definition 25.6 *We define* explicit_OrderedField_rationalp *to be*

$$\lambda x.\exists n \in Z.\exists m \in Npos.m * x = n$$

of type $\iota \to o$ *identified by the following information:*

Pure Object Id:	*bacdefb214268e55772b42739961f5ea4a517ab077ed4b168f45f4c3d4703ec4*
Pure Object Address:	*TMS9Jk69fwzyerTVyhC5cLftFrz8QwxR86G*
Theory Object Id:	*6449a4f053483876d454bb93a1566cd7b4d24332ae6e750097943c2e903e0426*
Theory Object Address:	*TMdsiVayo81K6oYHmqnGdSi5QMPLt3MnzCJ*

Let Q be $\{x \in R | \mathsf{explicit\_OrderedField\_rationalp}\ x\}$.

`explicit_OrderedField_Npos_props`

Theorem 25.36

$$\begin{gathered}\mathsf{explicit\_OrderedField} {\to} \forall p{:}o. \\ (Npos \subseteq R \to \mathsf{explicit\_Nats}\ Npos\ one\ (\lambda m.m + one) \to one \in Npos \to \\ (\forall m \in Npos.m + one \neq one) \to \\ (\forall m \in Npos.\forall q : \iota \to o.q\ one \to (\forall n \in Npos.q\ (n + one)) \to q\ m) \to \\ (\forall nm \in Npos.\mathsf{explicit\_Nats\_one\_plus}\ Npos\ one\ (\lambda m.m + one)\ n\ m = n + m) \to \\ (\forall nm \in Npos.\mathsf{explicit\_Nats\_one\_mult}\ Npos\ one\ (\lambda m.m + one)\ n\ m = n * m) \to \\ (\forall nm \in Npos.n + m \in Npos) \to \\ (\forall nm \in Npos.n * m \in Npos) \to p) \\ {\to} p.\end{gathered}$$

The proposition is identified by the following information:

Pure Prop Id:	*70d4a40bcc65e646c44adc246e6e8230f11d26380216c98eef3ce0f533c5b967*
Pure Prop Address:	*TMWVNiuNPNFnodbziNN8nLxyxLMnKWrd8wJ*
Theory Prop Id:	*ccf61651d2adec6894aedbbc427dceb7844e2399f21269514640c7e6094ad427*
Theory Prop Address:	*TMFByiQaDpiDUkPeJbt5Bp3Vmn1hJbuuYP1*

`explicit_OrderedField_Z_props`

Theorem 25.37

$$\begin{gathered}\mathsf{explicit\_OrderedField} {\to} \forall p{:}o. \\ ((\forall n \in Npos.{-n} \in Z) \to \\ zero \in Z \to Npos \subseteq Z \to Z \subseteq R \to \\ (\forall n \in Z.\forall q : o.({-n} \in Npos \to q) \to (n = zero \to q) \to (n \in Npos \to q) \to q) \to \\ one \in Z \to {-one} \in Z \to \\ (\forall m \in Z.{-m} \in Z) \to \\ (\forall nm \in Z.n + m \in Z) \to \\ (\forall nm \in Z.n * m \in Z) \to p) \\ {\to} p.\end{gathered}$$

The proposition is identified by the following information:

Pure Prop Id:	*c4e83c38fecfc73fc1d09e7d004296072c59b1e4f2df931d758d29e6794ea856*
Pure Prop Address:	*TMXcg17hazxWWPbWnFd6LixgqsqD72qWt13*
Theory Prop Id:	*5d739c339801f054de73227f43deae05f5146a6f72e3301a34806a65741eee8f*
Theory Prop Address:	*TMdyXYYsofQoayXp7LxqNscqFCZMBXVaVYV*

`explicit_OrderedField_Q_props`

Theorem 25.38

$$\begin{gathered}\text{explicit\_OrderedField}\to\forall p:o.\\(Q\subseteq R\to\\(\forall x\in Q.\forall q:o.(x\in R\to\forall n\in Z.\forall m\in Npos.m*x=n\to q)\to q)\to\\(\forall x\in R.\forall n\in Z.\forall m\in Npos.m*x=n\to x\in Q)\to p)\\\to p.\end{gathered}$$

The proposition is identified by the following information:

Pure Prop Id:	*469ebe93febae512d65ef077bb259d3d61a42e332d5dc0f218b4ef51c641e3e4*
Pure Prop Address:	*TMTZyHSuTABLn2UE5U2gcU2Ezo5LNbr4p3D*
Theory Prop Id:	*d268c341664267b91610963ba224b4a1a9c0f9f442a16df66f09d45352a4151a*
Theory Prop Address:	*TMds236CmEKbqpnTaTJdR2MqiniGBAQsAPi*

Definition 25.7 explicit_Reals *is the opaque object of type o identified by the following information:*

Pure Object Id:	*2c81615a11c9e3e301f3301ec7862ba122acea20bfe1c120f7bdaf5a2e18faf4*
Pure Object Address:	*TMNQkXLgjhMYMywv7BnJ8Buh3QQLNR7YL5B*
Theory Object Id:	*f814a2e6f2e99b382ad540cb48300cde1d9995f4b6b2b5af7856369fd2644732*
Theory Object Address:	*TMRfnwEvCdNWYa7FnDpZ9E82K56Dx1FvxWm*

`explicit_Reals_I`

Theorem 25.39

$$\begin{gathered}\text{explicit\_OrderedField}\to\\(\forall xy\in R.zero<x\to zero\leq y\to\exists n\in N.y\leq n*x)\to\\(\forall ab\in R^N.(\forall n\in N.a\ n\leq b\ n\wedge a\ n\leq a\ (n+one)\wedge b\ (n+one)\leq b\ n)\to\\\exists x\in R.\forall n\in N.a\ n\leq x\wedge x\leq b\ n)\\\to\text{explicit\_Reals}.\end{gathered}$$

The proposition is identified by the following information:

Pure Prop Id:	*409e2333061c2a93e2645163cc79fc5d5af155c8a5d27793194dc767da89abbb*
Pure Prop Address:	*TMV9L8eBVS4dZTimiRu7xTnjDG7k6PkcS8x*
Theory Prop Id:	*5c0c700ed594f122d6868607cbbbb42d933731fe7421678006f15345427a750c*
Theory Prop Address:	*TMFrB18DrFskuCZs1RC1v2N6mv4m2xbxyyh*

`explicit_Reals_E`

Theorem 25.40

$$\begin{array}{c}\forall q : o. \\ (\mathsf{explicit\_Reals} \to \mathsf{explicit\_OrderedField} \to \\ (\forall xy \in R.zero < x \to zero \leq y \to \exists n \in N.y \leq n * x) \to \\ (\forall ab \in R^N.(\forall n \in N.a\ n \leq b\ n \wedge a\ n \leq a\ (n + one) \wedge b\ (n + one) \leq b\ n) \to \\ \exists x \in R.\forall n \in N.a\ n \leq x \wedge x \leq b\ n) \\ \to q) \\ \to \mathsf{explicit\_Reals} \to q.\end{array}$$

The proposition is identified by the following information:

Pure Prop Id:	*21d90e2433a10558f7145bc3be6f01fa8745c28bffc25ad66bd1a18358b3b240*
Pure Prop Address:	*TMGdZwHMdLeLHxRbHQCHJtCbKDNcLtG3W8L*
Theory Prop Id:	*4da66ca8d1ae9ed2c059a4f5bc10b24ce6225436b72100092ed91505c18552ff*
Theory Prop Address:	*TMXm1gKZKK9VncpZdpgFvZ65WUBmGNKHtic*

`explicit_Reals_characteristic_0`

Theorem 25.41

$$\mathsf{explicit\_Reals} \to \forall n \in \mathsf{omega}.\mathsf{nat\_primrec}\ one\ (\lambda\_r.plus\ one\ r)\ n \neq zero.$$

The proposition is identified by the following information:

Pure Prop Id:	*0212c2930b33db49e6a4620cb445fdddf720a725cafcc5cdfd73aba5653545a7*
Pure Prop Address:	*TMQD99B4xAaxuUtBVEmerL9nB3AVPpeVrb5*
Theory Prop Id:	*a77c5ab95a85405cab2dc8eb8cf6de3db3512e2461a8c18fdbf2c5586157d584*
Theory Prop Address:	*TMcXpA2UzrnRUBX4icewcaFFnK1yHVkVbAY*

Chapter 26

Rings II

Definition 26.1 *We define* CRing_with_id_carrier *to be* $\lambda Rs.Rs\ 0$ *of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*f0bb69e74123475b6ecce64430b539b64548b0b148267ea6c512e65545a391a1*
Pure Object Address:	*TMY8sbcPiVs9VL311KPHpbm67U6sLM9JvF7*
Theory Object Id:	*9211e41eaf81bb8c152618cbe35230be60d736fcd5684c0b7bdc5ccc1ff70181*
Theory Object Address:	*TMZQE6tPrFvJMaBcLrcMn85ZgscSeFbm4pT*

Definition 26.2 *We define* CRing_with_id_plus *to be* $\lambda Rs.$decode_b $(Rs\ 1)$ *of type* $\iota \to \iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*6968cd9ba47369f21e6e4d576648c6317ed74b303c8a2ac9e3d5e8bc91a68d9d*
Pure Object Address:	*TMYTM1QQGqgVyRM9unR8CLL2scwRHYvV3c3*
Theory Object Id:	*ed06aa34b4112f8507370b46c1c5c7de7f2f9af4f24b831d45893ad3066e994e*
Theory Object Address:	*TMRiY7KU6BpU28KD19iwgBDLsQ5XyfpcbYL*

Definition 26.3 *We define* CRing_with_id_mult *to be* $\lambda Rs.$decode_b $(Rs\ 2)$ *of type* $\iota \to \iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*4b4b94d59128dfa2798a2e72c886a5edf4dbd4d9b9693c1c6dea5a401c53aec4*
Pure Object Address:	*TMQSTL5b71ddbjnkjnGXfRZ61uXWmMGwJHD*
Theory Object Id:	*7eb39ed695c795e4451a9252d011d377d0f4bdf0bfe6193f7a31ceb58f687a40*
Theory Object Address:	*TMaJmzoi8UnDNTAXrAvZxYbTeEdKV3viygg*

Definition 26.4 *We define* CRing_with_id_zero *to be* $\lambda Rs.Rs\ 3$ *of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*61f7303354950fd70e647b19c486634dc22d7df65a28d1d6893007701d7b3df4*
Pure Object Address:	*TMJzMmYjL1HdXDWMSRf7c775tmGDc6osaRL*
Theory Object Id:	*b74fb66a398c6c647d135ea9283353963f66d301d2c1ee126b203679f2071094*
Theory Object Address:	*TMHQo21kvxiD8yab3ek4ATBz2PBfwvqsmaJ*

Definition 26.5 *We define* CRing_with_id_one *to be* $\lambda Rs.Rs\ 4$ *of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*57e734ae32adddc1f13e75e7bab22241e5d2c12955ae233ee90f53818028312a*
Pure Object Address:	*TMV8UDBZRqv4jocV6zt17aP89PcYugzQkoU*
Theory Object Id:	*b2fa78f7f58b95f4bc4f46ac268c3a959090897e56338c64bd84a73bc5fc2551*
Theory Object Address:	*TMWFb9uZk2JEesS9quTe9a6Ww4ND9VcbBJA*

26.1 CRing_with_id

Let $Rs : \iota$ be given. Assume the following.

$$\mathsf{CRing\_with\_id}\ Rs \tag{26.1}$$

Let $R : \iota$ be CRing_with_id_carrier Rs. Let $zero : \iota$ be CRing_with_id_zero Rs. Let $one : \iota$ be CRing_with_id_one Rs. **Notation.** We use $+$ as a right associative infix operator corresponding to applying term CRing_with_id_plus Rs. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term CRing_with_id_mult Rs.

`CRing_with_id_eta`

Theorem 26.1

$$Rs = \mathsf{pack\_b\_b\_e\_e}\ R\ (\mathsf{CRing\_with\_id\_plus}\ Rs)\ (\mathsf{CRing\_with\_id\_mult}\ Rs)\ zero\ one.$$

The proposition is identified by the following information:

Pure Prop Id:	*19895bd57673dd78940963d6fcc41b374e3914364fcfea397ea66c6fbf664993*
Pure Prop Address:	*TMGYeUaTmA7FDSErSMoGpiYWt7ZcZFFdt71*
Theory Prop Id:	*d44bbbcb20f869f08519c1b4ae75e3aa5154362f4fff11b557d1e7203da89831*
Theory Prop Address:	*TMJMAQYbxKT8R2VWNpVMaASpWXQHbY8dnP7*

`CRing_with_id_explicit_CRing_with_id`

Theorem 26.2

$$\mathsf{explicit\_CRing\_with\_id}\ R\ zero\ one\ (\mathsf{CRing\_with\_id\_plus}\ Rs)\ (\mathsf{CRing\_with\_id\_mult}\ Rs).$$

The proposition is identified by the following information:

Pure Prop Id:	*a3ff783737e4d8958b2aa7415fe17473670a67c35da28139dff7bae3f4e94735*
Pure Prop Address:	*TMVhXNKTXiit8PdcpXCyihn8b4FTsC3k2ej*
Theory Prop Id:	*56ddec707e12204d0adbd96c68e0500a77d81ad8852f7ce843d1a89f076bed97*
Theory Prop Address:	*TMEtWQ6ajg8K49oJHzYUKgUUcZG8mV9k9CD*

`CRing_with_id_zero_In`

Theorem 26.3 $zero \in R$. *The proposition is identified by the following information:*

Pure Prop Id:	*d1c133446d084c259b4e26e8455730b6db88c0480b50cfe8c1548ebaac397411*
Pure Prop Address:	*TMUd78nxJNekmpXHWS1gxBmHFVqBbVXqURF*
Theory Prop Id:	*42b83047b5470f202412a31f5819c2bcfd6ce9af636f261d3171012400a8362c*
Theory Prop Address:	*TMHL6R5QUphtzjmu3bzsfeJdq6c3Yemfgg6*

CRing_with_id_one_In

Theorem 26.4 *$one \in R$. The proposition is identified by the following information:*

Pure Prop Id:	*2fa10721fd9208ab1e1eda4402ab18280e5076b055a04f23501ee00c86ae3bd6*
Pure Prop Address:	*TMSn9V5PkmSZNhoiJE1BXRDWi4AyqGo5tEs*
Theory Prop Id:	*a53700e2c16d4d39a135c22a89b655a707721c850f4b02f791f7b5962324450e*
Theory Prop Address:	*TMHiL2AUU8VvaHXP7gKgkrGeZd2daCDT76Y*

CRing_with_id_plus_clos

Theorem 26.5 *$\forall xy \in R.x + y \in R$. The proposition is identified by the following information:*

Pure Prop Id:	*66b3347c3b4cc8181cb587a10e6fdddaf8f1db0dfaedebf691206c43b5454f81*
Pure Prop Address:	*TMaPdKX5KfZy52M1fsgS698CBznuSXjAAQq*
Theory Prop Id:	*d2fae7d02aba4468a0d3e7ae865a7175216a2064cc2a83eac8524bf761ae8665*
Theory Prop Address:	*TMSuXgDfq9vWRJXbJvdfjSGgSqjU13XUvKZ*

CRing_with_id_mult_clos

Theorem 26.6 *$\forall xy \in R.x * y \in R$. The proposition is identified by the following information:*

Pure Prop Id:	*ef950216b478ee5ac786dcd5b26724cf2b7cca1507f263cf09aa4d17c6a4cfdc*
Pure Prop Address:	*TMRK1cL4HUwW4gVfA7fWYbzoNAqvrXFqkd6*
Theory Prop Id:	*81665379575211b266779e952c3b4b18fd01a04f7da8eee59fb234fffb1db011*
Theory Prop Address:	*TMbGpmv6AAcSZfaNEN3rm5PX6sRS59HttJi*

CRing_with_id_plus_assoc

Theorem 26.7 *$\forall xyz \in R.x + (y + z) = (x + y) + z$. The proposition is identified by the following information:*

Pure Prop Id:	*454cc66248a33dc6e34bc2d7c3406f6ea1034419dd0abe5e4915b38af1bdd1da*
Pure Prop Address:	*TMQjX1emD1dnLgUKfE941oVUGCfDQyMUpXT*
Theory Prop Id:	*f2fe8f25a1fc149692ba6c28a8f4d1838e62305907ba78cf398ef5c4730d85f2*
Theory Prop Address:	*TMca3P6Uj8MFLwDfmKJw24JLqCBdBiTo2ge*

CRing_with_id_plus_com

Theorem 26.8 *$\forall xy \in R.x + y = y + x$. The proposition is identified by the following information:*

Pure Prop Id:	*e1ec1adb066b85b12e0506e4fc204093d7e6ada60496967a6728d34eff569554*
Pure Prop Address:	*TMPUNCr5oiXd8PKxvprAqJ3n8qdBby2zkwy*
Theory Prop Id:	*6b52cfd60b1677666cfc2a418ff72124083d95553587c923415f05e9d2660cfc*
Theory Prop Address:	*TMYoTWDw5hq6S5t6qWKJAQSbseTkUH3rGXK*

`CRing_with_id_zero_L`

Theorem 26.9 $\forall x \in R.zero + x = x$. *The proposition is identified by the following information:*

Pure Prop Id:	*f35d5e2cb4c5f2f6924a8a216db8a83b2419462902ed4931aafb8448de773de7*
Pure Prop Address:	*TMMv4dCHoLPd63XLsyBEpxxQatyYjuAN6Zd*
Theory Prop Id:	*f987b8eb4b1d1c03ffa91d2acf41d423fbb463f0f6ff54054d915245384b9664*
Theory Prop Address:	*TMZiPp7aZHNZCiWPSvz3S9vEUNYuY1ef5Ce*

`CRing_with_id_plus_inv`

Theorem 26.10 $\forall x \in R.\exists y \in R.x + y = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*39fc1073cf2b1717e43d5e37e722ec6dc5e2762de4efea8b60fcd83168259e8b*
Pure Prop Address:	*TMJppCMEt4Wjhgfzc6osCgs8pyrPW7A2t3q*
Theory Prop Id:	*8dbe3e89f0ed03c869c48edbd9729c9e9e07f83c48e1fc41cf4ce63726a55703*
Theory Prop Address:	*TMGEihSCgFbz19W1WMiKWFqirbHZRxBnwEk*

`CRing_with_id_mult_assoc`

Theorem 26.11 $\forall xyz \in R.x * (y * z) = (x * y) * z$. *The proposition is identified by the following information:*

Pure Prop Id:	*fa25d807da6b86aa2872ac8f4b6ce9af0dde8c126b739a180e2292a5ad0b7dc0*
Pure Prop Address:	*TMTGLRHnb5NytZEW9BtTjmjgzSoYUdX3Lq4*
Theory Prop Id:	*09949174d310e1abf6b516643ecbf69fb2cf7e3fb36f2b3cc266eaffaa5cca61*
Theory Prop Address:	*TMUmkRGj1nHwhjroNxwFzc3LpQcb2T4SgJr*

`CRing_with_id_mult_com`

Theorem 26.12 $\forall xy \in R.x * y = y * x$. *The proposition is identified by the following information:*

Pure Prop Id:	*8fc88e1ed947c8d1d020e3f1b2c9ae9d85ce481324a851dcaf7ac3e82d522fb3*
Pure Prop Address:	*TMUSB3uan5mCMRByLcZb9Vf6XGJLEqvGAx2*
Theory Prop Id:	*993072235b78d59978f6d2d7cf20206ad064f19386577afc365ea4ffde99e012*
Theory Prop Address:	*TMbZ4NWw66ZTcKKd4JXSjKenmXtKidd92WT*

`CRing_with_id_one_neq_zero`

Theorem 26.13 $one \neq zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*cedcbebb970be20c670e00da3c0b87ab77a367e70926a4ed9f074b2be9008982*
Pure Prop Address:	*TMEvCCzeCPquYLSEU7q1oMEzpWzipFajv3Q*
Theory Prop Id:	*ec7109346aee40725228c175e17b9274815d864e0391c1d1a02806855544bd46*
Theory Prop Address:	*TMX7e8dpinoTbP3eGKDcZggchnnnzddqYNo*

`CRing_with_id_one_L`

Theorem 26.14 $\forall x \in R. one * x = x$. *The proposition is identified by the following information:*

Pure Prop Id:	*a7a0f95242895f6eb543a18d764276740cde1e7c97815da94792785f6d715289*
Pure Prop Address:	*TMMDR8jBbmxxtSxnqMkUtFxddjJTP4p9AoR*
Theory Prop Id:	*47ab0ad2e91b36e068e15b85e2ae4556a99d2d2131d1a80be5fbc3485fcd84a8*
Theory Prop Address:	*TMZnbveBuCsuGB8UqSdxmfU8pxPepufVw7K*

`CRing_with_id_distr_L`

Theorem 26.15 $\forall xyz \in R. x * (y + z) = x * y + x * z$. *The proposition is identified by the following information:*

Pure Prop Id:	*3c94070ae4c81515b83396b01a1abf18f8fb887ede6f676ce628a4dd5c373902*
Pure Prop Address:	*TMRDgjKhiZf81J84szN54xfkhAS79YaDiGY*
Theory Prop Id:	*aca955a68afb3f852001bc0ad5cef20f02c96080f226c19c097a0acaeca82731*
Theory Prop Address:	*TMYojyDz1DaZZMWZU9guJSxD1TDFN4qyKXy*

Definition 26.6 *We define* CRing_with_id_omega_exp *to be*

$$\lambda x.\mathsf{nat\_primrec}\ one\ (\lambda kr. x * r)$$

of type $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*b079d3c235188503c60077e08e7cb89a17aa1444006788a8377caf62b43ea1ea*
Pure Object Address:	*TMQDSzK4zGei1WaFxwYxfp5T1fYD9frJ1iE*
Theory Object Id:	*5d35489a6173669c2175e151eba6c7c0d9bd86c956b8a3cbee0b8aa433d69a7a*
Theory Object Address:	*TMQgksRuPWDCMf6N4hRujxQQ6p1EcFejvED*

Notation. We use superscripts as notation corresponding to applying term CRing_with_id_omega_exp.

`CRing_with_id_omega_exp_0`

Theorem 26.16 $\forall x. x^0 = one$. *The proposition is identified by the following information:*

Pure Prop Id:	*8b9c8bf0cedf602ad64ec9fda9404cd08eaea17ebf9730500f9d73a231c12db0*
Pure Prop Address:	*TMKBtqa6CgMy36XtRCUkiY5qZML3T3omNvz*
Theory Prop Id:	*64d60c82466552e43c260e08ac60fa038064ba97a7b299601bb5ac348d15e92d*
Theory Prop Address:	*TMU64mwxKSqZr95cwqeqp8gLtWdfbUdGauU*

`CRing_with_id_omega_exp_S`

Theorem 26.17 $\forall x. \forall n \in \mathsf{omega}. x^{(\mathsf{ordsucc}\ n)} = x * x^n$. *The proposition is identified by the following information:*

Pure Prop Id:	*14d493dd404e3f9509a3156f2272e3c51abbd469b9e5a348093995321cdc02c7*
Pure Prop Address:	*TMaPKnDcmp98SwqmHGsP6Upmm4LVc7HBYeY*
Theory Prop Id:	*b501f7792ce7723a2549037b7664ccee4a1f7319ecbe7ec95d81d30ee5d64db7*
Theory Prop Address:	*TMWJS21wHv8n2fKeHv1VWF6sSsyo6xbsLgo*

CRing_with_id_omega_exp_1

Theorem 26.18 $\forall x \in R.x^1 = x$. *The proposition is identified by the following information:*

Pure Prop Id:	*730a1c203f7c249e3caf69aa7f389915ff525f30f92b1343d270b5df775e11ff*
Pure Prop Address:	*TMWRcHBBm5BHyxkidxbDUj99jsLbDdcXpMk*
Theory Prop Id:	*d2a867dea7b48e2f9e3089f5db74c3d8470de6f670f9209a78e953d06761fa8e*
Theory Prop Address:	*TMZwVw12XXo388yfUGycKftBC8osJcNPCHv*

CRing_with_id_omega_exp_clos

Theorem 26.19 $\forall x \in R.\forall n \in \mathsf{omega}.x^n \in R$. *The proposition is identified by the following information:*

Pure Prop Id:	*9a2b5725819918b6748494bfa537f50d0ca70b39d8c1e8ac3ac748fbeccec3f9*
Pure Prop Address:	*TMXNCAZsnHcpmBbcTyxVYrdZgESRVaFBJ13*
Theory Prop Id:	*a996c7c3b925951b4e2d7bbb01fcc9986e58880420715d9c42b7d108ffc16761*
Theory Prop Address:	*TMc79AxZxe6P15NYTfHWGFMuKcAwNRAnJsW*

Definition 26.7 *We define* CRing_with_id_eval_poly *to be*

$$\lambda ncsx.\mathsf{nat\_primrec}\ zero\ (\lambda mr.cs\ m * x^m + r)\ n$$

of type $\iota \to \iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*b84ca72faca3a10b9b75c38d406768819f79488fbad8d9bee8b765c45e35823c*
Pure Object Address:	*TMUwUn45GuDKgSXWDvDMPcovuhY4uP1dZdG*
Theory Object Id:	*808af0dde5838cc9e9caab3e4c3a545899005ebfaec703c548a438c00a522768*
Theory Object Address:	*TMTHaSMuk9FXH4zmYumMT1hCMdrHTXWYL3V*

CRing_with_id_eval_poly_clos

Theorem 26.20

$$\forall n \in \mathsf{omega}.\forall cs \in R^n.\forall x \in R.\mathsf{CRing\_with\_id\_eval\_poly}\ n\ cs\ x \in R.$$

The proposition is identified by the following information:

Pure Prop Id:	*e579be29fc0b739e1e606a16df30fd76865f842f7859988a490019039a140bfb*
Pure Prop Address:	*TMSCLopfevPZB8YYCtqzjREB3sftpBvTih7*
Theory Prop Id:	*b40a29975dbc63c22e581f918f9362b2bd0b03c27af7e307e9ac129426d92073*
Theory Prop Address:	*TMZXvJWRJta8nKp7RX1mQcytfayt5Sk89bB*

Chapter 27

Explicit Reals II and Fields

27.1 explicit_Reals

Let $R : \iota$ be given. Let $zero, one : \iota$ be given. Let $plus, mult : \iota \to \iota \to \iota$ be given. **Notation.** We use $+$ as a right associative infix operator corresponding to applying term $plus$. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term $mult$. **Notation.** We use $--$ as a prefix operator corresponding to applying term $\textsf{explicit\_Field\_minus}\ R\ zero\ one\ plus\ mult$. Let $leq : \iota \to \iota \to o$ be given. **Notation.** We use $\leq$ as an infix operator corresponding to applying term leq.

Let N be $\{n \in R | \textsf{natOfOrderedField\_p}\ R\ zero\ one\ plus\ mult\ leq\ n\}$.

Let $Npos$ be $\{n \in N | n \neq zero\}$.

Let Z be $\{n \in R | -n \in Npos \lor n = zero \lor n \in Npos\}$.

Let Q be $\{x \in R | \textsf{explicit\_OrderedField\_rationalp}\ R\ zero\ one\ plus\ mult\ leq\ x\}$.

`explicit_OrderedField_explicit_Field_Q`

Theorem 27.1

$$\textsf{explicit\_OrderedField}\ R\ zero\ one\ plus\ mult\ leq \to \textsf{explicit\_Field}\ Q\ zero\ one\ plus\ mult.$$

The proposition is identified by the following information:

Pure Prop Id:	*af6a3ba60f48d142602c8b680df2961966f1dca1044d4270f1f29c8a72c59c21*
Pure Prop Address:	*TMZT73ZmoriPCPWsoj29MFJq3aK7fa41ff9*
Theory Prop Id:	*63d4c7bfde12c187c511272b653382138f0591cd1d246d9947331032fefa5c86*
Theory Prop Address:	*TMGnn2Z2r22We9QuK5hXYB6c9XHixx14xDr*

`explicit_OrderedField_sub`

Theorem 27.2

$$\mathsf{explicit\_OrderedField}\ R\ zero\ one\ plus\ mult\ leq \rightarrow \forall R' \subseteq R.$$
$$zero \in R' \rightarrow one \in R' \rightarrow (\forall xy \in R'.x + y \in R') \rightarrow (\forall x \in R'.-x \in R') \rightarrow (\forall xy \in R'.x * y \in R')$$
$$\rightarrow (\forall x \in R'.x \neq zero \rightarrow \exists y \in R'.x * y = one) \rightarrow$$
$$\mathsf{explicit\_OrderedField}\ R'\ zero\ one\ plus\ mult\ leq.$$

The proposition is identified by the following information:

Pure Prop Id:	*1bc19ee4bcca58ce8b4534af0101d211b4fc533dc4db649a1bb010477c1d0998*
Pure Prop Address:	*TMF49guKUWG2dHumcM9HEv55USiZpXFGiDg*
Theory Prop Id:	*9966383d1240a949659da39081fa3506cca4e377a161c6ab8ff8c05865a2eab3*
Theory Prop Address:	*TMLR4J2wofUrAHouvRF1dws9BDPmZefs7gr*

`explicit_Reals_sub`

Theorem 27.3

$$\mathsf{explicit\_OrderedField}\ R\ zero\ one\ plus\ mult\ leq \rightarrow \forall R' \subseteq R.$$
$$zero \in R' \rightarrow one \in R' \rightarrow (\forall xy \in R'.x + y \in R') \rightarrow (\forall x \in R'.-x \in R') \rightarrow (\forall xy \in R'.x * y \in R')$$
$$\rightarrow (\forall x \in R'.x \neq zero \rightarrow \exists y \in R'.x * y = one) \rightarrow$$
$$\mathsf{explicit\_OrderedField}\ R'\ zero\ one\ plus\ mult\ leq.$$

The proposition is identified by the following information:

Pure Prop Id:	*1bc19ee4bcca58ce8b4534af0101d211b4fc533dc4db649a1bb010477c1d0998*
Pure Prop Address:	*TMF49guKUWG2dHumcM9HEv55USiZpXFGiDg*
Theory Prop Id:	*9966383d1240a949659da39081fa3506cca4e377a161c6ab8ff8c05865a2eab3*
Theory Prop Address:	*TMLR4J2wofUrAHouvRF1dws9BDPmZefs7gr*

27.1.1 explicit_Reals_Q_min_props

Let $R' : \iota$ be given.

Let N' be $\{n \in R' | \mathsf{natOfOrderedField\_p}\ R'\ zero\ one\ plus\ mult\ leq\ n\}$.

Let $Npos'$ be $\{n \in N' | n \neq zero\}$.

Let Z' be

$$\{n \in R' | \ \mathsf{explicit\_Field\_minus}\ R'\ zero\ one\ plus\ mult\ n \in Npos' \vee n = zero \vee n \in Npos'\}.$$

Let Q' be $\{x \in R' | \mathsf{explicit\_OrderedField\_rationalp}\ R'\ zero\ one\ plus\ mult\ leq\ x\}$.

`explicit_Reals_Q_min_props`

Theorem 27.4

$$\text{explicit\_OrderedField } R \; zero \; one \; plus \; mult \; leq \to R' \subseteq R \to$$
$$\text{explicit\_Field } R' \; zero \; one \; plus \; mult \to \forall p : o.$$
$$((\forall x \in R'.\text{explicit\_Field\_minus } R' \; zero \; one \; plus \; mult \; x = -x) \to$$
$$(\forall x \in R'.-x \in R') \to$$
$$N = N' \to Npos = Npos' \to Z = Z' \to Q = Q' \to p)$$
$$\to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*31679e3e73614c3b4e276ed2c828816866f95e9a6a5e2f4721f319e80f473912*
Pure Prop Address:	*TMLPtuuQw2TanZmHDBpjYzPS2V2V5uY5Rv5*
Theory Prop Id:	*2ead4f461950b2019d11d00ddc1a62977823328b7794ed17307fb66dde3d0ccf*
Theory Prop Address:	*TMU33ZE5CuhCrg8XK3QNyMjHM54qa6M4HFy*

27.1.2 Q is Minimal as a Subfield of R

`explicit_Reals_Q_min`

Theorem 27.5

$$\text{explicit\_OrderedField } R \; zero \; one \; plus \; mult \; leq \to \forall R' \subseteq R.$$
$$\text{explicit\_Field } R' \; zero \; one \; plus \; mult \to Q \subseteq R'.$$

The proposition is identified by the following information:

Pure Prop Id:	*aa9362c28b55f5ede37059e938f69a0bdc1f5a2c40ebc1e5233c99544f022cfc*
Pure Prop Address:	*TMTBsyn4r1w2A2v1wey6f5Xapri8dMAVYuY*
Theory Prop Id:	*4e94b5ab940a9c05d8ab92b169c0a80ff166d8866174805a6eccc7b684604233*
Theory Prop Address:	*TMURjNJo39iyQq3Vy1vrzEGuvvAuAkto3iD*

27.2 explicit_Field_transfer

Let $R : \iota$ be given. Let $zero, one : \iota$ be given. Let $plus, mult : \iota \to \iota \to \iota$ be given. **Notation.** We use $+$ as a right associative infix operator corresponding to applying term $plus$. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term $mult$. Let $R' : \iota$ be given. Let $zero', one' : \iota$ be given. Let $plus', mult' : \iota \to \iota \to \iota$ be given. **Notation.** We use $\oplus$ as a right associative infix operator corresponding to applying term $plus'$. **Notation.** We use $\times$ as a right associative infix operator corresponding to applying term $mult'$. Let $f : \iota \to \iota$ be given.

`explicit_Field_transfer`

Theorem 27.6

$$\mathsf{explicit\_Field}\ R\ zero\ one\ plus\ mult \to \mathsf{bij}\ R\ R'\ f \to$$
$$f\ zero = zero' \to f\ one = one' \to (\forall xy \in R. f\ (x+y) = f\ x \oplus f\ y) \to$$
$$(\forall xy \in R. f\ (x*y) = f\ x \times f\ y) \to$$
$$\mathsf{explicit\_Field}\ R'\ zero'\ one'\ plus'\ mult'.$$

The proposition is identified by the following information:

Pure Prop Id:	*ef2cec44c7db17e7db32410fd90c52ba3fa20b7b18117cf540a99f8cbc44c289*
Pure Prop Address:	*TMH4MjCpSyBbECQsWMxidZBXrRgD3jxBeiR*
Theory Prop Id:	*ac0144b31bec8f30802ed9e79cf8dd729ee2b3186952c3cb53f3e3d279b5d6e2*
Theory Prop Address:	*TMHe1ubYe2eN5E7tR9x9YatesmwdpCoqbZC*

27.3 explicit_Field_RepIndep2

Let $R : \iota$ be given. Let $zero, one : \iota$ be given. Let $plus, mult : \iota \to \iota \to \iota$ be given. **Notation.** We use $+$ as a right associative infix operator corresponding to applying term $plus$. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term $mult$. Let $plus', mult' : \iota \to \iota \to \iota$ be given. **Notation.** We use $\oplus$ as a right associative infix operator corresponding to applying term $plus'$. **Notation.** We use $\times$ as a right associative infix operator corresponding to applying term $mult'$. Assume the following.

$$\forall ab \in R. a + b = a \oplus b \tag{27.1}$$

Assume the following.

$$\forall ab \in R. a * b = a \times b \tag{27.2}$$

`explicit_Field_repindep`

Theorem 27.7

$$\mathsf{explicit\_Field}\ R\ zero\ one\ plus\ mult \Leftrightarrow \mathsf{explicit\_Field}\ R\ zero\ one\ plus'\ mult'.$$

The proposition is identified by the following information:

Pure Prop Id:	*54840abeaa24e09c7beaa1936a5defc6c4ea07ae3fb67135a78d8477a50986cc*
Pure Prop Address:	*TMVPNiv37XT25SiNai41zYPSyiPfKQk6XPN*
Theory Prop Id:	*e0eee3acfed1c638458013101d378b56d17393576fd269927418992a1b7bcbc9*
Theory Prop Address:	*TMRdcfuetQSDQ7zq6iYdygkjYWKXS2v6SE2*

27.4 Fields Encoded as Sets

Definition 27.1 *We define* Field *to be*

$$\lambda F.\mathsf{struct\_b\_b\_e\_e}\ F \wedge \\ \mathsf{unpack\_b\_b\_e\_e\_o}\ F \\ (\lambda Q\ plus\ mult\ zero\ one.\mathsf{explicit\_Field}\ Q\ zero\ one\ plus\ mult)$$

of type $\iota \to o$ identified by the following information:

Pure Object Id:	*c7b84fda1101114e9994926a427c957f42f89ae1918086df069cb1a030519b46*
Pure Object Address:	*TMaCGKsNBVewLEqMyTdCeiKd5uAgK8kFPjY*
Theory Object Id:	*622c30c9ddf18abf87387ddef4f26b6be7b92f631ff5e6b4f5eabaac5839554b*
Theory Object Address:	*TMNqKgkVbs1Np95FXHhyjgcT3UAD21waXUL*

`Field_unpack_eq`

Theorem 27.8

$$\forall R.\forall plus\ mult : \iota \to \iota \to \iota.\forall zero\ one. \\ \mathsf{unpack\_b\_b\_e\_e\_o}\ (\mathsf{pack\_b\_b\_e\_e}\ R\ plus\ mult\ zero\ one) \\ (\lambda R\ plus\ mult\ zero\ one.\mathsf{explicit\_Field}\ R\ zero\ one\ plus\ mult) \\ = \mathsf{explicit\_Field}\ R\ zero\ one\ plus\ mult.$$

The proposition is identified by the following information:

Pure Prop Id:	*0da03c63995752d95dbfb4f0ee50676ef20f4ec29763384de9d7a7d83ec6a852*
Pure Prop Address:	*TMG8uPxmuweznLRMtKvhzoTZwhtZUZ4zZ9d*
Theory Prop Id:	*8e102fafcd372a62e72fe55218203e2c116e3761df378e4406f47cf88eb7c593*
Theory Prop Address:	*TMbAKTnaoDqhKZZ3k6KRGTBNAbKxCGKqeLg*

`Field_is_CRing_with_id`

Theorem 27.9 $\forall F.\mathsf{Field}\ F \to \mathsf{CRing\_with\_id}\ F$. *The proposition is identified by the following information:*

Pure Prop Id:	*26b551faed44acd5217dd94eab68e99a166869a3dfd0d404203d4386512d45c8*
Pure Prop Address:	*TMH41wnx33D5QctmfmPNsPdyGr5hZzzsYVV*
Theory Prop Id:	*e3b51a97808ddcd82d668dd97352b7379f2a20e8f4731608e340953d0b7c4cf0*
Theory Prop Address:	*TMQA24cveY5wc5Ys2Z97RVQgobE2zAHEJMR*

27.5 explicit_OrderedField_transfer

Let $R : \iota$ be given. Let $zero, one : \iota$ be given. Let $plus, mult : \iota \to \iota \to \iota$ be given. **Notation.** We use $+$ as a right associative infix operator corresponding to applying term $plus$. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term $mult$. Let $leq : \iota \to \iota \to o$ be given. **Notation.** We use $\leq$ as an infix operator corresponding to applying term leq. Let $R' : \iota$ be given. Let $zero', one' : \iota$ be given. Let $plus', mult' : \iota \to \iota \to \iota$ be given. **Notation.** We use $\oplus$ as a right associative infix operator corresponding to applying term $plus'$. **Notation.** We use $\times$ as a right associative infix operator corresponding to applying term $mult'$. Let $leq' : \iota \to \iota \to o$ be given. Let $f : \iota \to \iota$ be given.

`explicit_OrderedField_transfer`

Theorem 27.10

$$\mathsf{explicit\_OrderedField}\ R\ zero\ one\ plus\ mult\ leq \to \mathsf{bij}\ R\ R'\ f \to f\ zero = zero' \to f\ one = one' \to (\forall xy \in R.f\ (x + y) = f\ x \oplus f\ y) \to (\forall xy \in R.f\ (x * y) = f\ x \times f\ y) \to (\forall xy \in R.x \leq y \Leftrightarrow leq'\ (f\ x)\ (f\ y)) \to \mathsf{explicit\_OrderedField}\ R'\ zero'\ one'\ plus'\ mult'\ leq'.$$

The proposition is identified by the following information:

Pure Prop Id:	*d8a397eb2d41fc3df32161a85d5f88f8e1a1f828c3996623964e0cef70186f61*
Pure Prop Address:	*TMPpLE4XGZTmn26HRXT5ka3vz3Konq3gJec*
Theory Prop Id:	*5e1306d3daaaa9e52a4c86d25354e58525141369b79522baa7699df2d0fe77ea*
Theory Prop Address:	*TMau3sL5KTvBtt4AUk19sXEx1vw2u1rpj2V*

27.6 Selectors for Fields

Definition 27.2 *We define* Field_carrier *to be* $\lambda Fs.Fs\ 0$ *of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*f0bb69e74123475b6ecce64430b539b64548b0b148267ea6c512e65545a391a1*
Pure Object Address:	*TMY8sbcPiVs9VL311KPHpbm67U6sLM9JvF7*
Theory Object Id:	*9211e41eaf81bb8c152618cbe35230be60d736fcd5684c0b7bdc5ccc1ff70181*
Theory Object Address:	*TMZQE6tPrFvJMaBcLrcMn85ZgscSeFbm4pT*

Definition 27.3 *We define* Field_plus *to be* $\lambda Fs.\mathsf{decode\_b}\ (Fs\ 1)$ *of type* $\iota \to \iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*6968cd9ba47369f21e6e4d576648c6317ed74b303c8a2ac9e3d5e8bc91a68d9d*
Pure Object Address:	*TMYTM1QQGqgVyRM9unR8CLL2scwRHYvV3c3*
Theory Object Id:	*ed06aa34b4112f8507370b46c1c5c7de7f2f9af4f24b831d45893ad3066e994e*
Theory Object Address:	*TMRiY7KU6BpU28KD19iwgBDLsQ5XyfpcbYL*

Definition 27.4 *We define* Field_mult *to be* $\lambda Fs.$decode_b $(Fs\ 2)$ *of type* $\iota \to \iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*4b4b94d59128dfa2798a2e72c886a5edf4dbd4d9b9693c1c6dea5a401c53aec4*
Pure Object Address:	*TMQSTL5b71ddbjnkjnGXfRZ61uXWmMGwJHD*
Theory Object Id:	*7eb39ed695c795e4451a9252d011d377d0f4bdf0bfe6193f7a31ceb58f687a40*
Theory Object Address:	*TMaJmzoi8UnDNTAXrAvZxYbTeEdKV3viygg*

Definition 27.5 *We define* Field_zero *to be* $\lambda Fs.Fs\ 3$ *of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*61f7303354950fd70e647b19c486634dc22d7df65a28d1d6893007701d7b3df4*
Pure Object Address:	*TMJzMmYjL1HdXDWMSRf7c775tmGDc6osaRL*
Theory Object Id:	*b74fb66a398c6c647d135ea9283353963f66d301d2c1ee126b203679f2071094*
Theory Object Address:	*TMHQo21kvxiD8yab3ek4ATBz2PBfwvqsmaJ*

Definition 27.6 *We define* Field_one *to be* $\lambda Fs.Fs\ 4$ *of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*57e734ae32adddc1f13e75e7bab22241e5d2c12955ae233ee90f53818028312a*
Pure Object Address:	*TMV8UDBZRqv4jocV6zt17aP89PcYugzQkoU*
Theory Object Id:	*b2fa78f7f58b95f4bc4f46ac268c3a959090897e56338c64bd84a73bc5fc2551*
Theory Object Address:	*TMWFb9uZk2JEesS9quTe9a6Ww4ND9VcbBJA*

Definition 27.7 Field_minus *is the opaque object of type* $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*9072a08b056e30edab702a9b7c29162af6170c517da489a9b3eab1a982fdb325*
Pure Object Address:	*TMSqYJKBR2z1AqDFGucgVbSMyDGaQ2vHXdt*
Theory Object Id:	*15e172a9ace52b84fbc8c26210b492b9383ddf56708a72b2b40566b8848bddcf*
Theory Object Address:	*TMN39qFAba46WDLPG5mn6arc4RJWRpLYRxY*

27.7 Field

Let $Fs : \iota$ be given. Assume the following.

$$\mathsf{Field}\ Fs \tag{27.3}$$

Let $F : \iota$ be Field_carrier Fs. Let $zero : \iota$ be Field_zero Fs. Let $one : \iota$ be Field_one Fs. **Notation.** We use $+$ as a right associative infix operator corresponding to applying term Field_plus Fs. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term Field_mult Fs. **Notation.** We use $--$ as a prefix operator corresponding to applying term Field_minus Fs.

`Field_eta`

Theorem 27.11

$$Fs = \mathsf{pack\_b\_b\_e\_e}\ F\ (\mathsf{Field\_plus}\ Fs)\ (\mathsf{Field\_mult}\ Fs)\ zero\ one.$$

The proposition is identified by the following information:

Pure Prop Id:	*d8b340ddeffe429e1fbbc93f3ec159869e95202bb0548fa01ab79999624db595*
Pure Prop Address:	*TMcWSufBdeKRFXPYdLCveXwHxkWQ1G8Ku3s*
Theory Prop Id:	*9308bd595b390a905ea8a71bb9251cf9b4ee7546ed5158a721de4d6eec1991fd*
Theory Prop Address:	*TMHvjr39A4G916GqBKMAajZKdrtBFnoqgcY*

`Field_explicit_Field`

Theorem 27.12

$$\mathsf{explicit\_Field}\ F\ zero\ one\ (\mathsf{Field\_plus}\ Fs)\ (\mathsf{Field\_mult}\ Fs).$$

The proposition is identified by the following information:

Pure Prop Id:	*63dbf45fdb4fede703bd1ce1ee23508eef7fbbd8640d566b8ed583677d390af1*
Pure Prop Address:	*TMYH7ErtGiAdfW9CrDDJ623kNniytKGVcjx*
Theory Prop Id:	*cb57dbd436ae765672438d8ff24477520b196a981db8feb13f2698511c06c6f2*
Theory Prop Address:	*TMLZFScv2FGGQMRvkH7kAbY42toRcVj4bqQ*

`Field_zero_In`

Theorem 27.13 $zero \in F$. *The proposition is identified by the following information:*

Pure Prop Id:	*5d2a12ae03ea09b1b00364de5be96f9417eba1807387eef0593c4686271f8b43*
Pure Prop Address:	*TMV5NCa6Wx7gnzRhWo5AeVT8cTwstJid2DP*
Theory Prop Id:	*124d417e36ffc4ecd962d65565094a8b6a908ebccf5fab90ad3c52962d4c8899*
Theory Prop Address:	*TMJTmqQWrFwKLaFx3qyrzQjDtPmc6sNEAEL*

`Field_one_In`

Theorem 27.14 $one \in F$. *The proposition is identified by the following information:*

Pure Prop Id:	*44022c6821b37c9bd64d3412ecf0090eea66e80eebb2def47c0ea8452f269ff2*
Pure Prop Address:	*TMNtPhTBrbXJwXqr9MRjQjBCeakFLxdm4rA*
Theory Prop Id:	*54c548fa567255c0074c7815ff32044eca656b312bcbb87e75ab72ac4e037c4c*
Theory Prop Address:	*TMaxye1k4K8yC8XLPLqCw4nFNFVXQMfGNmy*

`Field_plus_clos`

Theorem 27.15 $\forall xy \in F.x + y \in F$. *The proposition is identified by the following information:*

Pure Prop Id:	*6cb9ab847c2ae424b9e4a853861f83b1519ca940a43ca706c3f58543aa68e6f8*
Pure Prop Address:	*TMSL8rTfsH4EGhTbnt4mmV2f3XC1kRDSuhc*
Theory Prop Id:	*1404ed2bc6fc9cb6d24916d53178afcbbae9052d74dbddd08abb219ef3cc6988*
Theory Prop Address:	*TMU5Koik3TBZQ4TJ7EcUGYpqbJxJUvF15eS*

`Field_mult_clos`

Theorem 27.16 $\forall xy \in F.x * y \in F$. *The proposition is identified by the following information:*

Pure Prop Id:	*90491b3584ca5d345c64ad706ac87ca12bdae4cf2f13a8b47ed70d427919cb6a*
Pure Prop Address:	*TMXeFVMheHpzd5M1At9SBCuvJfvr5e1F4vX*
Theory Prop Id:	*865f20b7a4b6f98b4c54586f84b21600d7e07243207713b57fa40e2ec5eda5ff*
Theory Prop Address:	*TMLt6bowMxotb2XT7CkV7ag3q8KNznCkvSg*

`Field_plus_assoc`

Theorem 27.17 $\forall xyz \in F.x + (y + z) = (x + y) + z$. *The proposition is identified by the following information:*

Pure Prop Id:	*49e71c4d43cf0fdaf6727db033c4e9e44f74cc50c7f11e8f287078c9de4186d6*
Pure Prop Address:	*TMctJ6QSciEPYRwyuAGW91t49FWX5iwGskB*
Theory Prop Id:	*5dac6c802dc58a4009410054756eb7533311da03f206d571ca13b54b7de62f48*
Theory Prop Address:	*TMF9ZdDcMZBL4YbPNXREiZ4z6Ej4bQZGiAt*

`Field_plus_com`

Theorem 27.18 $\forall xy \in F.x + y = y + x$. *The proposition is identified by the following information:*

Pure Prop Id:	*3b445936b40abf6f0f573b3695af272cd8f98957cd9977a5b0ca3f6dadd52baf*
Pure Prop Address:	*TMcSdkyySZ2JMLtoyeiiuVnq14co8TxbPgp*
Theory Prop Id:	*27a761d026aec44ac48a86b672c56e46b77dfbf7622acf1151a03c5a486af018*
Theory Prop Address:	*TMM9QF6dQ6HnJnFroaJHaH635ak1Jn8Jx1o*

`Field_zero_L`

Theorem 27.19 $\forall x \in F.zero + x = x$. *The proposition is identified by the following information:*

Pure Prop Id:	*e0daa0e57cb9935241117668f353fd724bcb61730885 71de6e8928a74ded74c9*
Pure Prop Address:	*TMTonQi48JamuRgckkERNUD5uLVijoYmgL3*
Theory Prop Id:	*37d0579521bcd82e296de371d607f6dbe718d874c53486483fbe1d86059ae1e2*
Theory Prop Address:	*TMUpYjiRJxYVxfan4X4iVYUVm3AfpaMR9Tf*

`Field_plus_inv`

Theorem 27.20 $\forall x \in F.\exists y \in F.x + y = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*919ce52a6adadf0cc8094ecbe60076ee5858155672dd27ab2180e7e6c7bbe1f6*
Pure Prop Address:	*TMN39v7z6H2CoUPdUhfYEYjvy3Zw4eENYPq*
Theory Prop Id:	*262504b07a3f4e1940e4234afa8b6cb0e82cce09ab141e4e0d8b9675a6e62edd*
Theory Prop Address:	*TMJdMzeW7T3rZffNfiYy8hAcNdvCNvWPow9*

`Field_mult_assoc`

Theorem 27.21 $\forall xyz \in F.x * (y * z) = (x * y) * z$. *The proposition is identified by the following information:*

Pure Prop Id:	*08143128b200fbed930f0fff6e90f4cb05098f19ae900fd1f1cb0beb563e2c09*
Pure Prop Address:	*TMc9dXbETfWeBa2b8YnDKbhLw7uaVyQboHe*
Theory Prop Id:	*bad75ea2350024fa534d6b728d0eba5cf28e28fbe493d52077a0cc7bedc82c9d*
Theory Prop Address:	*TMLXQH9iTPeEeNTAWWLcpo5cJTA4AgEWJuq*

`Field_mult_com`

Theorem 27.22 $\forall xy \in F.x * y = y * x$. *The proposition is identified by the following information:*

Pure Prop Id:	*b067a98764ad614c2eab7437092942718dbd9dd9a388f583ce6d32e68bd7dd64*
Pure Prop Address:	*TMPegBnUirvhxDN9mWgopPcPibU15GBD2aT*
Theory Prop Id:	*95c74fc682d34ffea07fca550e5dfd7573581c19dd74de76bca295caa9119383*
Theory Prop Address:	*TMSy5iWokESs8xWZnGcamGMUMENTfP5k3Jh*

`Field_one_neq_zero`

Theorem 27.23 $one \neq zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*0cd6b9f7a2a8864851fc03b62eca322da80ca6a538426cf27bba6634e3656726*
Pure Prop Address:	*TMbSs4K7qkmEkJCui8RQ5ucU6Bxp9AhXSiP*
Theory Prop Id:	*266cd947601e72d416352cfdc17094f13320ebf353611367683f3cf2c946d097*
Theory Prop Address:	*TMUvPmKRiKPF1BAM9e9AAYevwEYsUoNceXf*

`Field_one_L`

Theorem 27.24 $\forall x \in F.one * x = x$. *The proposition is identified by the following information:*

Pure Prop Id:	*f43915f1f2955af2eb66b15dad875d94d73d7b7b9834e7c9adb0f55df5881fe9*
Pure Prop Address:	*TMXMDwpgqy9TopEv1E4DfdDaXfqz9tA6hLH*
Theory Prop Id:	*013427804b41d47918f24b864b24aab70dfc31113ba98edcd336123495101 78b*
Theory Prop Address:	*TMZ8XZoWUvugHdtZXDcG1tmeGAV5J4gRkxN*

`Field_mult_inv_L`

Theorem 27.25 $\forall x \in F.x \neq zero \to \exists y \in F.x * y = one$. *The proposition is identified by the following information:*

Pure Prop Id:	*159c70606656e548df27dc9f2989d005a799bc59dfc48530c9aaef4d88bfe55a*
Pure Prop Address:	*TMPGyXrYchHMpQoxBotw4fTsy9qkcod1cXb*
Theory Prop Id:	*99224d179a2cb454d745622ccfd66d3d60660e4bde41c61da98aa4dd3e938ee4*
Theory Prop Address:	*TMH8ZcNMHvhfoVar9hvT3VD2o3DW9jyeJdr*

`Field_distr_L`

Theorem 27.26 $\forall xyz \in F.x * (y + z) = x * y + x * z$. *The proposition is identified by the following information:*

Pure Prop Id:	*250384aaa9ecf10fbcf4ebefa3dc489b41d3cb10f656500151d49a9053cfe4f9*
Pure Prop Address:	*TMZiuCgz7mgpvcfSTTHjcdzaaeZFaZEUrv8*
Theory Prop Id:	*0ebf99ce21204a4f57fb5783b2803ac92906cc63624dd2cc67cc70a1353aeee0*
Theory Prop Address:	*TMUjioKdcupXefDDTsQSWE2Vw19Nna8245U*

Definition 27.8 Field_div *is the opaque object of type* $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*33f36e749d7a3683affaed574c634802fe501ef213c5ca5e7c8dc0153464ea3e*
Pure Object Address:	*TMR6i3AfWJEnAVs3vStqGHLoyVAtt7HuhVZ*
Theory Object Id:	*3a617adfbc7761f4bff8c1a67cc5a2811928c372828b963f2809216d145f0056*
Theory Object Address:	*TMKD8yCQcW3giMss49AdhftfjbmKvDhxqNd*

Notation. We use $\div$ as an infix operator corresponding to applying term Field_div.

`Field_div_prop`

Theorem 27.27 $\forall x \in F.\forall y \in F \setminus \{zero\}.x \div y \in F \wedge x = y * (x \div y)$. *The proposition is identified by the following information:*

Pure Prop Id:	*320029a8925ea4f20053afa631f90353310f8bd2a02fc1312664ae3d5e617bab*
Pure Prop Address:	*TMLCRGM73NJSReYA8S8dFeUvsVHD5BezMmw*
Theory Prop Id:	*11f8a74d8832813bb0bf2e47adbaecaec90b4bb4ffc2917b900e65c5fdfcd21a*
Theory Prop Address:	*TMFtSZaRxbLf7cg7s9eC7F24ERy3YpttcjS*

`Field_div_clos`

Theorem 27.28 $\forall x \in F.\forall y \in F \setminus \{zero\}.x \div y \in F$. *The proposition is identified by the following information:*

Pure Prop Id:	*ae15934f0050805acbad83a7ea9b08d39343e7275a2aa0544983f8c3029b68ab*
Pure Prop Address:	*TMLj2rQAk7mQRGGwfxY9ab6r8GZpdh5nryZ*
Theory Prop Id:	*f08d2cba3fd57f474e2e6a9ed26207be75354dfc72cde6a65408f169f301435a*
Theory Prop Address:	*TMLDYbyYNgVKinoxufwLvutrLNRfnk66vDt*

`Field_mult_div`

Theorem 27.29 $\forall x \in F.\forall y \in F \setminus \{zero\}.x = y * (x \div y)$. *The proposition is identified by the following information:*

Pure Prop Id:	*1bda12fe605e6023cf81fa8393926be18b348502472ef7b4c757036a807eff68*
Pure Prop Address:	*TMb6WJncGKrL2byhXB2YuUpprymEHFctWHV*
Theory Prop Id:	*c8ef92ac56a58978069a954b8be3acacbe73d63049ec632444d403ecbafb062b*
Theory Prop Address:	*TMEsHD4dd1bPW6M8yUdXaW4UYoSXZQW37fk*

`Field_div_undef1`

Theorem 27.30 $\forall xy.x \notin F \to x \div y = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*20e2690180e6250c71da49027a7a363d806e4fdad713709dbf99337aaa83d216*
Pure Prop Address:	*TMWhJ43K252biCbrmZvzzeVwqqpjcQVZBxU*
Theory Prop Id:	*b5ccab25818eb5893b4528f4adc070775587f1a1b252b703a355452ef3ec25ef*
Theory Prop Address:	*TMS9ggt9Tn8AYLjGKxejwDFj1op6j9f7TWs*

`Field_div_undef2`

Theorem 27.31 $\forall xy.y \notin F \to x \div y = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*08ea037ff82421470289f5932634b1ce136fcb6d1f8706fa6883561e9bff8d34*
Pure Prop Address:	*TMUWRvAmiES3iybNvXn7sUoxZwKmagyQ8rC*
Theory Prop Id:	*6c03f1015d94454404a03499287892345ea00012a20efb26d3b80fcf490734cd*
Theory Prop Address:	*TMLn5yRrKaaMpTjWyKDfNinNPu78LWX738s*

`Field_div_undef3`

Theorem 27.32 $\forall x.x \div zero = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*5c48b82d4fc362fe344e205c36ca6878bd5138f2eb8555efdd2826a31ede70e2*
Pure Prop Address:	*TMcjdraN7C77qSyG5w9GJM3oWt4kL3HUmCG*
Theory Prop Id:	*24e90d5e4e5f1d4012652a7c973b6fdadb4b7a94e33c52e4c0945440a41f612e*
Theory Prop Address:	*TMUNnuEej7hLE5MrRKfW9mQkKWFqHyvwbkp*

Notation. We use superscripts as notation corresponding to applying term CRing_with_id_omega_exp Fs.

`Field_omega_exp_0`

Theorem 27.33 $\forall x.x^0 = one$. *The proposition is identified by the following information:*

Pure Prop Id:	*4362c28e2db5e3383addcc627e5b9fc5c9d8d6e0b00cb0d2216109cbb0b3cfb0*
Pure Prop Address:	*TMNwr244Poo2trbPpcVQmtxpagskYic6goM*
Theory Prop Id:	*4527b98489253d272cf17c4b7c4e4690a8ebdb756ee228c39e35e2396e350909*
Theory Prop Address:	*TMKmcrh5e3aKCAfTomskajMoU8zcUuiFdRz*

`Field_omega_exp_S`

Theorem 27.34 $\forall x.\forall n \in \mathsf{omega}.x^{(\mathsf{ordsucc}\ n)} = x * x^n$. *The proposition is identified by the following information:*

Pure Prop Id:	*d2e5103c06f38d501b5624ac09de58fe1199ed0428e0099e2cecd857b56c9463*
Pure Prop Address:	*TMRxJHnciqiQFbYupzsqKnpymvXkgoFKbz2*
Theory Prop Id:	*414637f793aeaef595d4415aea905c8712dd128bdf3888d6a1b8b79c783f993f*
Theory Prop Address:	*TMXUtYW7Vcrt9RLnqAwfxxVBZXyfpj7hTyU*

`Field_omega_exp_1`

Theorem 27.35 $\forall x \in F.x^1 = x$. *The proposition is identified by the following information:*

Pure Prop Id:	*351bec92d644d3d3f57736c30184720a55a3206dd32c5c5de668a1a15d21cf70*
Pure Prop Address:	*TMKtTCkPJ5sHx3gK7y8wuSN5Rfkt5hunA6A*
Theory Prop Id:	*97fa394baba04b04edef47f65fc4ec793503e5777b288b978d77b4c9bb64fe45*
Theory Prop Address:	*TMTi7WjdmiDN2vrsRDu17s69nAUYnNDuowU*

`Field_omega_exp_clos`

Theorem 27.36 $\forall x \in F.\forall n \in \mathsf{omega}.x^n \in F$. *The proposition is identified by the following information:*

Pure Prop Id:	*026ac1d1f5ef29832893f6c7944103acad2da742c648c31b129e3d067c5334f3*
Pure Prop Address:	*TMSeptsHCdwBLhRoQhW2RzWACc5328SueZM*
Theory Prop Id:	*8e8df3c69edbfdbdf46653edec4be7381ff02d8b6f0c54facfc9849a6b7ad1fb*
Theory Prop Address:	*TMJ98yveoAeohCchdDEX5eSGLgbMb5NtmQ4*

`Field_eval_poly_clos`

Theorem 27.37

$$\forall n \in \mathsf{omega}.\forall cs \in F^n.\forall x \in F.\mathsf{CRing\_with\_id\_eval\_poly}\ Fs\ n\ cs\ x \in F.$$

The proposition is identified by the following information:

Pure Prop Id:	*339c6079b932e105f62ee9a139090ec0c35d05a91b77324b7c9d00829f5e157a*
Pure Prop Address:	*TMUHU4Kdwck1DovJNNC1BznGxQ9LfqrpcD2*
Theory Prop Id:	*e3e5a122a52cb88471f32d0108c8ebfd779225b3f222aaa00e6b00105eb91521*
Theory Prop Address:	*TMWrCNHLicENtgPnEmebmghuFpCDVv9acP4*

`Field_plus_cancelL`

Theorem 27.38 $\forall xyz \in F.x + y = x + z \to y = z$. *The proposition is identified by the following information:*

Pure Prop Id:	*443b41e62519d1cf11ce79b28ce6902b1cb036b303feb0522d8233e77510b0da*
Pure Prop Address:	*TMQYEe7aS7C6puTrXwtgHP1HzLv6iA9FCYV*
Theory Prop Id:	*65814d60a21e4514adfe8122a71e77983e1c014e2364364e2b428a8e889110d1*
Theory Prop Address:	*TMPeQdMTjszQk8ZwjKbe2Yx9p1bhEuku2th*

`Field_plus_cancelR`

Theorem 27.39 $\forall xyz \in F.x + z = y + z \to x = y$. *The proposition is identified by the following information:*

Pure Prop Id:	*f16e614fd8bcca5468eff3f77a1ffdbdf61ce6894a9642616315fd8217fa6896*
Pure Prop Address:	*TMHxcrqScEQVs2BM9LqdwdJ73QaGHTHjtAw*
Theory Prop Id:	*5f414f4ccd4d5d42eb071a720d4cfd7f8e2a864a6d3ffecc438de4a083e92c4d*
Theory Prop Address:	*TMMUfLR9eAhuBx9dH38X5X7iGs36sxjomLw*

`Field_minus_eq`

Theorem 27.40

$$\forall x \in F.$$
$$-x = \mathsf{explicit\_Field\_minus}\ F\ zero\ one\ (\mathsf{Field\_plus}\ Fs)\ (\mathsf{Field\_mult}\ Fs)\ x.$$

The proposition is identified by the following information:

Pure Prop Id:	*de7b3a103710cd73b730d9ab58f9ad4f2d88bd767fec9ca2edba308880936b16*
Pure Prop Address:	*TMVexy4tBFivo588m7GQ84C5mQBJ4yZx98Z*
Theory Prop Id:	*f6684a8b3c3ff9441214e19ef1db049db0227f25e3db1d329cfd06250e47f513*
Theory Prop Address:	*TMGtBPcXh4HJHkfMAC1yckyY5RPTB4NACYW*

`Field_minus_undef`

Theorem 27.41 $\forall x.x \notin F \to -x = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*357d777053f15331bc00d78b8e12f89dc8130cc8c44f2c6131dcc55defa8ee0f*
Pure Prop Address:	*TMTwnPHTSFHEiMGLciYU9XDmEYP1U9Z2eZN*
Theory Prop Id:	*60ed459a7799cd3e857363ae83693d0f4826d792577f17db36e25c07793bc296*
Theory Prop Address:	*TMW6cjVsvwBeiH3JX9mV7y27yXLNaXoLkjr*

`Field_minus_clos`

Theorem 27.42 $\forall x \in F.-x \in F$. *The proposition is identified by the following information:*

Pure Prop Id:	*f1becfbfee0ef8b26de040c7f18230ec7527ddcf884ada3c8941b9b5c6de9b87*
Pure Prop Address:	*TMbwyWNKrfuYFG1TmqXNEzZJ4rHSH3Yvzwv*
Theory Prop Id:	*d590d7bb3e9f5cb4915a63f72350aaa482789cd650f74ceaa143f1b01be467b9*
Theory Prop Address:	*TMKy1fXBQm9Ci94udEjidssK8Da4zqwq6BX*

`Field_minus_R`

Theorem 27.43 $\forall x \in F.x + -x = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*5cb54808b5cc0bfe1a9faf501b5e357f53bff36ec590e8f602fed76ba642d4b3*
Pure Prop Address:	*TMR6vs3rpH6qDTpNibPSbTftrd3h5nzUygq*
Theory Prop Id:	*2391ef531b49ac2b610cb94e6726c81909131c9e4acf4d3aa6ee03f5bdeea688*
Theory Prop Address:	*TMMcagzy7PNZ6o6f6y6gxJVCHKFabzvbY1n*

`Field_minus_L`

Theorem 27.44 $\forall x \in F. -x + x = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*7c66759f9e4abb07e2159e5e1557c1602f8cb0f7f38d7aeae89ee98bbc859ebe*
Pure Prop Address:	*TMHN25GQoh4jYakdvcZYx96QdJXH514To2u*
Theory Prop Id:	*b5cf76ad592ab4b5a7384338d4297cf97c6981dd0a45ac579115230625384cc5*
Theory Prop Address:	*TMXuvPZ1qizoaTqpK2wbJgJGEjrtducR28f*

`Field_minus_invol`

Theorem 27.45 $\forall x \in F. --x = x$. *The proposition is identified by the following information:*

Pure Prop Id:	*e097d2ad2fa0d0d1bf0193b8e412f65b42e3e5c73733c262d1b7eeb1cbd57883*
Pure Prop Address:	*TMMMeGBCnj2s3dzeT7Yfa9bpUfU46psmpSe*
Theory Prop Id:	*7f6c7d184b0f20e8f912398c740d072ae44f16dc17d5358d415b7f2a5ccdece3*
Theory Prop Address:	*TMTzNG5pZe3QRegKNVBSsZp3PqJjsutAt1f*

`Field_minus_one_In`

Theorem 27.46 $-one \in F$. *The proposition is identified by the following information:*

Pure Prop Id:	*fbd93d93f1671451fec2d592c753cd960b2e1288c4241e1fad9a57ee7a022c02*
Pure Prop Address:	*TMHWKgvtYjVJyEYbaQY7sTTe4usgtgVj9LS*
Theory Prop Id:	*61a82a30b97b7f4fb97d4f6625d6e919a0010537baceadb9f9b571c21d50a9f8*
Theory Prop Address:	*TMSwuqi7dSFVmvwt5crCA1gi7nDJn439QVp*

`Field_zero_multR`

Theorem 27.47 $\forall x \in F. x * zero = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*45513e8074cff3c93f264cf92510bcaed6b453af70ec391191356622b3682292*
Pure Prop Address:	*TMPYu6VMYJksB1pVtcZhxA1ZtBtBiyAQA3q*
Theory Prop Id:	*fadc07fc14b79ffe08eb28c7b3dd0a9607e542c43123389eacca8922d9a3bbd6*
Theory Prop Address:	*TMWgQLa4SJqx34swEHxJoxG1BrHPgR8ESyy*

`Field_zero_multL`

Theorem 27.48 $\forall x \in F. zero * x = zero$. *The proposition is identified by the following information:*

Pure Prop Id: *5ee5eb5fde0181d92b2c7f6d7a2a896cd74e3a750cdabbb27d94b1bb4aab908c*
Pure Prop Address: *TMLc5uhw7hxR8sbQGdBpQkCtjeGhbq6hy1L*
Theory Prop Id: *3e7854299ba2aa2daea10889a8309962f1b1d6c32b7006fc1c40500925221173*
Theory Prop Address: *TMRcY5cpYprw5iQzhQcoGB6ndppCPbrhUtT*

`Field_minus_mult`

Theorem 27.49 $\forall x \in F. -x = (-one) * x$. *The proposition is identified by the following information:*

Pure Prop Id: *7cd676e4adffa30d0cf22cf26fcde8bfd203a61ee59f416b12fa62cfd93db1b7*
Pure Prop Address: *TMHF8RJu8ok4xK5Js75hyF7aTwTeYUktaWC*
Theory Prop Id: *775b246ad68e751222cbcb9a5e49e184ed3019fcce6a74f91edd30125e3f0098*
Theory Prop Address: *TMQzhW2ohogP8qPJxruSSb4HgNFe1kHyJaE*

`Field_minus_one_square`

Theorem 27.50 $(-one) * (-one) = one$. *The proposition is identified by the following information:*

Pure Prop Id: *f1d40df6baeeb2bbfbb7f411d9caeb48477cc6718458d7e70ca7bd1b7a78d1cc*
Pure Prop Address: *TMPY9Fe2gL8ytA1WVeKrmqRrbe6bXkndVWA*
Theory Prop Id: *4f370c1adbdb484572d2f537b4a62640f4e6192dcbbfc4d7dee46fb09f3433e6*
Theory Prop Address: *TMYd1s86jR7deZ3QCmxu2FzfXcVu8tMBNyU*

`Field_minus_square`

Theorem 27.51 $\forall x \in F.(-x) * (-x) = x * x$. *The proposition is identified by the following information:*

Pure Prop Id: *a55d0e9b98e66f6d5322b6523e3e708815bb201d618aa58f328931f5f07d0f37*
Pure Prop Address: *TMN6DCGTRZLbdyYaDBJ9hD7em3K6sLzKydd*
Theory Prop Id: *726593b03eefe34d9157a17b6e679b85bcf1865e151930e64fe462ee18a8a6a9*
Theory Prop Address: *TMNpHhjMNCtccTVy7oxGE8LEDQS3ikFDxRL*

`Field_minus_zero`

Theorem 27.52 $-zero = zero$. *The proposition is identified by the following information:*

Pure Prop Id: *f5214a9560f5568380d8be3826be07444fe57c851dd66aa09a5764efb8077718*
Pure Prop Address: *TMNcsvycuD4vkz4pfqQk1YGD8DMTSNJU4AH*
Theory Prop Id: *25e6829a5bf06f2eaecafec005fea75bb87bdbfe0e161ec9b16912fa023dc76d*
Theory Prop Address: *TMXQJxi91EWkUHXgdLamfSNazD2x5oL6iHv*

`Field_dist_R`

Theorem 27.53 $\forall xyz \in F.(x + y) * z = x * z + y * z$. *The proposition is identified by the following information:*

Pure Prop Id:	*deb7136831fde91f8aaab03eb0c31cdcadb1e244807de445094801e21f93cc47*
Pure Prop Address:	*TMdNLPJWkse1Z9urNENt7q5Qeyf5LwWd49a*
Theory Prop Id:	*10fe6a61397ce15f1f074447b70bfba8150ab8b2d77f7129a3ca0d5dda084ce2*
Theory Prop Address:	*TMNDDzYVTXFEChN6smH4YbceCPCtL4pBEz9*

`Field_minus_plus_dist`

Theorem 27.54 $\forall xy \in F.-(x+y) = -x + -y$. *The proposition is identified by the following information:*

Pure Prop Id:	*7d70513a02d66f0f56d24e5c8cab215e006349e8a3dbcd5fd887d9d1dcc56fe1*
Pure Prop Address:	*TMKE1EAuG9tUBJEqHNho1ojpdVMUyyVrRkp*
Theory Prop Id:	*cf09f8db6ef5dbb5d2ed9a9695a44c7323061de997ed2f25e7785eb1d93e4486*
Theory Prop Address:	*TMRjUdfwmx5YiiXEHvkgNMeZ615jeu3ixtf*

`Field_minus_mult_L`

Theorem 27.55 $\forall xy \in F.(-x) * y = -(x * y)$. *The proposition is identified by the following information:*

Pure Prop Id:	*58520e0833b7099a6940ba9408893dd9df17b42fbcadaeb89d0b3be3b1d4b459*
Pure Prop Address:	*TMF5FPzDp822hueufQ3yiZiRPZucSMkbB1N*
Theory Prop Id:	*09040b49c949f9afb231e21ee54bb76add27e0f1944d83e5dfdea037fb2b5e78*
Theory Prop Address:	*TMHvcKKRtR69YcV7eYTB3batvpaofCBGg4b*

`Field_minus_mult_R`

Theorem 27.56 $\forall xy \in F.x * (-y) = -(x * y)$. *The proposition is identified by the following information:*

Pure Prop Id:	*0f87ffe9baf0ac37844ca25d819bbf2e6b56af4a7acba178604fef6b2c73975b*
Pure Prop Address:	*TMNFhiGH735rxLAYHzqXjLVSVhiSUKYVJFj*
Theory Prop Id:	*d3e4735bbc06ae8a25cb40640c44781c9e83add0c02c7c02058ad54bddb5536e*
Theory Prop Address:	*TMbXkVYiquoTVr7qFbrQXZCThaRMHyD72Na*

`Field_square_zero_inv`

Theorem 27.57 $\forall x \in F.x * x = zero \to x = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*2b2bd6de108abc8d3794de6a31f5341040d00d973de79329c574098caed59e31*
Pure Prop Address:	*TMZKCqVzNkAqM9J9D53Y9aBiDZvqkPQkdVB*
Theory Prop Id:	*08c2f8793e760cef2edbc292e690c746158bd7259df3295852d68c7214236e65*
Theory Prop Address:	*TMUCadVDmPtHtGW9E9rogNfiQCpPtyuh4MU*

`Field_mult_zero_inv`

Theorem 27.58 $\forall xy \in F.x * y = zero \to x = zero \lor y = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*e191e1698c04a863c4f10266ada28dbd2db7490864c6c1c80f7dc9b6049a62ab*
Pure Prop Address:	*TMY8AV6XaQZR4ezqWu2BT6WM441CDJBthsX*
Theory Prop Id:	*fbeb3acafb828d177072b74d6982e4e3a3df773c746d4ea7168c3a03b93613bd*
Theory Prop Address:	*TMYaR1LATsRePnNzvKfW1CMi8VJXyAhwf3e*

27.8 Field2

Let $Fs : \iota$ be given. Let $Fs' : \iota$ be given. Let $F : \iota$ be Field_carrier Fs. Let $zero : \iota$ be Field_zero Fs. Let $one : \iota$ be Field_one Fs. **Notation.** We use $+$ as a right associative infix operator corresponding to applying term Field_plus Fs. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term Field_mult Fs. Let $F' : \iota$ be Field_carrier Fs'. Let $zero' : \iota$ be Field_zero Fs'. Let $one' : \iota$ be Field_one Fs'. **Notation.** We use $\oplus$ as a right associative infix operator corresponding to applying term Field_plus Fs'. **Notation.** We use $\times$ as a right associative infix operator corresponding to applying term Field_mult Fs'. **Notation.** We use $--$ as a prefix operator corresponding to applying term Field_minus Fs. **Notation.** We use $--$ as a prefix operator corresponding to applying term Field_minus Fs'.

Definition 27.9 subfield *is the opaque object of type o identified by the following information:*

Pure Object Id: *9216abdc1fcc466f5af2b17824d279c6b333449b4f283df271151525ba7c9aca*
Pure Object Address: *TMJhV7ybK4sNoy7dLu16XHkQMMh2u7ogDg9*
Theory Object Id: *8b767b8278051b976eaa45775446a5c3d3fb3b6c01e0d6652741dd499114ae43*
Theory Object Address: *TMGfnnZhzcvAyc6uD3itbd4JQrSmEALeWf1*

`subfield_I`

Theorem 27.59

$$\text{Field } Fs \rightarrow \text{Field } Fs' \rightarrow F \subseteq F' \rightarrow zero = zero' \rightarrow one = one' \rightarrow (\forall ab \in F. a + b = a \oplus b) \rightarrow (\forall ab \in F. a * b = a \times b) \rightarrow \text{subfield}.$$

The proposition is identified by the following information:

Pure Prop Id: *7e727ad2e5128678d00eab2e3f41718c941d248ab8bc80d9290e8e449722a6cf*
Pure Prop Address: *TMM7f9u7WrEFj8YHR1zoL7g1oxSnnpx4nuA*
Theory Prop Id: *d7fa7daaa1fd31a6790debe2a61072593e062f53926bea2d384140301fd0f3c5*
Theory Prop Address: *TMTAZrzxHvVF3ivGCdEm8NHia2yU7XfKPxD*

`subfield_E`

Theorem 27.60

$$\begin{gathered}\text{subfield} \rightarrow \forall p : o. \\ (\text{Field } Fs \rightarrow \text{Field } Fs' \rightarrow \\ F \subseteq F' \rightarrow zero = zero' \rightarrow one = one' \rightarrow \\ (\forall ab \in F. a + b = a \oplus b) \rightarrow (\forall ab \in F. a * b = a \times b) \rightarrow p) \\ \rightarrow p.\end{gathered}$$

The proposition is identified by the following information:

Pure Prop Id: *d7e94964874d4552f7612152bc7b4fe8077a730735f362aadb7346233bbbc7eb*
Pure Prop Address: *TMTNHkoYxk53iGUuEwSw3EVWoAuX1WUvdw8*
Theory Prop Id: *e75e56a8a51307bd8de2cf55f809fee0d2458291f278ceed9218a3d441845a43*
Theory Prop Address: *TMNqv7RamAPGivDz6Nh19aHNfkFEMuvESP4*

Definition 27.10 Field_Hom *is the opaque object of type* $\iota \to o$ *identified by the following information:*

Pure Object Id: *597c2157fb6463f8c1c7affb6f14328b44b57967ce9dff5ef3c600772504b9de*
Pure Object Address: *TMGp1283AAzgHauWyMFM6fwpTKhT2UvW44v*
Theory Object Id: *30bf3301fb81b8870c663685fc6bfa8593d92c4bb8d8baedadeac04223d7577b*
Theory Object Address: *TML6Zjm746Vd3N4sjVSwnNb4J5yLWD6e5Yj*

`Field_Hom_I`

Theorem 27.61

$$\forall g.\mathsf{Field}\ Fs \to \mathsf{Field}\ Fs' \to g \in F'^{F} \to g\ zero = zero' \to g\ one = one' \to \\ (\forall ab \in F.g\ (a+b) = g\ a \oplus g\ b) \to (\forall ab \in F.g\ (a*b) = g\ a \times g\ b) \to \\ \mathsf{Field\_Hom}\ g.$$

The proposition is identified by the following information:

Pure Prop Id: *31c68d4d1908803e35c198b4917b363aece27f736abc525d3be26b2bbfaf25ed*
Pure Prop Address: *TML1nC9DaTHq5nADe4PBSDz3pRDBKuM4coa*
Theory Prop Id: *12959a75af826745c76c8cacac56b223b08b5dfa4b785d1f4dbe42ad5c2234c4*
Theory Prop Address: *TMFvSuVBfh1siZrHxBxDNdzASV8SPhqhWGz*

`Field_Hom_E`

Theorem 27.62

$$\forall g.\mathsf{Field\_Hom}\ g \to \forall p : o. \\ (\mathsf{Field}\ Fs \to \mathsf{Field}\ Fs' \to g \in F'^{F} \to \\ g\ zero = zero' \to g\ one = one' \to \\ (\forall ab \in F.g\ (a+b) = g\ a \oplus g\ b) \to \\ (\forall ab \in F.g\ (a*b) = g\ a \times g\ b) \to \\ (\forall a \in F.g\ (-a) = -g\ a) \to \\ (\forall a \in F.g\ a = zero' \to a = zero) \to \\ (\forall ab \in F.g\ a = g\ b \to a = b) \to \\ (\forall a \in F.\forall n \in \mathsf{omega}.g\ (\mathsf{CRing\_with\_id\_omega\_exp}\ Fs\ a\ n) \\ = \mathsf{CRing\_with\_id\_omega\_exp}\ Fs'\ (g\ a)\ n) \\ \to p) \\ \to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*0ba6afb6588f84ed92332057634 16bbcb48104160ae81e7c83aaa749df56f30a*
Pure Prop Address:	*TMHyWBfPWgBD8fHLFmj2uHd8LiGwzKaPXbL*
Theory Prop Id:	*4bd91ea8df1144675fc91986c05fe9c6aa39bb2352629ee956d785fb210304d0*
Theory Prop Address:	*TMd7scvhecxt55fo9HQefDSbDkqviHLYMQk*

`Field_Hom_inj`

Theorem 27.63 $\forall g.\mathsf{Field\_Hom}\ g \rightarrow \forall ab \in F.g\ a = g\ b \rightarrow a = b.$ *The proposition is identified by the following information:*

Pure Prop Id:	*68a0d1b5c80c7bc045dfade09ce3a0bf9500817cee9a0014553c03d27df85b25*
Pure Prop Address:	*TMb9HZYE9ntGsPKXkYbtsoVw7iJz9L9VS4R*
Theory Prop Id:	*be594831fd63fc200596d913b2146b4087df4d853a5d72bbbec93372920c5c06*
Theory Prop Address:	*TMPDzSNyQDmjStN75qX6xofhPvz45wh64pJ*

27.9 Basic Results about Fields

`subfield_refl`

Theorem 27.64 $\forall Fs.\mathsf{Field}\ Fs \rightarrow \mathsf{subfield}\ Fs\ Fs.$ *The proposition is identified by the following information:*

Pure Prop Id:	*0eb1f1396b31417610ee278fb6a8b8d8cdd140a8eedf731088c361bd94a2e7f0*
Pure Prop Address:	*TMQXDmjHytbWQAqPJjgqXLMDq8SMfVBrMVy*
Theory Prop Id:	*463bacb9ffd8a7c1e580b20d409cb1f1185318cd2fcd9c728d149096a959c4f5*
Theory Prop Address:	*TMbHGwwWjrPBorTYyXRmvsBmdT2tyD2ziXc*

`subfield_tra`

Theorem 27.65

$$\forall FsFs'Fs''.\mathsf{subfield}\ Fs\ Fs' \rightarrow \mathsf{subfield}\ Fs'\ Fs'' \rightarrow \mathsf{subfield}\ Fs\ Fs''.$$

The proposition is identified by the following information:

Pure Prop Id:	*66345481ffe8c4fd4fbb8f011c765a0d87adab31bcd77023511f6539b528d18e*
Pure Prop Address:	*TMM11DSqtT24FeA4LM8aLCBtypoFCW7Y25E*
Theory Prop Id:	*98a2debbf6e60c86f06666f4d0a11ec1170a4b003c3f6f6ac2a0dbc3da02a741*
Theory Prop Address:	*TMGnVRkmmsKG3BTL5eNCc52GyUUvBzfgjLe*

Definition 27.11 Field_extension_by_1 *is the opaque object of type*

$$\iota \rightarrow \iota \rightarrow \iota \rightarrow o$$

identified by the following information:

Pure Object Id:	*15c0a3060fb3ec8e417c48ab46cf0b959c315076fe1dc011560870c5031255de*
Pure Object Address:	*TMQQtdP8BXq6yrNPjAqW2TErHDrTMXXAnHf*
Theory Object Id:	*937edba4a112338c9bd442b454c3d9930703191cb45b86271c395bc54d538d82*
Theory Object Address:	*TMKZ3FEYqeFEFtGCpBAd1kYajCJ4YnCDYCB*

`Field_extension_by_1_I`

Theorem 27.66

$$\forall Fs Fs' a.\mathsf{subfield}\ Fs\ Fs' \to$$
$$a \in \mathsf{Field\_carrier}\ Fs' \setminus \mathsf{Field\_carrier}\ Fs \to$$
$$(\forall Fs''.\mathsf{subfield}\ Fs\ Fs'' \to a \in \mathsf{Field\_carrier}\ Fs'' \to \mathsf{subfield}\ Fs'\ Fs'')$$
$$\to \mathsf{Field\_extension\_by\_1}\ Fs\ Fs'\ a.$$

The proposition is identified by the following information:

Pure Prop Id:	*b32e90df057fb63c1ad17c2a471f552dc5c42a3b49ce631d182cc17768c8ce16*
Pure Prop Address:	*TMTostbVVVe1Wc1NFks4m8cRXEwnEtFnycJ*
Theory Prop Id:	*d5557b3eb7f49b4ddec41d07f1d23e0b1ce4f8e7657a1c751d122b54af9fa41b*
Theory Prop Address:	*TMPm9E17AXXZrMjJG29R9UHEicjTbN2gajj*

`Field_extension_by_1_E`

Theorem 27.67

$$\forall Fs Fs' a.\mathsf{Field\_extension\_by\_1}\ Fs\ Fs'\ a \to \forall p : o.$$
$$(\mathsf{subfield}\ Fs\ Fs' \to a \in \mathsf{Field\_carrier}\ Fs' \setminus \mathsf{Field\_carrier}\ Fs \to$$
$$(\forall Fs''.\mathsf{subfield}\ Fs\ Fs'' \to a \in \mathsf{Field\_carrier}\ Fs'' \to \mathsf{subfield}\ Fs'\ Fs'')$$
$$\to p)$$
$$\to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*338a5505ed42a91cb5fa5ec6ab0cf4e76ba17cac12e25143d21eb1cd650c46da*
Pure Prop Address:	*TMNZzAKhVJ6yFrUHhoMKiupzbWxqEfPoy4J*
Theory Prop Id:	*beafbae6117b463b1efc7ff06cf528539b2a460a7d5e21e7589a7e1d18fa1642*
Theory Prop Address:	*TMFdxk4XGCyjeLqB2oApjE5fghNi2pqqZLN*

Definition 27.12 radical_field_extension *is the opaque object of type* $\iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*4b1aa61ecf07fd27a8a97a4f5ac5a6df80f2d3ad5f55fc4cc58e6d3e76548591*
Pure Object Address:	*TMchEBbJiTrZFrworejH2faUQ47yR2qjVUh*
Theory Object Id:	*816f6d770c8c6df637703bbfd5f2ee005e7474a4930bf0953ebe06dca050d7cf*
Theory Object Address:	*TMZ6XfW5b4iotXcF9javzfEe6WjNPU9qL1w*

`radical_field_extension_I`

Theorem 27.68

$$\forall Fs Fs'.\forall r \in \text{omega}.\forall Fseq.Fseq\ 0 = Fs \to Fseq\ r = Fs' \to$$
$$(\forall i \in \text{ordsucc}\ r.\text{Field}\ (Fseq\ i)) \to$$
$$(\forall i \in r.\exists a \in \text{Field\_carrier}\ (Fseq\ (\text{ordsucc}\ i)).\exists n \in \text{omega}.$$
$$\text{CRing\_with\_id\_omega\_exp}\ (Fseq\ (\text{ordsucc}\ i))\ a\ n \in \text{Field\_carrier}\ (Fseq\ i) \wedge$$
$$\text{Field\_extension\_by\_1}\ (Fseq\ i)\ (Fseq\ (\text{ordsucc}\ i))\ a)$$
$$\to \text{radical\_field\_extension}\ Fs\ Fs'.$$

The proposition is identified by the following information:

Pure Prop Id:	*4e422a257d6bf13415f526581580edf290299362d88de0af188ed0d16b294fc0*
Pure Prop Address:	*TMaTmGPiv6voz3uCpKzxGcrY1beeHKnr1k9*
Theory Prop Id:	*6bd73bdf40cf6c2ecf949365a2d93d0d579cef5aa7319b45b404b1739d97e812*
Theory Prop Address:	*TMb2ezs3LpjpMQYPEQvK9PUQE6aN6HjXb27*

`radical_field_extension_E`

Theorem 27.69

$$\forall Fs Fs'.\text{radical\_field\_extension}\ Fs\ Fs' \to \forall p : o.$$
$$(\text{Field}\ Fs \to \text{Field}\ Fs' \to \text{subfield}\ Fs\ Fs' \to$$
$$\forall r \in \text{omega}.\forall Fseq.Fseq\ 0 = Fs \to Fseq\ r = Fs' \to$$
$$(\forall i \in \text{ordsucc}\ r.\text{Field}\ (Fseq\ i)) \to$$
$$(\forall i \in \text{ordsucc}\ r.\forall j \in \text{ordsucc}\ i.\text{subfield}\ (Fseq\ j)\ (Fseq\ i)) \to$$
$$(\forall i \in r.\exists a \in \text{Field\_carrier}\ (Fseq\ (\text{ordsucc}\ i)).\exists n \in \text{omega}.$$
$$\text{CRing\_with\_id\_omega\_exp}\ (Fseq\ (\text{ordsucc}\ i))\ a\ n \in \text{Field\_carrier}\ (Fseq\ i) \wedge$$
$$\text{Field\_extension\_by\_1}\ (Fseq\ i)\ (Fseq\ (\text{ordsucc}\ i))\ a)$$
$$\to p)$$
$$\to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*04aa436465678920b287c7c3b096ec8a8ad9f9429cfbf92c410bebfde430692d*
Pure Prop Address:	*TMKr8e5rQhQ1BMfoiUXe73gTjPvh5qaip7V*
Theory Prop Id:	*89a63147cae765b2eabc9cc755f1a4a6c0b4b6c6791569bb6a8e0ed9bd00f0fd*
Theory Prop Address:	*TMYAut45JZbtAUu8e69Mdhf8jq8dSnpBN3M*

Definition 27.13 Field_automorphism_fixing *is the opaque object of type*

$$\iota \to \iota \to \iota \to o$$

identified by the following information:

Pure Object Id:	*55cacef892af061835859ed177e5442518c93eb7ee28697cde3deaec5eafbf01*
Pure Object Address:	*TMN78iPppPwcxR2NUi3dP5EVHayaxwFwfWr*
Theory Object Id:	*8e98c0485c8e7d57eb3687ec81bbf72649aea6f54b04d8a845c0664d3392f858*
Theory Object Address:	*TMFfKJ2CTJMaiuRS28eKxgU3frsxPZBoTum*

`Field_automorphism_fixing_I`

Theorem 27.70

$$\forall K F f.\mathsf{subfield}\ F\ K \rightarrow \mathsf{Field\_Hom}\ K\ K\ f \rightarrow (\forall y \in K\ 0.\exists x \in K\ 0.f\ x = y) \rightarrow (\forall x \in F\ 0.f\ x = x) \rightarrow \mathsf{Field\_automorphism\_fixing}\ K\ F\ f.$$

The proposition is identified by the following information:

Pure Prop Id:	*8f616dfe39b72caefcdc6aabcb73389242f81b1a7364f5444e6672294fc7a13d*
Pure Prop Address:	*TMSoh1Y12vBR6CW3KxCXhcWQZd5A1FZq1cU*
Theory Prop Id:	*b928875c73ac27a404acf23ab1966ddfcc532a64ee21fb30dd9d57314756729b*
Theory Prop Address:	*TMaXJQsntpr39fhBWDxXZwk8s6e8PSpYjTR*

`Field_automorphism_fixing_E`

Theorem 27.71

$$\forall K F f.\mathsf{Field\_automorphism\_fixing}\ K\ F\ f \rightarrow \forall p : o. (\mathsf{subfield}\ F\ K \rightarrow \mathsf{Field\_Hom}\ K\ K\ f \rightarrow (\forall y \in K\ 0.\exists x \in K\ 0.f\ x = y) \rightarrow (\forall x \in F\ 0.f\ x = x) \rightarrow p) \rightarrow p.$$

The proposition is identified by the following information:

Pure Prop Id:	*abbbec85a4745b7e0f7fca7af4f6c54dc3a65291a67132d9d9848d4338a72eef*
Pure Prop Address:	*TMa3L2FNtNGpWEaV5Ctdc4CtUWWGvbY4cRP*
Theory Prop Id:	*b2509c4f43b6dbea462f25fd32e67cd06f5e46262cb519d918cbb958ec386a98*
Theory Prop Address:	*TMHuLZV7jdk3tK6t9sNMSxXdPPK7H6k2VGW*

Definition 27.14 *We define* lam_comp *to be* $\lambda A f g.\lambda x \in A.f\ (g\ x)$ *of type* $\iota \rightarrow \iota \rightarrow \iota \rightarrow \iota$ *identified by the following information:*

Pure Object Id:	*29d9e2fc6403a0149dee771fde6a2efc8a94f848a3566f3ccd60af2065396289*
Pure Object Address:	*TMZBHh5NXVkgufPoqMzvVCpRFLW2WdfYiJJ*
Theory Object Id:	*f47cda05aa5549792c42c687ced0171e637154b32dbda20a3dd2065d950abd6f*
Theory Object Address:	*TMT1tRAwqVEGpkkvkUdLo93JTWkuSxofJxU*

Definition 27.15 *We define* lam_id *to be* $\lambda A.\lambda x \in A.x$ *of type* $\iota \rightarrow \iota$ *identified by the following information:*

Pure Object Id:	*6271864c471837aeded4c4e7dc37b9735f9fc4828a34ac9c7979945da815a3c7*
Pure Object Address:	*TMTjnAw5cKWKUVsHNJJeNN6CgmY8LpuZiYg*
Theory Object Id:	*86ccd74f57ceb4f87e01c0731683396877d457448dc5c342c35066e0a38f076b*
Theory Object Address:	*TMPRHK8Wyojj7r7mPHtDjeU6KLQUQfd4mis*

`lam_comp_exp_In`

Theorem 27.72 $\forall ABC.\forall f \in B^A.\forall g \in C^B.\mathsf{lam\_comp}\ A\ g\ f \in C^A$. *The proposition is identified by the following information:*

Pure Prop Id:	*a368c0a62f8223dcdf075df5f370b455c9b5fc97a3edf8807b478842be43c4f7*
Pure Prop Address:	*TMGg18zcz7ghZbezgKuURXy33FhGmmAWbXm*
Theory Prop Id:	*4fede2b31459edc9b94436567d1a7a2010f2eee7b82f8cd7fbbdb2b6ad196ec4*
Theory Prop Address:	*TMHzPAuFt6vTdQEr4t62NpVuDaya4x1nqZK*

`lam_id_exp_In`

Theorem 27.73 $\forall A.\mathsf{lam\_id}\ A \in A^A$. *The proposition is identified by the following information:*

Pure Prop Id:	*a328f177c58333c407842c44b36358151a17648de6780a0fe94204d0656910eb*
Pure Prop Address:	*TMG2mqqHqLB3YWR1XE3KpoehYhtcgwLGMsh*
Theory Prop Id:	*025740d37f99a1c2a8b9a6afd977237c7629216ef0f1579ec6fa5bc5a0b22968*
Theory Prop Address:	*TMdqvc3zfrpqPDraF5Ttxg4ZWaLTYaoRK8c*

`lam_comp_assoc`

Theorem 27.74

$$\forall AB.\forall f \in B^A.\forall gh.$$
$$\mathsf{lam\_comp}\ A\ h\ (\mathsf{lam\_comp}\ A\ g\ f) = \mathsf{lam\_comp}\ A\ (\mathsf{lam\_comp}\ B\ h\ g)\ f.$$

The proposition is identified by the following information:

Pure Prop Id:	*10d3c2043b57f47398d529842b2b4ef41092863b2ed292e467d739a5a5d48d05*
Pure Prop Address:	*TMGNEh1ULegqm5w1qTFWfgYNKdQ2N5PWfL3*
Theory Prop Id:	*51ef710d91519b6ae4f1e6c1240232977314be4a5e218f254330dc6c7b0e7fb2*
Theory Prop Address:	*TMX7HYhhWwwEmvKudmWCRyNh3vcLzY4xMPJ*

`lam_comp_id_L`

Theorem 27.75 $\forall AB.\forall f \in B^A.\mathsf{lam\_comp}\ A\ (\mathsf{lam\_id}\ B)\ f = f$. *The proposition is identified by the following information:*

Pure Prop Id:	*e63b276d423c5ffa382e3d24fff1281fc5d23cdc2de80558f7664c37ab8eed29*
Pure Prop Address:	*TMbj9gKjk5JQsbDmoPhorvNQU4Jq7DGrLwD*
Theory Prop Id:	*64750339f13b14b1eca32c23b90fa53b58a24aa24d334fb3e2b45602882f363c*
Theory Prop Address:	*TMNXXqZFWBCWcQvn8czxiCArUPkXVyGAsqm*

`lam_comp_id_R`

Theorem 27.76 $\forall AB.\forall f \in B^A.\mathsf{lam\_comp}\ A\ f\ (\mathsf{lam\_id}\ A) = f$. *The proposition is identified by the following information:*

Pure Prop Id: *0acec29b4958df486c134fef66fff2628d976311a127b9e66e47d31dce87a2b5*
Pure Prop Address: *TMMEAAXiHtjyoC9UixZUX89czksUW4L2AbP*
Theory Prop Id: *b8b0cd7ab3f2745fc54a89466dffe557184df20469638425a99e0d6af17cc7cf*
Theory Prop Address: *TMPrbzDNKGk5rH6cAEX3uuwsRXfYLypKbkq*

`Field_Hom_id`

Theorem 27.77 $\forall F.\mathsf{Field}\ F \to \mathsf{Field\_Hom}\ F\ F\ (\mathsf{lam\_id}\ (F\ 0))$. *The proposition is identified by the following information:*

Pure Prop Id: *e76108b7394a2d0ef4a23eeff5c2eff0a59bfd099b41a3efcf63c829da51cd0e*
Pure Prop Address: *TMMTreDckMCE8QUxnfjGRBucakRyKZDd5Uw*
Theory Prop Id: *0a051badc75ad444016ac67529bbfa1ebe66ca5459cd8875e236240cb319fd29*
Theory Prop Address: *TMbgZLG5jAx7kLyryCTxrpfmKu9WndjBQKz*

`Field_Hom_comp`

Theorem 27.78

$$\forall F F' F'' g h.\mathsf{Field\_Hom}\ F\ F'\ g \to \mathsf{Field\_Hom}\ F'\ F''\ h \to \mathsf{Field\_Hom}\ F\ F''\ (\mathsf{lam\_comp}\ (F\ 0)\ h\ g).$$

The proposition is identified by the following information:

Pure Prop Id: *f96e7b8972ab6ddc476b85f43c26f21233bb2d05a4284718f987e06e5bb0a539*
Pure Prop Address: *TMXHqDN9vGwVX2JmvbvZuV4Jx7aQ6ptioS9*
Theory Prop Id: *c7176150678884e662532193f41ddefed41f8c69c784867e0e5e7599cfccc3d7*
Theory Prop Address: *TMFHd96nKERMA8xrf6a4PPnaFSZAyhMgQkd*

Definition 27.16 *We define* Galois_Group *to be*

$$\lambda K F.\mathsf{pack\_b}\ \{f \in K\ 0^{K\ 0} | \mathsf{Field\_automorphism\_fixing}\ K\ F\ f\}\ (\mathsf{lam\_comp}\ (K\ 0))$$

of type $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id: *831192b152172f585e3b6d9b53142cba7a2d8becdffe4e6337c037c027e01e21*
Pure Object Address: *TMYqxuc9rAFqZk2CRVHyc2y7pfNsCZU6equ*
Theory Object Id: *ca458a9929d10327f6d82d25599b1037bc98c2bdf117e527489c6a4d6ffdd167*
Theory Object Address: *TMTJqgfvRfobXKQitP5Ljdprk53Cpq27TKH*

`Galois_Group_0`

Theorem 27.79

$$\forall K F. \mathsf{Galois\_Group}\ K\ F\ 0 = \{f \in K\ 0^{K\ 0} | \mathsf{Field\_automorphism\_fixing}\ K\ F\ f\}.$$

The proposition is identified by the following information:

Pure Prop Id:	*c937d2405bc02024b3013de5808dbe9e3504c142d376d7ff1ca27d63e12e562e*
Pure Prop Address:	*TMP5RNZvDusyH2t1cf286riba2LdRzc1YJv*
Theory Prop Id:	*efd193beaee2e86f35612249b45870a142e7e8c50719336225df8ef19e54b2d9*
Theory Prop Address:	*TMHr9KDkArCoQ1vA2dUxzDwknQj6pDMv7LB*

`Galois_Group_Group`

Theorem 27.80 $\forall FK.\mathsf{subfield}\ F\ K \to \mathsf{Group}\ (\mathsf{Galois\_Group}\ K\ F)$. *The proposition is identified by the following information:*

Pure Prop Id:	*23e81a79edfd40eff87623569e69d5d6bd17fea035e30db29d3fa252dc9292df*
Pure Prop Address:	*TMF8s32NGdaXaUxYsMdcKNoMmb1qtqso9ph*
Theory Prop Id:	*ed4e90cd2b4fa72539381db20b559cb765274456c2523b29855fe21eefb0bf54*
Theory Prop Address:	*TMZrV2L74MSsae3E5TMtqrcs8St33ZF5Hbo*

27.10 explicit_Reals_transfer

Let $R : \iota$ be given. Let $zero, one : \iota$ be given. Let $plus, mult : \iota \to \iota \to \iota$ be given. **Notation.** We use $+$ as a right associative infix operator corresponding to applying term $plus$. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term $mult$. Let $leq : \iota \to \iota \to o$ be given. **Notation.** We use $\leq$ as an infix operator corresponding to applying term leq. Let $R' : \iota$ be given. Let $zero', one' : \iota$ be given. Let $plus', mult' : \iota \to \iota \to \iota$ be given. **Notation.** We use $\oplus$ as a right associative infix operator corresponding to applying term $plus'$. **Notation.** We use $\times$ as a right associative infix operator corresponding to applying term $mult'$. Let $leq' : \iota \to \iota \to o$ be given. Let $f : \iota \to \iota$ be given.

`explicit_Reals_transfer`

Theorem 27.81

$$\begin{gathered}\mathsf{explicit\_Reals}\ R\ zero\ one\ plus\ mult\ leq \to \mathsf{bij}\ R\ R'\ f \to \\ f\ zero = zero' \to f\ one = one' \to (\forall xy \in R. f\ (x+y) = f\ x \oplus f\ y) \to \\ (\forall xy \in R. f\ (x*y) = f\ x \times f\ y) \to (\forall xy \in R. x \leq y \Leftrightarrow leq'\ (f\ x)\ (f\ y)) \to \\ \mathsf{explicit\_Reals}\ R'\ zero'\ one'\ plus'\ mult'\ leq'.\end{gathered}$$

The proposition is identified by the following information:

Pure Prop Id:	*eab1221701cf6d740b9b72782e98f152a3f7f2cdf16638d049fc649d996fc29c*
Pure Prop Address:	*TMYhyMVVJTvxXty5NxJVswTmqpdUcPwJ3PN*
Theory Prop Id:	*76534117383e5ea208dccdd5106698afc06e708ae05d009cb0e347dfd0cf2ae2*
Theory Prop Address:	*TMY35GcU6u4DUqWLoorNqpUAYwMdFLABjvj*

Chapter 28

Surreal Numbers II

28.1 SurrealArithmetic

Definition 28.1 minus_SNo *is the opaque object of type $\iota \to \iota$ identified by the following information:*

Pure Object Id:	*268a6c1da15b8fe97d37be85147bc7767b27098cdae193faac127195e8824808*
Pure Object Address:	*TMcxvtkwUHDLCnnXkVE7rvLpYFkkCr9t57h*
Theory Object Id:	*8a6d8513b8fae7a11b1033553a028661380ee0d23337db3b5de7c37c00bf7d3a*
Theory Object Address:	*TMNRx3zLagXCf6MQrfwBvyPXLa8o1ZnJoqf*

Notation. We use $--$ as a prefix operator corresponding to applying term minus_SNo. **Notation.** We use $\leq$ as an infix operator corresponding to applying term SNoLe.

`minus_SNo_eq`

Theorem 28.1

$$\forall x.\mathsf{SNo}\ x \to -x = \mathsf{SNoCut}\ \{-z | z \in \mathsf{SNoR}\ x\}\ \{-w | w \in \mathsf{SNoL}\ x\}.$$

The proposition is identified by the following information:

Pure Prop Id:	*c9793e4a852e2a5d73a3008d3c0a88124864f04508e5f6cd84920975e8bddfc6*
Pure Prop Address:	*TMFADt2W5uMVy5FYrgHvEE2vrdFR8Gnxb1b*
Theory Prop Id:	*570cba2e3229759ec994c333f1852d951e44a44817d76474c936f0b2783d17fb*
Theory Prop Address:	*TMQTpg4CNj1EB19CWqcr8oBSbG7cvoVZJuJ*

`minus_SNo_prop1`

Theorem 28.2

$$\forall x.\mathsf{SNo}\ x\rightarrow$$
$$\mathsf{SNo}\ (-x) \land (\forall u \in \mathsf{SNoL}\ x.-x < -u) \land (\forall u \in \mathsf{SNoR}\ x.-u < -x)\land$$
$$\mathsf{SNoCutP}\ \{-z|z \in \mathsf{SNoR}\ x\}\ \{-w|w \in \mathsf{SNoL}\ x\}.$$

The proposition is identified by the following information:

Pure Prop Id:	*7c0e82df70ecda3be7a8153a63670c1b82bc354a3b3bb658b240db5bf6079ccc*
Pure Prop Address:	*TMPFDUxTsU6Hgp2dqtAfHiUMzQdZC4s6Rpo*
Theory Prop Id:	*f0b75f0c13ab8217a5ebfacdc1aebc7bacaf4f0bcf65609bd5bdfc26f85fe25e*
Theory Prop Address:	*TMHp3P8GQ2PdxPN38fbjih9CXrrvAVxARmW*

`SNo_minus_SNo`

Theorem 28.3 $\forall x.\mathsf{SNo}\ x \rightarrow \mathsf{SNo}\ (-x)$. *The proposition is identified by the following information:*

Pure Prop Id:	*5cf7d9b9d43ef2ee9370e1aa6dded6d26ae3d971cbc3b561b6d07723f8215269*
Pure Prop Address:	*TMdc5VANKtJFyfviMQyb75gDTg48GudNzRZ*
Theory Prop Id:	*744cc2ee9e53d674ba772a0aa685c56eb8e460e86e64854f7ce0be53b0da5fff*
Theory Prop Address:	*TMQ1ogbeVfSeNvyG9vTPaMVZg2dqUSJyjSo*

`minus_SNo_Lt_contra`

Theorem 28.4 $\forall xy.\mathsf{SNo}\ x \rightarrow \mathsf{SNo}\ y \rightarrow x < y \rightarrow -y < -x$. *The proposition is identified by the following information:*

Pure Prop Id:	*5ce52d7493ef6b5572b747bf671cea2e6e6f80ceb28c95e2123521dd504b0b0f*
Pure Prop Address:	*TMYLjCwrhRZgQN5K5EwHjSFpA4jVNLAHCjn*
Theory Prop Id:	*3ebf6e39ebb99f5ddf5eee4fb81a248a684fd1274696b9741a23625f9df6dc52*
Theory Prop Address:	*TMPCextugz7QEf4NpPTNxs5QRszhndWEXZL*

`minus_SNo_Le_contra`

Theorem 28.5 $\forall xy.\mathsf{SNo}\ x \rightarrow \mathsf{SNo}\ y \rightarrow x \leq y \rightarrow -y \leq -x$. *The proposition is identified by the following information:*

Pure Prop Id:	*c1b2259be1bdf45a1f93fe5cc80752b95adc141e37aea0fc5974e675529dbab0*
Pure Prop Address:	*TMSdtEKoTTw4VsgYxJhGDbFrsEav7bgzwDz*
Theory Prop Id:	*f8c8964a3e52fb3b06f65d0148f443ef72cb1a1710977f309628d62a6dc762ff*
Theory Prop Address:	*TMR6iQfy2ketSGqusuoDGBAoZDHFdKZi5ei*

`minus_SNo_SNoCutP`

Theorem 28.6

$$\forall x.\mathsf{SNo}\ x\rightarrow\mathsf{SNoCutP}\ \{-z|z \in \mathsf{SNoR}\ x\}\ \{-w|w \in \mathsf{SNoL}\ x\}.$$

The proposition is identified by the following information:

Pure Prop Id:	*99c6fcde5ba109860366fc8eb324200f6b134a75dc50371fa98ecbf13fe8dd9f*
Pure Prop Address:	*TMPGzP3WvAeSXVX4PkvAXm9hswTwYn7Lx5a*
Theory Prop Id:	*5ad5e46892ec6dde1a8e632b515b72dd9624c207d92d7f5fc1b887ec45170a91*
Theory Prop Address:	*TMYA5Ns3A1StK4eq1kfygXeX7mudw4n946H*

minus_SNo_SNoCutP_gen

Theorem 28.7

$$\forall LR.\mathsf{SNoCutP}\ L\ R \rightarrow \mathsf{SNoCutP}\ \{-z|z \in R\}\ \{-w|w \in L\}.$$

The proposition is identified by the following information:

Pure Prop Id:	*e7650d8c5c9fb96894ef4334966c9c4043dc7b919ef49e218164db5101a12296*
Pure Prop Address:	*TMcYQN83fYBS39M2EEEbDtftNsYeWHCkMAi*
Theory Prop Id:	*687fea2cc86a5e6dc4aaff906beb9303fa12e95620a78db83430a8be9dfd1bfc*
Theory Prop Address:	*TMK1WeYsUqdVKtQ7kU3SZV4h8vJvGUyosuH*

minus_SNo_Lev_lem1

Theorem 28.8

$$\forall \alpha.\mathsf{ordinal}\ \alpha \rightarrow \forall x \in \mathsf{SNoS\_}\ \alpha.\mathsf{SNoLev}\ (-x) \subseteq \mathsf{SNoLev}\ x.$$

The proposition is identified by the following information:

Pure Prop Id:	*14986a810fa991813a940caff72106cdeb17e367c8dc9c6eeb88ab77ed7fb70c*
Pure Prop Address:	*TMGTTrCpJEdP2JL7ZKjHkPvFjc3gfmdeYX5*
Theory Prop Id:	*afc115b1b5f9ff0b6226e6d37a5708b45bf273bc776bf7816207943b0cf564fa*
Theory Prop Address:	*TMcrjfrThFM8M6pTTZtnJdFZBn72x9XkSzf*

minus_SNo_Lev_lem2

Theorem 28.9 $\forall x.\mathsf{SNo}\ x \rightarrow \mathsf{SNoLev}\ (-x) \subseteq \mathsf{SNoLev}\ x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*87902034fac50c9d369be3b80426e4d2a737cf936787a4c0f6c8ad74e0843878*
Pure Prop Address:	*TMbVFKSGdQJSr2kHyJUmNCjt7CTD1D6JKHf*
Theory Prop Id:	*19cf80ac01fc15c5bac2a1a546c9cfca35cf8eb2a455b3cf3fb66a2a7fe614f8*
Theory Prop Address:	*TMQtKJ6HCGKKXSHeLSJwRyRPcKmC4ZiLZK5*

minus_SNo_invol

Theorem 28.10 $\forall x.\mathsf{SNo}\ x \rightarrow --x = x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*44bfe89261da61b19b9a5cd41ede8430faa107c7822701da47ef38cb4549faa6*
Pure Prop Address:	*TMc77Sp2dpzatNwekdynQs9vhSPfJgGXk1U*
Theory Prop Id:	*cc64eeea172423c51379afadb3e01de9b7f2513ce938649ddcb32805f5bdafb0*
Theory Prop Address:	*TMTt3PRTf9CD5ikrtEqSt2QevQVtKjQrFbz*

`minus_SNo_Lev`

Theorem 28.11 $\forall x.\mathsf{SNo}\ x \rightarrow \mathsf{SNoLev}\ (-x) = \mathsf{SNoLev}\ x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*f65499af9040b6b3e15497cab281d7675bff566bb6d4fdd182cc0343568ae07e*
Pure Prop Address:	*TMHMPna5msJBVV2aj6dRhmCueZSUBa2VhT5*
Theory Prop Id:	*476ae5e0d40774e983e1d64325be7c212ae84435e983e476cb99925933ecd248*
Theory Prop Address:	*TMRdSSgdRywiPXPvi9bXyTwx2JCnJRrLkHS*

`minus_SNo_SNo_`

Theorem 28.12

$$\forall \alpha.\mathsf{ordinal}\ \alpha \rightarrow \forall x.\mathsf{SNo\_}\ \alpha\ x \rightarrow \mathsf{SNo\_}\ \alpha\ (-x).$$

The proposition is identified by the following information:

Pure Prop Id:	*84605fa359f5f8a80873f561cdcf5e371180d9965ea1157048ba66f9af585e90*
Pure Prop Address:	*TMZviY6yHgD2P1xafVTUWcBcjX291SfHuZq*
Theory Prop Id:	*14bd6d1aecca30a4c446e92cca90fa66fd7b0499acca03daf8cfa848740f428f*
Theory Prop Address:	*TMTsLyGVKuoMiH43ymXqPkKMWYDN3HyfXHd*

`minus_SNo_SNoS_`

Theorem 28.13

$$\forall \alpha.\mathsf{ordinal}\ \alpha \rightarrow \forall x.x \in \mathsf{SNoS\_}\ \alpha \rightarrow -x \in \mathsf{SNoS\_}\ \alpha.$$

The proposition is identified by the following information:

Pure Prop Id:	*12f1a5194619e18f763ea1432c60862d2c280bcc57c6b538e7af0b8563b3aef0*
Pure Prop Address:	*TMNu6dbAKA4NtYuwXCDn3Z2xwPtwvtj71gH*
Theory Prop Id:	*e10c92f39791767985ef5b9f5eed352a930cae37c683dfacbd6f47c7d2d23e93*
Theory Prop Address:	*TMVtCJL3NR1TVjUhDpc8LMfGugbda1XLCG2*

`minus_SNoCut_eq_lem`

Theorem 28.14

$$\forall v.\mathsf{SNo}\ v \rightarrow \forall L R.\mathsf{SNoCutP}\ L\ R \rightarrow v = \mathsf{SNoCut}\ L\ R \rightarrow -v = \mathsf{SNoCut}\ \{-z|z \in R\}\ \{-w|w \in L\}.$$

The proposition is identified by the following information:

Pure Prop Id:	*29069a9700032f41c27cbce5d097d02ca500af7402feea86befd364a812bea18*
Pure Prop Address:	*TMJJorcHcCvbte94Wk2mohN3ofdRRZi9USf*
Theory Prop Id:	*11eb03174b07f424338800b6644cd5921561dc680adb86b22f9254abda306195*
Theory Prop Address:	*TMRBupH2YERv2pxoxNHvbLaoKkUEWp3LmVu*

minus_SNoCut_eq

Theorem 28.15

$$\forall LR.\mathsf{SNoCutP}\ L\ R \to -\mathsf{SNoCut}\ L\ R = \mathsf{SNoCut}\ \{-z|z \in R\}\ \{-w|w \in L\}.$$

The proposition is identified by the following information:

Pure Prop Id:	*083ffe4c6522008e61c2b89e0e64c5db7b371419a23a3f14a519e0582688fb5b*
Pure Prop Address:	*TMaD6A1fin9xGjs9gJk55ba3XE4iuTh2W7Y*
Theory Prop Id:	*54736f85eb44e8ebcd3e07eea3d33c79842c2ce605f7a6384f4ba14b9eec7d07*
Theory Prop Address:	*TMGrpE8HLXgUszxGm966iTpad2EhS6XSckh*

minus_SNo_Lt_contra1

Theorem 28.16 $\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to -x < y \to -y < x$. *The proposition is identified by the following information:*

Pure Prop Id:	*226757cb052a4956017abe14687f3e1caa99a28fde325c1d779946bba7658526*
Pure Prop Address:	*TMXXUYTV6XCxwBUgA7MgZTzj6H9ahkRPPTo*
Theory Prop Id:	*29872961c38defbc42168eadfbb395ff6b34bb794dfd9e81bcce6ad6374039e9*
Theory Prop Address:	*TMEs4ZMsQaGiy3xaRcw9TpZk6KCqk5ye83u*

minus_SNo_Lt_contra2

Theorem 28.17 $\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to x < -y \to y < -x$. *The proposition is identified by the following information:*

Pure Prop Id:	*c6f2dabf5db1946ff00666f185293c94dc027bfbc3c6b34ab842300e6fc61e10*
Pure Prop Address:	*TMK1aCcPBp5isVNhukJBv5WwEVryhWR39im*
Theory Prop Id:	*f419684217fb1e0747adc0f778ba066f7018450793455dd8aaa40b5499c5cd992*
Theory Prop Address:	*TMEu6PgqrG75bbqAH6z2eo8taFDwoWi72LL*

minus_SNo_Lt_contra3

Theorem 28.18 $\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to -x < -y \to y < x$. *The proposition is identified by the following information:*

Pure Prop Id:	*30b6d6ce0519f672dafb8da416d1f08cc773f7718ea0c0e59fe2b674a8a28eb5*
Pure Prop Address:	*TMEiiHLSUJb67TTgfxay254xNKWCum3TL6U*
Theory Prop Id:	*20ebc3b1dcf4268fcbbda61e77e0b1aa0cd9de3c07e995cd027e1623022ab3a6*
Theory Prop Address:	*TMdv9HV4i4ia6bWB1AEuc8hYPUVNAMAF4i3*

minus_SNo_0

Theorem 28.19 $-0 = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*77bc6707cfcc559fd1f9509caeb31aaec5c6ea27e99983f5ed2a0b31aba16371*
Pure Prop Address:	*TMExypvZyPTdXH5VP42X3hVoCRLY5xqMGSS*
Theory Prop Id:	*3fcc57eb71cf1d99ef0ab04ed19f5bc54b596714fb2b6d70ce2f61b05445433c*
Theory Prop Address:	*TMS4JbQtTa5bFrxAcTD38DVaprpkd1UddT2*

SNo_momega

Theorem 28.20 SNo (−omega). *The proposition is identified by the following information:*

Pure Prop Id:	*9682bc4f449aab7850dab8b061240c04bbfa925a9af0c6d436b57571e88bf054*
Pure Prop Address:	*TMbs8VnGZaJLV9PMfUU3ZA9UMWppyWZCbpk*
Theory Prop Id:	*e72fb1696565b219d098fc491769ed67f15f1c716f9ba159513fa19c4a3cfb68*
Theory Prop Address:	*TMQGDzKedUx3YyBnazoAQzc2BJYMagTe9aU*

mordinal_SNo

Theorem 28.21 $\forall\alpha.$ordinal $\alpha \to$ SNo $(-\alpha)$. *The proposition is identified by the following information:*

Pure Prop Id:	*67157695184d77aa292d2e91ae058aab5728c1aa262ad91430d98e01a61ecee5*
Pure Prop Address:	*TMZCauMQgviQfBsquF7s2dXY5ovy2NZhJmB*
Theory Prop Id:	*533947d00d9f2f544511c3e8876c2d3bec39a85cf5de0507579619af91b1cfe3*
Theory Prop Address:	*TMdpAbWGcEmUBU6mDwF1QUJ8J3YQKqAnC2W*

mordinal_SNoLev

Theorem 28.22 $\forall\alpha.$ordinal $\alpha \to$ SNoLev $(-\alpha) = \alpha$. *The proposition is identified by the following information:*

Pure Prop Id:	*872283abfa6f0dfbb1d72e382868b2cf0992ece74aabc0727d30681dd1c82466*
Pure Prop Address:	*TMWf88dibCFMEwQnUShPjR7f5qzRTHihxjA*
Theory Prop Id:	*425f79bccb3002745cf4cfe3c4ee8e5f28fc81700abf759ffe8688c3ec559879*
Theory Prop Address:	*TMJYCVXHmoi2fzEKN7sFk63txQj7WCuRVYU*

mordinal_SNoLev_min

Theorem 28.23

$$\forall\alpha.\text{ordinal } \alpha \to \forall z.\text{SNo } z \to \text{SNoLev } z \in \alpha \to -\alpha < z.$$

The proposition is identified by the following information:

Pure Prop Id:	*c28e6646c6374a60e623edaf3e98581ff82a2d308bc362c7ed683cb7ef763c9b*
Pure Prop Address:	*TMGx6jRVew4w8A8QL13WubtT3gsqgBkjuKj*
Theory Prop Id:	*a11cfa7f908447229087de2b3b5ed724471c04c5ace74fef64304120331db0b2*
Theory Prop Address:	*TMcMHpMnYb8FFYn3kS1X4hRDwLCnUBPrPN2*

mordinal_SNoLev_min_2

Theorem 28.24

$$\forall\alpha.\mathsf{ordinal}\ \alpha \to \forall z.\mathsf{SNo}\ z \to \mathsf{SNoLev}\ z \in \mathsf{ordsucc}\ \alpha \to -\alpha \leq z.$$

The proposition is identified by the following information:

Pure Prop Id:	*2766c7ac03ddae43cd989f364bd3933e33016953b97822589263b8813a782baf*
Pure Prop Address:	*TMdLNMofLk9MRXKxrVStAmXcnfLmpPf5ZLD*
Theory Prop Id:	*a96d68eb40e4505dff800c19d3a67d927597a15491c19c2e7613c3653faa7592*
Theory Prop Address:	*TMUbQC9sotj3Sdmk1rPhzFc2uGHxwQ7fTKG*

minus_SNo_SNoS_omega

Theorem 28.25 $\forall x \in \mathsf{SNoS\_\ omega}.-x \in \mathsf{SNoS\_\ omega}$. *The proposition is identified by the following information:*

Pure Prop Id:	*daeba565dddc519103825c78e55714d95a0ae94548ea309e8d64ab2518ce1c95*
Pure Prop Address:	*TMHUVaZspynr2H3jcHXqWoRHirGbNS7NfFb*
Theory Prop Id:	*0c19f5cfaa9b340bbf0166314f1046d3ad7c8558b64304949986f6ee62b7946e*
Theory Prop Address:	*TMV7G2rh3rEAwiDk5QRMvAdQX4xEJpR6g26*

Definition 28.2 add_SNo *is the opaque object of type* $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*127d043261bd13d57aaeb99e7d2c02cae2bd0698c0d689b03e69f1ac89b3c2c6*
Pure Object Address:	*TMHBQoir8zBmsd18ceCptzsSPneX7e796Qh*
Theory Object Id:	*f62f9ad0a4b78ce9ea1f70abc3f81bb4efd2f55019f5ea81fec6bff7d056a9c4*
Theory Object Address:	*TMLfAbLP1JzEe3VPaGmScRvpGeCCLs2hBkM*

Notation. We use $+$ as a right associative infix operator corresponding to applying term add_SNo.

add_SNo_eq

Theorem 28.26

$$\begin{gathered}\forall x.\mathsf{SNo}\ x \to \forall y.\mathsf{SNo}\ y \to \\ x+y = \\ \mathsf{SNoCut} \\ (\{w+y \mid w \in \mathsf{SNoL}\ x\} \cup \{x+w \mid w \in \mathsf{SNoL}\ y\}) \\ (\{z+y \mid z \in \mathsf{SNoR}\ x\} \cup \{x+z \mid z \in \mathsf{SNoR}\ y\}).\end{gathered}$$

The proposition is identified by the following information:

Pure Prop Id:	*0c11282e349ac03ead817fec6965779c38db2bba16e6617397d7a00fec0025f2*
Pure Prop Address:	*TMUsqiTidRPvWsTgAU6Pe8thY6esLYajg8X*
Theory Prop Id:	*437a9c143d85d0cbb514a028f14f083cc0f8d79aa47f3997255c796243fc0fe7*
Theory Prop Address:	*TMGPPTS21hFunW6FDmNfEs1RLtzqupLkwJY*

add_SNo_prop1

Theorem 28.27

$$\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \\ \mathsf{SNo}\ (x+y) \wedge (\forall u \in \mathsf{SNoL}\ x.u+y < x+y) \wedge \\ (\forall u \in \mathsf{SNoR}\ x.x+y < u+y) \wedge \\ (\forall u \in \mathsf{SNoL}\ y.x+u < x+y) \wedge \\ (\forall u \in \mathsf{SNoR}\ y.x+y < x+u) \wedge \\ \mathsf{SNoCutP} \\ (\{w+y|w \in \mathsf{SNoL}\ x\} \cup \{x+w|w \in \mathsf{SNoL}\ y\}) \\ (\{z+y|z \in \mathsf{SNoR}\ x\} \cup \{x+z|z \in \mathsf{SNoR}\ y\}).$$

The proposition is identified by the following information:

Pure Prop Id:	*f3ece7fc5faeb9b5356d40372aef6e582964fa486d2e44e7f3eed3f05d52e3e0*
Pure Prop Address:	*TMcpTrmdaf7f5xyc6e4BpSWAmxNsmf3E5yS*
Theory Prop Id:	*5a879f7365529b11a3ffce943b27ada13bc1e46e2af4e2258090e38d55606b7f*
Theory Prop Address:	*TMRNpdhJ6PXJ334PSwBu4u3N4MRpHafuGvT*

SNo_add_SNo

Theorem 28.28 $\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ (x+y)$. *The proposition is identified by the following information:*

Pure Prop Id:	*c4a9a6e12a8ec5b5bebaa97ea0d920bd43eea74d9b3d0ffda15762cf19565969*
Pure Prop Address:	*TMPtcdp9k9YMXhBQVaR4jVorJnSfZBVvKki*
Theory Prop Id:	*a57095a9caa997f445ebde562cde8dd04d8e8a5fd2a57d1c3a9f8decf60ccdeb*
Theory Prop Address:	*TMXsdqtKxXjtQCrsnZQNbBCQsbgVDjWXpMf*

SNo_add_SNo_3

Theorem 28.29 $\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to \mathsf{SNo}\ (x+y+z)$. *The proposition is identified by the following information:*

Pure Prop Id:	*f6ef07d3f7fb2565f3fe4732b883d4499d8442b69d6ae7d977010d18d4d54ecf*
Pure Prop Address:	*TMZ4dtBp9Qizyu4nFxn7a2N9SQRVoE9Q4W8*
Theory Prop Id:	*f2d78c7d8aa64050 36ae26879362dfde9af9a15cdbbf11ba2bff0d754b7750ca*
Theory Prop Address:	*TMVox3RUbkj9ZYffbdvRh7znGFXNfLVwPFV*

SNo_add_SNo_4

Theorem 28.30

$$\forall xyzw.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to \mathsf{SNo}\ w \to \mathsf{SNo}\ (x+y+z+w).$$

The proposition is identified by the following information:

Pure Prop Id:	*d71f0eb42650c644bb0c4a2fbb29173cc3bf47fea9dd9401c0ba715333db1820*
Pure Prop Address:	*TMF4oHEkLVa8KD2NcVmcFum9JhucpudfEb1*
Theory Prop Id:	*42037e0c85566de52aba1c0f97268637a493ef459b07be2723bfca5dc7b13a97*
Theory Prop Address:	*TMXXq7JGxapVLMuBsmWzgDWpgqW72RtAMLV*

add_SNo_Lt1

Theorem 28.31 $\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to x < z \to x + y < z + y.$
The proposition is identified by the following information:

Pure Prop Id:	*92232f18c39390c575ed22b7c92927a25b5ade867b5e2ed5f2cdf7e509909805*
Pure Prop Address:	*TMLEq8tneuw32G568W1mBtwRTNNgQPCRYyz*
Theory Prop Id:	*cb0738c7872b30b23cb65bbcd1ce88351f2466b21c1b5cc15f5b44ae581e28e0*
Theory Prop Address:	*TMWTBYY2jenWVwUwvSromB8AgRWDVcJnbY6*

add_SNo_Le1

Theorem 28.32 $\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to x \leq z \to x + y \leq z + y.$
The proposition is identified by the following information:

Pure Prop Id:	*d8f37ed7006da1a1793317440a31b24b6f87f27cad9329c38e8a7ff9753d2dcc*
Pure Prop Address:	*TMaVRrrSjDXkuboEtM4VK6QYgB6UVbfBcaG*
Theory Prop Id:	*abcf7263a107c501b54d30826fb67a71bfd65aa0e1fa33d5cb94ba28bbd40502*
Theory Prop Address:	*TMR5TeFwaZGZUnBsDVqT64ZqwgMewHGgCDF*

add_SNo_Lt2

Theorem 28.33 $\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to y < z \to x + y < x + z.$
The proposition is identified by the following information:

Pure Prop Id:	*6ad46463d6f79423eff3ac9a3ac6201e44983b342ffb704d9cde5795efc8a7ec*
Pure Prop Address:	*TMZabfrosx2WbihjxmG74tgmZaLS7LCKpAU*
Theory Prop Id:	*6f0941dc05860da5ece42171247edca54127ab818e70661d52995088db287957*
Theory Prop Address:	*TMQWhSEEumkynNocoC5sSjUxscps15Jp64G*

add_SNo_Le2

Theorem 28.34 $\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to y \leq z \to x + y \leq x + z.$
The proposition is identified by the following information:

Pure Prop Id:	*14975eae6645de30652349b64a88f0b2e35b84bc21be97aafc239516dd628b87*
Pure Prop Address:	*TMdukf7SbuQ39yuQq8KCjup31RNEDuTLQnT*
Theory Prop Id:	*3f52ba378d30a817615bc0f6bf9697c00b83ce446cbe3de8fff3a6d92b6bd5de*
Theory Prop Address:	*TMGdyU2zNNkb5T6LeAwxX74oWd2P7WYJJ8c*

add_SNo_Lt3a

Theorem 28.35

$$\forall xyzw.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to \mathsf{SNo}\ w \to x < z \to y \leq w \to x + y < z + w.$$

The proposition is identified by the following information:

Pure Prop Id:	*b2215f63878ba2a6fc09f12939fd6fc51dc580691897e8323370abb12c70f7bd*
Pure Prop Address:	*TMYyEHQwicHmVL6MUyPu2QNJHpVu8pL5YVP*
Theory Prop Id:	*18ff91f30d21406ebb6b75675c0b1f7e8ad283cbf98775f53dd3e656a5fa522f*
Theory Prop Address:	*TMSmpE2jeB2cyTEuG7Rq8sykhPLAACRxekP*

`add_SNo_Lt3b`

Theorem 28.36

$$\forall xyzw.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to \mathsf{SNo}\ w \to x \leq z \to y < w \to x + y < z + w.$$

The proposition is identified by the following information:

Pure Prop Id:	*5b4f92cfc6a3acaa35dd069d515cd4c025daa3b2bca2a7add6810dc336b0235f*
Pure Prop Address:	*TMKXJb12jTxUrEE48Fsz3YxLGNM4wfBPXJW*
Theory Prop Id:	*8fe0d98eaddd0798a52530dbd10ab1d00f92e187620a37fb685f5830bd4c085c*
Theory Prop Address:	*TMKS8vtMN5qWoxjxHv7NEEavnk6emGyiiMz*

`add_SNo_Lt3`

Theorem 28.37

$$\forall xyzw.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to \mathsf{SNo}\ w \to x < z \to y < w \to x + y < z + w.$$

The proposition is identified by the following information:

Pure Prop Id:	*7141a86b460a8ecf3af113551c387396eea76973c00c0ae6d3a7a9f0806449bd*
Pure Prop Address:	*TMNriuSM92c9RucKEto1JSRuMZvj8bjdnr5*
Theory Prop Id:	*55bdabe93657d73da3016bb38681d52b49a61dca9be7e4b0f99461db17aaf692*
Theory Prop Address:	*TMKtJcmnivzZgTkMMwdV5vUEUxKGyMkrQFd*

`add_SNo_Le3`

Theorem 28.38

$$\forall xyzw.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to \mathsf{SNo}\ w \to x \leq z \to y \leq w \to x + y \leq z + w.$$

The proposition is identified by the following information:

Pure Prop Id:	*0be637ad324a3ea9caae189e47f4890cba93a45121b22ff1e10fad675f87f12c*
Pure Prop Address:	*TMSVKSkQGVqbboR9prtEBre18xSpVMapMrK*
Theory Prop Id:	*be66ca7a659e973fad15e883dbd8397986cfe73bd862e7b50ccf70a912a6e0b0*
Theory Prop Address:	*TMdD3NhDM7mVSeEXJrP8Vh8HeE3of9UjkEu*

add_SNo_SNoCutP

Theorem 28.39

$$\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNoCutP}\ (\{w+y|w\in\mathsf{SNoL}\ x\}\cup\{x+w|w\in\mathsf{SNoL}\ y\})\ (\{z+y|z\in\mathsf{SNoR}\ x\}\cup\{x+z|z\in\mathsf{SNoR}\ y\}).$$

The proposition is identified by the following information:

Pure Prop Id:	*e2b7d74c79711fae6a14f4d67a701081855be3bb26fd923a62a391e04160bfae*
Pure Prop Address:	*TMdKu7mpdNdzCSmXhFYuNuBjrmEsPPsqqGm*
Theory Prop Id:	*f86b1a41eabc3959fd8be49bbc8865a0009802c350299879c6b3a1eb97530c7d*
Theory Prop Address:	*TMR6c36epFgVd323wsqJDSNi9GvJ5dHX5bG*

add_SNo_SNoCutP_gen

Theorem 28.40

$$\forall LxRxLyRy.\mathsf{SNoCutP}\ Lx\ Rx \to \mathsf{SNoCutP}\ Ly\ Ry \to \mathsf{SNoCutP}\ (\{w+\mathsf{SNoCut}\ Ly\ Ry|w\in Lx\}\cup\{\mathsf{SNoCut}\ Lx\ Rx+w|w\in Ly\})\ (\{z+\mathsf{SNoCut}\ Ly\ Ry|z\in Rx\}\cup\{\mathsf{SNoCut}\ Lx\ Rx+z|z\in Ry\}).$$

The proposition is identified by the following information:

Pure Prop Id:	*075eaabfcc7432298a2db5f87aafbc0106915ca041c774d445bb94d0821d5824*
Pure Prop Address:	*TMS5jpucdv4wNPKAJszA5iBP9k5qK9fGjf2*
Theory Prop Id:	*9afab2f26e1613aa776c9706130a926c7e2da1ffe974ae51ddb5a8b57c102cf0*
Theory Prop Address:	*TMUA1C1f53xTBiuMvrMGVhLK8tVv3uALH6H*

add_SNo_com

Theorem 28.41 $\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to x+y = y+x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*548814e9730b34ca53c6149a1239b003d299813e4e1610c765a8f32771db006e*
Pure Prop Address:	*TMb6qo68fsewkpEkdhHBPyCDbKiaep9rQ9B*
Theory Prop Id:	*f017e50fae3edfeeaedc58af7e2473b3c3008dd0d65d3dd902ce9902cfcedaed*
Theory Prop Address:	*TMdQusEuxAaq4pjYGPyZyTsGeCDoe7ixEg5*

add_SNo_0L

Theorem 28.42 $\forall x.\mathsf{SNo}\ x \to 0+x = x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*107598652879bcb8baa57261d6be92d58926c75b0158561c7a0fbf824b5533ac*
Pure Prop Address:	*TMPpszvpgSas6jmAy2oPhZkMYNxdnAa9bCr*
Theory Prop Id:	*446bada3d91ea404046f63fd5ef3b8841131bc7c36182164979919c7306a98104*
Theory Prop Address:	*TMLtAihTfZ4tMgum6Ca1thK1rJp885zUT7m*

add_SNo_0R

Theorem 28.43 $\forall x.\mathsf{SNo}\ x \to x + 0 = x$. *The proposition is identified by the following information:*

Pure Prop Id:	*d8a186fe010013b2755c30ba0a202ddabfa4c545c10aac8b4e1364cac61cebd6*
Pure Prop Address:	*TMTawHrBTSYsj41yX9x5qiLExuLJBg499YB*
Theory Prop Id:	*1a7fa1896336c0b5037e49fd8da8f614bd60fec53a52508d1c24eb4c32d2c081*
Theory Prop Address:	*TMMmjuw778ta6y4jTCoCMS5fmu1gZ8B6AGF*

add_SNo_minus_SNo_linv

Theorem 28.44 $\forall x.\mathsf{SNo}\ x \to -x + x = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*42471917566847dbc969b69ad8eb5cfbb677e19d7005aaefa3ddbcd227fbc72c*
Pure Prop Address:	*TMPWBgm88UpCF22WujXVVkVtCu8iciY3QTW*
Theory Prop Id:	*44e251b28aec6132ef7e2d0a0c34be7ac5c327d1039a329806f84dd65df681f5*
Theory Prop Address:	*TML2RWMKucEcfgS9pHC3eNVJ6jgXjXCWKn2*

SNo_add_SNo_3c

Theorem 28.45 $\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to \mathsf{SNo}\ (x + y + -z)$. *The proposition is identified by the following information:*

Pure Prop Id:	*104eb97ceeb39012baf2633f2295fcd8488c5da863f883a4f0bd3508d7022b82*
Pure Prop Address:	*TMVNoU8QDePUm51AQC5k9QeJwu735iBxPvj*
Theory Prop Id:	*833be87302ff13765fe452b02d5b463172269368fd9039c3342e653a13738d59*
Theory Prop Address:	*TMPvYhSo4MBSTDX3QY6YVnoEkB94k85UJj6*

add_SNo_minus_SNo_rinv

Theorem 28.46 $\forall x.\mathsf{SNo}\ x \to x + -x = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*9e8c44fbf607e617cb13942672b1aaa08da477c7c043bd3445e12df7880842e3*
Pure Prop Address:	*TMYMYkykBhLq1apDzVyC2vJc6hiCZ65cEJD*
Theory Prop Id:	*b2f9cf9ffdd19a14a7d49bd3eecd70f6ba950f3bae905926575b58dbe69c5611*
Theory Prop Address:	*TMS4a619pMmyZvGY8DXex1wWCwRRFpTtwJ7*

add_SNo_ordinal_SNoCutP

Theorem 28.47

$$\forall\alpha.\mathsf{ordinal}\ \alpha\to\forall\beta.\mathsf{ordinal}\ \beta\to$$
$$\mathsf{SNoCutP}\ (\{x+\beta|x\in\mathsf{SNoS\_}\ \alpha\}\cup\{\alpha+x|x\in\mathsf{SNoS\_}\ \beta\})\ \mathsf{Empty}.$$

The proposition is identified by the following information:

Pure Prop Id:	*2823e439cb46c064666c0dd60fea0fc706e8f3518725e3e71e9fe27292df89a0*
Pure Prop Address:	*TMUPaueAJ3SYX5RiMrtRmYAsP3Gv5ibyiJr*
Theory Prop Id:	*507eab0935b7feec7b7116c01bd1ac27c74326982b1b48e41b6c44b87990749b*
Theory Prop Address:	*TMT1Gz8K4EguVmo5NB2Woi7sw5sQBm85KE2*

`add_SNo_ordinal_eq`

Theorem 28.48

$$\forall\alpha.\mathsf{ordinal}\ \alpha\to\forall\beta.\mathsf{ordinal}\ \beta\to$$
$$\alpha+\beta=\mathsf{SNoCut}\ (\{x+\beta|x\in\mathsf{SNoS\_}\ \alpha\}\cup\{\alpha+x|x\in\mathsf{SNoS\_}\ \beta\})\ \mathsf{Empty}.$$

The proposition is identified by the following information:

Pure Prop Id:	*cfe1f500f4a75d5fa0a4a8a1366f200cc39ec6e6179ec1c26eadd7b4611de086*
Pure Prop Address:	*TMPQvhtityMjoibgERv3aQLgbEgRcqm7yXL*
Theory Prop Id:	*ee8e7c5eccf8aee691d357e43b94c019382682307359 07d55299cc3a35954d02*
Theory Prop Address:	*TMQPK7b3tUhBwCLXYG48aj1mc232WYpgPJF*

`add_SNo_ordinal_ordinal`

Theorem 28.49

$$\forall\alpha.\mathsf{ordinal}\ \alpha\to\forall\beta.\mathsf{ordinal}\ \beta\to\mathsf{ordinal}\ (\alpha+\beta).$$

The proposition is identified by the following information:

Pure Prop Id:	*e9309ac99999e13ed9ba4be238444a0ade5ba236e0ed518f2e0b024d43d638d7*
Pure Prop Address:	*TMQJ6hARogVv6a8W1vmi4QLJdstmu7vvsko*
Theory Prop Id:	*ee9c238cf553597102f9b9ee4da351edf8550e6a9298c9f211be331552a2cb2b*
Theory Prop Address:	*TMRRB6iU5Y8ULDEdxSDS6ZdHUDYdozzKnx2*

`add_SNo_ordinal_SL`

Theorem 28.50

$$\forall\alpha.\mathsf{ordinal}\ \alpha\to\forall\beta.\mathsf{ordinal}\ \beta\to$$
$$\mathsf{ordsucc}\ \alpha+\beta=\mathsf{ordsucc}\ (\alpha+\beta).$$

The proposition is identified by the following information:

Pure Prop Id: d55bd8e80d14016f86e065ddbbdfb226571ce2f30e4aebe8b4e0aca9e5092472
Pure Prop Address: TMQnHkNidg5FWQLhSNenPvjDxCjnJdeF1bF
Theory Prop Id: 61ae57513d7783186f7327027f1807ee424c8d2af57399e9fb52c5685603f457
Theory Prop Address: TMcBJQnghsqzp4UQfnx4PBJi2B8VvW3agRn

add_SNo_ordinal_SR

Theorem 28.51

$$\forall \alpha.\text{ordinal } \alpha \to \forall \beta.\text{ordinal } \beta \to \alpha + \text{ordsucc } \beta = \text{ordsucc } (\alpha + \beta).$$

The proposition is identified by the following information:

Pure Prop Id: e7378a22af92b7c147bbd7898baff5455a0ef96fa00ffdf898122a3bf428bdd3
Pure Prop Address: TMXcxTEQ97P4n3PTFXCCqgWyQErB5tL9Za7
Theory Prop Id: 0c1a6df8d4077eb378e14555b16968f4b38ec67e6f97bbeefe5a78c70e15a227
Theory Prop Address: TMLPo24YiNr9FHf939LUvMLAynxujJ8Qhjg

add_SNo_ordinal_InL

Theorem 28.52

$$\forall \alpha.\text{ordinal } \alpha \to \forall \beta.\text{ordinal } \beta \to \forall \gamma \in \alpha.\gamma + \beta \in \alpha + \beta.$$

The proposition is identified by the following information:

Pure Prop Id: 09a829e8ab6f31d46768fdcb929196886d6328bcf64c285f93a561a65c1398aa
Pure Prop Address: TMRVsTCaSviQYts1RQeAwzNWGiuAUb5kYUF
Theory Prop Id: 5acda8957c70e073f723185142d6e08818fa09b36f05b1bd211a60f1ba14fdf9
Theory Prop Address: TMVTmvXxRgnniTCwKdyop3YHpKGaSbz6KZo

add_SNo_ordinal_InR

Theorem 28.53

$$\forall \alpha.\text{ordinal } \alpha \to \forall \beta.\text{ordinal } \beta \to \forall \gamma \in \beta.\alpha + \gamma \in \alpha + \beta.$$

The proposition is identified by the following information:

Pure Prop Id: d291f44187e6051da6ee1d562ded685ef47c231d1bf1f5d37d7dda55a8e5ca3f
Pure Prop Address: TMNJwMYCWAjhoPBNGGAFssvdn2KaxT6X5zC
Theory Prop Id: 0f42b5bcdea755e828207f260bca45c0356328488660727bd8cab53262b609b3
Theory Prop Address: TMSm47BPo5yfVb687cN4t62bbsfGbbbXCiA

add_nat_add_SNo

Theorem 28.54 $\forall nm \in \text{omega.add\_nat } n\ m = n + m.$ *The proposition is identified by the following information:*

Pure Prop Id: *1701fc5ed7579abb5b201f5ec8a96acd2ac68046ad3176c036d134dea5669afe*
Pure Prop Address: *TMXMKYNwgEJbbeSZcXdj5a87FYKoc4K9apt*
Theory Prop Id: *9df659a7b32d9f3afa10987ffbc9d9a84706cca922950d969a3d81f702e1b893*
Theory Prop Address: *TMGmoXE93EaXQanV3zMwj3Fp6HW7SWjG9xx*

add_SNo_In_omega

Theorem 28.55 $\forall nm \in \mathsf{omega}.n + m \in \mathsf{omega}$. *The proposition is identified by the following information:*

Pure Prop Id: *f45cbb9681681aaabac9270b640eb30171eb36f216ec7a99e5823ef45f94066a*
Pure Prop Address: *TMPib41CNAq7UTPnewz4ZBfR7FdVnYWEtRy*
Theory Prop Id: *eb6375250d942b2db2172dc601d939190506da07644a1cc5fdce38ab22ecf5fa*
Theory Prop Address: *TMP489nANG759rumydRqmyjQssxqD72fosB*

add_SNo_SNoL_interpolate

Theorem 28.56

$$\forall xy.\mathsf{SNo}\ x \rightarrow \mathsf{SNo}\ y \rightarrow \forall u \in \mathsf{SNoL}\ (x + y).$$
$$(\exists v \in \mathsf{SNoL}\ x.u \leq v + y) \vee (\exists v \in \mathsf{SNoL}\ y.u \leq x + v).$$

The proposition is identified by the following information:

Pure Prop Id: *97136316ef3ba2048fc844999ceaa34b07167a3712ce76f041d8382dbfc0b380*
Pure Prop Address: *TMFeNW7HkxH1nH7hkD7ZZwNKgZJpFXGy2jK*
Theory Prop Id: *3cafc181e5e0dd3820d6f9d73c0ed6639ab60e79224ef8a2c5119732a877cd30*
Theory Prop Address: *TMMVRDZraKVB3LvPdW6kCBnJker3CjaSPfz*

add_SNo_SNoR_interpolate

Theorem 28.57

$$\forall xy.\mathsf{SNo}\ x \rightarrow \mathsf{SNo}\ y \rightarrow \forall u \in \mathsf{SNoR}\ (x + y).$$
$$(\exists v \in \mathsf{SNoR}\ x.v + y \leq u) \vee (\exists v \in \mathsf{SNoR}\ y.x + v \leq u).$$

The proposition is identified by the following information:

Pure Prop Id: *928f71993ee178fd38105f25f4ad163aeeb97d5ac1bbb4c218f4c3b81d4cd25f*
Pure Prop Address: *TMX122A6CkJ17L11g6XTAB5Z87Xwqzq3F4H*
Theory Prop Id: *31fa9428aeddf723102cc0d5660dce0ce0e1e068f2bd5d6fd5d853418947a80e*
Theory Prop Address: *TMTKJ9smZabCsGAHoewvLNHykWiCKJuNchm*

add_SNo_assoc

Theorem 28.58

$$\forall xyz.\mathsf{SNo}\ x \rightarrow \mathsf{SNo}\ y \rightarrow \mathsf{SNo}\ z \rightarrow x + (y + z) = (x + y) + z.$$

The proposition is identified by the following information:

Pure Prop Id: *39b331b6381b6e5ee82cce99630525f4cb0ca03008347a4acf301031da6002ce*
Pure Prop Address: *TMUConoLia3kWzpgXw8RzijjEGkNtvGJecF*
Theory Prop Id: *bc15ad70cce2a78c1f054e4e77fa522dd72c07d6620fb831284863016cf6e3da*
Theory Prop Address: *TMFHBgSLqG7xXui9hnNbAKuSE67kqnChBbz*

add_SNo_cancel_L

Theorem 28.59 $\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to x+y = x+z \to y = z.$ *The proposition is identified by the following information:*

Pure Prop Id: *0140969cf9285d05d4b30bf21c910adfed405122d410120abd72c11f5ab69789*
Pure Prop Address: *TMV4roc7BLNMpAapZh7KA7NtFj11mPqdozE*
Theory Prop Id: *781256c452aceda44607ee4205e8ee86d1eea0b3f32743f53b342771142ce743*
Theory Prop Address: *TMHbLqftV2LcfDUv7HqxqMhm7CVsY7Ef2K8*

add_SNo_cancel_R

Theorem 28.60 $\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to x+y = z+y \to x = z.$ *The proposition is identified by the following information:*

Pure Prop Id: *3de43c040780ea5cd150c912ec06fac7c4f463b8b0a89ff1d2cba1c673d3b3d6*
Pure Prop Address: *TMLNfa9NDWM7B9HYGBe7xuczocSytHXKARC*
Theory Prop Id: *e62801806a6a72a797723a605192e13732faf66171abd8d9323db7972673a977*
Theory Prop Address: *TMVQQYhZPbjCpQtrWniV5GvvRBpj4GaPPzW*

minus_add_SNo_distr

Theorem 28.61 $\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to -(x+y) = (-x)+(-y).$ *The proposition is identified by the following information:*

Pure Prop Id: *57ec9da45362cd7c9418befe3fb2ca772e55318bd76cfac89139baf603dd198c*
Pure Prop Address: *TMQTvHftZnQzgoUvhX3DhSiB9EYGcjbMEVt*
Theory Prop Id: *da8d5a3c4332f78b540e7fb374f3ba393f34ebef39aa08f03a43e8a0eb0ec48f*
Theory Prop Address: *TMMrTspPpgWYYtPV5GfMyFKZxbTpr6YpBF8*

minus_add_SNo_distr_3

Theorem 28.62

$$\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to -(x+y+z) = -x + -y + -z.$$

The proposition is identified by the following information:

Pure Prop Id: *b074acb051db50b8aebd6c84b804f36ef95546c8c1d8b4a12fad41b6036b675d*
Pure Prop Address: *TMHG5Wzxu3ufeWeCBHAySJy5K1FsFRffozM*
Theory Prop Id: *94dbf3383e5ad97c01e0494f7ad58f05eb86370106ca98f801505d5cc17d8e8c*
Theory Prop Address: *TMGDtLEgxm1wM1omKuqmT7qds1DiL446e61*

add_SNo_Lev_bd

Theorem 28.63

$$\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNoLev}\ (x+y) \subseteq \mathsf{SNoLev}\ x + \mathsf{SNoLev}\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*0f257c03f4bd792cfafdf5b6628524966280c12bc90c0b7152a74c1c453b0a97*
Pure Prop Address:	*TMHLNim923vpzTQhz7fmKu2bm4ncVWUmWEE*
Theory Prop Id:	*0680effc26564a3378ad068703ae226003b02850f73e91331345c41424701bcf*
Theory Prop Address:	*TMKzYXGUEBsqkz6Pd1y7TGUmq9kreetmiSK*

add_SNo_SNoS_omega

Theorem 28.64 $\forall xy \in \mathsf{SNoS\_}\ \mathsf{omega}.x+y \in \mathsf{SNoS\_}\ \mathsf{omega}$. *The proposition is identified by the following information:*

Pure Prop Id:	*9311032f024de4324b4cbbbc04cbbc6f507e4664b92e5daf8d2b539deaa0d877*
Pure Prop Address:	*TMRdieTgkX8dUfZyNNYHypf4kTH3PoJW3Ms*
Theory Prop Id:	*6d16aa7a6b095f74f1b34091c0e9a002aa247761437c60a7c0261e4d1687a1d4*
Theory Prop Address:	*TMVjF1tKn3421u44CyHtBcSNvHoJBqRKST1*

add_SNo_minus_R2

Theorem 28.65 $\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to (x+y) + -y = x$. *The proposition is identified by the following information:*

Pure Prop Id:	*3e7b333f3cd5354e81de9e18e82adb1ea3101121fa6de8717d4d9dd7a8de2dfc*
Pure Prop Address:	*TMbjFdWsQkwJkFdFKsygecXhLiJUroenET2*
Theory Prop Id:	*91f4efb884c7f253c44c5fd69b6ef119bd466cbf203a587487e6a292ebc37f9e*
Theory Prop Address:	*TMdam15xQW7pR7unsMT5iH6SPdFATTi712m*

add_SNo_Lt1_cancel

Theorem 28.66 $\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to x+y < z+y \to x < z$. *The proposition is identified by the following information:*

Pure Prop Id:	*e5c11e1cc1b83566aacf4247c67eda38cbd881bcf48b4a30fd81d1ae9be3ae25*
Pure Prop Address:	*TMVGk6XaUoPX6ZGVPLEVZWzvzKkLektMYLP*
Theory Prop Id:	*2ec8aef4360c9ee2ffa1912943451de1852b1cf2af0400f6da375d69c467ae9e*
Theory Prop Address:	*TMLJkdMCCCn6YVEdM7uNU6PFwazJo7w6REd*

add_SNo_Lt2_cancel

Theorem 28.67 $\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to x+y < x+z \to y < z$. *The proposition is identified by the following information:*

Pure Prop Id:	*9ecfc82c2e1b3fbbf14127fd056f50b5f4f4058c7b9c2da324e8e11162871620*
Pure Prop Address:	*TMJyFpf59yM4gCj8nhUhoxe5MLMc9ZvK1hj*
Theory Prop Id:	*d4afbdcdb563fce45f9fdb7e6d26326d06598fc21bee88b552fb7439b946957b*
Theory Prop Address:	*TMF8QKcffVRxMAdSUMqMY8FSyHY2Pu3AYJc*

add_SNo_assoc_4

Theorem 28.68

$$\forall xyzw.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to \mathsf{SNo}\ w \to x + y + z + w = (x + y + z) + w.$$

The proposition is identified by the following information:

Pure Prop Id:	*b9919a859ce250ebed40a38e210d51ea56ad81bd3b548d2973c10fa4b77224fb*
Pure Prop Address:	*TMNn2vs2scHFb5sPooErQtoePq5skwF4Rqs*
Theory Prop Id:	*f2ec8cf0669d6b7f4ba80ac3f1c7fc3a38bc969d82b8ab446307e654aeb60211*
Theory Prop Address:	*TMJdJZYxC1SYeUZNH9CnoUzpc5nbzLd4E1K*

add_SNo_com_3_0_1

Theorem 28.69 $\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to x + y + z = y + x + z$. *The proposition is identified by the following information:*

Pure Prop Id:	*53adb5589660e895a568e553cecd4c833dfe11b8d4b0a59395733cf1473d5a4f*
Pure Prop Address:	*TMan13aAzSS4boU3k6BCV2m9HpcAqzjMqYs*
Theory Prop Id:	*3c9cc8b1863d8757dc461ba3222ea23daf40651a26eb8bbaa4eb74d8ce5ecb34*
Theory Prop Address:	*TMLUi74QYWaLowsgzo8D5KaFFmMGAF33Uov*

add_SNo_com_4_inner_flat

Theorem 28.70

$$\forall xyzw.\mathsf{SNo}\ y \to \mathsf{SNo}\ z \to \mathsf{SNo}\ w \to x + y + z + w = x + z + y + w.$$

The proposition is identified by the following information:

Pure Prop Id:	*353c422c4473fec6d253f64dc6011273518f6fd9b8666b1d04ac4b7a10b21676*
Pure Prop Address:	*TMYagwq4AL9eS2satFuJrRGsXJQ7UaQ2Y2Z*
Theory Prop Id:	*d32d7fe4fd859c93a5b059b561d61395eae0d2dc1771536899a51aa7290836c3*
Theory Prop Address:	*TMJ7ALWqQknTtDK5LC67JaovH5kH3hMjM3S*

add_SNo_com_3b_1_2

Theorem 28.71

$$\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to (x + y) + z = (x + z) + y.$$

The proposition is identified by the following information:

Pure Prop Id:	*2ccf2bcc11ca672b64552dc368e1746da109c4c422678991b3ab7c3011849772*
Pure Prop Address:	*TMPuoTqK9f79je6oZPXi6p9wEreuXeYKFpg*
Theory Prop Id:	*f2dd80a1f9d4848b30eb661596f9b7355f21ffc0ff2b67d04df7791c06f27387*
Theory Prop Address:	*TMXQdGRpcSTBjZ8pjJmoA2DCTzkK5tBAKzR*

add_SNo_com_4_inner_mid

Theorem 28.72

$$\forall xyzw.\mathsf{SNo}\ x \rightarrow \mathsf{SNo}\ y \rightarrow \mathsf{SNo}\ z \rightarrow \mathsf{SNo}\ w \rightarrow (x+y)+(z+w)=(x+z)+(y+w).$$

The proposition is identified by the following information:

Pure Prop Id:	*635cb763d6883b1df327b97d296852bd72be35ebf27ba3197cc20d58b5db1923*
Pure Prop Address:	*TMNBs9W7CDbVpEM2DJAD6LJaUJdzLs5mL2p*
Theory Prop Id:	*e25ee6de8e445a677f9f91027aeb3dc6ab5acb07b724880668b83cb66a9d287c*
Theory Prop Address:	*TMRqXvHGUrVc7Tj4T8TaayZuLEe7vs6QkAa*

add_SNo_rotate_3_1

Theorem 28.73 $\forall xyz.\mathsf{SNo}\ x \rightarrow \mathsf{SNo}\ y \rightarrow \mathsf{SNo}\ z \rightarrow x+y+z=z+x+y.$ *The proposition is identified by the following information:*

Pure Prop Id:	*51654e504536bbcc3c5f757f3dcad5e306f61cb3f3113fe097e1d1fbba5aac97*
Pure Prop Address:	*TMNHRTRCD9bsMPb3UbC8jvRrJ6N9ZH1Eh4N*
Theory Prop Id:	*340640be61f33faa3a7f153679d19bb2ce9291b5efec590860ba767ade73a9a3*
Theory Prop Address:	*TMQNLhVAcRPvXzedEwagyyteqQKc581dhD5*

add_SNo_rotate_4_1

Theorem 28.74

$$\forall xyzw.\mathsf{SNo}\ x \rightarrow \mathsf{SNo}\ y \rightarrow \mathsf{SNo}\ z \rightarrow \mathsf{SNo}\ w \rightarrow x+y+z+w=w+x+y+z.$$

The proposition is identified by the following information:

Pure Prop Id:	*4bc7aa1aee0011ef665476863c178f590fbe4fbe6699dd7ccb934845846d8dd3*
Pure Prop Address:	*TMaTstSL13vyKcGtLbTNn8eAVSGWCpDMRRP*
Theory Prop Id:	*ce0a6b44cbcebe4b78d9bcbc4db2e540564f76969de5d18f22a263048dfe8518*
Theory Prop Address:	*TMYrz7XuuFsQmh65FTDfV3W2dcW9j9WhNCe*

add_SNo_rotate_5_1

Theorem 28.75

$$\forall xyzwv.\mathsf{SNo}\ x \rightarrow \mathsf{SNo}\ y \rightarrow \mathsf{SNo}\ z \rightarrow \mathsf{SNo}\ w \rightarrow \mathsf{SNo}\ v \rightarrow$$
$$x+y+z+w+v=v+x+y+z+w.$$

The proposition is identified by the following information:

Pure Prop Id:	*4b4b645a9de69d196bcfac0f564a76aba4c24d5518f13324b64621276d543128*
Pure Prop Address:	*TMSSCZTFkPsvFM1mXZ7MP4aihGyFoBRfhdC*
Theory Prop Id:	*0714aabd91b09f4e30b2978cd4c05acb5975913fcf91782f98282a06ecc98e76*
Theory Prop Address:	*TMKsqf29EQpew5TASzktuUb1DHqMTYo9542*

add_SNo_rotate_5_2

Theorem 28.76

$$\forall xyzwv.\mathsf{SNo}\ x\to\mathsf{SNo}\ y\to\mathsf{SNo}\ z\to\mathsf{SNo}\ w\to\mathsf{SNo}\ v\to x+y+z+w+v=w+v+x+y+z.$$

The proposition is identified by the following information:

Pure Prop Id:	*33b5cc61253fa4c067f352dd7d4f385b800a18a8d891459904b242726be1c98b*
Pure Prop Address:	*TMNP66fS26FKAKuh9MEtRW9VDian3FYj9qr*
Theory Prop Id:	*fec9d019d82128a582a52e0bc0e64cfc44a05d4f84f9952d8cb92946b031856d*
Theory Prop Address:	*TMUguce5TL82mq83tgNMXbiPmFe3zcgvQFJ*

add_SNo_minus_SNo_prop1

Theorem 28.77 $\forall xy.\mathsf{SNo}\ x\to\mathsf{SNo}\ y\to -x+x+y=y.$ *The proposition is identified by the following information:*

Pure Prop Id:	*2cd65e577fae467be069f5ab9a2bfa88c578085e70deca674f146625df01bb31*
Pure Prop Address:	*TMNj1ZknUmnj8fyskPShRtyeBwzp2553NVN*
Theory Prop Id:	*598ee6b17ec7a3c47d15dffa959d384acc4d4d5bdbcd02790c13e2ff045e1440*
Theory Prop Address:	*TMKE2xaN2B6wD8vGVfwUJaufhki7EhAkGfE*

add_SNo_minus_SNo_prop2

Theorem 28.78 $\forall xy.\mathsf{SNo}\ x\to\mathsf{SNo}\ y\to x+-x+y=y.$ *The proposition is identified by the following information:*

Pure Prop Id:	*632a4d661bba9435289e33a84ddfd6e1ebe55256f17529d37fd7bf784a245260*
Pure Prop Address:	*TMFUMMG5NWfvwVqV1VzWhtid5D1cW3ENDqd*
Theory Prop Id:	*a8450da6ba6ffd91e010f90197162261ef21fb4e025e591e5137531e12d7ad70*
Theory Prop Address:	*TMJQtEivAHnWG6vm64j8jL7rdfeCLJv59AH*

add_SNo_minus_SNo_prop3

Theorem 28.79

$$\forall xyzw.\mathsf{SNo}\ x\to\mathsf{SNo}\ y\to\mathsf{SNo}\ z\to\mathsf{SNo}\ w\to(x+y+z)+(-z+w)=x+y+w.$$

The proposition is identified by the following information:

Pure Prop Id:	*038fdd7f7b41d5dbabb1c2b43f6ee18b5eb51c7b73b574795aa81a5f6e7185db*
Pure Prop Address:	*TMQQTF6qMromiAnZNHGPbLUmDnoRJJsw184*
Theory Prop Id:	*0d4f2459906cf0deac11ccfa5d8ce0300b8a0e2e2e35908d116f376387aeaf89*
Theory Prop Address:	*TMVtSJwaz4PKT2M9971YpjjmSqY2uXJQFDe*

add_SNo_minus_SNo_prop4

Theorem 28.80

$$\forall xyzw.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to \mathsf{SNo}\ w \to (x + y + z) + (w + -z) = x + y + w.$$

The proposition is identified by the following information:

Pure Prop Id:	*9303242681e65d4bb41186882b5827d87cfb757866a860dcd21e8d85c6540d00*
Pure Prop Address:	*TMZxz5UVrwqNzRN8eXdiojfqVbJhkXRzfpt*
Theory Prop Id:	*4c3067acc33707997f49c35d6a0ac8a1b7845a5ed0454d39acbfe19ba57d0c1f*
Theory Prop Address:	*TMWftxv2baJqYmzmKYPcnYgd25XrtWoXJST*

add_SNo_minus_SNo_prop5

Theorem 28.81

$$\forall xyzw.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to \mathsf{SNo}\ w \to (x + y + -z) + (z + w) = x + y + w.$$

The proposition is identified by the following information:

Pure Prop Id:	*bdfd3ff33cf2f7c3beec961e55cb8c70e3e72bedc8e24a5079ae13e57cef924e*
Pure Prop Address:	*TMbd9Mg6tPihAjiCRTdu2H9iHTJu8P6vdH9*
Theory Prop Id:	*eeb587de80f26e1b6307d8991dce2c6ed3d960c0a1571b8722cff655214df698*
Theory Prop Address:	*TMRmP8fuJtFdvTRXG6bJ2zsSkasVFLW8YiM*

add_SNo_minus_Lt1

Theorem 28.82 $\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to x + -y < z \to x < z + y.$
The proposition is identified by the following information:

Pure Prop Id:	*643876090ca9e8c818bb1d1517026337499a8a61a6383863a9f6b90713b953f8*
Pure Prop Address:	*TMHorPzXGJj6NLSDuVdotitbnKv8te63hAg*
Theory Prop Id:	*4033dc222689776035a38336a7f6ce157593e8786381729a207ecdc69f8508d1*
Theory Prop Address:	*TMGgmNN1c8BXxQunMKeC7ctpH6iBXxvzUXy*

add_SNo_minus_Lt1b

Theorem 28.83 $\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to x < z + y \to x + -y < z.$
The proposition is identified by the following information:

Pure Prop Id:	*5012b14b32060175e560f0d1e7e017da053c610000fa170aeaf365e6d6410d7c*
Pure Prop Address:	*TMGs3ZQCrCC6dtPSMkxhL3RGUTqUer9SdGs*
Theory Prop Id:	*0b5cca84918d73e36a13fd959569d5c6a5d4a05c85317235433f59a19bc601a2*
Theory Prop Address:	*TMPp2LmW4bmLHnLXjmQuZtQpoHAz4uTwF1i*

add_SNo_minus_Lt2

Theorem 28.84 $\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to z < x + -y \to z + y < x.$
The proposition is identified by the following information:

Pure Prop Id:	*76c543653b6da539ad0d8357582da1ab031b9383d2fae35a230e98de8946856e*
Pure Prop Address:	*TMFL2J2Xi9knH1EYexCMF24iaPSDRahCQtP*
Theory Prop Id:	*3621456ff64800b7e93dff3968e21ba0755569c11a30efbf0360787c625c56dc*
Theory Prop Address:	*TMSsbaxypo5H5nF8jvxTgrgpmxmVfGPHE3p*

add_SNo_minus_Lt2b

Theorem 28.85 $\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to z + y < x \to z < x + -y.$
The proposition is identified by the following information:

Pure Prop Id:	*24d1a6c7194f01d2e61624e193908bc5ed6870b188d549fdbfafb81f3e69660b*
Pure Prop Address:	*TMYLxBcGLaYBenPqsTbU48VLhTDu35ZvWin*
Theory Prop Id:	*68419210383aa804b0c97019a1190bf0a75fb45009b55d3f310cccb98a55f216*
Theory Prop Address:	*TMLRXmb9LNiNLw5Y1urKn6AQHWWwgKREUkY*

add_SNo_minus_Lt1b3

Theorem 28.86

$$\forall xyzw.\mathsf{SNo}\ x\to\mathsf{SNo}\ y\to\mathsf{SNo}\ z\to\mathsf{SNo}\ w\to x + y < w + z\to x + y + -z < w.$$

The proposition is identified by the following information:

Pure Prop Id:	*218e4f25517f8ff1478046e3df782ae832d9f7e33e0b433bf05b9643a0208e85*
Pure Prop Address:	*TMRSZGz7nE2AVHGmgMaepm9CAmfWzJ72Boq*
Theory Prop Id:	*8ea4a7cab08aeec1b407bd563474dd6766fbcf0794f34b125012f4f8d61f021f*
Theory Prop Address:	*TMRrXMsdvggvcKCwoxKF8kLUbznUq5NaVmd*

add_SNo_minus_Lt2b3

Theorem 28.87

$$\forall xyzw.\mathsf{SNo}\ x\to\mathsf{SNo}\ y\to\mathsf{SNo}\ z\to\mathsf{SNo}\ w\to w + z < x + y\to w < x + y + -z.$$

The proposition is identified by the following information:

Pure Prop Id:	*d9462b9396765bcb6e599c448b5b2a0e15bfeea62de5713637e71fe91bc14e06*
Pure Prop Address:	*TMG6X9g8zK9X5PbsVyTtDpJgN73SyRkZBJ2*
Theory Prop Id:	*4bf57517c0115e8dae375c95d1c1bf403694f4ee328c9dfd9c34f22eca40d62b*
Theory Prop Address:	*TMGSKHUM3KGisvSLGi1Zpy9MG4UtFxqFThU*

add_SNo_Lt_subprop2

Theorem 28.88

$$\forall xyzwuv.\mathsf{SNo}\ x\to\mathsf{SNo}\ y\to\mathsf{SNo}\ z\to\mathsf{SNo}\ w\to\mathsf{SNo}\ u\to\mathsf{SNo}\ v\to x + u < z + v\to y + v < w + u\to x + y < z + w.$$

The proposition is identified by the following information:

Pure Prop Id:	*848909edfa645bd860cf5cc6acec778e23e88c5a769fe3c959d4b3566fbb306b*
Pure Prop Address:	*TMLUrjDQwdJ3cSJVaNX11wZPbmHmYJm2F4A*
Theory Prop Id:	*7f1b50ad173ad970e6eee06a0d1dedb182caf1cfff89406b30003a7afc38c6c3*
Theory Prop Address:	*TMcJPm4yGSHS1kUSPRFgEHGenN7vG4V1Nzw*

`add_SNo_Lt_subprop3a`

Theorem 28.89

$$\forall xyzwua.\mathsf{SNo}\ x\to\mathsf{SNo}\ y\to\mathsf{SNo}\ z\to\mathsf{SNo}\ w\to\mathsf{SNo}\ u\to\mathsf{SNo}\ a\to x+z<w+a\to y+a<u\to x+y+z<w+u.$$

The proposition is identified by the following information:

Pure Prop Id:	*43322d509ad5f4223a3ea2d226e4b0ccb8c26b9a1175adae9d1d447e776bf7c0*
Pure Prop Address:	*TMMYB2kCLPC8xoSmU63JvGfNSPFzUYuK6pv*
Theory Prop Id:	*cde0c9b5a2910f46768762e1f3849c4e1b8cda57c9c1575aa1b9bf556baa627d*
Theory Prop Address:	*TMPNtwtWveLBYLorWYj4fRoEpqz56R9oTm5*

`add_SNo_Lt_subprop3b`

Theorem 28.90

$$\forall xywuva.\mathsf{SNo}\ x\to\mathsf{SNo}\ y\to\mathsf{SNo}\ w\to\mathsf{SNo}\ u\to\mathsf{SNo}\ v\to\mathsf{SNo}\ a\to x+a<w+v\to y<a+u\to x+y<w+u+v.$$

The proposition is identified by the following information:

Pure Prop Id:	*9d5c37e2e6a26c6dc29e00cf8767d02fc77b30d09cebed0550f159cb9d1334c6*
Pure Prop Address:	*TMakKb23RRtRLWKpGuoMNwnASjKertEftag*
Theory Prop Id:	*b197b177985ff32095d16adfad88ff6b97c3c9885a5a1cadc19c5cb553b4345d*
Theory Prop Address:	*TMduddfsYHhZALrRDzTwzypdKDVHK1k8VYh*

`add_SNo_Lt_subprop3c`

Theorem 28.91

$$\forall xyzwuabc.\mathsf{SNo}\ x\to\mathsf{SNo}\ y\to\mathsf{SNo}\ z\to\mathsf{SNo}\ w\to\mathsf{SNo}\ u\to\mathsf{SNo}\ a\to\mathsf{SNo}\ b\to\mathsf{SNo}\ c\to x+a<b+c\to y+c<u\to b+z<w+a\to x+y+z<w+u.$$

The proposition is identified by the following information:

Pure Prop Id:	*18461471e206fc8672b0e953edf289af2051148c543ab9c39752e75fd74b9902*
Pure Prop Address:	*TMLDVH1R1XQXWmhFyN7kxcnxZ8vV1jj1gge*
Theory Prop Id:	*f8cf2ba034b2ae66a187647fe084618a9c62057a7ab092702eb1355e51bfdb56*
Theory Prop Address:	*TMNUyCkvtK5vp56fDjYpDWz4tisomP2mVS5*

`add_SNo_Lt_subprop3d`

Theorem 28.92

$$\forall xywuvabc.\mathsf{SNo}\ x\to\mathsf{SNo}\ y\to\mathsf{SNo}\ w\to\mathsf{SNo}\ u\to\mathsf{SNo}\ v\to\mathsf{SNo}\ a\to\mathsf{SNo}\ b\to \mathsf{SNo}\ c\to x+a<b+v\to y<c+u\to b+c<w+a\to x+y<w+u+v.$$

The proposition is identified by the following information:

Pure Prop Id:	*a447c6d932bea68dbde54b06fe58a08cf250a32d1e9e252f9aa646d7d450a31f*
Pure Prop Address:	*TMGmodxxtB7938aypz74aGsuhYxLsXh57g5*
Theory Prop Id:	*a1520a522ad3deb92a6746488e5cfc1dfb1d1fd6684fe79e87bba20cda4756b6*
Theory Prop Address:	*TMLrgfSnAmTdRkGo9xWrngAvimcZVK9yU2d*

`ordinal_ordsucc_SNo_eq`

Theorem 28.93 $\forall\alpha.\mathsf{ordinal}\ \alpha\to\mathsf{ordsucc}\ \alpha=1+\alpha$. *The proposition is identified by the following information:*

Pure Prop Id:	*f0596d71a0270ee871aebb4abb6a468185b1e2d2deec29fda29af07ac94080e7*
Pure Prop Address:	*TMKvpBUtRCutc7AQ2jjzLG87zkaCypGHwqw*
Theory Prop Id:	*36bf6dd6ade137d7a6431b1d2f74c22803884fe3bcc9f6b223f90c9587bc1fde*
Theory Prop Address:	*TMM3U1W3pWWJb6gkvcZQNx2QDYDK4UxBhJx*

`add_SNo_omega_eps_Lt`

Theorem 28.94

$$\forall xy\in\mathsf{SNoS\_}\ \mathsf{omega}.x<y\to\exists n\in\mathsf{omega}.x+\mathsf{eps\_}\ n<y.$$

The proposition is identified by the following information:

Pure Prop Id:	*5a850b3a5f7e8e409aa7b3a8fde3eaf603b6c57ac0565694a8d02fbe81aa1496*
Pure Prop Address:	*TMSCHeh28B5U2XMhbESr5SKSXVCskfzuwZx*
Theory Prop Id:	*0afdf612e70b6faf98df67163f00af4cacf8720792da2450f3f84ab2b65e27b4*
Theory Prop Address:	*TMYHZueBTibUJWtZVQUjiVzfGCegt89U2VT*

Definition 28.3 mul_SNo *is the opaque object of type* $\iota\to\iota\to\iota$ *identified by the following information:*

Pure Object Id:	*48d05483e628cb37379dd5d279684d471d85c642fe63533c3ad520b84b18df9d*
Pure Object Address:	*TMSw7TH4yUrNvqkxFpRdKgaGjhbMXKDtVKM*
Theory Object Id:	*b14730ddd09967223586b7b73ba059d8fee46ef74b4d5c13b7276c084e8f3dea*
Theory Object Address:	*TMdQsgnzh6YiiBs9ki4XhdobvDFEYV9QYKG*

Notation. We use $*$ as a right associative infix operator corresponding to applying term mul_SNo.

`mul_SNo_eq`

Theorem 28.95

$$\forall x.\mathsf{SNo}\ x \to \forall y.\mathsf{SNo}\ y \to$$
$$x * y =$$
$$\mathsf{SNoCut}$$
$$(\{(w\ 0) * y + x * (w\ 1) + -(w\ 0) * (w\ 1) | w \in \mathsf{SNoL}\ x \times \mathsf{SNoL}\ y\} \cup$$
$$\{(z\ 0) * y + x * (z\ 1) + -(z\ 0) * (z\ 1) | z \in \mathsf{SNoR}\ x \times \mathsf{SNoR}\ y\})$$
$$(\{(w\ 0) * y + x * (w\ 1) + -(w\ 0) * (w\ 1) | w \in \mathsf{SNoL}\ x \times \mathsf{SNoR}\ y\} \cup$$
$$\{(z\ 0) * y + x * (z\ 1) + -(z\ 0) * (z\ 1) | z \in \mathsf{SNoR}\ x \times \mathsf{SNoL}\ y\}).$$

The proposition is identified by the following information:

Pure Prop Id:	*726bd964aa39e59e3a6693e9b7d4306b1accb9a6c62ae1891b050b131c4ea01f*
Pure Prop Address:	*TMSnEDskXoZEWD5U2DVgxtTRUp75JLmq8fL*
Theory Prop Id:	*3525c5e458ce4f160f6f38c46b032eb43534d43481b8d0ff0b1096daa93e9080*
Theory Prop Address:	*TMR2SBeeUCWzXPrVEFQprrZoRPjqi24Acwd*

`mul_SNo_eq_2`

Theorem 28.96

$$\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \forall p : o.$$
$$(\forall LR.(\forall u.u \in L \to$$
$$(\forall q : o.(\forall w0 \in \mathsf{SNoL}\ x.\forall w1 \in \mathsf{SNoL}\ y.u = w0 * y + x * w1 + -w0 * w1 \to q) \to$$
$$(\forall z0 \in \mathsf{SNoR}\ x.\forall z1 \in \mathsf{SNoR}\ y.u = z0 * y + x * z1 + -z0 * z1 \to q) \to q))$$
$$\to$$
$$(\forall w0 \in \mathsf{SNoL}\ x.\forall w1 \in \mathsf{SNoL}\ y.w0 * y + x * w1 + -w0 * w1 \in L) \to$$
$$(\forall z0 \in \mathsf{SNoR}\ x.\forall z1 \in \mathsf{SNoR}\ y.z0 * y + x * z1 + -z0 * z1 \in L) \to$$
$$(\forall u.u \in R \to$$
$$(\forall q : o.(\forall w0 \in \mathsf{SNoL}\ x.\forall z1 \in \mathsf{SNoR}\ y.u = w0 * y + x * z1 + -w0 * z1 \to q) \to$$
$$(\forall z0 \in \mathsf{SNoR}\ x.\forall w1 \in \mathsf{SNoL}\ y.u = z0 * y + x * w1 + -z0 * w1 \to q) \to q))$$
$$\to$$
$$(\forall w0 \in \mathsf{SNoL}\ x.\forall z1 \in \mathsf{SNoR}\ y.w0 * y + x * z1 + -w0 * z1 \in R) \to$$
$$(\forall z0 \in \mathsf{SNoR}\ x.\forall w1 \in \mathsf{SNoL}\ y.z0 * y + x * w1 + -z0 * w1 \in R) \to$$
$$x * y = \mathsf{SNoCut}\ L\ R \to p)$$
$$\to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*97f72a94a57b9c976cabb7db6e1c9e782baccf6d99699b64f549f38a4a8cb50c*
Pure Prop Address:	*TMLuSvvCfD9Ze7AKPowkWZPkeM2o9mvreGc*
Theory Prop Id:	*d69cc6521bcd2b18a3af424dfe57b3ca7eccc3c292bd43c9f65e2a98dcebcffe*
Theory Prop Address:	*TMRezz7usQBo1aanPbksRyAGNHbrpgWVes4*

mul_SNo_prop_1

Theorem 28.97

$$\forall x.\mathsf{SNo}\ x\to\forall y.\mathsf{SNo}\ y\to\forall p:o.$$
$$(\mathsf{SNo}\ (x*y)\to$$
$$(\forall u\in\mathsf{SNoL}\ x.\forall v\in\mathsf{SNoL}\ y.u*y+x*v<x*y+u*v)\to$$
$$(\forall u\in\mathsf{SNoR}\ x.\forall v\in\mathsf{SNoR}\ y.u*y+x*v<x*y+u*v)\to$$
$$(\forall u\in\mathsf{SNoL}\ x.\forall v\in\mathsf{SNoR}\ y.x*y+u*v<u*y+x*v)\to$$
$$(\forall u\in\mathsf{SNoR}\ x.\forall v\in\mathsf{SNoL}\ y.x*y+u*v<u*y+x*v)\to p)$$
$$\to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*3018e48e1b0b8923d2bc9e30faf03586dced5941b18f9a31b0e91b0d91716d40*
Pure Prop Address:	*TMdaGCethjpbbQ5f2zXiMzy1Jw6trWXkwEA*
Theory Prop Id:	*5e4d160028e2e0d69e57789a6c38f1230eca4d80e5f1e171fc8969764d28dd07*
Theory Prop Address:	*TMZ8JpWhmNRNmu7xFhYLa8nAtcJESfVjnAP*

SNo_mul_SNo

Theorem 28.98 $\forall xy.\mathsf{SNo}\ x\to\mathsf{SNo}\ y\to\mathsf{SNo}\ (x*y)$. *The proposition is identified by the following information:*

Pure Prop Id:	*7fa2e2dd6764040b3f888d899581ea742d829880456c04996546837d770370c5*
Pure Prop Address:	*TMX6qVheSX36EwKWKF211NtEBN8GMqFFQjT*
Theory Prop Id:	*9fd082b661154955f669084ede91b91cef6249e30cbec7c7a954ac883b5ca01b*
Theory Prop Address:	*TMTR3KoWZkq4VBDJf71KPEZUEwdxEBQ1SwZ*

mul_SNo_eq_3

Theorem 28.99

$$\forall xy.\mathsf{SNo}\ x\to\mathsf{SNo}\ y\to\forall p:o.$$
$$(\forall LR.\mathsf{SNoCutP}\ L\ R\to$$
$$(\forall u.u\in L\to$$
$$(\forall q:o.(\forall w0\in\mathsf{SNoL}\ x.\forall w1\in\mathsf{SNoL}\ y.u=w0*y+x*w1+-w0*w1\to q)\to$$
$$(\forall z0\in\mathsf{SNoR}\ x.\forall z1\in\mathsf{SNoR}\ y.u=z0*y+x*z1+-z0*z1\to q)\to q))$$
$$\to$$
$$(\forall w0\in\mathsf{SNoL}\ x.\forall w1\in\mathsf{SNoL}\ y.w0*y+x*w1+-w0*w1\in L)\to$$
$$(\forall z0\in\mathsf{SNoR}\ x.\forall z1\in\mathsf{SNoR}\ y.z0*y+x*z1+-z0*z1\in L)\to$$
$$(\forall u.u\in R\to$$
$$(\forall q:o.(\forall w0\in\mathsf{SNoL}\ x.\forall z1\in\mathsf{SNoR}\ y.u=w0*y+x*z1+-w0*z1\to q)\to$$
$$(\forall z0\in\mathsf{SNoR}\ x.\forall w1\in\mathsf{SNoL}\ y.u=z0*y+x*w1+-z0*w1\to q)\to q))$$
$$\to$$
$$(\forall w0\in\mathsf{SNoL}\ x.\forall z1\in\mathsf{SNoR}\ y.w0*y+x*z1+-w0*z1\in R)\to$$
$$(\forall z0\in\mathsf{SNoR}\ x.\forall w1\in\mathsf{SNoL}\ y.z0*y+x*w1+-z0*w1\in R)\to$$
$$x*y=\mathsf{SNoCut}\ L\ R\to p)$$
$$\to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*51793959fac9070d8e2d9b035efecb4ad9c7ae3ca6da33d55ab3bf797bef3297*
Pure Prop Address:	*TMaj5xJYDn5QmHnKGHqSMLP3XxL1wkJv3vo*
Theory Prop Id:	*720ba07dcca3e0725279be9e2c27ff2f73fa21fb411e18239df4e0915a5111b7*
Theory Prop Address:	*TMWEwehT67Ew9qx9JCyHJdWR4Zn2VcBGx6y*

`mul_SNo_Lt`

Theorem 28.100

$$\forall xyuv.\mathsf{SNo}\ x\to\mathsf{SNo}\ y\to\mathsf{SNo}\ u\to\mathsf{SNo}\ v\to u<x\to v<y\to u*y+x*v<x*y+u*v.$$

The proposition is identified by the following information:

Pure Prop Id:	*428b35f3bc30112249eab05959f8ad7ada3ebbb7987ced62d2fe3afde9af092b*
Pure Prop Address:	*TMLpp8YEH3knCvvN7eHZnAuNDCMtsQzd14d*
Theory Prop Id:	*c232011455c468168fd8e65dfaebb59e340331574a0089936e166d3a4fcf2f5d*
Theory Prop Address:	*TMWuSHJpH8gNAX4DUjm9Wzk6tp7vPQphtfr*

`mul_SNo_Le`

Theorem 28.101

$$\forall xyuv.\mathsf{SNo}\ x\to\mathsf{SNo}\ y\to\mathsf{SNo}\ u\to\mathsf{SNo}\ v\to u\leq x\to v\leq y\to u*y+x*v\leq x*y+u*v.$$

The proposition is identified by the following information:

Pure Prop Id:	*d9a12c5ce8058d66cd1876f20d85be2a5ceec00d4bf569f3282bbee6cb91faab*
Pure Prop Address:	*TMXSQ1sbRtw5pLGrB7wQb2i4sBb3bPWrE3x*
Theory Prop Id:	*acb62030c6b82e1dd945365185266c95047e40137c030a2512820a86905108f6*
Theory Prop Address:	*TMFCxz59dZUG7KrmMeKtLnrjcymT35p7rac*

`mul_SNo_SNoL_interpolate`

Theorem 28.102

$$\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \forall u \in \mathsf{SNoL}\ (x*y).$$
$$(\exists v \in \mathsf{SNoL}\ x.\exists w \in \mathsf{SNoL}\ y.u + v*w \leq v*y + x*w) \vee$$
$$(\exists v \in \mathsf{SNoR}\ x.\exists w \in \mathsf{SNoR}\ y.u + v*w \leq v*y + x*w).$$

The proposition is identified by the following information:

Pure Prop Id:	*7d1b52199f12ab9873941a7285388e03c30e7d2abddf759a627f6f95d4d06ea0*
Pure Prop Address:	*TMd7FoqCiPFwP8mZudWTt9WbKfWC2KEFNpJ*
Theory Prop Id:	*f460fc19e5496a545e8abbaf4188a0ddb17c95a931bb76236bbcc1ab7829c3da*
Theory Prop Address:	*TMQhwb7bstgkPJXZU2FBTPqwJhZ6onk8qEw*

`mul_SNo_SNoL_interpolate_impred`

Theorem 28.103

$$\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \forall u \in \mathsf{SNoL}\ (x*y).\forall p : o.$$
$$(\forall v \in \mathsf{SNoL}\ x.\forall w \in \mathsf{SNoL}\ y.u + v*w \leq v*y + x*w \to p) \to$$
$$(\forall v \in \mathsf{SNoR}\ x.\forall w \in \mathsf{SNoR}\ y.u + v*w \leq v*y + x*w \to p) \to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*395b74682e5fe85f5a9dd02048bafdd0dde83a774140d2e273cde97e801116d8*
Pure Prop Address:	*TMLLQmqPzFYwqdwY8PrqVGLuvrDLR95qgbq*
Theory Prop Id:	*ad6f9609616220556828655 32b9e4b9576d116539cdde8aee2b4846f9ef3628d*
Theory Prop Address:	*TMUjL5DBmrfFRCEH1KtPfFFFPNQ8LHK6fES*

`mul_SNo_SNoR_interpolate`

Theorem 28.104

$$\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \forall u \in \mathsf{SNoR}\ (x*y).$$
$$(\exists v \in \mathsf{SNoL}\ x.\exists w \in \mathsf{SNoR}\ y.v*y + x*w \leq u + v*w) \vee$$
$$(\exists v \in \mathsf{SNoR}\ x.\exists w \in \mathsf{SNoL}\ y.v*y + x*w \leq u + v*w).$$

The proposition is identified by the following information:

Pure Prop Id:	*e1594953c98071d2fbe62599ce28adefdbe6f718c6c29a167036427051a6c7ae*
Pure Prop Address:	*TMb5PPn6ZNjq3UQXjdnTK8NbeGxGWefm1uY*
Theory Prop Id:	*b67a3574b63d97373488eb186c785d77ae947f04968d568abb2eaa58e430ac851*
Theory Prop Address:	*TMUGZi51yni6VZeAy2PCP7s6JRSFQbEGocS*

mul_SNo_SNoR_interpolate_impred

Theorem 28.105

$$\forall xy.\mathsf{SNo}\ x\to\mathsf{SNo}\ y\to\forall u\in\mathsf{SNoR}\ (x*y).\forall p:o.\\ (\forall v\in\mathsf{SNoL}\ x.\forall w\in\mathsf{SNoR}\ y.v*y+x*w\leq u+v*w\to p)\to\\ (\forall v\in\mathsf{SNoR}\ x.\forall w\in\mathsf{SNoL}\ y.v*y+x*w\leq u+v*w\to p)\to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*e5cae53a35ffe5b399ffd05d7bd80447303ca0d61aa9feca19313ed6bd26663b*
Pure Prop Address:	*TMYf3j6h6sSAvpn3bX7i2vn59bPSs8V1Lyj*
Theory Prop Id:	*d1fff1afdd85cf834265b58f478397a8d6a7bb46decf4e5fa0e45dfaca997f73*
Theory Prop Address:	*TMGsDwijjDwd8s7TJ4BtBpPj7TywGA7uPvh*

mul_SNo_zeroR

Theorem 28.106 $\forall x.\mathsf{SNo}\ x\to x*0=0.$ *The proposition is identified by the following information:*

Pure Prop Id:	*d279f406aa4b69f7009533d48bdd2cde0236df1ead29105c36976e3dc31e0eea*
Pure Prop Address:	*TMbh4sw15c3nkGcdW9gjqchsqeGnE6CLJka*
Theory Prop Id:	*a4f4a68e9bdee9674a6a2323e1cb9216f16fac58db1b650aa99363d451432e40*
Theory Prop Address:	*TMX3UwQTHomopN15s51CQVuoHBm4nfFok7n*

mul_SNo_oneR

Theorem 28.107 $\forall x.\mathsf{SNo}\ x\to x*1=x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*1d2f1d16bda75bb4c0ed0799701f296b11ddee1eb905b339cd6220b0faf7d94c*
Pure Prop Address:	*TMPJYXDtPMCDcjFFQegpZzooLCy6nB6AnVt*
Theory Prop Id:	*5477a4ee853ab494ceac03d98e85ac4588ce7bdd59886ad227b471fce11f4d89*
Theory Prop Address:	*TMKdX9N9egsSZcy8hkKrdhm8AietaRG3z8M*

mul_SNo_com

Theorem 28.108 $\forall xy.\mathsf{SNo}\ x\to\mathsf{SNo}\ y\to x*y=y*x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*21f7d80766165f191325b0834d9de7e7be3a0dd1d07fa89c1f7a6020d3653a22*
Pure Prop Address:	*TMWdJKtXqZ53gSQC2BVHdRbKeugBVoakiVX*
Theory Prop Id:	*83d5fab1574f51f9c204a181c7dfc8be42d76f66ec97dbb3470cdfdc46326b24*
Theory Prop Address:	*TMdg6XgWdnxnzK7BDdwoHBgq7caePWS9hV1*

mul_SNo_minus_distr

Theorem 28.109 $\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to (-x) * y = -x * y$. *The proposition is identified by the following information:*

Pure Prop Id:	*9beaa01938101baaa11d2398155e5373cc0ae1d7998293c91f378b289f8e122f*
Pure Prop Address:	*TMRSczXWoW4GcLzauYywq4qjgTNRs8DVfKY*
Theory Prop Id:	*3d0cd1fa56b07fbeb5efae9a27e18be23a0f0160374b403e50685ba3228a818a*
Theory Prop Address:	*TMXe2FHAwQLWs8w3e4pZKSq3Ay1HjVYmAo9*

`mul_SNo_distrR`

Theorem 28.110

$$\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to (x + y) * z = x * z + y * z.$$

The proposition is identified by the following information:

Pure Prop Id:	*6a23e513510b3a7b5a3392d09ddfa8bf723376a9ef4220bd708cc688d619d3f6*
Pure Prop Address:	*TMEse8J6Aed49wCopbm1rWGiRVunCrST9NN*
Theory Prop Id:	*5f038253af67c067b7090c811a60073e88d35c5f778f03a6091776774636ffef*
Theory Prop Address:	*TMYL91kZXWG9iK5FrgphNrfwHqoEcLWbXTo*

`mul_SNo_distrL`

Theorem 28.111

$$\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to x * (y + z) = x * y + x * z.$$

The proposition is identified by the following information:

Pure Prop Id:	*4e7eba7fc4815da2d007f42429b70d6437766813fda9ea365ce905b2c09884fa*
Pure Prop Address:	*TMSEGG8Zev8b3WSFiMRZbgEmXP278T6o11U*
Theory Prop Id:	*2a483403dad7cd65bfa69608ac938c5560f66186e39f7cfeafc037b7cf113feb*
Theory Prop Address:	*TMXPqQpttxR52gPP1rKLuLFPGP1wExdFugR*

`mul_SNo_assoc`

Theorem 28.112

$$\forall xyz.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to \mathsf{SNo}\ z \to x * (y * z) = (x * y) * z.$$

The proposition is identified by the following information:

Pure Prop Id:	*71a504fd63097789a8be85f133188fe0e6cdb7c87cd67c75d8f488d15233773b*
Pure Prop Address:	*TMbdjdgGdSBzHFduhV2xBDmPQxmKwQt1tuM*
Theory Prop Id:	*d322c17c41c9815d6f808e736a3c46d7cf138d7fb9de4bfebcd6ce05e597e568*
Theory Prop Address:	*TMUDFiVX4hFhFSKJNGre9DQ2rsnsdEBsmWG*

`mul_nat_mul_SNo`

Theorem 28.113 $\forall nm \in \mathsf{omega}.\mathsf{mul\_nat}\ n\ m = n * m$. *The proposition is identified by the following information:*

Pure Prop Id:	*da39d396fdbb6dc3abd4d8b61394e2ae8cd83bfda154cafeaa7b0d1ee0288b3b*
Pure Prop Address:	*TMQtAZAzTub1xFgkzJYBwEnD6qz9UzUMmJp*
Theory Prop Id:	*eb633595c5ae47b0fcc4717134102599b1f7892572e59291ab05562597e9fb82*
Theory Prop Address:	*TMaXHxX9CkqxrxU1kbFsYKxJbdBZWsWsgbd*

`mul_SNo_In_omega`

Theorem 28.114 $\forall nm \in \mathsf{omega}.n * m \in \mathsf{omega}$. *The proposition is identified by the following information:*

Pure Prop Id:	*7b6023e5c720f7f5082b318b3189f8c277b1498a0c34be6e13516ce9bce0a8f1*
Pure Prop Address:	*TMbHqwvzDZAcjviA88kYptiX95LF6c3pw8m*
Theory Prop Id:	*8435a4b44154c9d6a5498ab9f17973fc5f75b3b8098f3884189ec529e738ffc5*
Theory Prop Address:	*TMLcnoowy4SKdgLinmXu8vq7b1VLk5mBvRB*

Definition 28.4 *We define* diadic_open *to be*

$$\lambda X.X \subseteq \mathsf{SNoS\_\ omega} \wedge$$
$$\forall x \in X.\exists n \in \mathsf{omega}.\forall y \in \mathsf{SNoS\_\ omega}.x + -\mathsf{eps\_\ } n < y \to y < x + \mathsf{eps\_\ } n \to y \in X$$

of type $\iota \to o$ identified by the following information:

Pure Object Id:	*16a206dc52b4bb55b4a6c92782a08f06f95014669558c00b452a0ba01b887b1c*
Pure Object Address:	*TMXnCV5QN4u4VL3jUYWi1Q5TKvaYVFiyMAy*
Theory Object Id:	*f8473685b008a719f4b168e65e6b1cd5168f93a2f789ae2758a850f2b196b7db*
Theory Object Address:	*TMFiXJkdoLrmEDAbDLC8bw1WgubyoT6m1rW*

`diadic_open_I`

Theorem 28.115

$$\forall X \subseteq \mathsf{SNoS\_\ omega}.$$
$$(\forall x \in X.\exists n \in \mathsf{omega}.\forall y \in \mathsf{SNoS\_\ omega}.x + -\mathsf{eps\_\ } n < y \to y < x + \mathsf{eps\_\ } n \to y \in X)$$
$$\to \mathsf{diadic\_open}\ X.$$

The proposition is identified by the following information:

Pure Prop Id:	*690598e6fd4ea25bb12e3b584dc4b13fe6c60343f666f02f1641e57297d2d891*
Pure Prop Address:	*TMc3CK4vFU9yhYePJb36m6beBT5HxUzg5Dz*
Theory Prop Id:	*c974cede2afec12b64c094aa129dc1b1d1a692b8afe4e7954cb66d9d2f5c2b77*
Theory Prop Address:	*TMHeah5uo3Soo2iroPYRB2ntKum4R7fb8uT*

Definition 28.5 *We define* SNoL_omega *to be* $\lambda x.\{y \in \mathsf{SNoS\_\ omega} | y < x\}$ *of type $\iota \to \iota$ identified by the following information:*

Pure Object Id: *5f401f6d84aeff20334d10330efdc9bb3770d636d6c18d803bce11a26288a60d*
Pure Object Address: *TMS5SZr3PAStrSpayzuJN2wkhBQBnxtsCKv*
Theory Object Id: *2e3a675cbb0cd13246446f4a4b5819f9d0cd3a852470bdf73b86e4c9fa5f8aac*
Theory Object Address: *TMTDoEUQNzUxLj7ehiQCYBDF5vvzDtYJBZ7*

Definition 28.6 *We define* SNoR_omega *to be* $\lambda x.\{y \in \text{SNoS\_ omega}|x < y\}$ *of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id: *d24599d38d9bd01c24454f5419fe61db8cc7570c68a81830565fbfa41321648f*
Pure Object Address: *TMcZ1fLri43RwrwbvUWagasgXfWF6BJHVSg*
Theory Object Id: *255a0c77460ecd33846983d06bff6d5687f5c12703ee773bd7ac29ee80876aee*
Theory Object Address: *TMZoQpNUd5MbGrEDKDP95Eu98wX9TZSnriY*

`diadic_open_SNoL_omega_I`

Theorem 28.116

$$\forall z.\text{SNo } z \to (\forall x \in \text{SNoL\_omega } z.\exists n \in \text{omega}.x + \text{eps\_ } n < z) \to \text{diadic\_open (SNoL\_omega } z).$$

The proposition is identified by the following information:

Pure Prop Id: *f034141f72d6c4eb35d80a2119f7efda32b3a2da588de397b25001b9c4cc72d9*
Pure Prop Address: *TMZo5wo73TruUc3HVgkyKRi1Lm1wR18VgRA*
Theory Prop Id: *7fb6824bab5f2011e10de03e8eea40ef57b1d040bc5a7600c6e49a17598be7f1*
Theory Prop Address: *TMKVw1RPWUuG98CDGajHDtbjjBEGV18MFAE*

`diadic_open_SNoR_omega_I`

Theorem 28.117

$$\forall z.\text{SNo } z \to (\forall x \in \text{SNoR\_omega } z.\exists n \in \text{omega}.z < x + -\text{eps\_ } n) \to \text{diadic\_open (SNoR\_omega } z).$$

The proposition is identified by the following information:

Pure Prop Id: *76200c785beb8c7d3f1ca8ae3c4cf9b46875e0cfc933a9cb2834cf5dfa366c76*
Pure Prop Address: *TMVnsa7wcgr6WYawj3FNHQPu1WxbQwjWAQf*
Theory Prop Id: *58c46af97ae7d614a76a0d68417ebdde3cae5408341ae09dd687c865366186f0*
Theory Prop Address: *TMbr7DWfJLuoLhotsiMYBsLoSh3C8or2K85*

Definition 28.7 *We define* real *to be*

$$\{x \in \text{SNoS\_ (ordsucc omega)}| \text{SNoL\_omega } x \neq 0 \wedge \text{SNoR\_omega } x \neq 0 \wedge \text{diadic\_open (SNoL\_omega } x) \wedge \text{diadic\_open (SNoR\_omega } x)\}$$

of type ι *identified by the following information:*

Pure Object Id: *1bd1ac4ee8852db6f99d372f1c27ef21bca430b1c9c4c4d184e017f6a4d8d1b4*
Pure Object Address: *TMKMHojzxJK5coFqxQsQtHCgmSHKPCW9Vm9*
Theory Object Id: *87ab7c83c1184f9f51c4168a5ec6a5ea9da4a8e2acb1fc82e0f19a7afc1617c4*
Theory Object Address: *TMYohVoF9p6nAvkXe1FijCMbejrCeRTmmEN*

`real_I`

Theorem 28.118

$$\forall x \in \mathsf{SNoS\_}\ (\mathsf{ordsucc\ omega}).\mathsf{SNoL\_omega}\ x \neq 0 \to \mathsf{SNoR\_omega}\ x \neq 0 \to \mathsf{diadic\_open}\ (\mathsf{SNoL\_omega}\ x) \to \mathsf{diadic\_open}\ (\mathsf{SNoR\_omega}\ x) \to x \in \mathsf{real}.$$

The proposition is identified by the following information:

Pure Prop Id: *09fffc7b9e5d0b1d7f15cd56209324363defbb3ebe9c272e83a66e2590b7bd24*
Pure Prop Address: *TMUWdsBKSGjUkuMEbeQNbUqTtaf4Jfjg27G*
Theory Prop Id: *594245216a8c68c4e6457b47578cb5437b65d219679a5836cb77b5ff224d6c9a*
Theory Prop Address: *TMGSVE4T9JHVEamf5z5kbyL8PNooxb94NMt*

`real_E`

Theorem 28.119

$$\forall x \in \mathsf{real}.\forall p : o.\ (x \in \mathsf{SNoS\_}\ (\mathsf{ordsucc\ omega}) \to \mathsf{SNoL\_omega}\ x \neq 0 \to \mathsf{SNoR\_omega}\ x \neq 0 \to \mathsf{diadic\_open}\ (\mathsf{SNoL\_omega}\ x) \to \mathsf{diadic\_open}\ (\mathsf{SNoR\_omega}\ x) \to p) \to p.$$

The proposition is identified by the following information:

Pure Prop Id: *44069e352f5036a86127073abddc29a0ac5cf4eb6f4d81d6ec3a0209c2755103*
Pure Prop Address: *TMHRqA5xCUMEW6aiM8SkmMPejfbXcp8AuW2*
Theory Prop Id: *e2acb3ab0b286d7acb20f49882f08eb34a7bbd287abb4ee34541635153010085*
Theory Prop Address: *TMbuQMxV1gs1YZpGC9dAoKbCjDcsEGPDtg5*

`Subq_real_SNoS_ordsucc_omega`

Theorem 28.120 $\mathsf{real} \subseteq \mathsf{SNoS\_}\ (\mathsf{ordsucc\ omega})$. *The proposition is identified by the following information:*

Pure Prop Id: *78ab2f997523a88bf0249dfd2d26b12d3641ee1e1b29b0fe0b6ecbe2bcb9ba5e*
Pure Prop Address: *TMTqU7cPcHtqdsT73PkgeBYFon4NVibV1rP*
Theory Prop Id: *2858a333c0f86c675cb7a2fd1e8d4029fc7b67d5a2e36fcb017031c2a22d4db9*
Theory Prop Address: *TMKo5QEXCMn8QagibutbBTzBZSUDAxobgfW*

`Subq_SNoS_omega_real`

Theorem 28.121 $\forall x \in \mathsf{SNoS\_\ omega}.x \in \mathsf{real}$. *The proposition is identified by the following information:*

Pure Prop Id:	*db92898b58c469e9ad58b8b3729b507e8cadb7c632fb9757b051ec3b47524b67*
Pure Prop Address:	*TMJL6KqW8mXtGGFbki5ao4x1Pu9AbFw59T2*
Theory Prop Id:	*7315809dd9bd2ef3a3617333ce251afbd32d098f391ab09694db0d0a3ed52701*
Theory Prop Address:	*TMYg3xroKjVdTtceUrNgH6Jq8ZdKuyS3gD1*

`SNoCutP_SNoL_SNoR_omega`

Theorem 28.122

$$\forall x.\mathsf{SNo}\ x \to \mathsf{SNoCutP}\ (\mathsf{SNoL\_omega}\ x)\ (\mathsf{SNoR\_omega}\ x).$$

The proposition is identified by the following information:

Pure Prop Id:	*15b221ba0c4ae0e05ed8cc21a7cdf61ac684525faddd1c477ed29fb7cbd9ee7b*
Pure Prop Address:	*TMQ4q89ASduA9c9ftASiNBHz8pQAiMs6fvH*
Theory Prop Id:	*354d7d3952bf1c176fc5b59a37d8d3fde5ecdce30261e4d7e68896d0d7a6e87a*
Theory Prop Address:	*TMLgzevvcXAVWAzGL5wtnbtt749rGGQR3aV*

`SNoS_ordsucc_omega_SNoL_SNoR_omega`

Theorem 28.123

$$\forall x \in \mathsf{SNoS\_\ (ordsucc\ omega)}.$$
$$x = \mathsf{SNoCut}\ (\mathsf{SNoL\_omega}\ x)\ (\mathsf{SNoR\_omega}\ x).$$

The proposition is identified by the following information:

Pure Prop Id:	*0e7b9203cc682cd4470f749df0d7773bb56f7d49172a681a31039c99f4c6190e*
Pure Prop Address:	*TMHfAHcvWTSrs5RmhM3t5QLf3y1Ps4dbWMu*
Theory Prop Id:	*1df44e924ecdb42a015d2758e58cb9ae0cb4cf2f67ebce9729fc086bad0fbdd4*
Theory Prop Address:	*TMHNAhMLb3WzEVoWuDPHTbhjyEcTzN9mZJx*

`real_SNoL_SNoR_omega`

Theorem 28.124

$$\forall x \in \mathsf{real}.x = \mathsf{SNoCut}\ (\mathsf{SNoL\_omega}\ x)\ (\mathsf{SNoR\_omega}\ x).$$

The proposition is identified by the following information:

Pure Prop Id:	*e3f60c2768e445b24135a40107035ad21c03de45130bcda31e71da16caa4c99c*
Pure Prop Address:	*TMKK4pjw5QfzGoVXPooBhrwXjMz5y7zwC83*
Theory Prop Id:	*11dba32c06250858d053b157288593cb4e9cc3b5e1cac811216e1643ff3caecb*
Theory Prop Address:	*TMad5zGLRZhwq8fTQPe55TZwskUMePg3kMb*

real_ex_diad_Lt

Theorem 28.125 $\forall x \in \mathsf{real}.\exists w \in \mathsf{SNoS\_\ omega}.w < x$. *The proposition is identified by the following information:*

Pure Prop Id:	*8a1a82ac356e457f7d6fea412cc602e7bf973ae5a3d1fe392acbd6183e111060*
Pure Prop Address:	*TMRmEwRJSZMnW9FxQdrwZVkpY4X3hZtgEsF*
Theory Prop Id:	*95fe1677ab448626991245b1f6c72063712786eb5fc15e18c8ad861be5a10d24*
Theory Prop Address:	*TMWnkcpDQgaZXznHwxbP51Zpm4HsdCijkgK*

real_ex_diad_Gt

Theorem 28.126 $\forall x \in \mathsf{real}.\exists z \in \mathsf{SNoS\_\ omega}.x < z$. *The proposition is identified by the following information:*

Pure Prop Id:	*edbfff85c909fe0b9cc4f471ef0cb3f4b241c89ff5fdc0aac267048126f79d55*
Pure Prop Address:	*TMWbbSgeg81TLqCRrR2wnZxFJM2VyEnhoKs*
Theory Prop Id:	*8b82781e914ba17ea6121ff32dd4355d61bc2d5e58cf93f4dfc19f6be51aa543*
Theory Prop Address:	*TMVen1Z8YEbACtaXxe8EHzxX42Fe1HVxEn9*

mul_SNo_pos_pos

Theorem 28.127 $\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to 0 < x \to 0 < y \to 0 < x * y$. *The proposition is identified by the following information:*

Pure Prop Id:	*fda2502eea941a875df72ec6015f1570649b018b98f8184cd3c875919ea4099f*
Pure Prop Address:	*TMMBQ62TWFg4vbNhYgkxMEhq8L4wrwpNeH4*
Theory Prop Id:	*d984dcdb63a86f7c983729aa4499bd351e39b1649325ebc42012cca4a4f9dda6*
Theory Prop Address:	*TMRy9goqWBW3Lrva8p4Hh4yhTMAF2u9LrbY*

mul_SNo_pos_neg

Theorem 28.128 $\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to 0 < x \to y < 0 \to x * y < 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*4dec1e722a076d06ad4666c4d9e70af174b210cfaebca7581e05f1d86d354df3*
Pure Prop Address:	*TMPctCrMmwtqdsDYtwTpmW8cn8ReSGEXW1d*
Theory Prop Id:	*697e77b4240d0fbd5154af82b65814d654a79f225b48542bc4a3d1b906b094d7*
Theory Prop Address:	*TMbmWHtMpjybVdPJPF2YcxQPAiySRhKRtU7*

mul_SNo_neg_pos

Theorem 28.129 $\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to x < 0 \to 0 < y \to x * y < 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*6f7ff58caeaa194bad28d270130f11e1db31bb6ab731d5ea0051c78e8fe144e4*
Pure Prop Address:	*TMHNbtt4BUkPSLXdqKhLABgy4oEcpaoGeQF*
Theory Prop Id:	*6e7adce5ea23f07343a9f2808aa1618cc2fef1e19abfb8217fb9154a7c79b8f6*
Theory Prop Address:	*TMX5KmwY5bfi7Ub8rgxQwDCpNPeAkJVtZsN*

`mul_SNo_neg_neg`

Theorem 28.130 $\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to x < 0 \to y < 0 \to 0 < x * y$. *The proposition is identified by the following information:*

Pure Prop Id:	*86d604810bc69f81920bb16b33d0acc3bedea0cfb10d12b4458a7d1fafe8a9d9*
Pure Prop Address:	*TMaUyk4nPJDDrgihhHeRFKvQ2fABVtnDW2a*
Theory Prop Id:	*03517a8b5c20769332200cc5606cad376758ee52bb6086d3523cb789f19cee77*
Theory Prop Address:	*TMZsT4fPodxbAqp3EhReFUneq4rHkVaHEZS*

`mul_SNo_nonzero`

Theorem 28.131 $\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to x \neq 0 \to y \neq 0 \to x * y \neq 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*62b0712aafda95c0ed43facad05fea37b6fb484147b8453d916f90fe017d591f*
Pure Prop Address:	*TMGU71yz2XNvrMWpSrZ2VaZAXfAZoXYkpsg*
Theory Prop Id:	*edd14a5db38e09a96214c426f3ca626167e570af71109dca57dc26ce0cdfae63*
Theory Prop Address:	*TMH1iV5T59LgjrgGemBxMfYGnwBeTHPSpxr*

`minus_SNo_restr_SNo`

Theorem 28.132

$$\forall x.\mathsf{SNo}\ x \to \forall \alpha \in \mathsf{SNoLev}\ x. \\ (-x) \cap \mathsf{SNoElts\_}\ \ \alpha = -(x \cap \mathsf{SNoElts\_}\ \ \alpha).$$

The proposition is identified by the following information:

Pure Prop Id:	*6e2d93e0dc17abea886b929fe5995e073d3fdcf446b9f988050524e8d71bd48f*
Pure Prop Address:	*TMG933hdiaf584T7p9ZFRvPn9oiCEWieGQW*
Theory Prop Id:	*33a9b7427401c079a1d2742931e0769dcffafc114bd1bf1548f1b32dbd6c8dc6*
Theory Prop Address:	*TMYmCxYkd92jjewaiC7sVwLTus2QgwYLten*

`minus_SNo_exactly1of2`

Theorem 28.133

$$\forall x.\mathsf{SNo}\ x \to \forall \alpha \in \mathsf{SNoLev}\ x.\mathsf{exactly1of2}\ (\alpha \in x)\ (\alpha \in -x).$$

The proposition is identified by the following information:

Pure Prop Id:	*8867febe718634f83190f3f8c3dab70b7c761d564ddd1acb9cadda95bf892f1b*
Pure Prop Address:	*TMbbMHfSdx97M7MMua7mnYgvKSs2grfdonk*
Theory Prop Id:	*c2e4600c71315f1fd85398873234adada26e385afa312ce802aceb4cd7e1c2b3*
Theory Prop Address:	*TMZRH9eUQQW9BQLZhXvbWDnscfgzh7sfosN*

`minus_SNo_In`

Theorem 28.134 $\forall x.\mathsf{SNo}\ x \to \forall \alpha \in \mathsf{SNoLev}\ x.\alpha \in x \to \alpha \notin -x$. *The proposition is identified by the following information:*

Pure Prop Id:	*634ffccfb2da872d1912604afc7221a4c2f97936c287ab1c4253071e1dfcfc61*
Pure Prop Address:	*TMRJdHvUjKuqbmun53qwo4QgnKCJpLgQbyt*
Theory Prop Id:	*6ad85181a4e76fe3485b7a68f6c450251768534fcb5c6074cb42e2acf9661987*
Theory Prop Address:	*TMHfvPTQWHBMA7BghiU2NwAbmLn8jDpkF9g*

minus_SNo_nIn

Theorem 28.135 $\forall x.\mathsf{SNo}\ x \to \forall \alpha \in \mathsf{SNoLev}\ x.\alpha \notin x \to \alpha \in -x$. *The proposition is identified by the following information:*

Pure Prop Id:	*42b82e1e669aebbf602bf5113f1d414db866cfe089bda3a8f91d980976cd7959*
Pure Prop Address:	*TMcrVxUeXZKH9hKHxpogFheX7RW2QbWzvcH*
Theory Prop Id:	*c24e982ace7dff228ddfbb28d50b4cddc7e5842b4c70d4190e10a022c64bebc0*
Theory Prop Address:	*TMbXvwL69SpXNMdrP7cnTzE8ykxUg2LWeVZ*

real_minus_SNo

Theorem 28.136 $\forall x \in \mathsf{real}.-x \in \mathsf{real}$. *The proposition is identified by the following information:*

Pure Prop Id:	*5ba504335f94f103b4e4f4b4128f81159e7d57c6f6e51ddf1aac116c6bc3d844*
Pure Prop Address:	*TMRu8g6RVBXBFKY32BG4WQRsE2JJUCAdD8A*
Theory Prop Id:	*54413c793a2a6cf0fcb3da3a35724c098c7e8b441474a0aac138fbd28ee3cb5c*
Theory Prop Address:	*TMJ5SdaCycBTmAK8QT7hmjqUxcKbkFRkZSH*

Definition 28.8 *We define* div_SNo *to be*

$$\lambda xy.\mathsf{if}\ y = 0\ \mathsf{then}\ 0\ \mathsf{else}\ \mathsf{Eps\_i}\ (\lambda z.\mathsf{SNo}\ z \wedge z * y = x)$$

of type $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*16510b6e91dc8f8934f05b3810d2b54c286c5652cf26501797ea52c33990fa93*
Pure Object Address:	*TMJtKhWrCvxZo4EySoUGxRLHGe8kxsrf5wV*
Theory Object Id:	*41b026a24f3f123a0f6f2e71ea4d39925a2b48bf6dedd1c902de3b86264323f1*
Theory Object Address:	*TMXu1QtbM1e9R4bcCcqrQqVE2nbX4D1QdbG*

Definition 28.9 *We define* exp_SNo_nat *to be*

$$\lambda nm : \iota.\mathsf{nat\_primrec}\ 1\ (\lambda\_\ r.n * r)\ m$$

of type $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*cc51438984361070fa0036749984849f690f86f00488651aabd635e92983c745*
Pure Object Address:	*TMHssmmer5vtejxULVomtdegStRPMUKFbiM*
Theory Object Id:	*ea81040eaa779da7e35317f42e478d807fb38e0dd71814599673b64763a57fba*
Theory Object Address:	*TMPrqwjmsGkTo9k9ojpFo3twgRABMuqrPhD*

Notation. We use superscripts as notation corresponding to applying term exp_SNo_nat.

28.2 Complex Surreals

Definition 28.10 *We define* CSNo *to be*

$$\lambda z.\exists x.\mathsf{SNo}\ x \wedge \exists y.\mathsf{SNo}\ y \wedge z = \mathsf{SNo\_pair}\ x\ y$$

of type $\iota \to o$ *identified by the following information:*

Pure Object Id:	*c35281fa7c11775a593d536c7fec2695f764921632445fa772f3a2a45bdf4257*
Pure Object Address:	*TMUD5RtiGmKRpxwvJihKA6DLDFMXAMhjUgo*
Theory Object Id:	*9f6440ba5a12befd32f7a9856b6769fa208407cf0c9b4747cb6036bb0d6250a3*
Theory Object Address:	*TMLizygxxxRc6MNrrbecR4regbF9SnDnoL1*

CSNo_I

Theorem 28.137

$$\forall xy.\mathsf{SNo}\ x{\to}\mathsf{SNo}\ y{\to}\mathsf{CSNo}\ (\mathsf{SNo\_pair}\ x\ y).$$

The proposition is identified by the following information:

Pure Prop Id:	*a092484c55b8291c75d0d36b92ada8b7272277bdbed2f8e85dd4647982d82a11*
Pure Prop Address:	*TMLuqWx8Kn7Zr4K7kCY3vBtreAy6N2Gq5Ho*
Theory Prop Id:	*b22a0800c64b20ca0a19ed9cf3cf1103cba24b4506c039ffdc78edc941d716bc*
Theory Prop Address:	*TMSb6wFZKXobEGqJEzsAV83ZDCU7SEHmi81*

CSNo_E

Theorem 28.138

$$\forall z.\mathsf{CSNo}\ z{\to}\forall p:\iota \to o.$$
$$(\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to z = \mathsf{SNo\_pair}\ x\ y \to p\ (\mathsf{SNo\_pair}\ x\ y)){\to}p\ z.$$

The proposition is identified by the following information:

Pure Prop Id:	*d92da0243181c2c853df14146584be693f357edce1fe0d6dc91b3a43175cd9a5*
Pure Prop Address:	*TMF2YxYA3n45UKAAPKZdwQUZ5pkopFy84Zo*
Theory Prop Id:	*fdbc8d36959b5172de4f1d46ecd65e45290e18a238b0d3274dd0238e86f92e5a*
Theory Prop Address:	*TMREupmRGKvMjoTNRuLkffjgfQYrtqyaxmd*

SNo_CSNo

Theorem 28.139 $\forall x.\mathsf{SNo}\ x \to \mathsf{CSNo}\ x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*56121e05e33db2f68607348644f875d4c59e36314e4c7aade2ed7acf3bc097ea*
Pure Prop Address:	*TMKSgGTcxX9dqXoEQ6Z8PkWivenDVtKxQTL*
Theory Prop Id:	*d50db589d56adb9c8fd5479df494050c31aaa3429ec352b511a7b0a13aeea945*
Theory Prop Address:	*TMR4DcyLohCpAbSXv9WH4Xcg5MLahA6HKCW*

28.3 Complex

Definition 28.11 *We define* Complex_i *to be* SNo_pair 0 1 *of type* ι *identified by the following information:*

Pure Object Id: *d0c55cfc8a943f26e3abfa84ecab85911a27c5b5714cd32dcda81d104eb92c6e*
Pure Object Address: *TMRLLQsBvKWihFburT51dAV1GsLtPYwc9Zk*
Theory Object Id: *df0e4e103bfa2441c4757b4d5b0e56da78eeec142d6eb59cb57bf6f96d041b2e*
Theory Object Address: *TMWUiSAowU8u6wDqUE8abxMdTuajU7UtstH*

Let i be Complex_i.

SNo_Complex_i

Theorem 28.140 CSNo i. *The proposition is identified by the following information:*

Pure Prop Id: *0b8add253d6a14045898089017 4b902430843878d8fbe0e4e45711dca51045e1*
Pure Prop Address: *TMTBCkaSu5M84GaWqNVMA2i4e55BjcK87XM*
Theory Prop Id: *139a531c852859624509b40bb3d55cf6dc1c47b1c91e4e56557dd5eca9352643*
Theory Prop Address: *TMMieXaSc5pjuKyowxbkxfTqbz2F6SwKDZS*

Definition 28.12 *We define* CSNo_Re *to be*

$$\lambda z.\mathsf{Eps\_i}\ (\lambda x.\mathsf{SNo}\ x \wedge \exists y.\mathsf{SNo}\ y \wedge z = \mathsf{SNo\_pair}\ x\ y)$$

of type $\iota \to \iota$ *identified by the following information:*

Pure Object Id: *9481cf9deb6efcbb12eccc74f82acf453997c8e75adb5cd83311956bcc85d828*
Pure Object Address: *TMbtVDb4DspXgdVe6YzPV3kDEEEwHY6ZCot*
Theory Object Id: *78e2fa14ee2f6baf7a5e87660241882e064e475595b0b4d3abb32b22e16d4844*
Theory Object Address: *TMcoCaQ7XUy1bDRwyD8B1Hc9XvedDZEaj9Z*

Definition 28.13 *We define* CSNo_Im *to be*

$$\lambda z.\mathsf{Eps\_i}\ (\lambda y.\mathsf{SNo}\ y \wedge z = \mathsf{SNo\_pair}\ (\mathsf{CSNo\_Re}\ z)\ y)$$

of type $\iota \to \iota$ *identified by the following information:*

Pure Object Id: *5dad3f55c3f3177e2d18188b94536551b7bfd38a80850f4314ba8abb3fd78138*
Pure Object Address: *TMZBTXQFnkALw8eLrVDkBW9kDUBUNtJ9KV1*
Theory Object Id: *995ec6771de10483744c42c2860b06ba0be54c9d0335cccb1a962e90378575b5*
Theory Object Address: *TMFEq3qfdWzjBymvGvoarLmQpoH3fgwe9yD*

Let $Re : \iota \to \iota$ be CSNo_Re. Let $Im : \iota \to \iota$ be CSNo_Im. Let $pa : \iota \to \iota \to \iota$ be SNo_pair.

CSNo_Re1

Theorem 28.141

$$\forall z.\mathsf{CSNo}\ z \to \mathsf{SNo}\ (Re\ z) \wedge \exists y.\mathsf{SNo}\ y \wedge z = pa\ (Re\ z)\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*d1afc4d126dd465be260002c44748dfd6b52c42ef68b99e2aca9d39589415eff*
Pure Prop Address:	*TMVD9LSTuGahX5mYEQHbkD3Aj4r6YjvDqav*
Theory Prop Id:	*dda4fdb9868a65730eda625dcc1b25d74a368899640ab43b19a56dee292fd951*
Theory Prop Address:	*TMdw63sz9FuEG3u34nTUC9xRWJSnjKoK5iM*

CSNo_Re2

Theorem 28.142 $\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to Re\ (pa\ x\ y) = x$. *The proposition is identified by the following information:*

Pure Prop Id:	*b2daba3d81b7c6ff9fbeddbdc673af753aea0654c6f0beb2e4150cec6aa3ff36*
Pure Prop Address:	*TMbd79jCDur1j41vTRXKVgXL6tDG3dczCch*
Theory Prop Id:	*c4a4431b7dc74e1c59f2d3f78882e9511898bb365f4441eb504d35a102c73f5d*
Theory Prop Address:	*TMTPxuYePSpdrjC2hZR9YA4mNhhCM2obwvy*

CSNo_Im1

Theorem 28.143

$$\forall z.\mathsf{CSNo}\ z \to \mathsf{SNo}\ (Im\ z) \wedge z = pa\ (Re\ z)\ (Im\ z).$$

The proposition is identified by the following information:

Pure Prop Id:	*74e0f37f75ccb3555e54f4af34c332761bbf40f68215121dd3426a5fc1d57563*
Pure Prop Address:	*TMbMFKS5usWyvTh28krP3yJ58GAo85ggE5u*
Theory Prop Id:	*f920bd6c9ae2ef46e0da84205614a41dca1aa86484bc96ed97046e5cb2dcc706*
Theory Prop Address:	*TMSHmgJJG9JSYax4SjR2fRxfC1xbTdvaFHp*

CSNo_Im2

Theorem 28.144 $\forall xy.\mathsf{SNo}\ x \to \mathsf{SNo}\ y \to Im\ (pa\ x\ y) = y$. *The proposition is identified by the following information:*

Pure Prop Id:	*2bc6171b4c1d4c23d2b8257ee568deebce1bdeeb5abadafad24f46402844bec8*
Pure Prop Address:	*TMPfMNHVuYrdTndLNgbw7ssKmmTUSmVQhuq*
Theory Prop Id:	*8be49e5c0e40a6664121bff48d6f103b94ded1a85e8276f4270b2fecc4365f2d*
Theory Prop Address:	*TMUySj1Kdt4rWdt8CvGuPiFauFwdwqGuBrt*

CSNo_ReR

Theorem 28.145 $\forall z.\mathsf{CSNo}\ z \to \mathsf{SNo}\ (Re\ z)$. *The proposition is identified by the following information:*

Pure Prop Id:	*2d8bafb4716aba45ba78940b437d476181b1832c88e0fdf73d1f44139d745aa6*
Pure Prop Address:	*TMHWBdpBEx7e2XaqsRv4PexZFWK6mxntB5L*
Theory Prop Id:	*9eab1faa688e3e8a126798d1a4feec2d230170ca2c79e1f6a7cac4413ce087e6*
Theory Prop Address:	*TMLjD3HKc5VtY2zV8JTsHHEn8qrJqupKsD9*

CSNo_ImR

Theorem 28.146 $\forall z.\mathsf{CSNo}\ z \to \mathsf{SNo}\ (Im\ z)$. *The proposition is identified by the following information:*

Pure Prop Id:	*ab80bc51455c8401ea6cfc9fb8b50fce5202d296b70813e18ab3b4578724d01b*
Pure Prop Address:	*TMHM4TWxDfhFey1F4K4ZfTkNgJ3aXPsHVnx*
Theory Prop Id:	*f91d5d3724c3d6ffc90c6787c4369da0ef974adfbd2841122b8eea32364c3543*
Theory Prop Address:	*TMaeNJLGGokLzoVMSdnkmyBJB7tx3aSEAk3*

CSNo_ReIm

Theorem 28.147 $\forall z.\mathsf{CSNo}\ z \to z = pa\ (Re\ z)\ (Im\ z)$. *The proposition is identified by the following information:*

Pure Prop Id:	*445a11e5d038263fe95c6356046a0e1e52288c7a48a5d5c22900215dc15818bf*
Pure Prop Address:	*TMbfkg4MdezPBN2Y4Qjkj5qcWqmWQ3YK3MM*
Theory Prop Id:	*520c8f693aef23e47d75aa0230b2dc952db5beee2eec2007112e6239da337856*
Theory Prop Address:	*TMPQ5WLn58Hodm558JuPKepWo3qKYLgWAfo*

CSNo_ReIm_split

Theorem 28.148

$$\forall zw.\mathsf{CSNo}\ z \to \mathsf{CSNo}\ w \to Re\ z = Re\ w \to Im\ z = Im\ w \to z = w.$$

The proposition is identified by the following information:

Pure Prop Id:	*1b6f8a62f72e786e4c9bf9f10ef1b663021c7dd462f9001118924d0c3eb05d56*
Pure Prop Address:	*TMF22319Si8r5cP9dtLLXWhca498DPYaPfW*
Theory Prop Id:	*e59d00e026008d1f4bb97a82ee1e249a3a6533f81c95228512a948d73a5948a1*
Theory Prop Address:	*TMQQTkXaTqRkMfBzratppu8SFd9kJhSw1K9*

Notation. We use $--$ as a prefix operator corresponding to applying term minus_SNo. **Notation.** We use $+$ as a right associative infix operator corresponding to applying term add_SNo. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term mul_SNo.

Definition 28.14 *We define* minus_CSNo *to be* $\lambda z.pa\ (-Re\ z)\ (-Im\ z)$ *of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*9c138ddc19cc32bbd65aed7e98ca568d6cf11af8ab01e026a598657906119 8b7*
Pure Object Address:	*TMLB9tLVVqyKeYKxjvFwaspknDz9iJsXi8C*
Theory Object Id:	*e44642b3fe572281d00e78cdd9551e412cd2f9c70b5057cc266234cae61b142c*
Theory Object Address:	*TMcRv5uzwYDFcF46XsKgFNN7RdY9tuZbeEK*

Definition 28.15 *We define* add_CSNo *to be* $\lambda zw.pa\ (Re\ z + Re\ w)\ (Im\ z + Im\ w)$ *of type* $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*30acc532eaa669658d7b9166abf687ea3e2b7c588c03b36ba41be23d1c82e448*
Pure Object Address:	*TMEg3b8dv5i7oFXmXe8scTrFUcM8PzJ4zpW*
Theory Object Id:	*b095bc500f522f93df919a70a2f70782f5980aa96ce262ebe8358b56594a3c2f*
Theory Object Address:	*TMaA7uRAA9rzybfgoRTMJhUqq2bqKoX4Wx4*

Definition 28.16 *We define* mul_CSNo *to be* $\lambda zw.pa\ (Re\ z * Re\ w + -(Im\ z * Im\ w))\ (Re\ z$ *of type* $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*e40da52d94418bf12fcea785e96229c7cfb23420a48e78383b914917ad3fa626*
Pure Object Address:	*TMG6o42Tv7okyH5PRhCzCahxTbhTrJK9kr3*
Theory Object Id:	*9567b4289023a19d5a345a5792fabad48f116c4488984243ea731c878bfe55aa*
Theory Object Address:	*TMKcJrsKgvdh8ShstdBRhZw6Tr7gLxdNL7q*

Definition 28.17 *We define* div_CSNo *to be*

$$\lambda xy.\text{if } y = 0 \text{ then } 0 \text{ else Eps\_i } (\lambda z.\text{CSNo } z \wedge \text{mul\_CSNo } z\ y = x)$$

of type $\iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*98e51ea2719a0029e0eac81c75004e4edc85a0575ad3f06c9d547c11b354628c*
Pure Object Address:	*TMcjyPavg5d3bDQmhU1GhdJgnZiMVguCn1Q*
Theory Object Id:	*144bb6125865eeb3fbeec27048fa8deb33d515116b6f667efc6200b817c7e681*
Theory Object Address:	*TMTPDUmndBUSQQFuYNMpTVmCmdY9FA3BZtX*

`CSNo_minus_CSNo`

Theorem 28.149 $\forall z.\text{CSNo } z \to \text{CSNo } (\text{minus\_CSNo } z)$. *The proposition is identified by the following information:*

Pure Prop Id:	*7ee6ad0a8aa89406d6b84b29278fb17651271 52d723155b4a1c9d6f6c0fea591*
Pure Prop Address:	*TMKJwYdVTJY7k1e8hMuzVYgoKvGEEEBByvU*
Theory Prop Id:	*309c1df653ed60931ce0009531575a033315729a35f70a72363bc4dc5a0b3d22*
Theory Prop Address:	*TMFr3QPLTA3mRgec65aLawX7v5PhREet73h*

`SNo_Re`

Theorem 28.150 $\forall x.\text{SNo } x \to Re\ x = x$. *The proposition is identified by the following information:*

Pure Prop Id:	*62c082375447ec523dbbb2ba00907a038d5e29d38b336bb7ebcfe5b9350e5b40*
Pure Prop Address:	*TMUR6cn1xni6GYWDsvMbvTJXUokaMMtL4U4*
Theory Prop Id:	*b4d75e236f52950d7d4585f33566679757107a6a43067a557b1b954b9c81d095*
Theory Prop Address:	*TMcerjrejE4PH4juW3mMCPrs95Q3KqvPrwh*

`SNo_Im`

Theorem 28.151 $\forall x.\mathsf{SNo}\ x \to Im\ x = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*79fa36b0f877aa0ebc7f2b10cd692fd78dbc9d6316fc4e53d0c61bab998ecf30*
Pure Prop Address:	*TMZ4xEKBB6n896d4JKvevEKTFR67E7xpLGS*
Theory Prop Id:	*045ef70364115765ca4038bb6cadac8b50b511da827132c78a50ec9ae776e935*
Theory Prop Address:	*TMGyhJYv437WcR94jiMRHMwX2AwFbxWRWtp*

`Re_0`

Theorem 28.152 $Re\ 0 = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*b67351949a46cdf795a5f976b27f975352929b68d5d5087280de8114752abcf2*
Pure Prop Address:	*TMU3hZhd9p6dUuXdcCT5sFV8f9rU1SLuLvg*
Theory Prop Id:	*4d3237f583747a05847601b9321cb843fa14501467c9f468a5a30405685ab1cf*
Theory Prop Address:	*TMPVj3k4kQSu82UN1JnuwzTMbxZYQkycJGG*

`Im_0`

Theorem 28.153 $Im\ 0 = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*9c8b687bb8d2707b766077cc814cd6418fa32bda09f73340d3b7538ba73166a2*
Pure Prop Address:	*TMQ5sK61iFZnh5NvkYvBS6MNzDSzr4Q2mQx*
Theory Prop Id:	*c115486aac15de92b350f5674a5898e90fd8a9590b7695815ace4de38d425d98*
Theory Prop Address:	*TMR2c3XQQEW8f7iwU4myNJae1rZMRsRSBpo*

`Re_1`

Theorem 28.154 $Re\ 1 = 1$. *The proposition is identified by the following information:*

Pure Prop Id:	*88a2aa81be9b1d2f724a88816564d021c38a577fdbf421180a4c9a6b8d80afc7*
Pure Prop Address:	*TMdGDE8JarQ1jbtW4A8oVRGior9g3VHcxwo*
Theory Prop Id:	*a28b943c82a02609fdf4f3d340fd99c6f99f7fd4e1f24cf07c819165b82ba877*
Theory Prop Address:	*TMR387PaXEC5B2kz1ZyEUXJc9eNwtkfqgH2*

`Im_1`

Theorem 28.155 $Im\ 1 = 0$. *The proposition is identified by the following information:*

Pure Prop Id:	*3b8fb5faff6bf916e912016e794c0366b38e4279d8fe1cdeeee5aa0afc03878d*
Pure Prop Address:	*TMZBLCABN62himZDekvtrQPidyv7fcLegSk*
Theory Prop Id:	*1ca93cd098ccdc1ee3d42ee3ff16671c864fc5e68c4548d64594dc3f72490d3f*
Theory Prop Address:	*TMXR9fNVoFqUzmgKeV3ME2ptYRiTSH3E7yW*

`Re_i`

Theorem 28.156 *Re $i = 0$. The proposition is identified by the following information:*

Pure Prop Id:	*8f12f0decf716ca445164b75da58be3f2a7555cb429371632eb027a71508c1d1*
Pure Prop Address:	*TMK96CbwKZMULThVCgvnQBDjGHq6LXfbzfA*
Theory Prop Id:	*bfd076483d6cacc95ee0f7ff33c3ceda810e4fef403a422c0ecef2104636ffdf*
Theory Prop Address:	*TMWVmowUnC9iG9jmJkW8uaTKvWjDDPGhJaq*

`Im_i`

Theorem 28.157 *Im $i = 1$. The proposition is identified by the following information:*

Pure Prop Id:	*1795ca8f39eb6b9aa0312ca4743f676b7dad188738ed6523cb5b0e265558330b*
Pure Prop Address:	*TMUhvnCxhioSCB7KmHbHrUrhDXFyjVzSfy7*
Theory Prop Id:	*6a700bf595245199596fb74a524dc20a87be2fa234d75a9425a486afde75e739*
Theory Prop Address:	*TMV23ugJNe6xxnjymVV5jwq448H5U6cUMNz*

`add_SNo_add_CSNo`

Theorem 28.158 $\forall xy.$SNo $x \rightarrow$ SNo $y \rightarrow x + y =$ add_CSNo x y. *The proposition is identified by the following information:*

Pure Prop Id:	*7f45ee359a31d432da4768def1fc35bea04f7baa2682362d567be00cc83e019c*
Pure Prop Address:	*TMPEXTCLw1a9KLNu4LjTEYeAiXpBwtm8LYN*
Theory Prop Id:	*a9fb7d4440806134531895d72ae0d8c65453fa88d1b4a12beeccef25e3784bcc*
Theory Prop Address:	*TMYERyzMgccEbP9urACTzpKQZ6NqHoJHCpt*

`CSNo_add_CSNo`

Theorem 28.159

$$\forall zw.\text{CSNo } z \rightarrow \text{CSNo } w \rightarrow \text{CSNo } (\text{add\_CSNo } z\ w).$$

The proposition is identified by the following information:

Pure Prop Id:	*e57a7792a9adb12eaa7d6f4b276a0d7c46de087f9e650c63fa726890a32a5ea5*
Pure Prop Address:	*TMTfa7wPMhSgYfANj3ojEMJAqmSXEbR5wUi*
Theory Prop Id:	*9f039b5cd4b098a77ad130de4441f49110a2a2055cf0ea279af6db8406a33f15*
Theory Prop Address:	*TMRZouHFioSkxKnrxrt5ocqpagQGyQhPBxr*

`add_CSNo_0L`

Theorem 28.160 $\forall z.$CSNo $z \rightarrow$ add_CSNo 0 $z = z$. *The proposition is identified by the following information:*

Pure Prop Id:	*e96f34c9cab6bff68069bd60d532d81645e33b9518db1f62bf0603eb2cb2b881*
Pure Prop Address:	*TMVPvy3qhQSp4Z8EZHLGefCshoKUigGYBXD*
Theory Prop Id:	*9130336e1f65fce4abcbac21206949f4a80b64433ce8b55a026e39164bbe17ae*
Theory Prop Address:	*TMJn8298DRnHjSHEgaMHnaaNn1aon8VyNao*

add_CSNo_0R

Theorem 28.161 $\forall z.\mathsf{CSNo}\ z \to \mathsf{add\_CSNo}\ z\ 0 = z$. *The proposition is identified by the following information:*

Pure Prop Id:	*013f9d9fcb6813d280a54b0c238a0259bbf245f8c19e895056f52e4c3adeb31e*
Pure Prop Address:	*TMMjUSCaFbW2xfSEQbTD6ikQsQfwYk4GFkA*
Theory Prop Id:	*c676dd1073460165d7f272c397af95f64013350b90182b6212b134b88768602a*
Theory Prop Address:	*TMSwnX5gYRWnQPynxcvBXkQVYgYNF2643Z5*

add_CSNo_minus_CSNo_linv

Theorem 28.162

$$\forall z.\mathsf{CSNo}\ z \to \mathsf{add\_CSNo}\ (\mathsf{minus\_CSNo}\ z)\ z = 0.$$

The proposition is identified by the following information:

Pure Prop Id:	*87ef9663f32d233b6c1ad44f942d450a76582c2e2ac1a28a691747d6debc0bb4*
Pure Prop Address:	*TMU75d85SiBY1nyEa7XDpioTuTb27YfaBGF*
Theory Prop Id:	*08935706a424eeef4a646b98c7dc667515cfe98f6a4e99488cf85e717d7bf96d*
Theory Prop Address:	*TMWn3sP7p5YyoTSSYF8fydsrBPLkb9tjRKH*

add_CSNo_minus_CSNo_rinv

Theorem 28.163

$$\forall z.\mathsf{CSNo}\ z \to \mathsf{add\_CSNo}\ z\ (\mathsf{minus\_CSNo}\ z) = 0.$$

The proposition is identified by the following information:

Pure Prop Id:	*350994f849f5d91eeb37d84462edfab4a847176479ae191377608bb65c929618*
Pure Prop Address:	*TMRirTWCrP5cVCz5MNiHfJ1PfNDmQMnbMjY*
Theory Prop Id:	*268ed1aa6845aff2a06bc069ee87b3e46e06fa4ca30e8fe25c18f5b8541b8d69*
Theory Prop Address:	*TMH6YAu7Qh6P2UjC47FxTcP6vhxmhZgf3V8*

minus_SNo_minus_CSNo

Theorem 28.164 $\forall x.\mathsf{SNo}\ x \to -x = \mathsf{minus\_CSNo}\ x$. *The proposition is identified by the following information:*

Pure Prop Id:	*95399dcec725f3eaf0d48ead791b18326af59f257c823337fdccdeb08631a78c*
Pure Prop Address:	*TMc7tz6FA1Zoz96tsmZ9t1mi6UymeB4f5gM*
Theory Prop Id:	*23ad035b79e39c7c63924d15360f357008dcb6ef5d26f79c98ed7ee083b32498*
Theory Prop Address:	*TMYnJa2FrK92ftPPxTYDAiWjQqyzmtH1euw*

28.4 Complex II

Notation. We use $--$ as a prefix operator corresponding to applying term minus_CSNo. **Notation.** We use $+$ as a right associative infix operator corresponding to applying term add_CSNo. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term mul_CSNo. **Notation.** We use $\div$ as an infix operator corresponding to applying term div_CSNo.

Definition 28.18 *We define* int *to be* $\mathsf{omega} \cup \{-n | n \in \mathsf{omega}\}$ *of type* ι *identified by the following information:*

Pure Object Id:	*4daffb669546d65312481b5f945330815f8f5c460c7278516e497b08a82751e5*
Pure Object Address:	*TMTGgpvNpXMeoR3NwZFd8y2xpBzTUSKgwbU*
Theory Object Id:	*50d206e99485ce72d7138ea33476258564bafcdd8979a0e0edf31a56964bb534*
Theory Object Address:	*TMYfYSNfpYrjF2bVUtReGwX4aiKctLkRVZM*

Definition 28.19 *We define* rational *to be*

$$\mathsf{ReplSep2}\ \mathsf{int}\ (\lambda\_.\mathsf{omega})\ (\lambda num\ den.den \neq 0)\ (\lambda num\ den.num \div den)$$

of type ι *identified by the following information:*

Pure Object Id:	*604584444cba0be7f74a050669bc3716573bcc6798d790d2a2e1d8203a2fd4d1*
Pure Object Address:	*TMHgCpKhfWk6RuSojtz9kE5WKRRiXLgKQKG*
Theory Object Id:	*cab0d274c7daec544f49542150159453f1d3ebe31677ddce588712380bdb3ffb*
Theory Object Address:	*TMJ8qVVQVeuLH1QKNaSQ3YvEGoSB176XS43*

Definition 28.20 *We define* Sum *to be*

$$\lambda mnf.\mathsf{nat\_primrec}\ 0\ (\lambda kr.\mathsf{if}\ k \in m\ \mathsf{then}\ 0\ \mathsf{else}\ f\ k + r)\ (\mathsf{ordsucc}\ n)$$

of type $\iota \to \iota \to (\iota \to \iota) \to \iota$ *identified by the following information:*

Pure Object Id:	*d8ee26bf5548eea4c1130fe0542525ede828afda0cef2c9286b7a91d8bb5a9bd*
Pure Object Address:	*TMFy5sQDCD3u6eVG9houLmF2KaGQyzqJsCp*
Theory Object Id:	*81ddb08445e3612684413209359bef93d8b0cf5c8394caeca364d20434961abd*
Theory Object Address:	*TMcbn2wMDtNVYfjf2vBVxsgPGPuUtbFsxBs*

Definition 28.21 *We define* Prod *to be*

$$\lambda mnf.\mathsf{nat\_primrec}\ 1\ (\lambda kr.\mathsf{if}\ k \in m\ \mathsf{then}\ 1\ \mathsf{else}\ f\ k * r)\ (\mathsf{ordsucc}\ n)$$

of type $\iota \to \iota \to (\iota \to \iota) \to \iota$ *identified by the following information:*

Pure Object Id:	*db3d34956afa98a36fc0bf6a163888112f6d0254d9942f5a36ed2a91a1223d71*
Pure Object Address:	*TMdN8KB95hDDy7J7CZ25zug8XWiYBFRqyCp*
Theory Object Id:	*da66d8ac9780f328f37f0b84d5376c156ffd8cf8b9eaf38731c6ac83d286cb0e*
Theory Object Address:	*TMSX8CAGQArDkpR8V6Cb73nTcPCTkLUYXtA*

28.5 Int

Notation. We use + as a right associative infix operator corresponding to applying term add_SNo. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term mul_SNo. **Notation.** We use $--$ as a prefix operator corresponding to applying term minus_SNo. **Notation.** We use $\div$ as an infix operator corresponding to applying term div_SNo.

int_SNo_cases

Theorem 28.165

$$\forall p : \iota \to o.(\forall n \in \mathsf{omega}.p\ n) \to (\forall n \in \mathsf{omega}.p\ (-n)) \to \forall x \in \mathsf{int}.p\ x.$$

The proposition is identified by the following information:

Pure Prop Id:	*2504c05a08587fe0873ed45685efc417625f0a904281d653d757d643896f9a70*
Pure Prop Address:	*TMUEfaEhnmjqzhHDct4aN4nYsgTMNHexc5Z*
Theory Prop Id:	*a32835e1d8cbee7ebb59bff60b86d8d343a78a5a5b0a1a988fe8ae0752f9b0a9*
Theory Prop Address:	*TMcPpJsLS8zQSHciyZymBNG61rHAnJoGbht*

Subq_omega_int

Theorem 28.166 omega $\subseteq$ int. *The proposition is identified by the following information:*

Pure Prop Id:	*c213ff287d87049b1e6a47a232f87c366800922741a9eeadb1d3ac2fbadaf052*
Pure Prop Address:	*TMbZKRVRA2o1bkPt9NF2BWMQLKKJzDaq77o*
Theory Prop Id:	*c3d99d24216bf364de569b6a9cd1de13b46ceb52421e78a517df07524d1308a5*
Theory Prop Address:	*TMJyopLK6GuuCWqhP1XKASZh9iSwtdLiMs2*

int_minus_SNo_omega

Theorem 28.167 $\forall n \in \mathsf{omega}.-n \in \mathsf{int}$. *The proposition is identified by the following information:*

Pure Prop Id:	*a66fb27a7b2af57722c6537d3983b55a12cc28475f1d8b8d9bdb1d857010e7af*
Pure Prop Address:	*TMHaC3837eh4SUn1jL2MVZSY2HYJVykA2q8*
Theory Prop Id:	*1b4d50647e615e454ab958cf206bb3f4b349286064ed07e205cde10ee49537b0*
Theory Prop Address:	*TMYLKPRR9oEXo9N8Z2YBDanBt7x7VLvfePj*

int_minus_SNo

Theorem 28.168 $\forall x \in \mathsf{int}.-x \in \mathsf{int}$. *The proposition is identified by the following information:*

Pure Prop Id:	*66d7a7b7f8768657be1ea35e52473cc5e1846e635153a280e3783a8275062773*
Pure Prop Address:	*TMSXiWmTFpmbC8vtTafxAwFeYuNrgZJj4vW*
Theory Prop Id:	*daaadd58a6cfb975a0d9200d19c663526ca0a4951284ec16a5e11ae2e3fd7492*
Theory Prop Address:	*TMV6sHDGtUc6XzQK9XLPBGFqCcMjQdPpFHz*

int_add_SNo_lem

Theorem 28.169 $\forall n \in \mathsf{omega}.\forall m.\mathsf{nat\_p}\ m \to -n + m \in \mathsf{int}$. *The proposition is identified by the following information:*

Pure Prop Id:	*6c976be5ae7c4eec61e1190f4b65a1cc39ebfb81542ae63578b69be42c01a06a*
Pure Prop Address:	*TMLvPoJ9aN37XYyR84rMi7xqr8n8tsuhtfM*
Theory Prop Id:	*6fbb03f9ad1cac7ae41730f8a527dac94e9c83e918287c89f3787c4a3faac1a5*
Theory Prop Address:	*TMPwH3A6N2tb87tNY5vbkqqydAuD5QR3EyF*

int_add_SNo

Theorem 28.170 $\forall xy \in \mathsf{int}.x + y \in \mathsf{int}$. *The proposition is identified by the following information:*

Pure Prop Id:	*02609f82bf442d61fb8c6410818e7e4cbf8c67c9ea08bffcbf8a77c06b0f784a*
Pure Prop Address:	*TMFDVQawMT1qNmvZaWsWLAFRwNsZDvdEWd7*
Theory Prop Id:	*a0aa5766d0cdd5590d613a2f72f025acfa458bc0cda8293f060de4d24de253c5*
Theory Prop Address:	*TMag1esGeGDfnX5JdbJHct4N6L993dNSRki*

int_mul_SNo

Theorem 28.171 $\forall xy \in \mathsf{int}.x * y \in \mathsf{int}$. *The proposition is identified by the following information:*

Pure Prop Id:	*3f000c087708670f2a9497d0587231f65beb85593b7ebdaf7909fe5e7d4b8c27*
Pure Prop Address:	*TMbhP3nSug9dzpwnhRm7VfyD7KAzF5WfghL*
Theory Prop Id:	*eefdfbfec3e66c305c9c9bc9c837268558851ac36fad12ee4b371f995afc9fd5*
Theory Prop Address:	*TMbSE7nyU8X9azWYyGBkvTt8zRNBARSeYkH*

Definition 28.22 *We define* divides_int *to be*

$$\lambda mn.m \in \mathsf{int} \land n \in \mathsf{int} \land \exists k \in \mathsf{int}.m * k = n$$

of type $\iota \to \iota \to o$ identified by the following information:

Pure Object Id:	*af5f6e65098725dbbf9014383760c67fe7961b5486f9b4fa4d3289ed424f0c1b*
Pure Object Address:	*TMX9HJKhzi9AdyH6nXyJVdE4nKLJvyF9yNE*
Theory Object Id:	*5fa28bc7b02e3ca96f01706f3aa25b382b44df0d71c91f7a7c68aea0935e9273*
Theory Object Address:	*TMT9KA9dVEU4Cq2ursSu7Ep2iNaWdDT8u12*

Definition 28.23 *We define* equiv_int_mod *to be*

$$\lambda mkn.m \in \mathsf{int} \land k \in \mathsf{int} \land n \in \mathsf{omega} \setminus 1 \land \mathsf{divides\_int}\ (m + -k)\ n$$

of type $\iota \to \iota \to \iota \to o$ identified by the following information:

Pure Object Id:	*aea710807b7901288ae25daacdd1e52348dfbe79c9a58fabb5e91c68e15a1581*
Pure Object Address:	*TMHGhKGHRdgbdvTgpBF85dWTWtVw1PHGZZM*
Theory Object Id:	*a0bc48ffd10d4e9112ad9d98af3c6392a81e8ef3d019ce4b8ec86f77a5a49a7d*
Theory Object Address:	*TMWqX7aoEj8tNfrfptrVaGaaFkknJoVdiPo*

Definition 28.24 *We define* coprime_int *to be*

$\lambda ab.a \in \mathsf{int} \wedge b \in \mathsf{int} \wedge \forall x \in \mathsf{omega} \setminus 1.\mathsf{divides\_int}\ x\ a \to \mathsf{divides\_int}\ x\ b \to x = 1$

of type $\iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*48ca77268e35362c65956d7fd4145d07598e7a91e7ef4a1e6af043bc1202b37f*
Pure Object Address:	*TMRPbZHuWZc21V3zuKPZMGa165KDXe2rvuU*
Theory Object Id:	*76fc5b183d6bea5474424b2b6616dbb8c29591fb620aa364408cc43266fa15ec*
Theory Object Address:	*TMXXHSe9mRiHSQ717podNGPQLawz24s1P1q*

28.6 Packing Two Operations, a Relation and Two Constants

Definition 28.25 pack_b_b_r_e_e *is the opaque object of type*

$$\iota \to (\iota \to \iota \to \iota) \to (\iota \to \iota \to \iota) \to (\iota \to \iota \to o) \to \iota \to \iota \to \iota$$

identified by the following information:

Pure Object Id:	*8efb1973b4a9b292951aa9ca2922b7aa15d8db021bfada9c0f07fc9bb09b65fb*
Pure Object Address:	*TMFRc8E8tg6JtPF2W7a5hcH7dKma4rhhjpe*
Theory Object Id:	*a33bdffb44829c458189fcf9c1fc93f1abfb0007a9318b33fa18bab49932b69b*
Theory Object Address:	*TMUoWdaNvqJBEvaP7AZFqh425UCXiK49xiT*

pack_b_b_r_e_e_0_eq

Theorem 28.172

$$\forall SX.\forall f : \iota \to \iota \to \iota.\forall g : \iota \to \iota \to \iota.\forall R : \iota \to \iota \to o.\forall c : \iota.\forall d : \iota.$$
$$S = \mathsf{pack\_b\_b\_r\_e\_e}\ X\ f\ g\ R\ c\ d \to X = S\ 0.$$

The proposition is identified by the following information:

Pure Prop Id:	*f5a2203f23e7dd04fe8ca0e3fb972d0098d3812a6f2a49e5016460d7db993bf5*
Pure Prop Address:	*TMRGip4djr41vccrTDQXNBbxCWrX8VbHcqJ*
Theory Prop Id:	*2c10ca5a1162991d07d3c553c21901d5abd7b71cb1ca8c024781096deed06657*
Theory Prop Address:	*TMQjRgtSbPqLcPdbvF5gpcTyqr1VGnTtfgw*

pack_b_b_r_e_e_0_eq2

Theorem 28.173

$$\forall X.\forall f : \iota \to \iota \to \iota.\forall g : \iota \to \iota \to \iota.\forall R : \iota \to \iota \to o.\forall c : \iota.\forall d : \iota.$$
$$X = \mathsf{pack\_b\_b\_r\_e\_e}\ X\ f\ g\ R\ c\ d\ 0.$$

The proposition is identified by the following information:

Pure Prop Id:	*7136b27641c7cc4e1a647a898e4ebf0836d4299f8da51ec9709fc246183a9280*
Pure Prop Address:	*TMXU7ECTAkVR3MxsrCyo2KrDJ4eLHVp18Z2*
Theory Prop Id:	*eb82f1296debb50aec472ad719325c4d415f74e68c24cbfeb6eb48e13b06fe27*
Theory Prop Address:	*TMHkCAK4V2154ECdQyfXBNV3CMQ6datiFHq*

`pack_b_b_r_e_e_1_eq`

Theorem 28.174

$$\forall SX.\forall f : \iota \to \iota \to \iota.\forall g : \iota \to \iota \to \iota.\forall R : \iota \to \iota \to o.\forall c : \iota.\forall d : \iota.$$
$$S = \mathsf{pack\_b\_b\_r\_e\_e}\ X\ f\ g\ R\ c\ d \to \forall xy \in X.$$
$$f\ x\ y = \mathsf{decode\_b}\ (S\ 1)\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*ccc17a15fee3ff8e50e0a429d8fc3703e0d4322e680a51e8331926734edd0a78*
Pure Prop Address:	*TMGv6tf3d83HWkjLGc4Bjfk2yL6bP1Y69Ea*
Theory Prop Id:	*b982a46d670799d817f2c72da1c2d0fb616888b041e52fcb1bdd4cd4f36e4a15*
Theory Prop Address:	*TMPFdacX88nXYB1oHJX48p69cohuPHPDyHj*

`pack_b_b_r_e_e_1_eq2`

Theorem 28.175

$$\forall X.\forall f : \iota \to \iota \to \iota.\forall g : \iota \to \iota \to \iota.\forall R : \iota \to \iota \to o.\forall c : \iota.\forall d : \iota.\forall xy \in X.$$
$$f\ x\ y = \mathsf{decode\_b}\ (\mathsf{pack\_b\_b\_r\_e\_e}\ X\ f\ g\ R\ c\ d\ 1)\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*e738dbc94ff336d55dbd404f3523f553aed8269c125ce07c558108d936f8bf8e*
Pure Prop Address:	*TMczpSJ93XaFjvH4vijE4cc2RjpSvLCBnZ1*
Theory Prop Id:	*3048f770c8c092a3cc8c55bb78a7d82f19565aac3ff6652ef9d55e6c0cfb0962*
Theory Prop Address:	*TMGxoR9fiBCzsbqU2uTbeELHpxWJM9ikYSB*

`pack_b_b_r_e_e_2_eq`

Theorem 28.176

$$\forall SX.\forall f : \iota \to \iota \to \iota.\forall g : \iota \to \iota \to \iota.\forall R : \iota \to \iota \to o.\forall c : \iota.\forall d : \iota.$$
$$S = \mathsf{pack\_b\_b\_r\_e\_e}\ X\ f\ g\ R\ c\ d \to \forall xy \in X.$$
$$g\ x\ y = \mathsf{decode\_b}\ (S\ 2)\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*a822390210aa0eeae11feeec870412a9d3f167449db2716772c8d85d36fde3ad*
Pure Prop Address:	*TMGuyMyeVJR3BDF1pJNw6KGpyy6D5ZNkVqb*
Theory Prop Id:	*2d8185611c6998b74706354379f7e37a3983d2acac54f2a80803b05199954f8f*
Theory Prop Address:	*TMNWRvZ8TbLk6nt1L5aUcRmKEgigkF4bzbC*

pack_b_b_r_e_e_2_eq2

Theorem 28.177

$$\forall X.\forall f :\iota \to \iota \to \iota.\forall g :\iota \to \iota \to \iota.\forall R :\iota \to \iota \to o.\forall c :\iota.\forall d :\iota.\forall xy \in X.$$
$$g\ x\ y = \mathsf{decode\_b}\ (\mathsf{pack\_b\_b\_r\_e\_e}\ X\ f\ g\ R\ c\ d\ 2)\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*0904eb84a994cf26e71f4541979476936ad04090940f84ea7e3ae166191f386f*
Pure Prop Address:	*TMZgHEgS5WRZZnmn2WtWxSCUaL6S8KAKuEU*
Theory Prop Id:	*514b7ec403837110f3c264073db0858ef3b66bcb89b034df3d99771f31f409f1*
Theory Prop Address:	*TMPsaDHiTWNr1DquvQg49N7jxY2zPJAhBQD*

pack_b_b_r_e_e_3_eq

Theorem 28.178

$$\forall SX.\forall f :\iota \to \iota \to \iota.\forall g :\iota \to \iota \to \iota.\forall R :\iota \to \iota \to o.\forall c :\iota.\forall d :\iota.$$
$$S = \mathsf{pack\_b\_b\_r\_e\_e}\ X\ f\ g\ R\ c\ d \to \forall xy \in X.$$
$$R\ x\ y = \mathsf{decode\_r}\ (S\ 3)\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*20b01269edbb5f190d76d15ed12389468b65b4f3a49a6747fc5a644de89692c4*
Pure Prop Address:	*TMZCKwwQH2ns9uknt6pfpXLnMeYPTdrkKsA*
Theory Prop Id:	*543a595f6d03cb89cc51280f13fd9710e66da890237a5f792e79701620885eb7*
Theory Prop Address:	*TMPLwektPtu4AjrXp4ghumX9jTbcGU7yeaD*

pack_b_b_r_e_e_3_eq2

Theorem 28.179

$$\forall X.\forall f :\iota \to \iota \to \iota.\forall g :\iota \to \iota \to \iota.\forall R :\iota \to \iota \to o.\forall c :\iota.\forall d :\iota.\forall xy \in X.$$
$$R\ x\ y = \mathsf{decode\_r}\ (\mathsf{pack\_b\_b\_r\_e\_e}\ X\ f\ g\ R\ c\ d\ 3)\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*84e5f9f9566add41b035a90f33b58ab881576661927e7112435c4cc63f5b9a82*
Pure Prop Address:	*TMcKPp5Wtf1GqSMtPaoCwtuAz9G543UZXY7*
Theory Prop Id:	*e6c65fe282c292809ca69c9ff83b219fff807abed4e4977ed7b6ca1bd889aeb5*
Theory Prop Address:	*TMH676J4Qexg1jep9yQTFXu4vNi1TJ1qrZX*

pack_b_b_r_e_e_4_eq

Theorem 28.180

$$\forall S X.\forall f :\iota \to \iota \to \iota.\forall g :\iota \to \iota \to \iota.\forall R :\iota \to \iota \to o.\forall c :\iota.\forall d :\iota.$$
$$S = \mathsf{pack\_b\_b\_r\_e\_e}\ X\ f\ g\ R\ c\ d \to c = S\ 4.$$

The proposition is identified by the following information:

Pure Prop Id:	*7c411fa1b904b5c366996dc15f10c49f1ca9b84a3441266671af3c8907575779*
Pure Prop Address:	*TMSSAAEDYMUWREf5eqjEN4jmyCnfvyeobwE*
Theory Prop Id:	*4e1693db4de27914c071907d430af0401ef3bd6ac24367ac8791b6c5c512091e*
Theory Prop Address:	*TMGzMNkeS7o5At8847MUjvdcHvigbTpaypB*

pack_b_b_r_e_e_4_eq2

Theorem 28.181

$$\forall X.\forall f :\iota \to \iota \to \iota.\forall g :\iota \to \iota \to \iota.\forall R :\iota \to \iota \to o.\forall c :\iota.\forall d :\iota.$$
$$c = \mathsf{pack\_b\_b\_r\_e\_e}\ X\ f\ g\ R\ c\ d\ 4.$$

The proposition is identified by the following information:

Pure Prop Id:	*79c12fb164791187a04849be1011349dce3e15aad6a3329faac347fc335089f5*
Pure Prop Address:	*TMFqB5mPoMCs8LrcpMtffiyDeommry4W3Wf*
Theory Prop Id:	*11df0febe6f1e3560b5041849da4dd2bdc05c8527c22c5617386567a92e57fa6*
Theory Prop Address:	*TMct6V7NrbQ2y9kAcrjnEM2WVDGaQZrcgjK*

pack_b_b_r_e_e_5_eq

Theorem 28.182

$$\forall S X.\forall f :\iota \to \iota \to \iota.\forall g :\iota \to \iota \to \iota.\forall R :\iota \to \iota \to o.\forall c :\iota.\forall d :\iota.$$
$$S = \mathsf{pack\_b\_b\_r\_e\_e}\ X\ f\ g\ R\ c\ d \to d = S\ 5.$$

The proposition is identified by the following information:

Pure Prop Id:	*94b3c3a63e1902ef3037b87eaa7fcbbae94338cd1163cceeed376f1eb8ec7ef1*
Pure Prop Address:	*TMHxq3wxovqDHYziUjPoyA45c2sAhm2UNoq*
Theory Prop Id:	*cc915a2d36f6cc79c293654aab8fd3adb2808299e9b173a1c39bf3cdac0dfe5c*
Theory Prop Address:	*TMSYohXBC39TSSsVBLMnYazqxd2k9SYkjSg*

pack_b_b_r_e_e_5_eq2

Theorem 28.183

$$\forall X.\forall f :\iota \to \iota \to \iota.\forall g :\iota \to \iota \to \iota.\forall R :\iota \to \iota \to o.\forall c :\iota.\forall d :\iota.$$
$$d = \mathsf{pack\_b\_b\_r\_e\_e}\ X\ f\ g\ R\ c\ d\ 5.$$

The proposition is identified by the following information:

Pure Prop Id:	*b0d3a42291179839984699106279142ceeb8c26475897616c224628ed6d51873*
Pure Prop Address:	*TMcWSqvc4wwoPFxLaHnaqahY7ZNdALEXxz1*
Theory Prop Id:	*29ae629ba4f8c9f4ea56fe01e58c7e5adc9a3f4032832a6256e4172bc1857db1*
Theory Prop Address:	*TMSx1WE5sJ7RC3YQth9f6oafrR3W5T8cUAW*

pack_b_b_r_e_e_inj

Theorem 28.184

$$\forall X X'.\forall f f' : \iota \to \iota \to \iota.\forall g g' : \iota \to \iota \to \iota.\forall R R' : \iota \to \iota \to o.\forall c c' : \iota.\forall d d' : \iota.$$
$$\mathsf{pack\_b\_b\_r\_e\_e}\ X\ f\ g\ R\ c\ d = \mathsf{pack\_b\_b\_r\_e\_e}\ X'\ f'\ g'\ R'\ c'\ d'$$
$$\to$$
$$X = X' \wedge$$
$$(\forall x y \in X.f\ x\ y = f'\ x\ y) \wedge$$
$$(\forall x y \in X.g\ x\ y = g'\ x\ y) \wedge$$
$$(\forall x y \in X.R\ x\ y = R'\ x\ y) \wedge$$
$$c = c' \wedge d = d'.$$

The proposition is identified by the following information:

Pure Prop Id:	*41bdc7cea27f40d41bd47e10070a9221fac56ee80059ed6ef1145c640f0a0696*
Pure Prop Address:	*TMZzLqefgu7rLgWmGbXgEsJw9KyJ9HfnEfv*
Theory Prop Id:	*030a267e64cc2ab4e90b427a38635de36741b1d1f5f749729223836ad895b355*
Theory Prop Address:	*TMFdLKT9s9pybGa29WS8rRsZtaZEB878Tqc*

pack_b_b_r_e_e_ext

Theorem 28.185

$$\forall X.\forall f f' : \iota \to \iota \to \iota.\forall g g' : \iota \to \iota \to \iota.\forall R R' : \iota \to \iota \to o.\forall c.\forall d.$$
$$(\forall x y \in X.f\ x\ y = f'\ x\ y) \to (\forall x y \in X.g\ x\ y = g'\ x\ y) \to$$
$$(\forall x y \in X.R\ x\ y \Leftrightarrow R'\ x\ y) \to$$
$$\mathsf{pack\_b\_b\_r\_e\_e}\ X\ f\ g\ R\ c\ d = \mathsf{pack\_b\_b\_r\_e\_e}\ X\ f'\ g'\ R'\ c\ d.$$

The proposition is identified by the following information:

Pure Prop Id:	*65aff8c9424ee01cd07c15ae87a00a524f02d18e2ec143a5bba84026587bf7f3*
Pure Prop Address:	*TMcK19CLntYiEUmEtDidiA4HpkWUA29DCY8*
Theory Prop Id:	*37b535ddadbc59098fcbd9eea3333e24e8d8ba92e8fa6adac43561bcb663a641*
Theory Prop Address:	*TMSrpevWvuK3PWAB7qQDUur4T35U2y5PcFh*

Definition 28.26 *We define* struct_b_b_r_e_e *to be*

$$\lambda S.\forall q : \iota \to o.(\forall X : \iota.\forall f : \iota \to \iota \to \iota.(\forall x y \in X.f\ x\ y \in X) \to$$
$$\forall g : \iota \to \iota \to \iota.(\forall x y \in X.g\ x\ y \in X) \to$$
$$\forall R : \iota \to \iota \to o.\forall c : \iota.c \in X \to \forall d : \iota.d \in X \to$$
$$q\ (\mathsf{pack\_b\_b\_r\_e\_e}\ X\ f\ g\ R\ c\ d))$$
$$\to q\ S$$

of type $\iota \to o$ identified by the following information:

Pure Object Id: *d542f60aebdbe4e018abe75d8586699fd6db6951a7fdc2bc884bfb42c0d77a22*
Pure Object Address: *TMXf978YVAAD5ZQex2ymHckFjvhRj8VqJG9*
Theory Object Id: *4b668479c87844857651881d76dfae59eab34bb3676e61f76f6dd64b6060cf5d*
Theory Object Address: *TMKS3b6mms1vofrtcap5oMtL9Kdzd24UwRB*

pack_struct_b_b_r_e_e_I

Theorem 28.186

$$\forall X.\forall f : \iota \to \iota \to \iota.(\forall xy \in X.f\ x\ y \in X) \to \forall g : \iota \to \iota \to \iota.(\forall xy \in X.g\ x\ y \in X) \to \forall R : \iota \to \iota \to o.\forall c : \iota.c \in X \to \forall d : \iota.d \in X \to \mathsf{struct\_b\_b\_r\_e\_e}\ (\mathsf{pack\_b\_b\_r\_e\_e}\ X\ f\ g\ R\ c\ d).$$

The proposition is identified by the following information:

Pure Prop Id: *837b01cc10cf77ecfed9c05cb135bfee5d5785cb3eeb155cd67420a1e8117277*
Pure Prop Address: *TMdKBXs47VAuzq32DPCspAGpMJxWtP7HZgn*
Theory Prop Id: *4e1d4bac0d2464119ef3ae868cd30477053be4e52d11f44a4826bb32b543f78a*
Theory Prop Address: *TMRfKTdq9BR53wNTCPty6ckkNQRPzNXtwXR*

pack_struct_b_b_r_e_e_E1

Theorem 28.187

$$\forall X.\forall f : \iota \to \iota \to \iota.\forall g : \iota \to \iota \to \iota.\forall R : \iota \to \iota \to o.\forall c : \iota.\forall d : \iota. \mathsf{struct\_b\_b\_r\_e\_e}\ (\mathsf{pack\_b\_b\_r\_e\_e}\ X\ f\ g\ R\ c\ d) \to \forall xy \in X.f\ x\ y \in X.$$

The proposition is identified by the following information:

Pure Prop Id: *33666944863c7073c5d24b456627c47939a6c292049de068c2dd0a2696024881*
Pure Prop Address: *TMbqCqWUV4Qr8yRDenPHW1SCXnMHwtzsEqR*
Theory Prop Id: *bce7cd0660e462218e87e9b6e9756ae8795d7e47a2034af9f01a233591cd4516*
Theory Prop Address: *TMTcgTJU3oXQrtzipjvwHThz1WBYVpT6Tr8*

pack_struct_b_b_r_e_e_E2

Theorem 28.188

$$\forall X.\forall f : \iota \to \iota \to \iota.\forall g : \iota \to \iota \to \iota.\forall R : \iota \to \iota \to o.\forall c : \iota.\forall d : \iota. \mathsf{struct\_b\_b\_r\_e\_e}\ (\mathsf{pack\_b\_b\_r\_e\_e}\ X\ f\ g\ R\ c\ d) \to \forall xy \in X.g\ x\ y \in X.$$

The proposition is identified by the following information:

Pure Prop Id: *8d7f819bc88f04c8fb38f9c45656b24be62467c1111be1ab6669a362d2c42bd9*
Pure Prop Address: *TMFq66jAxt6ToaMefzyqpZmo2jxWBS1L5KS*
Theory Prop Id: *a642313376049cb0e12a05cfd42eb6fe35a06fa7ac25ab566e3a668a2f094fa5*
Theory Prop Address: *TMKRXGcsc5vFBxDbQ738RFuErE5UAsvbXrX*

pack_struct_b_b_r_e_e_E4

Theorem 28.189

$$\forall X.\forall f :\iota \to \iota \to \iota.\forall g :\iota \to \iota \to \iota.\forall R :\iota \to \iota \to o.\forall c :\iota.\forall d :\iota.$$
$$\mathsf{struct\_b\_b\_r\_e\_e}\ (\mathsf{pack\_b\_b\_r\_e\_e}\ X\ f\ g\ R\ c\ d) \to$$
$$c \in X.$$

The proposition is identified by the following information:

Pure Prop Id:	*fe2c180cc0cf649c36df0e3a2f9dabf02603a9d163927935d608712fe908c0a0*
Pure Prop Address:	*TMKkUcfVuzqdAAqixhrJdaDtwAiwP588iFb*
Theory Prop Id:	*4343532cd958d270bb174fb9ef0d486e82bcb7068472964cad87b3aa59268c36*
Theory Prop Address:	*TMVkwckBqwmZ3ptsoBAtwgWpGJnBf1HEKJX*

pack_struct_b_b_r_e_e_E5

Theorem 28.190

$$\forall X.\forall f :\iota \to \iota \to \iota.\forall g :\iota \to \iota \to \iota.\forall R :\iota \to \iota \to o.\forall c :\iota.\forall d :\iota.$$
$$\mathsf{struct\_b\_b\_r\_e\_e}\ (\mathsf{pack\_b\_b\_r\_e\_e}\ X\ f\ g\ R\ c\ d) \to$$
$$d \in X.$$

The proposition is identified by the following information:

Pure Prop Id:	*1eaba4d18e5ef9249157efebe811892c11f22eaccbdfaabe7207d0eed3610753*
Pure Prop Address:	*TMN8VR6KEzNcbMfJUKCFWE9VrqsSAghpm2a*
Theory Prop Id:	*d7fee779a369a0d4bd81cd98ec4b107047599cd8daebdc70621edcfba45ab303*
Theory Prop Address:	*TMRKtcvjctfdAsskw9TbK95G7FLTa6F6w99*

struct_b_b_r_e_e_eta

Theorem 28.191

$$\forall S.\mathsf{struct\_b\_b\_r\_e\_e}\ S \to$$
$$S =$$
$$\mathsf{pack\_b\_b\_r\_e\_e}\ (S\ 0)\ (\mathsf{decode\_b}\ (S\ 1))\ (\mathsf{decode\_b}\ (S\ 2))$$
$$(\mathsf{decode\_r}\ (S\ 3))\ (S\ 4)\ (S\ 5).$$

The proposition is identified by the following information:

Pure Prop Id:	*08c6efad770174c53c57bf1abbcff7a0b238338e1a0a913fdb1e12ac817545eb*
Pure Prop Address:	*TMGztHkuyt42zcYRkYwSMfmeqnNcm8EkxuB*
Theory Prop Id:	*3a4c426298d981750f4f7847d710686bde28aba7b880e1e16dc568078e493ab5*
Theory Prop Address:	*TMdFRkJgJNNXSyM8pAQoyEqPBnQPHMehaWr*

Definition 28.27 unpack_b_b_r_e_e_i *is the opaque object of type*

$$\iota \to (\iota \to (\iota \to \iota \to \iota) \to (\iota \to \iota \to \iota) \to (\iota \to \iota \to o) \to \iota \to \iota \to \iota) \to \iota$$

identified by the following information:

Pure Object Id:	*89fb5e380d96cc4a16ceba7825bfbc02dfd37f2e63dd703009885c4bf3794d07*
Pure Object Address:	*TMV6sraHemVG7Q2TgN6oCsJmkmebyJuYVNd*
Theory Object Id:	*76f5f924237f946f8a80d7200b117ad49d05a912a831233242659e6f501bf978*
Theory Object Address:	*TMRP69PDPah6ZkTeGTDbp6iVPip8vSRTsB8*

unpack_b_b_r_e_e_i_eq

Theorem 28.192

$$\begin{array}{c}\forall \Phi : \iota \to (\iota \to \iota \to \iota) \to (\iota \to \iota \to \iota) \to (\iota \to \iota \to o) \to \iota \to \iota \to \iota. \\ \forall X. \forall f : \iota \to \iota \to \iota. \forall g : \iota \to \iota \to \iota. \\ \forall R : \iota \to \iota \to o. \forall c : \iota. \forall d : \iota. \\ (\forall f' : \iota \to \iota \to \iota. (\forall xy \in X. f\ x\ y = f'\ x\ y) \to \\ \forall g' : \iota \to \iota \to \iota. (\forall xy \in X. g\ x\ y = g'\ x\ y) \to \\ \forall R' : \iota \to \iota \to o. (\forall xy \in X. R\ x\ y \Leftrightarrow R'\ x\ y) \to \\ \Phi\ X\ f'\ g'\ R'\ c\ d = \Phi\ X\ f\ g\ R\ c\ d) \\ \to \\ \mathsf{unpack\_b\_b\_r\_e\_e\_i}\ (\mathsf{pack\_b\_b\_r\_e\_e}\ X\ f\ g\ R\ c\ d)\ \Phi = \Phi\ X\ f\ g\ R\ c\ d.\end{array}$$

The proposition is identified by the following information:

Pure Prop Id:	*b58f007e91d43a2c7a09dc0b595961936ec7da158803e7b98c8e478584d206ef*
Pure Prop Address:	*TMPqkpAFSVWhtBYDqJsL7JfE4qYGQvWKVkp*
Theory Prop Id:	*9a7986d0258cb1c3270b60b6b454888095d2aa2a2a53d845b4756902a36876f7*
Theory Prop Address:	*TMQfsPU4J48cc3XAFwauAcYbj5YSbwJVxJw*

Definition 28.28 unpack_b_b_r_e_e_o *is the opaque object of type*

$$\iota \to (\iota \to (\iota \to \iota \to \iota) \to (\iota \to \iota \to \iota) \to (\iota \to \iota \to o) \to \iota \to \iota \to o) \to o$$

identified by the following information:

Pure Object Id:	*b3a2fc60275daf51e6cbe3161abaeed96cb2fc4e1d5fe26b5e3e58d0eb93c477*
Pure Object Address:	*TMP75qXVTx8SpwHV1y6wefKnbFqVV7VPCV7*
Theory Object Id:	*fc0d55fe8a09aad6902204ac4550a6671cc57b3b29e4c36503dea3e8cc818737*
Theory Object Address:	*TMQVwZxiv8LXLzFAAtQUdESpWctYhNPKuLQ*

unpack_b_b_r_e_e_o_eq

Theorem 28.193

$$\forall \Phi : \iota \to (\iota \to \iota \to \iota) \to (\iota \to \iota \to \iota) \to (\iota \to \iota \to o) \to \iota \to \iota \to o.$$
$$\forall X. \forall f : \iota \to \iota \to \iota. \forall g : \iota \to \iota \to \iota.$$
$$\forall R : \iota \to \iota \to o. \forall c : \iota. \forall d : \iota.$$
$$(\forall f' : \iota \to \iota \to \iota. (\forall xy \in X. f\ x\ y = f'\ x\ y) \to$$
$$\forall g' : \iota \to \iota \to \iota. (\forall xy \in X. g\ x\ y = g'\ x\ y) \to$$
$$\forall R' : \iota \to \iota \to o. (\forall xy \in X. R\ x\ y \Leftrightarrow R'\ x\ y) \to$$
$$\Phi\ X\ f'\ g'\ R'\ c\ d = \Phi\ X\ f\ g\ R\ c\ d)$$
$$\to$$
$$\mathsf{unpack\_b\_b\_r\_e\_e\_o}\ (\mathsf{pack\_b\_b\_r\_e\_e}\ X\ f\ g\ R\ c\ d)\ \Phi = \Phi\ X\ f\ g\ R\ c\ d.$$

The proposition is identified by the following information:

Pure Prop Id:	*b4ebc419abea98259a9b2fc6ee430a9eb6b66901e01778798caef327518ab73c*
Pure Prop Address:	*TMWy9NbJuezT2C2YvSVs4gNoW6RxDCu36cJ*
Theory Prop Id:	*308ccf9882d7cb03559534a600b368145e2c7062734d3ea9574a2b00c5a6bbd1*
Theory Prop Address:	*TMdarJDcWV77zy8PF3SrMpz5YVBUqiWCp7G*

Definition 28.29 OrderedFieldStruct *is the opaque object of type* $\iota \to o$ *identified by the following information:*

Pure Object Id:	*98aa98f64160bebd5622706908b639f9fdfe4056f2678ed69e5b099b8dd7b888*
Pure Object Address:	*TMEzWdp7RFmZMEiX1EsMMaPEcXjJJfCtETX*
Theory Object Id:	*1de6a120bae49aa4021a9a6626425398f0c8d34c17a449aca28902eba9cfe754*
Theory Object Address:	*TMWMkhjM35KDgsppiv3Up8VXcf7pNB4BC6Q*

28.7 explicit_OrderedField_RepIndep2

Let $R : \iota$ be given. Let $zero, one : \iota$ be given. Let $plus, mult : \iota \to \iota \to \iota$ be given. Let $leq : \iota \to \iota \to o$ be given. **Notation.** We use $+$ as a right associative infix operator corresponding to applying term $plus$. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term $mult$. Let $plus', mult' : \iota \to \iota \to \iota$ be given. Let $leq' : \iota \to \iota \to o$ be given. **Notation.** We use $\oplus$ as a right associative infix operator corresponding to applying term $plus'$. **Notation.** We use $\times$ as a right associative infix operator corresponding to applying term $mult'$. Assume the following.

$$\forall ab \in R. a + b = a \oplus b \tag{28.1}$$

Assume the following.

$$\forall ab \in R. a * b = a \times b \tag{28.2}$$

Assume the following.

$$\forall ab \in R. leq\ a\ b \Leftrightarrow leq'\ a\ b \tag{28.3}$$

`explicit_OrderedField_repindep`

Theorem 28.194

$$\mathsf{explicit\_OrderedField}\ R\ zero\ one\ plus\ mult\ leq$$
$$\Leftrightarrow \mathsf{explicit\_OrderedField}\ R\ zero\ one\ plus'\ mult'\ leq'.$$

The proposition is identified by the following information:

Pure Prop Id:	*aa6280ff9ecfb388daf95ebbc769d81f90cc6039e5299709d31d625891887057*
Pure Prop Address:	*TMbsFvtTvqkK3SszQVqXzQ1UJEsUnVYiMZL*
Theory Prop Id:	*be25ce3a19ac950fdaa6ea42fe6fcb818e428c8dc8aae4b4fbe47fa06d08e029*
Theory Prop Address:	*TMdEcCwhu3D2kVgdgGVo1Cmg5W1sXuLX3HR*

28.8 Unpacking Ordered Fields

`OrderedFieldStruct_unpack_eq`

Theorem 28.195

$$\forall R.\forall plus\ mult : \iota \to \iota \to \iota.\forall leq : \iota \to \iota \to o.\forall zero\ one.$$
$$\mathsf{unpack\_b\_b\_r\_e\_e\_o}\ (\mathsf{pack\_b\_b\_r\_e\_e}\ R\ plus\ mult\ leq\ zero\ one)$$
$$(\lambda R\ plus\ mult\ leq\ zero\ one.\mathsf{explicit\_OrderedField}\ R\ zero\ one\ plus\ mult\ leq)$$
$$= \mathsf{explicit\_OrderedField}\ R\ zero\ one\ plus\ mult\ leq.$$

The proposition is identified by the following information:

Pure Prop Id:	*0f64cee1eaa56826e0a1df23c3c822f54eeeaf3467661 43d43f9cd8aaa5bcc05*
Pure Prop Address:	*TMcaQWXF265MqyQdZxoLdbgCWynQwDq1QFz*
Theory Prop Id:	*6fe0fe98ce1675b6b7d6602c3080c16c71b29a61d4b004d43b5855aad4d9a43f*
Theory Prop Address:	*TMXMSqH6fsZZwHid44hbcR3wDdTqU7Dm9HQ*

Definition 28.30 *We define* RealsStruct *to be*

$$\lambda R.\mathsf{struct\_b\_b\_r\_e\_e}\ R \wedge$$
$$\mathsf{unpack\_b\_b\_r\_e\_e\_o}\ R$$
$$(\lambda R\ plus\ mult\ leq\ zero\ one.\mathsf{explicit\_Reals}\ R\ zero\ one\ plus\ mult\ leq)$$

of type $\iota \to o$ identified by the following information:

Pure Object Id:	*9971dcf75f0995efe4245a98bcdf103e4713261d35432146405052f36f8234bf*
Pure Object Address:	*TMJjjbmFf1HjinfhpdrL5vryqPNmEPLfGtv*
Theory Object Id:	*3d1a749dfea9bb7211b105adb25ae072c40841648e843f5cbc5483cd7b1257cd*
Theory Object Address:	*TMQydd7NRsjg2p9tCMw9pgNmAnEKZZD616o*

28.9 explicit_Reals_RepIndep2

Let $R : \iota$ be given. Let $zero, one : \iota$ be given. Let $plus, mult : \iota \to \iota \to \iota$ be given. Let $leq : \iota \to \iota \to o$ be given. **Notation.** We use $+$ as a right associative infix operator corresponding to applying term $plus$. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term $mult$. Let $plus', mult' : \iota \to \iota \to \iota$ be given. Let $leq' : \iota \to \iota \to o$ be given. **Notation.** We use $\oplus$ as a right associative infix operator corresponding to applying term $plus'$. **Notation.** We use $\times$ as a right associative infix operator corresponding to applying term $mult'$. Assume the following.

$$\forall ab \in R.a + b = a \oplus b \tag{28.4}$$

Assume the following.

$$\forall ab \in R.a * b = a \times b \tag{28.5}$$

Assume the following.

$$\forall ab \in R.leq\ a\ b \Leftrightarrow leq'\ a\ b \tag{28.6}$$

`explicit_Reals_repindep`

Theorem 28.196

$$\mathsf{explicit\_Reals}\ R\ zero\ one\ plus\ mult\ leq \Leftrightarrow \mathsf{explicit\_Reals}\ R\ zero\ one\ plus'\ mult'\ leq'.$$

The proposition is identified by the following information:

Pure Prop Id:	*e6777aed3a48a1eed98a6aa68ec40260a6fd7f77f99e3b2dfc8fd4213fbf51d5*
Pure Prop Address:	*TMHRoUeAQwKBzV5DNUEnEtSGLyRQBCWi2oz*
Theory Prop Id:	*e7a9656809ed03974b3a3e7265ecbe46e51f37029b92e73c457801fb7c611216*
Theory Prop Address:	*TMav3v4dLGD2kzFGsBAHjnvy2SFfYuJuB5m*

28.10 Unpacking Explicit Reals

`RealsStruct_unpack_eq`

Theorem 28.197

$$\begin{gathered}\forall R.\forall plus\ mult : \iota \to \iota \to \iota.\forall leq : \iota \to \iota \to o.\forall zero\ one.\\ \mathsf{unpack\_b\_b\_r\_e\_e\_o}\ (\mathsf{pack\_b\_b\_r\_e\_e}\ R\ plus\ mult\ leq\ zero\ one)\\ (\lambda R\ plus\ mult\ leq\ zero\ one.\mathsf{explicit\_Reals}\ R\ zero\ one\ plus\ mult\ leq)\\ = \mathsf{explicit\_Reals}\ R\ zero\ one\ plus\ mult\ leq.\end{gathered}$$

The proposition is identified by the following information:

Pure Prop Id:	*d753a2caecebad53fe06dc1e4aa5fc9c806b9df39093b6b31f3dfd7d2790d31f*
Pure Prop Address:	*TMKP2fsixYdw8yYkEFeLRmmhRgMFNqBiCkE*
Theory Prop Id:	*1e4afd0f43b0a314315c5e7b2a4354fc8792a577d40f5ddbfe8054c2fd4bf78c*
Theory Prop Address:	*TMHSDGonYJvmsYAvpMXetztjTkxJWrGWgqS*

Definition 28.31 *We define* RealsStruct_carrier *to be* $\lambda Rs.Rs\ 0$ *of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*f0bb69e74123475b6ecce64430b539b64548b0b148267ea6c512e65545a391a1*
Pure Object Address:	*TMY8sbcPiVs9VL311KPHpbm67U6sLM9JvF7*
Theory Object Id:	*9211e41eaf81bb8c152618cbe35230be60d736fcd5684c0b7bdc5ccc1ff70181*
Theory Object Address:	*TMZQE6tPrFvJMaBcLrcMn85ZgscSeFbm4pT*

Definition 28.32 *We define* RealsStruct_plus *to be* $\lambda Rs.$decode_b $(Rs\ 1)$ *of type* $\iota \to \iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*6968cd9ba47369f21e6e4d576648c6317ed74b303c8a2ac9e3d5e8bc91a68d9d*
Pure Object Address:	*TMYTM1QQGqgVyRM9unR8CLL2scwRHYvV3c3*
Theory Object Id:	*ed06aa34b4112f8507370b46c1c5c7de7f2f9af4f24b831d45893ad3066e994e*
Theory Object Address:	*TMRiY7KU6BpU28KD19iwgBDLsQ5XyfpcbYL*

Definition 28.33 *We define* RealsStruct_mult *to be* $\lambda Rs.$decode_b $(Rs\ 2)$ *of type* $\iota \to \iota \to \iota \to \iota$ *identified by the following information:*

Pure Object Id:	*4b4b94d59128dfa2798a2e72c886a5edf4dbd4d9b9693c1c6dea5a401c53aec4*
Pure Object Address:	*TMQSTL5b71ddbjnkjnGXfRZ61uXWmMGwJHD*
Theory Object Id:	*7eb39ed695c795e4451a9252d011d377d0f4bdf0bfe6193f7a31ceb58f687a40*
Theory Object Address:	*TMaJmzoi8UnDNTAXrAvZxYbTeEdKV3viygg*

Definition 28.34 *We define* RealsStruct_leq *to be* $\lambda Rs.$decode_r $(Rs\ 3)$ *of type* $\iota \to \iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*e59503fd8967d408619422bdebda4be08c9654014309b60880a3d7acf647e2b4*
Pure Object Address:	*TMVpPCt6DnVyyzfVCfyhFErhhmLuHJWfLF5*
Theory Object Id:	*e66070fe509c0522857fd1dec315953e7466e1fff9feba36b2b4e9f1f60f7ede*
Theory Object Address:	*TMGzX1JqbMSdTKAjW1hbwGwKwhVDCcRKEhA*

Definition 28.35 *We define* RealsStruct_zero *to be* $\lambda Rs.Rs\ 4$ *of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*57e734ae32adddc1f13e75e7bab22241e5d2c12955ae233ee90f53818028312a*
Pure Object Address:	*TMV8UDBZRqv4jocV6zt17aP89PcYugzQkoU*
Theory Object Id:	*b2fa78f7f58b95f4bc4f46ac268c3a959090897e56338c64bd84a73bc5fc2551*
Theory Object Address:	*TMWFb9uZk2JEesS9quTe9a6Ww4ND9VcbBJA*

Definition 28.36 *We define* RealsStruct_one *to be* $\lambda Rs.Rs\ 5$ *of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*7aa92281bc38360837d18b9ec38ff8359d55a8c6ef4349c5777aab707e79589b*
Pure Object Address:	*TMG8fYn75tfBRqBJzcYzTPSyLmxrzMatNuC*
Theory Object Id:	*e902384fa204aa548a7192aca2937826cba222865eef88ccb0f0dd747aff3878*
Theory Object Address:	*TMF5LB2qHkXJqUPWHWovgb3u9mde2rLtjtB*

Definition 28.37 Field_of_RealsStruct *is the opaque object of type* $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*e1df2c6881a945043093f666186fa5159d4b31d68340b37bd2be81d10ba28060*
Pure Object Address:	*TMYju5xqA4jiDon5Vrw64253Ybc4eYYipFW*
Theory Object Id:	*ef27c4f9f215b0e5024d128458fefa23faae900a3a796c7655191a5180aa8fd2*
Theory Object Address:	*TMS4LA1a2HRZdid71r9n3J7xvnqcWXoPRYU*

`Field_of_RealsStruct_0`

Theorem 28.198

$$\forall Rs.\text{Field\_of\_RealsStruct}\ Rs\ 0 = \text{RealsStruct\_carrier}\ Rs.$$

The proposition is identified by the following information:

Pure Prop Id:	*e3591bbc5c24313d727f99859f33bddaa24ed93f5e16fde0a5a27460f4deaede*
Pure Prop Address:	*TMGpDdtdmCUs1jrGPV3mZk171Ru5G3eeAgA*
Theory Prop Id:	*cb1fd7ac2423a50aaf869926ca1d1306d77aef6c4d3fa9f561447ef07b391cb8*
Theory Prop Address:	*TMNmZN2BcVU3Bvv3THemmMQphYBi7VZEiK4*

`Field_of_RealsStruct_1`

Theorem 28.199

$$\forall Rs.\forall xy \in \text{RealsStruct\_carrier}\ Rs.$$
$$\text{Field\_of\_RealsStruct}\ Rs\ 1\ x\ y = \text{RealsStruct\_plus}\ Rs\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*a29ae8db462729b7f524ef65c8169e19aa80b02b813f1538b50a4ea71cac00e4*
Pure Prop Address:	*TMH9MkMZY6Nfy8CtfBXnKbGTTMMDYtF8dhg*
Theory Prop Id:	*f4fd5c9274754011767f3f3a1963005a161b0e3450a8605917bb74939f42093a*
Theory Prop Address:	*TMHPEQeFskKiibC12Eg5RfK8N4RmZgBPc6M*

`Field_of_RealsStruct_2`

Theorem 28.200

$$\forall Rs.\forall xy \in \text{RealsStruct\_carrier}\ Rs.$$
$$\text{Field\_of\_RealsStruct}\ Rs\ 2\ x\ y = \text{RealsStruct\_mult}\ Rs\ x\ y.$$

The proposition is identified by the following information:

Pure Prop Id:	*144feb0ed55ae0791a3276243986374c09700d3d3d141752481d9130cc01a231*
Pure Prop Address:	*TMQiqH6UGKCC2AY2aZmhwjBD1uigXrL9U5A*
Theory Prop Id:	*b9ac92481e26e620bc1d7f8daba3ccb4dcffbac3703a966f7f6175c0996df7a2*
Theory Prop Address:	*TMHw73bPqVnCDmcBgyog1NWbZ2kKZNjpM2F*

`Field_of_RealsStruct_3`

Theorem 28.201

$$\forall Rs.\mathsf{Field\_of\_RealsStruct}\ Rs\ 3 = \mathsf{RealsStruct\_zero}\ Rs.$$

The proposition is identified by the following information:

Pure Prop Id:	*3a0c651ac07600c96cb2b4b797025bc57c3f63713cb95551e7ff7570600dd9a6*
Pure Prop Address:	*TMP2bK3Zy9P941RMigpsJeoKpTUvZdbu6jt*
Theory Prop Id:	*39ddb5b76ae60cb54984057402ef494c4fa1400d58d80b904f425a9a5e3609e3*
Theory Prop Address:	*TMLDSxVyVzr1FPB6KncZtioj1PDrXC6M8SJ*

`Field_of_RealsStruct_4`

Theorem 28.202

$$\forall Rs.\mathsf{Field\_of\_RealsStruct}\ Rs\ 4 = \mathsf{RealsStruct\_one}\ Rs.$$

The proposition is identified by the following information:

Pure Prop Id:	*f8ef8a9601b742e71cf054c41b20f6ec0dee405cf311308f2196ca2ae8742478*
Pure Prop Address:	*TMHKLv75keSDVRMA8BVfAdqxSSSWoVHCjjK*
Theory Prop Id:	*79ba04006dd069d9bc477280db9ba7c3bf231bcc112d5aaa271be90cf51e8d30*
Theory Prop Address:	*TMJTgymdjGtaV8G45xSwQDuSpRLtCjKY7LG*

28.11 RealsStruct

Let $Rs : \iota$ be given. Assume the following.

$$\mathsf{RealsStruct}\ Rs \tag{28.7}$$

Let $R : \iota$ be $\mathsf{RealsStruct\_carrier}\ Rs$. Let $zero : \iota$ be $\mathsf{RealsStruct\_zero}\ Rs$. Let $one : \iota$ be $\mathsf{RealsStruct\_one}\ Rs$. **Notation.** We use $+$ as a right associative infix operator corresponding to applying term $\mathsf{RealsStruct\_plus}\ Rs$. **Notation.** We use $*$ as a right associative infix operator corresponding to applying term $\mathsf{RealsStruct\_mult}\ Rs$. **Notation.** We use $\leq$ as an infix operator corresponding to applying term $\mathsf{RealsStruct\_leq}\ Rs$.

`RealsStruct_eta`

Theorem 28.203

$$Rs = \mathsf{pack\_b\_b\_r\_e\_e}\ R\ (\mathsf{RealsStruct\_plus}\ Rs)\ (\mathsf{RealsStruct\_mult}\ Rs)\ (\mathsf{RealsStruct\_leq}\ Rs)\ zero\ one.$$

The proposition is identified by the following information:

Pure Prop Id: *17af86f27285e29398960ca5e9e3f6f45b69b7a9bdf827de09c0c669b7c75c6c*
Pure Prop Address: *TMWmf9GomThUEUvw4iwAFWDwRvBCKuKUJ81*
Theory Prop Id: *f1cea2ca03fd17235452510cf22a8e1709b9373fee61b9d92e62557ba14e4287*
Theory Prop Address: *TMbTji4uUoLpyNMKCgYne4Fdo2wyrpD2a2z*

`RealsStruct_explicit_Reals`

Theorem 28.204

$$\text{explicit\_Reals}\ R\ zero\ one\ (\text{RealsStruct\_plus}\ Rs)\ (\text{RealsStruct\_mult}\ Rs)\ (\text{RealsStruct\_leq}\ Rs).$$

The proposition is identified by the following information:

Pure Prop Id: *d3877b9d93ebdb9c81bfb64ef324b904ac578f720458bd89b59d747bab8321ec*
Pure Prop Address: *TMbsdad9BHngHtYG5Eu56f41hLLyMhQ4wz9*
Theory Prop Id: *e96bfd03b3e326ebe235a70e085a6663531d7408960525dca468e6023497d823*
Theory Prop Address: *TMMM29TxtZ5QApu738fkTwRqJ5AxhqnNU1q*

`Field_of_RealsStruct_is_CRing_with_id`

Theorem 28.205

$$\text{CRing\_with\_id}\ (\text{Field\_of\_RealsStruct}\ Rs).$$

The proposition is identified by the following information:

Pure Prop Id: *30ac8336eb76436c0b3b234f78de119af4f8aae671ed6946516746919bd40b57*
Pure Prop Address: *TMRwfFMfrGHNJdjEebj6AWUAsps8wxbBciR*
Theory Prop Id: *8d5700aa263b7f06933873e9cd11f836e00e0138000b4d39f89edcf89a4b5b69*
Theory Prop Address: *TMYnS1DaswnJWAykA7gfcHRYUno5P7ZH1xt*

Definition 28.38 *We define* RealsStruct_lt *to be* $\lambda xy.x \leq y \wedge x \neq y$ *of type* $\iota \to \iota \to o$ *identified by the following information:*

Pure Object Id: *3a46d8baa691ebb59d2d6c008a346206a39f4baae68437520106f9673f41afc6*
Pure Object Address: *TMZU8qJtXFES85dykXqXncY6GKaYJb1UkwA*
Theory Object Id: *1fb825962c4e3c207604d053fa6fdb0b78c4215dab034f2c783d256d7e08a3f3*
Theory Object Address: *TMcanZ7iZKUNsAhX6fjudSebaubcUhfXhth*

Notation. We use $<$ as an infix operator corresponding to applying term RealsStruct_lt.

`explicit_Field_of_RealsStruct`

Theorem 28.206

$$\text{explicit\_Field}\ R\ zero\ one\ (\text{RealsStruct\_plus}\ Rs)\ (\text{RealsStruct\_mult}\ Rs).$$

The proposition is identified by the following information:

Pure Prop Id: *d59c3f2e6f4d6724d3005e0bee526d84560fc597c159862117cd50437503db9a*
Pure Prop Address: *TMYsjqWb3PES4DyoFitce9WwQU2Rt5WHX1i*
Theory Prop Id: *744fcb4f7b03a77f3f942ae46ec1dc0b6e66ac611a343175773867714b070aeb*
Theory Prop Address: *TMairfquc4KvRjg7Xc3pANWUih3uqyq8R5j*

`explicit_OrderedField_of_RealsStruct`

Theorem 28.207

$$\text{explicit\_OrderedField}\ R\ zero\ one\ (\text{RealsStruct\_plus}\ Rs)\ (\text{RealsStruct\_mult}\ Rs)\ (\text{RealsStruct\_leq}\ Rs).$$

The proposition is identified by the following information:

Pure Prop Id: *70df0d2f2fe64ff20992d794cafe8f45b02a15058699156c2ded116e9b0962e6*
Pure Prop Address: *TMUxQacVa4YowWpPGUq5zVNjPj5dQafMF9G*
Theory Prop Id: *afda6089023853c8239e43e59fcb8eb5e74e269802cfec8e72ac2071ca787656*
Theory Prop Address: *TMUSiQQFDe6Uiw3xmogsw37fkZT92vjPKrA*

`RealsStruct_OrderedField`

Theorem 28.208 OrderedFieldStruct Rs. *The proposition is identified by the following information:*

Pure Prop Id: *3149fc73f3626eb90d8ffbc6259b266c543eaec7594c6c6c6503c7eab6bc76d6*
Pure Prop Address: *TMQXpz1uzda7WjR5D59goy6Npidv1Fh3ego*
Theory Prop Id: *638a457d1b7e3a0558d1e4a8349534f593bad615e24d92349d002ae3afb8f22e*
Theory Prop Address: *TMGNWDRJRSAseCPGm2L1ufW5qeZCWTf1RTZ*

`Field_of_RealsStruct_1f`

Theorem 28.209

$$(\lambda xy : \iota.\text{Field\_of\_RealsStruct}\ Rs\ 1\ x\ y) = \text{RealsStruct\_plus}\ Rs.$$

The proposition is identified by the following information:

Pure Prop Id: *7affbde70101a9fb21231dde40bcdaf417f643cdb83ddc82db6176651037aac3*
Pure Prop Address: *TMe11GXyXEF6MGDABPzVWHtkhaqqa9Fkk9T*
Theory Prop Id: *fe164da026cbaa682ef9fe1136650ba3b8eb25224ebc8d70d1da787a38e26a69*
Theory Prop Address: *TMPP93FXoNZbKN3CDWyZUbmssxCbWqaUC2x*

`Field_of_RealsStruct_2f`

Theorem 28.210

$$(\lambda xy : \iota.\text{Field\_of\_RealsStruct}\ Rs\ 2\ x\ y) = \text{RealsStruct\_mult}\ Rs.$$

The proposition is identified by the following information:

Pure Prop Id:	*f983dc416a105a88e0acb3737c63f1e0d73c3c64c4520277a6e8d01d56bc243c*
Pure Prop Address:	*TMGdwUUczg7R1h1T84CWJM1aykiAmFN7fNu*
Theory Prop Id:	*5715158dd199c38a59bbf5c3c0b1f41806ecc17815efa3fc7d133801a3cddea5*
Theory Prop Address:	*TMMmVJkECh7QnA46fbLjGoemcoeBWqcFETd*

explicit_Field_of_RealsStruct_2

Theorem 28.211

$$\text{explicit\_Field}\ R\ (\text{Field\_of\_RealsStruct}\ Rs\ 3)\ (\text{Field\_of\_RealsStruct}\ Rs\ 4)\ (\text{decode\_b}\ (\text{Field\_of\_RealsStruct}\ Rs\ 1))\ (\text{decode\_b}\ (\text{Field\_of\_RealsStruct}\ Rs\ 2)).$$

The proposition is identified by the following information:

Pure Prop Id:	*cedc05dfb417fc06394a10542007a2f8ce55cc82ac0946b195b2c73fdfe9ade2*
Pure Prop Address:	*TMNT1KjdwrJ9LD8Vnb1Xouv3qSy7pD9ivJ4*
Theory Prop Id:	*edd0df751b28b776ab98f509eb6e9906b9ee240bf87903c0e4f8d10355c8de2c*
Theory Prop Address:	*TMYv4TivoSVD8m2uZbe9Vwkjd9899mo2iPo*

Field_Field_of_RealsStruct

Theorem 28.212 Field (Field_of_RealsStruct Rs). *The proposition is identified by the following information:*

Pure Prop Id:	*0ec36b9dac36f4bf382d817f90965f789c23b007329c1f2a14b0014a91d92067*
Pure Prop Address:	*TMZyZoKXReKnGyDuwVdpxzHCHi3LLUXkp3d*
Theory Prop Id:	*23c5218dab13d451320a4c008e8443b47261ff382e01d4a9eafa9b5032fd81d1*
Theory Prop Address:	*TMR5Ubk1J3EVtZScLio9b8k4kf9rtQ6ncTX*

RealsStruct_zero_In

Theorem 28.213 *$zero \in R$. The proposition is identified by the following information:*

Pure Prop Id:	*005b3674102dca79680f3c66d82d0a566ec5ce1fbc87acc22a9a6ca9ebebf54b*
Pure Prop Address:	*TMMBmyfDd54QzeVfnKcj1zfp6a52pbqPYQi*
Theory Prop Id:	*7813991fa3eec14bade682c678cb4927bd388c9ac91fba2e332415aa6f370b7f*
Theory Prop Address:	*TMcHAxvwy4L9sxYGy2zcQMnxHoHXr9htE8A*

RealsStruct_one_In

Theorem 28.214 *$one \in R$. The proposition is identified by the following information:*

Pure Prop Id:	*1b33553dbfb8dd1c45ebfbfd7313edc634050f17c1b4547ca0b73c8154450951*
Pure Prop Address:	*TMVFbWxvhhJfevVpKgMa2zFT5nstpC3SnzU*
Theory Prop Id:	*04a7a006f820df56470471035fcc3ca727dad5b96142181cf37a3fa7867815f3*
Theory Prop Address:	*TMQ7UNpiYfxP56HgBYDN27D2wfmUQoY6GXz*

RealsStruct_plus_clos

Theorem 28.215 $\forall xy \in R.x + y \in R.$ *The proposition is identified by the following information:*

Pure Prop Id:	*d59fc8a19b30608e6133175e65740cbf406831dcc29d41398580efd3565b0304*
Pure Prop Address:	*TMH8yov9d8x8DhvNqVPDNBwvVUPZbFnQY4N*
Theory Prop Id:	*d047b1443a58c7267b47a6534919e45b97226c58a703aad09b0fcb5a5e1649e2*
Theory Prop Address:	*TMMHW3MKm5vVqaUcstZAAHdswmLybKpv7jC*

RealsStruct_mult_clos

Theorem 28.216 $\forall xy \in R.x * y \in R.$ *The proposition is identified by the following information:*

Pure Prop Id:	*b521d3f1983a1d40a297548001ef03c96963cfbb20d3944063f920b35cb4f799*
Pure Prop Address:	*TMZECePNUD8jFwZt6i2o5f1wf4rK94xSn89*
Theory Prop Id:	*cd4e02b42f650e37ee913b313e6820d2ad0116d56cfd6e5ef595d72f2bc261d6*
Theory Prop Address:	*TMbyKuyhM9cra9Gm7KfgFoAyUN1YdzMLr52*

RealsStruct_plus_assoc

Theorem 28.217 $\forall xyz \in R.x + (y + z) = (x + y) + z.$ *The proposition is identified by the following information:*

Pure Prop Id:	*f3b3129567a8db11a6326406f56b94419e2d9818eac0f622df322d09400a6bcb*
Pure Prop Address:	*TMJzLB6CoD1vut7yT35PBx1oo2PXUVc7Zig*
Theory Prop Id:	*8f95762ccd9f93efb2b813048852b496d88e5771e670e7c8330941bc2870621b*
Theory Prop Address:	*TMTfndBh736RkfuCoAVb5EfY859TyC26KS3*

RealsStruct_plus_com

Theorem 28.218 $\forall xy \in R.x + y = y + x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*59c94c4de8b2476d2510dd4b09ddc432083d2bf7b7fb33b2024269e7cfc66637*
Pure Prop Address:	*TMduLqT57Q1QdbQNsUt6QSDeqMnyE1sLKgk*
Theory Prop Id:	*fe9682e564377a859f1f577491e17a474459086d1c02b7e016b4b3d5d708413c*
Theory Prop Address:	*TMdBQiCvauMBPkmwE1yJizNxoC35UqZBmtT*

RealsStruct_zero_L

Theorem 28.219 $\forall x \in R.zero + x = x.$ *The proposition is identified by the following information:*

Pure Prop Id:	*941604f00ed1e86b155b1b1cc752ab549a904f9ec545ac4e5e0501edea6bf9ae*
Pure Prop Address:	*TMVWFuU6oaPoeDy7jmijYCvqgjMhDBfnAp6*
Theory Prop Id:	*a028d8ecce4c4d08176730f488a7aa5d9e462795f24bc924a2776fee82844e1e*
Theory Prop Address:	*TMTKfSnfXd5nWfTXC5qtkJxtoSvW9Z1vQDx*

RealsStruct_mult_assoc

Theorem 28.220 $\forall xyz \in R.x * (y * z) = (x * y) * z$. *The proposition is identified by the following information:*

Pure Prop Id:	*a3751ec23c16fcb964095d434bd62fbff3f54ea6f367daa73dd6af90e7e15285*
Pure Prop Address:	*TMVx6KqoHebZKqDiuAYv9dmMFLDaUM7Cqcs*
Theory Prop Id:	*9964f7eae81893737f8b8d14d0e37c2cbbffc7ba0883b2c17e9015b758de91f5*
Theory Prop Address:	*TMHd9hTsZZPRjz6qz2H9V4kWSaeReQd6WyF*

RealsStruct_mult_com

Theorem 28.221 $\forall xy \in R.x * y = y * x$. *The proposition is identified by the following information:*

Pure Prop Id:	*917b61b0f80ba1c680830ac638f010b6d7ab9948b61b6bf2fa194f1559175899*
Pure Prop Address:	*TMYHM6VL7di1iR2UetLF2MsoT7BqwWRXefM*
Theory Prop Id:	*56ee11d2a162693703d6398a6fa3b44e437c11cbc2034fd1d87a3616b79c425f*
Theory Prop Address:	*TMM9uq46vBF85FpZjdJoCTa8ouco6ezSTf2*

RealsStruct_one_neq_zero

Theorem 28.222 $one \neq zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*a175b20ec46a263f0237a1f588ae8b080a36967feae498f91cceed32f1f07bfa*
Pure Prop Address:	*TMNKMBM19aBBHJYg384yRYAbPN53WVbc5ns*
Theory Prop Id:	*52ab44a448083c215a50ab8191899ef26e1fc7da0b367bd95ecbe48ce8e0c5ac*
Theory Prop Address:	*TMPjniRmWmW6zxw8tiEspuizDHoc7XRo3VJ*

RealsStruct_one_L

Theorem 28.223 $\forall x \in R.one * x = x$. *The proposition is identified by the following information:*

Pure Prop Id:	*25b4a10707d2aab2f4bcc2756fdd2e3abf00c1f705db1132de37a2fc6ea6f637*
Pure Prop Address:	*TMMvdyQpdr8L3T42os9auY5Nm8FUhCGnw76*
Theory Prop Id:	*a03ccb9f27af94b22542ec92ba2b8ba628d78ad3d57905ee68cf0d731ae618ef*
Theory Prop Address:	*TMXFnDHcsSjoFcQzEX5A9GawZJ2M6iZkEV4*

RealsStruct_distr_L

Theorem 28.224 $\forall xyz \in R.x * (y + z) = x * y + x * z$. *The proposition is identified by the following information:*

Pure Prop Id:	*7f277f99738c86e4cd0c17ce27f55256d4b83bac8c3ec324b94d179eb523992c*
Pure Prop Address:	*TMMJoedUhPVrXxiACByyi6RA7wzPg2bP8AE*
Theory Prop Id:	*80b5a5b1ce589268084c8d0ff372432a6afa48599f754381e6ecf72024d11b73*
Theory Prop Address:	*TMUbkahieeSRSSdoPifi8CERkncX96vdrCA*

`RealsStruct_leq_refl`

Theorem 28.225 $\forall x \in R. x \leq x$. *The proposition is identified by the following information:*

Pure Prop Id:	*868cecbda36c1ae46534769ebbdb84ef47482569c02f1c634ef3b569a7a12c94*
Pure Prop Address:	*TMUGH3okNJP2QDn6HWyznB2itjVp93X4N2M*
Theory Prop Id:	*16a3fb53c4f1ddec154a070ed94ab00c6d6f96a8618fb21ee9e7cf9b7c48e5b1*
Theory Prop Address:	*TMNhQCrwQkgbWcYiN9bZNZmiH341kRj81fx*

`RealsStruct_leq_tra`

Theorem 28.226 $\forall xyz \in R. x \leq y \to y \leq z \to x \leq z$. *The proposition is identified by the following information:*

Pure Prop Id:	*09ea87b2c8bb2a8ef1d5e4e3413f6ed17268c09df43a45348a5029aef49aec25*
Pure Prop Address:	*TMWjFSH117sB2fX5RBJDyEzKLmyYTS2S17B*
Theory Prop Id:	*31d22cb5ee22963edf60c65a1536469fb0185ae68587cf700c23504bb704d4a9*
Theory Prop Address:	*TMUUyEqPQ9BoJF9RsDg9cXEhU8SBZvmB7dv*

`RealsStruct_leq_antisym`

Theorem 28.227 $\forall xy \in R. x \leq y \to y \leq x \to x = y$. *The proposition is identified by the following information:*

Pure Prop Id:	*13d82edccef1b70d008353ac64c4821740a0880f70e0e680d356e6d9d079c525*
Pure Prop Address:	*TMHzbKmyZCrxTuS3pfvy16rZJqPRa2zbWR4*
Theory Prop Id:	*0aa45faf2f614bd2d02eaabb71f298ba2905b9e906d084f527121abf9d477ea5*
Theory Prop Address:	*TMNKwtGqpbPqDk86iVJJmWZZzeTFCETXo97*

`RealsStruct_leq_linear`

Theorem 28.228 $\forall xy \in R. x \leq y \vee y \leq x$. *The proposition is identified by the following information:*

Pure Prop Id:	*8492e0db1af1c27447e1da622672c190501a41929ea1a5d824d99bf5c02e2cd4*
Pure Prop Address:	*TMJpogMX6MJSnbVEpoyobdqAxQAvxSa6593*
Theory Prop Id:	*1f84f249a4223c91c6d7a2a24ab7e57bb4917d1c8b23d477391ddfe94e41e9a0*
Theory Prop Address:	*TMN3dF6E7pq8oupzx7bGMNiMPz5ir8mq112*

`RealsStruct_leq_plus`

Theorem 28.229 $\forall xyz \in R. x \leq y \to x + z \leq y + z$. *The proposition is identified by the following information:*

Pure Prop Id:	*8a4ab12e1315b912d8d64a150d7f14264405d48f8b09b19e281e9f7f5babc606*
Pure Prop Address:	*TMcTUgJ6Y2J84VUxaEX5jmfrXQaXknJHnsS*
Theory Prop Id:	*37b323e19944cb2094fbf59f40ee6dc75941a5e8638cd782e29f1595ba2feb8d*
Theory Prop Address:	*TMVdpCNvU6P7MxmUMqGyRQUPygZTUwsQj7w*

RealsStruct_lt_leq

Theorem 28.230 $\forall xy \in R.x < y \to x \leq y$. *The proposition is identified by the following information:*

Pure Prop Id:	*034e19d8815d7cac710f94e707f400dcc18e80e521efb31f4f07c41cc7eb551e*
Pure Prop Address:	*TMRG27Re6ZfnLLrivhufYbciXkVrRqvBff7*
Theory Prop Id:	*8b5db0b565ec35e47e13bb761e5bf3d54bc087a8eea8a3da96873a105ea5b67d*
Theory Prop Address:	*TMRAfQdUMEz2W8xXfP2Fp3jgBADZyt3d5q9*

RealsStruct_lt_irref

Theorem 28.231 $\forall x \in R.\neg(x < x)$. *The proposition is identified by the following information:*

Pure Prop Id:	*718b2a7630e43c2aa1a7ba78859688706a27263cc0e14ac10b19d2502d90d0be*
Pure Prop Address:	*TMT3hWZ2i8mzcX5AV3nDJbM74z3kxuqnWTT*
Theory Prop Id:	*2c2cd678622bfb74b4ccf1f72f60b44aaeda5e1673f8577488821e96d3567025*
Theory Prop Address:	*TMGLCxFW2fPYxLxs6VnBV5otswJNkwPZmso*

RealsStruct_lt_leq_asym

Theorem 28.232 $\forall xy \in R.x < y \to \neg(y \leq x)$. *The proposition is identified by the following information:*

Pure Prop Id:	*036716b7031d9e9545f672350877cdd3cd606d761131cc3ce1039205806864f1*
Pure Prop Address:	*TMJhZqMGmM2ycUqQT2qouBhNpEHfUJNVWVH*
Theory Prop Id:	*5cf7bdfe85d03b1346539e85cb1f326864fbad71343355b85a957631353d7744*
Theory Prop Address:	*TMVm3xj3nwvohAUcnSzL6EYJStDpJiCLKGX*

RealsStruct_leq_lt_asym

Theorem 28.233 $\forall xy \in R.x \leq y \to \neg(y < x)$. *The proposition is identified by the following information:*

Pure Prop Id:	*2cf72677c60b93322ea19a0826dab959dc3147d59f5d9d3fe5aa8b7174ee2ba1*
Pure Prop Address:	*TMPymWMasfNM2Rji1AUDQcQM4LrH9uWx3Va*
Theory Prop Id:	*eee248a189252e629bc193acfb88c7bba2acdb08489e25f8f10c512171761595*
Theory Prop Address:	*TMcmYMME185KoSKgmuzQKpMQLXMeqigGBdb*

RealsStruct_lt_asym

Theorem 28.234 $\forall xy \in R.x < y \to \neg(y < x)$. *The proposition is identified by the following information:*

Pure Prop Id:	*d64488d8f76ef862d95e7112eece4f8a3f3c67e4b05abe79c8e71640fdc45d88*
Pure Prop Address:	*TMViyVGCbgDmrEXs1auawYkVTQNQjDYXtNt*
Theory Prop Id:	*95a9252b4e98f9f989fdb3147e32cecbad9ba3739992d1cda07bac3cf57c3017*
Theory Prop Address:	*TMSE7BJFjaB53kpm9JzcemAjmx17n4miSVo*

`RealsStruct_lt_leq_tra`

Theorem 28.235 $\forall xyz \in R.x < y \to y \leq z \to x < z$. *The proposition is identified by the following information:*

Pure Prop Id:	*27e379d62bccda6def02d503ec80dce5751e1b6c06ea0598c0e4f12313a0c0a4*
Pure Prop Address:	*TMXQk3yFeunwa2tgkwDRkTAiC72gnc1kEbQ*
Theory Prop Id:	*0ad1d49b39850f537c0804836720afa6e45354a2506c9efdf082c5b1daef964a*
Theory Prop Address:	*TMYGBds2bCxGu39zPdjB8uTHjqLijWHi2KU*

`RealsStruct_leq_lt_tra`

Theorem 28.236 $\forall xyz \in R.x \leq y \to y < z \to x < z$. *The proposition is identified by the following information:*

Pure Prop Id:	*a7fac80d5d695102c89d549a0ac57e21a1da6b01abfff6821d8671d05ac5bfe9*
Pure Prop Address:	*TMWRFzWY9Ks96UCp5hHDSSwUHcp2kHabMiZ*
Theory Prop Id:	*17dbb16615d06d0e91aeb16bd8d187a094d35156dfc10c9a0cb26df2949b1a10*
Theory Prop Address:	*TMTNZLqjCyTkXHcJDFnDct7KWxJxFjE1gTZ*

`RealsStruct_lt_tra`

Theorem 28.237 $\forall xyz \in R.x < y \to y < z \to x < z$. *The proposition is identified by the following information:*

Pure Prop Id:	*db51e413c8dae5e58659c817cad674fc9603959ac8908d0d3173b2f974bf800b*
Pure Prop Address:	*TMbyGzMwnbYf7Ef87fxfadSAny6Pk5qsDtt*
Theory Prop Id:	*a20edd4147dc9434a2c2a9b874d21178efbdb0a783f8456ad92be39a4e9cc999*
Theory Prop Address:	*TMcywXpnEowovbp6p7WFLjPSpRAsCXmnz7r*

`RealsStruct_lt_trich_impred`

Theorem 28.238

$$\forall xy \in R.\forall p : o.(x < y \to p) \to (x = y \to p) \to (y < x \to p) \to p.$$

The proposition is identified by the following information:

Pure Prop Id:	*94de406929eca9f0fbfb4a2208de46b59aa03b0f73d1c028a0551f7cff85ac9c*
Pure Prop Address:	*TMPYj9ARJvgSpTSMs4ztnxZZeSPKhuoUoz5*
Theory Prop Id:	*f9cff737117c67a0fa5ded42ee2d5363ce2958c66b29c20fc47c132e3eb9efdd*
Theory Prop Address:	*TMF5ZzdDFzFLMP7TYe4pY4t96duPiB7Gv7f*

`RealsStruct_lt_trich`

Theorem 28.239 $\forall xy \in R.x < y \lor x = y \lor y < x$. *The proposition is identified by the following information:*

Pure Prop Id:	*9b61418aa95cad0821b7b70b1ea6c48b07b043fc6317f9c66ece033f5aa59bf6*
Pure Prop Address:	*TMboWZEUUV8p5yiVECUimfWvDD1YMWBhHDh*
Theory Prop Id:	*65d4e63207e42cdaab1884c4d8cdfd112007705d830e2a0ccb44f59f328c8223*
Theory Prop Address:	*TMbrq68wUob61FbNk4jqcD83WaJgVRjAJoe*

`RealsStruct_leq_lt_linear`

Theorem 28.240 $\forall xy \in R.x \leq y \vee y < x$. *The proposition is identified by the following information:*

Pure Prop Id:	*b5d0e7fc3cc6bce2c2a17bc42c6f157d981ae1ed7c08efd6ac7ca23f0356d3f0*
Pure Prop Address:	*TMXnJJbrRwtujq4xzLDYXsAr5wYR1aqSr18*
Theory Prop Id:	*7c7a8089b86844735c6b2d3f4a519d71e25477cdc5a960264b601472a84efadc*
Theory Prop Address:	*TMEmCCr1R4uUv68FnD6w9CyYspAKC65Udc3*

Notation. We use $--$ as a prefix operator corresponding to applying term Field_minus (Field_of_RealsStruct Rs).

`RealsStruct_minus_eq`

Theorem 28.241

$$\begin{array}{c}\forall x \in R.\\ -x =\\ \text{explicit\_Field\_minus } R \text{ (Field\_of\_RealsStruct } Rs \text{ 3)}\\ \text{(Field\_of\_RealsStruct } Rs \text{ 4)}\\ \text{(decode\_b (Field\_of\_RealsStruct } Rs \text{ 1))}\\ \text{(decode\_b (Field\_of\_RealsStruct } Rs \text{ 2))} \ x.\end{array}$$

The proposition is identified by the following information:

Pure Prop Id:	*47bc6dfa278b5babc8da67e3a1e0b527ec1d23809b12328023a26ca5c7249583*
Pure Prop Address:	*TMYZAfb32ApJmMaB3PnEm7Vj4oTBY46NBAk*
Theory Prop Id:	*66a3bbfbbb3361050ff6013b6efced8992a5edba8cbd9899558ae7cc61dbb8a6*
Theory Prop Address:	*TMS87Bxs4mXuUt9jfG6JifxZNzvF7zzmfJt*

`RealsStruct_minus_clos`

Theorem 28.242 $\forall x \in R.-x \in R$. *The proposition is identified by the following information:*

Pure Prop Id:	*fdd8e0f6e300511edced7df68d989a615e67aa1bc7de750f9a619a82debfc807*
Pure Prop Address:	*TMWr5eXBuGtfE7oLQbomsUzc6fjwmfid258*
Theory Prop Id:	*b7c738e349ab2d6f6e8ad276adafcf81dc9aca3628028a9f35cb26fc2d5a34ef*
Theory Prop Address:	*TMVd1PVi5iMBiKL7rtZswTGDLj8iQbjZCSh*

`RealsStruct_minus_R`

Theorem 28.243 $\forall x \in R. x + -x = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*09095b964336914 85a96edf8101177ad9b033cef032350fa0ce5069fae93de9f*
Pure Prop Address:	*TMLSWdagYu788EzMjhuZP6Qm21mcP7gao1q*
Theory Prop Id:	*d7fd806b6ae1b4b46446aada0c9c8d453b985568eb38cc1ba86372ba219d557d*
Theory Prop Address:	*TMMMVeZuEoKYrJsCZYGX7isAfmKNaheKNC6*

`RealsStruct_minus_L`

Theorem 28.244 $\forall x \in R. -x + x = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*98ac5d2df769fab37af56d31b7a7b0a24ac969c2c72d9889077705d5c24ceabf*
Pure Prop Address:	*TMKzv2RXJzXuZKiMMKDdny2ePWZafZ7U5ZX*
Theory Prop Id:	*23218f089518e8a515bb2026967940f961d86643db6282c71fb89bbac1b93ce39*
Theory Prop Address:	*TMLhepvAwS8gbHygkWi1J1B7BRAUEkXdFje*

`RealsStruct_plus_cancelL`

Theorem 28.245 $\forall xyz \in R. x + y = x + z \to y = z$. *The proposition is identified by the following information:*

Pure Prop Id:	*5ff5748a79dfce201548e0ea8e7fc3e8ed66fae54c0079d8409b468ec617420e*
Pure Prop Address:	*TMXHN9o2PJFBMaNmp83BfCNYAonHHWiGFzX*
Theory Prop Id:	*ff6953bf63e902e9883d98d6d01c20077d6e823216570f791adbe886fb641882*
Theory Prop Address:	*TMcrA1J9U6fRWwNyvbHM8Pvu7xQqVhBf1Vp*

`RealsStruct_minus_eq2`

Theorem 28.246

$$\forall x \in R. -x = \textsf{explicit\_Field\_minus}\ R\ zero\ one\ (\textsf{RealsStruct\_plus}\ Rs)\ (\textsf{RealsStruct\_mult}\ Rs)\ x.$$

The proposition is identified by the following information:

Pure Prop Id:	*e8ae10f25c6af12d1958559a4f188545a0cb40752c041c3f8d11eecc9bcd7199*
Pure Prop Address:	*TMTjWE5eDPyLEbX1dfg7EizwEtFGXzUQkDt*
Theory Prop Id:	*a1211f03836b5eee939c778de3c1018416ca8ed1d3ee2b73d2266429b7c3e3e2*
Theory Prop Address:	*TMaFALex8WPZq7Zo2cyTerKK2nzDvTo1quC*

`RealsStruct_plus_cancelR`

Theorem 28.247 $\forall xyz \in R. x + z = y + z \to x = y$. *The proposition is identified by the following information:*

Pure Prop Id:	baf30bc9e3bd74b52dd88cb1eea019d239e8ac999133603bb6c0ffe240f0585d
Pure Prop Address:	TMJ8DeDPrVjLKi4AWwmCV15PJt4zh9Qea36
Theory Prop Id:	a5736933cee0a9aaae630453df6f585065561f78e9b7df048786845e7dddfdc1
Theory Prop Address:	TMRycJqeTxMBEZCvL8J2Hpzh8k9oRbHARWu

RealsStruct_minus_invol

Theorem 28.248 $\forall x \in R.--x = x$. *The proposition is identified by the following information:*

Pure Prop Id:	08c894cf89246f6d453037ab7581b8a4980b918261d561476b8d1c569c9972f4
Pure Prop Address:	TMZQm7b2b6rZPGFcUA5n5jUA5xm2TjgdNtJ
Theory Prop Id:	58988336a2e2c10ce0f8c945854017783f1fdb55c1a40daf0cdb8f1a17da8153
Theory Prop Address:	TMExht3DFRB3GYdQvpgYztYDMSMkBYJGNpx

RealsStruct_minus_one_In

Theorem 28.249 $-one \in R$. *The proposition is identified by the following information:*

Pure Prop Id:	bd1d4f7909be8f917eab926743f5502922a8a810a93cbd00d6412952a908801c
Pure Prop Address:	TMWc1PSLG2857QLtbTuiWKgfqKfe8q9nLcz
Theory Prop Id:	39bedc18953a944c01c33894b19ee20e55993b517d311d2daa46cd1165c05d3b
Theory Prop Address:	TMR42g5o3sV9VSPe54YRCdkizri2QPBCjT6

RealsStruct_zero_multR

Theorem 28.250 $\forall x \in R.x * zero = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	e8baa64a646894666f2b0b82b5918f7ed9318f8f8eee4f59dbbfba3c20d9ff21
Pure Prop Address:	TMPdW3hMJTztdGUdkDRakYTSjbabprAc9Z6
Theory Prop Id:	88201a486135db74001498754cf27b8685565cca99912ee888c9482032e13640
Theory Prop Address:	TMX7vtUzERwfALngARjEZ8VQG9g4HjUN3xS

RealsStruct_zero_multL

Theorem 28.251 $\forall x \in R.zero * x = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	751db0caf92b3fe28e704ad517da9de59bb6d3b21c4e04d1f9b4a77d601f5c33
Pure Prop Address:	TMH5oywcudUDCPXjnkAtprfxnT8Mx3M4znP
Theory Prop Id:	15727a520c07b04ff12d5865f874e26ce2c8daf0a2613a88bd45313469d1bb82
Theory Prop Address:	TMU6ughQfPFJrRB8wkmoxv7fyit1ENwF82x

RealsStruct_minus_mult

Theorem 28.252 $\forall x \in R.-x = (-one) * x$. *The proposition is identified by the following information:*

Pure Prop Id:	*d488fae83059b5b9d5f6500d2855e253c82b63de6ffcbcc7a9a08a9e90364a54*
Pure Prop Address:	*TMYWs7B7tsm6BLxQZXktxsTeJ7uiY9hTMj7*
Theory Prop Id:	*221fb8b61d0263c90b7a1b788de3aeb55c2d1c2061e15c7cee6e1abf97657d4b*
Theory Prop Address:	*TMXHd2hW4LazoJxHtfBqjWYQHGqzvfC1yW7*

`RealsStruct_minus_one_square`

Theorem 28.253 $(-one) * (-one) = one$. *The proposition is identified by the following information:*

Pure Prop Id:	*cab4aa58bf0267bee8cf3c2536213ef66d714059ef2218d7e6f1728f28f8ee99*
Pure Prop Address:	*TMPnEwkVaRzmRUkEJ1woHFG1DZk5uMWQaEU*
Theory Prop Id:	*701edc74dbc3b46a566cc9743166e0ab300f6a34691eebe4339c03df61b41a75*
Theory Prop Address:	*TMQvuBviqrrzJPYbWJNSTnSuiYbaqYvrUro*

`RealsStruct_minus_square`

Theorem 28.254 $\forall x \in R.(-x) * (-x) = x * x$. *The proposition is identified by the following information:*

Pure Prop Id:	*95c7432ee6391c9809080d41eed33f798d0d274f825cb91bde1c81c29c2d0fb2*
Pure Prop Address:	*TMb7xqhMnUxosCkgLM9VrW3WNqF4se2wayn*
Theory Prop Id:	*93bb8340ea11964abd12d112440b8853a2dbc1264d1238cd0571b528ea47232d*
Theory Prop Address:	*TMaYV6vuKLDWHanfeHhxV3h8HcDTY2T7wrM*

`RealsStruct_minus_zero`

Theorem 28.255 $-zero = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*a65ac7f737bccdfc3753590b8bce334772b3b6919a1922509b3d00063820c713e*
Pure Prop Address:	*TMTqKgfPgRZixBhk5JsKaM8ZeTqva2dbwrH*
Theory Prop Id:	*31d61336fe6999362705f5feadf4f373b68c3c731a34d56603a78e289700a6fd*
Theory Prop Address:	*TMYiSLxX166trdrKLyA51oNquqKN3M3YbqS*

`RealsStruct_dist_R`

Theorem 28.256 $\forall xyz \in R.(x + y) * z = x * z + y * z$. *The proposition is identified by the following information:*

Pure Prop Id:	*862fd843939bd3632046f172b4ceaf3d4e6b2698b28cb880f14e41743a84cf8f*
Pure Prop Address:	*TMGHAMzokV28j6GZ5fDvRnCYLM1h8jYg2jh*
Theory Prop Id:	*ce3537a1b799f75f41b619dd337ad2faa0ab92ffa22088d81b931f86215f45cb*
Theory Prop Address:	*TMc4SuM9sH5ftzoRUZc5jyapjLYJmbwrTMH*

`RealsStruct_minus_plus_dist`

Theorem 28.257 $\forall xy \in R.-(x + y) = -x + -y$. *The proposition is identified by the following information:*

Pure Prop Id:	*bbbec91faa7e9e78adb2960de270de19dc98814dd4909c4e5f8279063acf2549*
Pure Prop Address:	*TMEpdMaKhWF6yhGZvGSnnsfdxW3JE2HhaX7*
Theory Prop Id:	*e102afbd1d47d23a82a7d28fb39b18fed4f9d1368508a7d3169ed416e1e3ecac*
Theory Prop Address:	*TMXSKFTjeXzq6D3jVPWKqcXCcbSQehf31Mv*

`RealsStruct_minus_mult_L`

Theorem 28.258 $\forall xy \in R.(-x) * y = -(x * y)$. *The proposition is identified by the following information:*

Pure Prop Id:	*87a9ee20b9e8f4696f85fe723b6595c3796cd59ace6d3cc05a2f8d4d42e69893*
Pure Prop Address:	*TMccDiAHv34RsKwiFv5Ze6HutuaTy3ivq71*
Theory Prop Id:	*145734116522a560429224bd70db12b8ab512947aa0e49a218ed22e31a0e7111*
Theory Prop Address:	*TMTFyy4RHjnYmm6dmcA1XLwxqwZju5r6roQ*

`RealsStruct_minus_mult_R`

Theorem 28.259 $\forall xy \in R.x * (-y) = -(x * y)$. *The proposition is identified by the following information:*

Pure Prop Id:	*1dfc6deb04af278e1661d3df5a3bdea0daab232c9302727cd688b9c76e430dbc*
Pure Prop Address:	*TMXtdQduGF54FwdCwf1pD3PBBy4rK8RihuY*
Theory Prop Id:	*8cd9e0d875d43b1ddc9c929156b184121abe145cc15e0d102566046cd722f439*
Theory Prop Address:	*TMSbRFVgjqs3EKV1FxZLbyQFWKwbK5ZxJ6C*

`RealsStruct_mult_zero_inv`

Theorem 28.260 $\forall xy \in R.x * y = zero \to x = zero \vee y = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*52e521e4ea57136109ff9c68732a22d7e0fce2c81fe7e0333f27725db442259e*
Pure Prop Address:	*TMSLZXtw9SNRfYRMKsJi6bSZm5LUozCpXP8*
Theory Prop Id:	*ca0065eef41293f5e9cd0d9a5017dcd5aa4f3456b0df6f2e56a71ae1b77b12ff*
Theory Prop Address:	*TMT3MLk7YqUD9iX8atswcLFDca5cKNVBF5B*

`RealsStruct_square_zero_inv`

Theorem 28.261 $\forall x \in R.x * x = zero \to x = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*8aa0cd6d0c435ca74fff47f90162b7c2c3d6b793b431dc4b57459872f365c838*
Pure Prop Address:	*TMaEmRhRXsGfs29zHkD7Vcbp4X5kNcSAapY*
Theory Prop Id:	*d4c42484e082d273bd7d0b5e37d7d640a925b2221ce532e2e8bf25c52fb9e418*
Theory Prop Address:	*TMNUaVt1RUxY5s8TcGo5CqbLi9uuLiivewh*

`RealsStruct_minus_leq`

Theorem 28.262 $\forall xy \in R.x \leq y \to -y \leq -x$. *The proposition is identified by the following information:*

Pure Prop Id: *f49a3223293a899b90e3f0e2b2ccb4c7c143e078381e7b41b18f20349f7da516*
Pure Prop Address: *TMGUQNAZwd9CygmSnCMdCF9RyBCaAERrmJ3*
Theory Prop Id: *56adcd399875c337ec8eb70665c36638054ac2e7b9ce43b5d38d0fd6d09d41fa*
Theory Prop Address: *TMMV3gE1eKPX49sr7atFfzx8pAxwxmwav9U*

`RealsStruct_square_nonneg`

Theorem 28.263 $\forall x \in R.zero \leq x * x$. *The proposition is identified by the following information:*

Pure Prop Id: *1f7e7600170f3d0edf48eb23103a7e83310e8bcbbb489afe89d2a550424bc433*
Pure Prop Address: *TMX92WctujuKb5AX5z7JnBAbwpRUF1F1Yrn*
Theory Prop Id: *116beb08289e1dae25c8f46eb411554c9409ef2e3486a7b37ca72edeeaa82c18*
Theory Prop Address: *TMHDYtHgvcDVHrwczd4cftPoYYYa2bG9w9x*

`RealsStruct_sum_squares_nonneg`

Theorem 28.264 $\forall xy \in R.zero \leq x * x + y * y$. *The proposition is identified by the following information:*

Pure Prop Id: *55dc2f3f44253c88b10d4b601b85eff9350d5214097a99dfdfbb9ed33ba8e291*
Pure Prop Address: *TMFA9PQVdmjfUj74vevkFTa7eHQkM1Vw17x*
Theory Prop Id: *e739ff92218b151c98245ecc8ad2ef578e165ca921eee5b84841fa4b11c3c743*
Theory Prop Address: *TMSDysdaJ68HZgUqpWWzxnY91V1BU9Eprtp*

`RealsStruct_sum_nonneg_zero_inv`

Theorem 28.265

$$\forall xy \in R.zero \leq x \rightarrow zero \leq y \rightarrow x + y = zero \rightarrow x = zero \wedge y = zero.$$

The proposition is identified by the following information:

Pure Prop Id: *a1f474df8b36a92e0b58e7c0515850cfcf881b7ddb216e72d98b55d27f765f6b*
Pure Prop Address: *TMQqUDVayaoT53ydRzz8cF1HJtqHjdQhbha*
Theory Prop Id: *789223bfb56d1d985a4558cb85e56b74044e5dfa7a47009d14691aa59d1c5e97*
Theory Prop Address: *TMSUNEckz8DH6CXDWbVK7G39vzxmKYfFe5u*

`RealsStruct_sum_squares_zero_inv`

Theorem 28.266 $\forall xy \in R.x * x + y * y = zero \rightarrow x = zero \wedge y = zero$. *The proposition is identified by the following information:*

Pure Prop Id: *64bfc4ce2dead3e98b809e881e851e8a7b8d4a5992eef4830fcbb2460cb33ae9*
Pure Prop Address: *TMb4XesQGWXCWm9yHHnypZ9bBes3f2tF3D7*
Theory Prop Id: *c2b9c84106f6aa374eab5069dd30d750baf9110b257653b6b4701e79424f840f*
Theory Prop Address: *TMcj8zjWByWFUKfZtiUpAAS1zrBiXhL76KL*

`RealsStruct_leq_zero_one`

Theorem 28.267 *zero* $\leq$ *one. The proposition is identified by the following information:*

Pure Prop Id:	*75a9e5ea0655f6a7b81f9562d536cc979c304fee58c31e40455766da8a0b8491*
Pure Prop Address:	*TMdQ75HNiDrTsVaaDbMqwf25EtKrdnktWAd*
Theory Prop Id:	*fc79c3810891aeb6dd0f786541364fa69b5d2c6cc14b14c578108424308f783f*
Theory Prop Address:	*TMWFE2LGFJYTijtjpZrCpn7C7MW7CXxMUks*

Definition 28.39 RealsStruct_N *is the opaque object of type* ι *identified by the following information:*

Pure Object Id:	*5e5ba235d9b0b40c084850811a514efd2e7f76df45b4ca1be43ba33c41356d3b*
Pure Object Address:	*TMbcMfzuQfUfq11QLM6oT11DkR9A7LKHyxL*
Theory Object Id:	*49868cd01ccaaa52966256a5255beef5cfc5d2b96374f80d5069a1170c2c4c78*
Theory Object Address:	*TMKcBDt3UC9DqpmqhzUAWU2YJ3EuMHE5yEh*

Let N be RealsStruct_N.

`RealsStruct_Arch`

Theorem 28.268 $\forall xy \in R.zero < x \to zero \leq y \to \exists n \in N.y \leq n * x$. *The proposition is identified by the following information:*

Pure Prop Id:	*3d2ba33455345a9c0ba6e7d86b15bcead0c25d5b93f9dfdb5a52b615ac9babfb*
Pure Prop Address:	*TMWCft8x2BTk4nokUerxJK2X11JdeiVv565*
Theory Prop Id:	*1f0d5fe981bc213e8f88cb7c74a9bf90e20b53b02288f897c78dfc099ab3dd61*
Theory Prop Address:	*TMZGPa1mwgbVzkFdodAv6RjSmb9g3Bhh3hq*

`RealsStruct_Compl`

Theorem 28.269

$$\forall ab \in R^N.(\forall n \in N.a\ n \leq b\ n \wedge a\ n \leq a\ (n + one) \wedge b\ (n + one) \leq b\ n) \to \exists x \in R.\forall n \in N.a\ n \leq x \wedge x \leq b\ n.$$

The proposition is identified by the following information:

Pure Prop Id:	*6a20f5ecaafcc7e432b7c8ac16d2d71517ad446546abe4f692d40edd5f175aab*
Pure Prop Address:	*TMSCeZV9tj3Dc22WnMpqqUpBYCFvYAsqBkc*
Theory Prop Id:	*bf5231ea148f4c4a8e2c8b3b3d3ac74a7a34a6f05496134d190b71e9d903d2eb*
Theory Prop Address:	*TMZEYVRwMZfXMAwxS5gUKffKiFvE5u1BQFx*

`RealsStruct_natOfOrderedField`

Theorem 28.270 explicit_Nats N *zero* $(\lambda m.m + one)$. *The proposition is identified by the following information:*

Pure Prop Id:	*6b43192d1dea3e3fc85a6591e66c6a726082864fd516780b26a754b19d6118ef*
Pure Prop Address:	*TMK4NwKLMJh9YDNuNjVZk7onpb3hJRVZcqP*
Theory Prop Id:	*49002473fe506d6b9c9c98fcd03504abba2716d63b6a552bc1c8b880dc2511ed*
Theory Prop Address:	*TMUHoaa1stUMPCnj1neeSaXtn79MZ9CQ2cT*

Definition 28.40 *We define* RealsStruct_Npos *to be* $\{n \in N | n \neq zero\}$ *identified by the following information:*

Pure Object Id:	*18dcd68b13ac8ef27a4d8dfec8909abfa78ffbb33539bea51d3397809f365642*
Pure Object Address:	*TMXGAJt8GRxM5LRMTFpsj1piXUY7f9nuK7P*
Theory Object Id:	*f5d38003115cfc06f11ecf0699e5b6ecab522d53550fce92c2ecdc20371c12e5*
Theory Object Address:	*TMMXxyZsUkoJFF2GqJHfHwyksqYKy5LeJWe*

Let $Npos$ be RealsStruct_Npos.

`RealsStruct_PosNats_natOfOrderedField`

Theorem 28.271 explicit_Nats $Npos$ one $(\lambda m.m + one)$. *The proposition is identified by the following information:*

Pure Prop Id:	*608937f38d90199679a353df12d26e9c24924a2849bcb600a52f62801cf8c8b1*
Pure Prop Address:	*TMH3THbHi69Lk6JSBHKSnCQykxJcsLKkScy*
Theory Prop Id:	*7be7ef55dfa7ee734330d79a5f088736665cccd7e5615ce026a8457ca9690f34*
Theory Prop Address:	*TMUwcaA4tPYCgfSCsdNGLV1J2Wj2UjJKYcT*

Definition 28.41 RealsStruct_Z *is the opaque object of type* ι *identified by the following information:*

Pure Object Id:	*736c836746de74397a8fa69b6bbd46fc21a6b3f1430f6e4ae87857bf6f077989*
Pure Object Address:	*TMZnqja29W4HpFDK7otUSfGC3Eomh9qtUB8*
Theory Object Id:	*0610d20138c846183403e57b8540fd87c9759a321e2a636ed8b592691ebcc810*
Theory Object Address:	*TMUL7MYE83ADM6VaAkyBjtEUt5rRp6pVBu3*

Let Z be RealsStruct_Z.

Definition 28.42 RealsStruct_Q *is the opaque object of type* ι *identified by the following information:*

Pure Object Id:	*255c25547da742d6085a5308f204f5b90d6ba5991863cf0b3e4036ef74ee35a2*
Pure Object Address:	*TMQNHtJRR1iyQCK2yVPcXNmgjvYt4LJWjPH*
Theory Object Id:	*0a4db083012656ded8a542881fa97da3998c2c25649b64f9c88525f48f727d53*
Theory Object Address:	*TMHUMp4nRyqMcV6TWWc9HRr7HJygDDY4KSh*

Let Q be RealsStruct_Q.

`RealsStruct_Npos_props`

Theorem 28.272

$$\begin{gathered}\forall p:o.\\ (Npos \subseteq R \to\\ \textsf{explicit\_Nats}\ Npos\ one\ (\lambda m.m+one) \to\\ one \in Npos \to\\ (\forall m \in Npos.m+one \neq one) \to\\ (\forall m \in Npos.\forall q:\iota \to o.q\ one \to (\forall n \in Npos.q\ (n+one)) \to q\ m) \to\\ (\forall nm \in Npos.\textsf{explicit\_Nats\_one\_plus}\ Npos\ one\ (\lambda m.m+one)\ n\ m = n+m) \to\\ (\forall nm \in Npos.\textsf{explicit\_Nats\_one\_mult}\ Npos\ one\ (\lambda m.m+one)\ n\ m = n*m) \to\\ (\forall nm \in Npos.n+m \in Npos) \to (\forall nm \in Npos.n*m \in Npos) \to p)\\ \to p.\end{gathered}$$

The proposition is identified by the following information:

Pure Prop Id:	*ac462c732947010e996e2195f9457d22c91ea98709503c70dfd886bb0ff05a86*
Pure Prop Address:	*TMUvjp5vzLmJS9vokz3WxDwgYtiFC9YfGot*
Theory Prop Id:	*a2de5a1838f8c2dedd49986ce8e9b16887f479df3ea69491847fdc1ff939d7e0*
Theory Prop Address:	*TMLHJgX12h6ob8HcMEAHsRmu8Sv6z8hA4Vg*

`RealsStruct_Npos_R`

Theorem 28.273 *$Npos \subseteq R$. The proposition is identified by the following information:*

Pure Prop Id:	*008a7ec9f81b901d173d4bb67cd3b9f8fd59e4f1d6086a46e01df9b0379a24ac*
Pure Prop Address:	*TMJ4VGyP4MLVJHuj3qvzEBau5k5S6d87mJp*
Theory Prop Id:	*fdd42c49268c6b0f74f093dd11460a4c3f87341db2ac930dc62f4e370c959def*
Theory Prop Address:	*TMFUveydbCfNR11n2X6XYqhXH9GGB3eCa7E*

`RealsStruct_one_Npos`

Theorem 28.274 *$one \in Npos$. The proposition is identified by the following information:*

Pure Prop Id:	*060e811ee11a2fdd622c33063c392284ab0e32d1798a3c22f69b0e2cc87e0bb3*
Pure Prop Address:	*TMVQ8Vzx4rfk9PgrXA5DC8bupwA7Rc71ciY*
Theory Prop Id:	*adbd044729f5641d3c3402b8dae736e674669ab85664a907a855a441004cb866*
Theory Prop Address:	*TMV8pdo8evmeKAvbv1pzGuyB3FWK7iaDugd*

`RealsStruct_Z_props`

Theorem 28.275

$$\begin{gathered}\forall p : o. \\ ((\forall n \in Npos. -n \in Z) \to \\ zero \in Z \to Npos \subseteq Z \to Z \subseteq R \to \\ (\forall n \in Z. \forall q : o.(-n \in Npos \to q) \to (n = zero \to q) \to (n \in Npos \to q) \to q) \to \\ one \in Z \to -one \in Z \to \\ (\forall m \in Z. -m \in Z) \to \\ (\forall nm \in Z. n + m \in Z) \to \\ (\forall nm \in Z. n * m \in Z) \to p) \\ \to p.\end{gathered}$$

The proposition is identified by the following information:

Pure Prop Id:	*017fe94adff66febe9607cfa8e37ab6ccbb7b674ee4fb54a05fdde026adcf09d*
Pure Prop Address:	*TMSzNh6NafcQ7JF4kYewkBLEW5kuczQKhgd*
Theory Prop Id:	*5f0a7e775f83d554f276621ef295027944bb5559b7afbb223b8ddfc6b44e546b*
Theory Prop Address:	*TMQPzwy4S512aDpt1mBbU3fhUfvi6C5QvWZ*

RealsStruct_neg_Z

Theorem 28.276 $\forall n \in Npos. -n \in Z$. *The proposition is identified by the following information:*

Pure Prop Id:	*f9e62d63a307ece18bbd80aa48a23f6f4d17cc93e2d4ced458eb45a3505e2639*
Pure Prop Address:	*TMZmeurK9YG3FXNye9gkxqjf5hSGk8queat*
Theory Prop Id:	*4d43ebd17d60a85f9c79d798f7b371ae521f4a7bff8c51313295f609080b380a*
Theory Prop Address:	*TMbm6VFrtyUyFrUCb5N5iv3gJ2ebnXd96kk*

RealsStruct_zero_Z

Theorem 28.277 $zero \in Z$. *The proposition is identified by the following information:*

Pure Prop Id:	*e63b5bd24a1e20be4e359c4763b33e6b62e4af0ee2eb346b98a874d8b75cdf72*
Pure Prop Address:	*TMZZPadL877aUv7uy4BJ4ZVoUgXHzpSDS2p*
Theory Prop Id:	*d617fb71b2447bb5050839c73d379af276584315cac62d0fc944a5d871c873b7*
Theory Prop Address:	*TMQkD9oz6MNGEZp6SBtWuDTTfrftxMyra1p*

RealsStruct_Npos_Z

Theorem 28.278 $Npos \subseteq Z$. *The proposition is identified by the following information:*

Pure Prop Id:	*3b82b4daba391f17879dad1f437cec7ff368e07aaae1e064ae35d652ed72ec7b*
Pure Prop Address:	*TMHtwXjtrWbWqU2d751pwq11Lj48hsv1V9i*
Theory Prop Id:	*654198ba4994329078fbb885ead1adc3e09120276bfd4095fccaf00cd2c50a5f*
Theory Prop Address:	*TMQdysxG2P8Lkp3CTWLhBkp2LqNBQWWr6Ak*

RealsStruct_Z_R

Theorem 28.279 *$Z \subseteq R$. The proposition is identified by the following information:*

Pure Prop Id:	*afb222581fed3232153f5f2d44da3bbf0ce181c57357472cf15a82a3d810681e*
Pure Prop Address:	*TMPwERt3rdFJJxsDnxdhcQgF2MpMFsf3i1V*
Theory Prop Id:	*9b0ab1796c671927fbee291f23eb65871f651953d2f84aa13bc10861a90fcac1*
Theory Prop Address:	*TMGkAuVyoZcR48zuQfsrquFkDgtyiNcdENZ*

RealsStruct_Q_props

Theorem 28.280

$$\begin{gathered}\forall p : o. \\ (Q \subseteq R \rightarrow \\ (\forall x \in Q. \forall q : o.(x \in R \rightarrow \forall n \in Z. \forall m \in Npos. m * x = n \rightarrow q) \rightarrow q) \rightarrow \\ (\forall x \in R. \forall n \in Z. \forall m \in Npos. m * x = n \rightarrow x \in Q) \rightarrow p) \\ \rightarrow p.\end{gathered}$$

The proposition is identified by the following information:

Pure Prop Id:	*df79085a1a2f4f031815dcd379e51112884d05ae157fcb74a8357af5dbe034eb*
Pure Prop Address:	*TMHAByNoEuG6wgDbba13PkCr4En34QBgFU2*
Theory Prop Id:	*61a4917a561c1dba953134725332117c4984e25d8b4082687743fbe846b3ddbb*
Theory Prop Address:	*TMFXPuxaie6AGgwAYXkFdSq3ph8iYboGfCE*

RealsStruct_Q_R

Theorem 28.281 *$Q \subseteq R$. The proposition is identified by the following information:*

Pure Prop Id:	*b58f7fc652d381159596cca7859b3e14f253ce70a6c3e82c028fd369e27be76d*
Pure Prop Address:	*TMPV2a7aaKXBnCmnZWeiVvEJgyUM2fJ74rB*
Theory Prop Id:	*2ff2bd5001c8ea913870f94bda57e521ab15c3861a373ea631397059ee56519a*
Theory Prop Address:	*TMExUYP3PJpDU1qNcRiaxE1KgojHSA3GBtE*

RealsStruct_Z_Q

Theorem 28.282 *$Z \subseteq Q$. The proposition is identified by the following information:*

Pure Prop Id:	*c1725ba9fa1a646ac94926c0e35963caee38e306b5ea5c0b8035f91f3edf3415*
Pure Prop Address:	*TMZAUPnvZW22Vu1BjruFvkzzd99KRdyCUdr*
Theory Prop Id:	*a45e948eb32abcb85b27b170a391ad1b87a9608ce8a8b0a2ca2474cc443dc97e*
Theory Prop Address:	*TMNdjiF5RG9zcoL2KHoDujJUXF1wJoxRKBu*

Notation. We use $\div$ as an infix operator corresponding to applying term Field_div (Field_of_RealsStruct Rs).

`RealsStruct_div_clos`

Theorem 28.283 $\forall x \in R.\forall y \in R \setminus \{zero\}.x \div y \in R.$ *The proposition is identified by the following information:*

Pure Prop Id:	*ec4148787b44fb5fb3cdf6e61529dd3330a1e03298e8bbb7f08346f738f7db84*
Pure Prop Address:	*TMJcMfeW6RzhudXxfGUAHHUfTerg3RCmgMh*
Theory Prop Id:	*d6d8aa472b7b2bd90954c9998de9b5a34d99b67efd39be56f8b6218f52cd2666*
Theory Prop Address:	*TMJZqMkKWwmAcVqoAbsN2xJM74ofu6EF33P*

`RealsStruct_mult_div`

Theorem 28.284 $\forall x \in R.\forall y \in R \setminus \{zero\}.x = y * (x \div y).$ *The proposition is identified by the following information:*

Pure Prop Id:	*e2c1b9283ef45d7416d60f2ca96fe1d920af9c0d51788f43831368a679fa3ada*
Pure Prop Address:	*TMX4EqbemoaxGX4DUARDb4VZRc5ZLJMnF3R*
Theory Prop Id:	*b7ef46712de5c007a18453e0201b1b57274c06e0a53cd5a6ca5b05df985c4296*
Theory Prop Address:	*TMds3wgEHVh1PGSrKhMkyaYaGRFs33AcAF5*

`RealsStruct_div_undef1`

Theorem 28.285 $\forall xy.x \notin R \to x \div y = 0.$ *The proposition is identified by the following information:*

Pure Prop Id:	*6dfdbeeddd4c9724f299edd163a3a6873b04737b480b6dc8622825fe39c1c4dc*
Pure Prop Address:	*TMc3no3gvLt4aCZ3tswEMYfjDATjdvWyatp*
Theory Prop Id:	*351a0c5a15c5c52ea191c34690d189cbd566f2471f004c24cbe5973f3f13e435*
Theory Prop Address:	*TMKJ4U4k2tPg572kNK69NYJYjNPh6PL3NV7*

`RealsStruct_div_undef2`

Theorem 28.286 $\forall xy.y \notin R \to x \div y = 0.$ *The proposition is identified by the following information:*

Pure Prop Id:	*f6f4785d660ace5156add885c383a28b7bc3a99a73d8631604e4197e578adf19*
Pure Prop Address:	*TMWAnh5soQNHhi3Ykqn6R2ibResPkSHppAi*
Theory Prop Id:	*38a3ac5076648eec3d5a9c2e50962482b5cbee9811940dc2427c63819c1ca1f1*
Theory Prop Address:	*TMH1mYFWp41cUUu5C87Kq6Db727u6UDJH3K*

`RealsStruct_div_undef3`

Theorem 28.287 $\forall x.x \div zero = 0.$ *The proposition is identified by the following information:*

Pure Prop Id:	*36b6fd6a7618c15898b200e117ddef4e3fe1f0afb35443199399f43ed8036266*
Pure Prop Address:	*TMch6Tu6CKzcYBgw8HNboBazjtiajd7yott*
Theory Prop Id:	*269a3881564868f7961f0c10bad584f0aaaf8295d8a8d2960f2f7fd1e55d6fd8*
Theory Prop Address:	*TMM4gQM96yMEzmMaUmB5ZXKT5bDGqjMZCBZ*

Notation. We use superscripts as notation corresponding to applying term CRing_with_id_omega_exp (Field_of_RealsStruct Rs).

`RealsStruct_omega_exp_0`

Theorem 28.288 $\forall x.x^0 = one$. *The proposition is identified by the following information:*

Pure Prop Id:	*4f6df1679e5256f3dee7aa33e1d3edf948a982563cef834884ba4637ff642cfd*
Pure Prop Address:	*TMPSGSuE8LXDV4V4pvTtsJEu5FNyH1pBH5q*
Theory Prop Id:	*e9266c9ac7734a0908ed30b0412c09ac2fdda52a8a7deb8bfa1f329e7fb57079*
Theory Prop Address:	*TMTnnud9giEgqcthVyEpMdJL9R2n8gzrP43*

`RealsStruct_omega_exp_S`

Theorem 28.289 $\forall x.\forall n \in \mathsf{omega}.x^{(\mathsf{ordsucc}\ n)} = x * x^n$. *The proposition is identified by the following information:*

Pure Prop Id:	*02959608ed47b98e7731a645df3547cd11df919eee7b8db3daea36611099ac2b*
Pure Prop Address:	*TMVd93sgsKDaad1CxNYVmDVHrLcidvdTG4V*
Theory Prop Id:	*f566c18daaed324edc80d61391474e1103d351c36f7ff173e024f2c1aafb71d1*
Theory Prop Address:	*TMPtaovCUa2PSiq9AjqGhtJZYWsZ7ChGjWW*

`RealsStruct_omega_exp_1`

Theorem 28.290 $\forall x \in R.x^1 = x$. *The proposition is identified by the following information:*

Pure Prop Id:	*68407b69e24d974f37306277e9073efc244d1ef9140fa0f4a8f4dd184e3a5a2a*
Pure Prop Address:	*TMSx7vspXi4hq9Zfc3jM1CSRCdp3Zc9NGRU*
Theory Prop Id:	*0c96c808e943389302e04e16c0d612284ef50232f566ab8f9ee19bc68a0f29ee*
Theory Prop Address:	*TMKrzNRVaVw4oMu8nZ96vwqWBiBXmMPztXd*

`RealsStruct_omega_exp_clos`

Theorem 28.291 $\forall x \in R.\forall n \in \mathsf{omega}.x^n \in R$. *The proposition is identified by the following information:*

Pure Prop Id:	*eddf030a23ad3950942cdc2fbcd92ac5593fa9938ea1aae2da771538223f24df*
Pure Prop Address:	*TMNd3EMECuhQHTRvrFDgzibwvZjig6hvURt*
Theory Prop Id:	*ce17ebf726a60585318ead733cb7b5ab80320f5ea8b27994da5e5e3d5ce67456*
Theory Prop Address:	*TMYuofpdnGFN8PpDrLruYivMbNZMsjVDEPo*

Definition 28.43 RealsStruct_abs *is the opaque object of type $\iota \to \iota$ identified by the following information:*

Pure Object Id: *fcf3391eeb9d6b26a8b41a73f701da5e27e74021d7a3ec1b11a360045a5f13ca*
Pure Object Address: *TMMiiibxSu8KWC8MJw2qM5UvvwMNYkqgGwE*
Theory Object Id: *480d2ba6a3a558d8d5642b2e68a88cdc65a67cf6bfe18dd0574c2b3d39d69d05*
Theory Object Address: *TMS4FH6cQhUENopB1oi9sMyEapmb4SckEPg*

`RealsStruct_abs_clos`

Theorem 28.292 $\forall x \in R.$RealsStruct_abs $x \in R$. *The proposition is identified by the following information:*

Pure Prop Id: *616c66d879b5e2f3a98bb920d8521cc473629f43225754cc970c502c02b4d70b*
Pure Prop Address: *TMUJhPP2L2evvXGruzGDgRSyLBqRLkJsvew*
Theory Prop Id: *5a1facaf2b22cf8313c47ab5d00dd1d44a7f68dcc498ee31e588070e06e5ee9a*
Theory Prop Address: *TMPi25yLHBrYYUawSVuNTAdtuM1J4ztYFgp*

`RealsStruct_abs_nonneg_case`

Theorem 28.293 $\forall x \in R.zero \leq x \to$ RealsStruct_abs $x = x$. *The proposition is identified by the following information:*

Pure Prop Id: *883e07037d0bc45c4917edc7b4cb14370ef1994f68ceb7ca887b0471e7efe2ce*
Pure Prop Address: *TMMrHyd4NfDirtPZFyor6Lq39dp81qYpayF*
Theory Prop Id: *6d71adbe5160105d6dc17d08068cceebbdc2513ee44ef22f79096d4f8800b735*
Theory Prop Address: *TMcT4sXVUoiheLTy9BFaCAh8cxDxrU7E51w*

`RealsStruct_abs_neg_case`

Theorem 28.294 $\forall x \in R.x < zero \to$ RealsStruct_abs $x = -x$. *The proposition is identified by the following information:*

Pure Prop Id: *e8df199bfa3379375d084412f37f48b94a51f0ab84bb93db7eee356000d6afef*
Pure Prop Address: *TMF7zwqh96P1YidqPVMDC2qQyYrNC6tSu2M*
Theory Prop Id: *dc229d0f2268fec24b49f688c19646ae7a2e4a681cd5a1de6e740b17385573ba*
Theory Prop Address: *TMFDA2267ptXsetvPjSP9B3kKZxjxiVe5eW*

`RealsStruct_abs_nonneg`

Theorem 28.295 $\forall x \in R.zero \leq$ RealsStruct_abs x. *The proposition is identified by the following information:*

Pure Prop Id: *3bae2c2e1b39e5830ba31107cc5578a7b625b83fec87496a6ae294027caaff03*
Pure Prop Address: *TMFabfpzFQBWazTGbdRKSFsh8H6hm1abtqe*
Theory Prop Id: *701a7053ca066eed86f0442240fc617b8c699f3d2e403833bad55bfc42fbe7db*
Theory Prop Address: *TMK2MLY5rofcEXZj5urmPDkpnsWbeYS4r1w*

`RealsStruct_abs_zero_inv`

Theorem 28.296 $\forall x \in R.$RealsStruct_abs $x = zero \to x = zero$. *The proposition is identified by the following information:*

Pure Prop Id:	*1c1613207ea57ba5de38eb117ab32b134385a248879098c9d978b0898f59be2f*
Pure Prop Address:	*TMaZstUdzVeUwd6dG1Lk3yajQ9jPpSDmhtx*
Theory Prop Id:	*480222f2b6e6e15591c55f3c22675fc071131e50cca95468ad1137f79bfc20cc*
Theory Prop Address:	*TMSwQ4V24ucAD2y6Y2NzBFzWw24QhLA9R2Q*

`RealsStruct_dist_zero_eq`

Theorem 28.297 $\forall xy \in R.\mathsf{RealsStruct\_abs}\ (x + -y) = zero \to x = y$. *The proposition is identified by the following information:*

Pure Prop Id:	*0342d3a1b4534f4505f3b6ec468772cbfcd3cabf927b470703dcc7905876e881*
Pure Prop Address:	*TMSygnAwf3Z1Nmyt9mu1mUiAZNnmpc8dwZS*
Theory Prop Id:	*d65f00cc24e6beb44968f1f8cd4eca4f2c99cc6a6e3fad31d5ee330fc5cba95b*
Theory Prop Address:	*TMG37mDdnRwQ9CZaHY6Sj3WmgU6SMiTvErN*

Definition 28.44 *We define* $\mathsf{RealsStruct\_divides}$ *to be* $\lambda mn.\exists k \in Npos.m * k = n$ *of type* $\iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*9b1a146e04de814d3707f41de0e21bf469d07ef4ce11ae2297f43acd3cb43219*
Pure Object Address:	*TMSgvVAsMPWzeya7w14QXMLpm3oR2sg2Eaw*
Theory Object Id:	*26b0fdc348372e6f151cfb0d9a85fba282630eaa98d67d43c3e14d87ae4d7907*
Theory Object Address:	*TMEjvmG5htm9fo5j3Pi2G5irTZNBxF7JyLe*

Definition 28.45 *We define* $\mathsf{RealsStruct\_Primes}$ *to be*

$$\{n \in N | one < n \wedge \forall m \in Npos.\mathsf{RealsStruct\_divides}\ m\ n \to m = one \vee m = n\}$$

identified by the following information:

Pure Object Id:	*96c4c45b8c7f933c096a5643362b6a5eeb1d9e4d338d9854d4921130e6c8cdbe*
Pure Object Address:	*TMZiAsgfRmCK6xaRVxCPcc2VzGp7DAXMkTD*
Theory Object Id:	*4f229104c32c2be9f21a21d40bf2c8848e077881ee9d18ea39563dd5055f3409*
Theory Object Address:	*TMRZKCzNYB8mn6YTzXbD5KApbcoYvpYmMSv*

Definition 28.46 *We define* $\mathsf{RealsStruct\_coprime}$ *to be*

$$\lambda mn.\forall k \in Npos.\mathsf{RealsStruct\_divides}\ k\ m \to \mathsf{RealsStruct\_divides}\ k\ n \to k = one$$

of type $\iota \to \iota \to o$ *identified by the following information:*

Pure Object Id:	*4857fdedbdc33f33b2c088479c67bbbceae1afd39fca032cb085f9f741cedca6*
Pure Object Address:	*TMbzphBsd7GLnrNR8Mi8GuGapc1Xgdxr2y6*
Theory Object Id:	*112ea9cf8a89d5ad797d290cd1c0b26c9e1d6d41ecead75d5c7c7c91bd7ae48a*
Theory Object Address:	*TMX9ZHV8iS7puN2vhGXqUyRwbsJ5YunXJLK*

Let Qs be $\mathsf{pack\_b\_b\_e\_e}\ Q\ (\mathsf{RealsStruct\_plus}\ Rs)\ (\mathsf{RealsStruct\_mult}\ Rs)\ zero\ one$.

`Field_RealsStruct_Q`

Theorem 28.298 Field Qs. *The proposition is identified by the following information:*

Pure Prop Id:	*8b600c18b435f30873008e1fd2e1a7754844b34c5ce89afb02dc5257a7d031f0*
Pure Prop Address:	*TMLDM8xjNHCEb7tkdmX1V9EKpmqGB7zvxKT*
Theory Prop Id:	*b55584754578f3ef26cef3b37955ee96ee21f6de68eca6b9d14e62a3e2800837*
Theory Prop Address:	*TMH6n1ZFLmtKDSDJVw8XnR3Do4UFL4NSg1Q*

Definition 28.47 *We define* RealsStruct_omega_embedding *to be*

$$\text{nat\_primrec}\ zero\ (\lambda\_r.r + one)$$

of type $\iota \to \iota$ *identified by the following information:*

Pure Object Id:	*7e897f0ce092b0138ddab8aa6a2fc3352b0c70d9383c22f99340c202779c7a39*
Pure Object Address:	*TMFS8k2m5BMHnDtneuBVexHr4VeDPJZVm2S*
Theory Object Id:	*8d1b1d11f8d7a646ada866cb891c4f97971a895e92e965d05e6be6a0bdc0a0ed*
Theory Object Address:	*TMLEHo8yiHeRBmSAVenhHZym5uhudBeWgVA*

Let $emb : \iota \to \iota$ be RealsStruct_omega_embedding.

`RealsStruct_omega_embedding_N`

Theorem 28.299 $\forall n \in \text{omega}.emb\ n \in N$. *The proposition is identified by the following information:*

Pure Prop Id:	*25cb31475191d8d41f82ac1cc047f3940af8421d025ff7a7c78fb4dc2ffe1cbe*
Pure Prop Address:	*TMPtZMNGEv6XXfgBKcBe8kTdZ3eDJkq7yCv*
Theory Prop Id:	*9c35b850fb3d8b0adb59d8c3695495c52fc50505407deb7cccb631635d035d6a*
Theory Prop Address:	*TMXJrPBAAoNfUBWRhgmzF7gfY3kLs2rzR2q*