Learning-based Statistical and Symbolic Guidance in Theorem Proving

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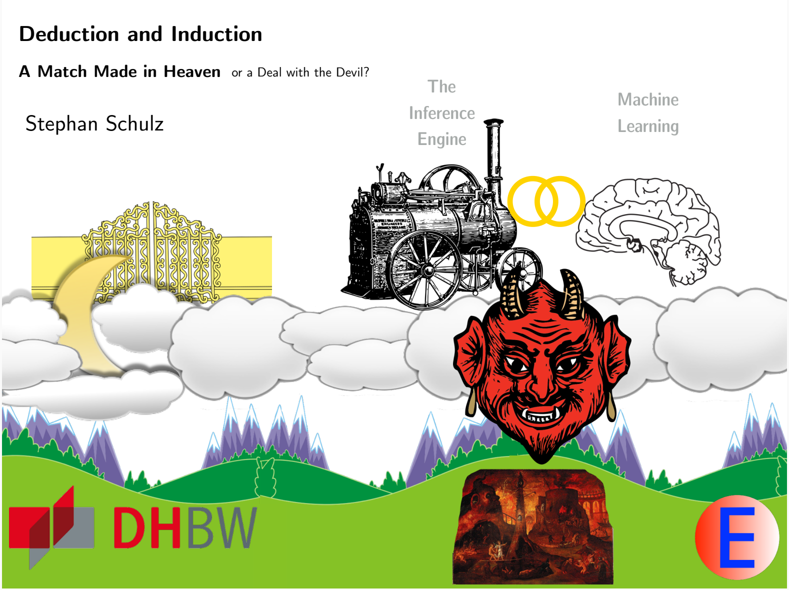
Intro - Stephan Schulz at AITP'16



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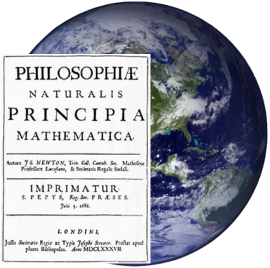
Theorem Proving: Big Picture

Real World Problem



Formalized Problem

$$X : human(X) \quad mortal(X)$$
$$X : philosopher(X) \quad human(X)$$
$$philosopher(socrates)$$
$$\stackrel{?}{\models}$$
$$mortal(socrates)$$

ATP

Proof Search

Proof

or

Countermodel

or

Timeout

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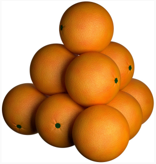
Big Example: The Flyspeck project

- Kepler conjecture (1611): The most compact way of stacking balls of the same size in space is a pyramid.

$$V = \frac{\pi}{\sqrt{18}} \approx 74\%$$



- Formal proof finished in 2014
- 20000 lemmas in geometry, analysis, graph theory
- All of it at `https://code.google.com/p/flyspeck/`
- All of it computer-understandable and verified in HOL Light:
- `polyhedron s /\ c face_of s ==> polyhedron c`
- However, this took 20 – 30 person-years!
- Our automation can now do about 45% of the lemmas

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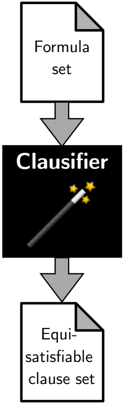
Contradiction and Saturation

- Proof by contradiction
  - Assume negation of conjecture
  - Show that axioms and negated conjecture imply falsity
- Saturation
  - Convert problem to Clause Normal Form
  - Systematically enumerate logical consequences of axioms and negated conjecture
  - Goal: Explicit contradiction (empty clause)
- Redundancy elimination
  - Use contracting inferences to simplify or eliminate some clauses



Search control problem: How and in which order do we enumerate consequences?

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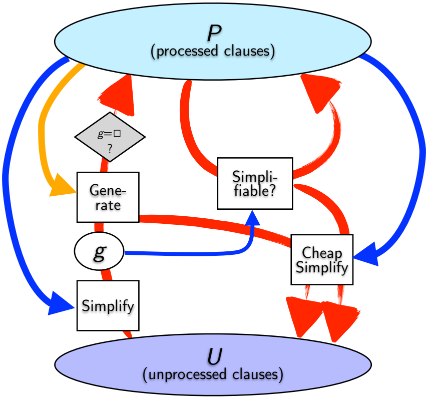
The Given-Clause Algorithm



- Aim: Move everything from U to P
- Invariant: All generating inferences with premises from P have been performed
- Invariant: P is interreduced
- Clauses added to U are simplified with respect to P

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Low-level ATP guidance: Prover9 hints

- The Prover9 community (ADAM workshop): non-associative algebra, 20-50k long proofs by Prover9 and Waldmeister
- Prover9 hints strategy (Bob Veroff): extract hints from easier proofs to guide more difficult proofs
- To get good hints Bob wants as little conjecture-based inferences as possible:
- Get an “essentially forward proof” by various Prover9 setting
- Exploration to get good hints (not really automated yet)
- Our recent work: use machine learning to select good hints for a problem

P9 Example (Bob Veroff)

```
list(given_selection).

 % high

 part(Hha,high,hint_age,hint & weight < 500 & hint_age < 200000)
      = 500.

 part(Hw, high, weight, hint & weight < 500) = 25.
 part(Ha, high, age,    hint & weight < 500) = 5.
 part(Hr, high, random, hint & weight < 500) = 5.

 % -false instead of true in case no truth value
 part(Wf, low, weight, false) = 1.
 part(Wnf, low, weight, -false) = 100.

 % just in case something isn't covered
 part(TheRest, low, weight, all) = 1.

end_of_list.
```

High-level ATP guidance: Premise Selection

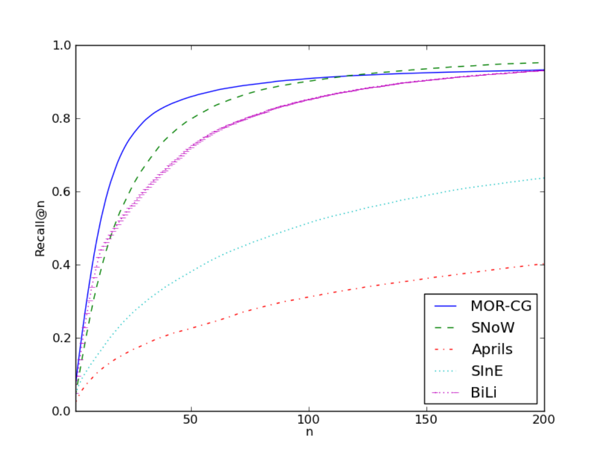
- Can existing ATPs be used over large math libraries?
- Is good premise selection for proving a new conjecture possible at all?
- Or is it a mysterious power of mathematicians? (Penrose, intuition?)
- Or should we use some complete exhaustive human-designed algorithms?
- Today: Premise selection is not a mysterious property of mathematicians!
- Complete human-engineering is inferior to learning from a large corpus of proofs

Example system: Mizar Proof Advisor (2003)

- train naive-Bayes fact selection on all previous Mizar/MML proofs (50k)
- input features: conjecture symbols; output labels: names of facts
- recommend relevant facts when proving new conjectures
- First results over the whole Mizar library in 2003:
  - about 70% coverage in the first 100 recommended premises
  - chain the recommendations with strong ATPs to get full proofs
  - about 14% of the Mizar theorems were then automatically provable (SPASS)
- Today's methods: about 45-50%
- My bet: at least 80% in 20 years
- `http://ai4reason.org/aichallenges.html`

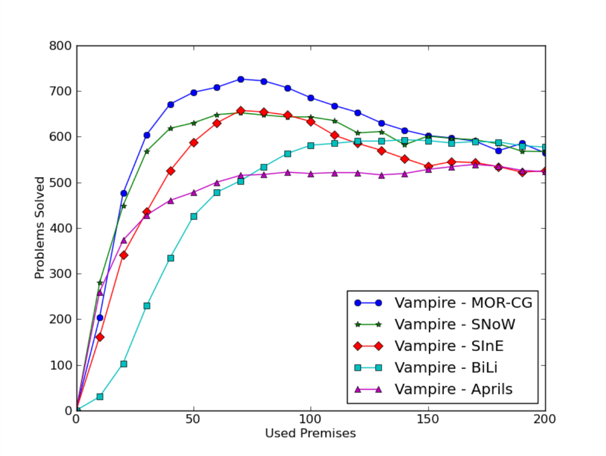
ML Evaluation of methods on MPTP2078 – recall

- Coverage (recall) of facts needed for the Mizar proof in first *n* predictions
- MOR-CG – kernel-based, SNoW - naive Bayes, BiLi - bilinear ranker
- SINe, Aprils - heuristic (non-learning) fact selectors



ATP Evaluation of methods on MPTP2078

- Number of the problems proved by ATP when given *n* best-ranked facts
- Good machine learning on previous proofs really matters for ATP!



Recent Improvements and Additions

- Semantic features encoding term matching/unification [IJCAI'15]
- Distance-weighted k-nearest neighbor, TF-IDF, LSI, better ensembles (MePo)
- Matching and transfering concepts and theorems between libraries (Gauthier & Kaliszyk) – allows “superhammers”, conjecturing, and more
- Lemmatization – extracting and considering millions of low-level lemmas
- First useful CoqHammer (Czajka & Kaliszyk 2016), 40%–50% reconstruction/ATP success on the Coq standard library
- Neural sequence models, definitional embeddings (Google Research)
- Hammers combined with statistical tactical search: TacticToe (HOL4)

Summary of Features Used

- From syntactic to more semantic:
- Constant and function symbols
- Walks in the term graph
- Walks in clauses with polarity and variables/skolems unified
- Subterms, de Bruijn normalized
- Subterms, all variables unified
- Matching terms, no generalizations
- terms and (some of) their generalizations
- Substitution tree nodes
- All unifying terms
- Evaluation in a large set of (finite) models
- LSI/PCA combinations of above
- Neural embeddings of above

Feature Statistics

- MPTP2078 and MML1147 – 4.5k and 150k formulas

Method	Speed (sec)		Number of features		Learning and prediction (sec)	
	MPTP2078	MML1147	total	unique	knn	naive Bayes
SYM	0.25	10.52	30996	2603	0.96	11.80
TRM_α	0.11	12.04	42685	10633	0.96	24.55
TRM_0	0.13	13.31	35446	6621	1.01	16.70
$\mathrm{MAT}_\varnothing$	0.71	38.45	57565	7334	1.49	24.06
MAT_r	1.09	71.21	78594	20455	1.51	39.01
MAT_l	1.22	113.19	75868	17592	1.50	37.47
MAT_1	1.16	98.32	82052	23635	1.55	41.13
MAT_2	5.32	4035.34	158936	80053	1.65	96.41
$\mathrm{MAT}_\cup$	6.31	4062.83	180825	95178	1.71	112.66
PAT	0.34	64.65	118838	16226	2.19	52.56
ABS	11	10800	56691	6360	1.67	23.40
UNI	25	N/A	1543161	6462	21.33	516.24

Low-level guidance for tableau: Machine Learning Connection Prover (MaLeCoP)

- MaLeCoP: put the AI methods inside a tableau ATP (J. Otten - leanCoP)
- the learning/deduction feedback loop runs across problems and inside problems
- The more problems/branches you solve/close, the more solutions you can learn from
- The more solutions you can learn from, the more problems you solve
- first prototype (2011): very slow learning-based advice (1000 times slower than inference steps)
- already about 20-time proof search shortening on MPTP Challenge compared to leanCoP
- second version (2015): Fairly Efficient MaLeCoP (= FEMaLeCoP)
- about 15% improvement over untrained leanCoP on the MPTP problems
- Recently Monte Carlo search (M. Faerber: MonteCop)
- Reinforcement learning (in progress)

Low-level guidance for superposition: ENIGMA

- Train a fast classifier (LIBLINEAR) distinguishing good and bad generated clauses
- Plug it into a superposition prover (E prover) as a clause evaluation heuristic
- ENIGMA: Efficient learNing-based Inference Guiding MAchine
- input: positive and negative examples (good/bad clauses as feature vectors)
- output: model (a vector of feature weights)
- evaluation of a clause feature vector: dot product with the model
- Combine it with various ways with more standard (common-sense) guiding methods
- Very recent work, 86% improvement of the best E tactic on the AIM 2016 CASC benchmark
- About 90% precision in predicting good/bad clauses
- Similar work using (much slower) neural guidance by Google (70-80% precision)

Other guidance for ATPs

- Knowledge base of abstracted lemmas from previous proofs in E (drawing analogies between different theories)
- nearest-neighbor guidance: ConjectureRelativeSymbolWeight in E
- further symbol weighting based on axiom relevance in E
- semantic (model-based) guidance: Prover9
- Waldmeister: theory recognition, optimization of term orderings, etc.
- Our recent work: search for good term orderings in Vampire
- Ongoing work for iProver, SMTs: do not enumerate instances but try the most probable ones

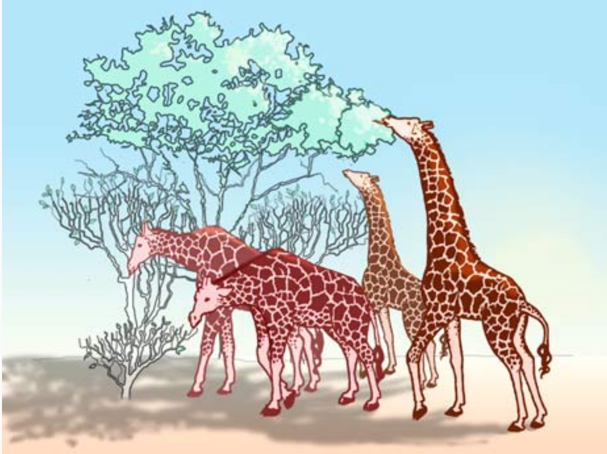
Large-theory Lemmatization and Conjecturing

- Over 1B low-level lemmas in Flyspeck
- 1.5M-7M higher-level lemmas in MML and Flyspeck
- Define fast preprocessing methods to extract the most important ones:
- PageRank, recursive dependency count, recursive use count, etc.
- Use the most important lemmas together with the toplevel theorems - helps by 5-20% (needs more evaluations)
- Conjecturing: guessing the intermediate lemmas in longer proofs
- Currently by learning statistical theory analogies and using probabilistic grammars

BliStr: Blind Strategymaker

- Problem: how do we put all the sophisticated ATP techniques together?
- E.g., Is conjecture-based guidance better than proof-trace guidance?
- Grow a population of diverse strategies by iterative local search and evolution!
- Dawkins: The Blind Watchmaker

BliStr: Blind Strategymaker



- The strategies are like giraffes, the problems are their food
- The better the giraffe specializes for eating problems unsolvable by others, the more it gets fed and further evolved

BliStr: Blind Strategymaker

- Use clusters of similar solvable problems to train for unsolved problems
- Interleave low-time training with high-time evaluation
- Thus co-evolve the strategies and their training problems
- In the end, learn which strategy to use on which problem
- Recent improvements: BliStrTune – hierarchical approach
- Combine search for low-level and high-level parameters in a loop
- Include multiple ENIGMA models

The E strategy with longest specification in Jan 2012

```
G-E--_029_K18_F1_PI_AE_SU_R4_CS_SP_S0Y:

--definitional-cnf=24 --simplify-with-unprocessed-units --tstp-in
--split-aggressive --split-clauses=4 --split-reuse-defs
--simul-paramod --forward-context-sr --destructive-er-aggressive
--destructive-er --prefer-initial-clauses -winvfreqrank -c1 -Ginvfreq
-F1 --delete-bad-limit=150000000 -WSelectMaxLComplexAvoidPosPred
-H'(
4 * ConjectureGeneralSymbolWeight(
       SimulateSOS,100,100,100,50,50,10,50,1.5,1.5,1),
3 * ConjectureGeneralSymbolWeight(
       PreferNonGoals,200,100,200,50,50,1,100,1.5,1.5,1),
1 * Clauseweight(PreferProcessed,1,1,1),
1 * FIFOWeight(PreferProcessed))'
-s --print-statistics --print-pid --resources-info --memory-limit=192
```

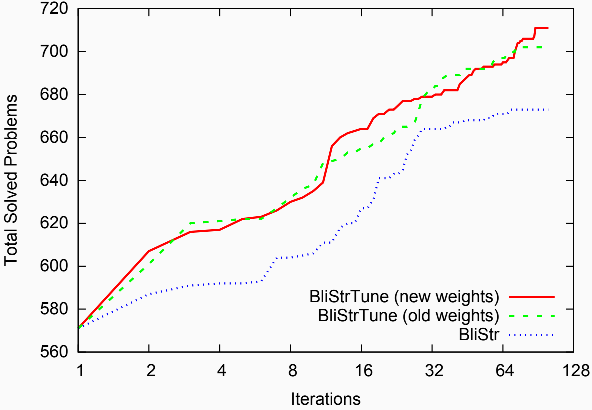
Its clause evaluation heuristic

```
G-E--_029_K18_F1_PI_AE_SU_R4_CS_SP_S0Y:

4 * ConjectureGeneralSymbolWeight(
      SimulateSOS,100,100,100,50,50,10,50,1.5,1.5,1),
3 * ConjectureGeneralSymbolWeight(
      PreferNonGoals,200,100,200,50,50,1,100,1.5,1.5,1),
1 * Clauseweight(PreferProcessed,1,1,1),
1 * FIFOWeight(PreferProcessed)
```

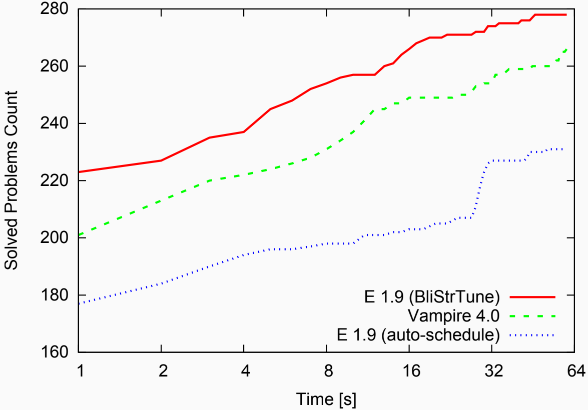
The E strategy with longest specification in May 2014

```
atpstr_my_c7bb78cc4c665670e6b866a847165cb4bf997f8a:

6 * ConjectureGeneralSymbolWeight(PreferNonGoals,100,100,100,50,50,1000,100,1.5,1.5,1)
8 * ConjectureGeneralSymbolWeight(PreferNonGoals,200,100,200,50,50,1,100,1.5,1.5,1)
8 * ConjectureGeneralSymbolWeight(SimulateSOS,100,100,100,50,50,50,50,1.5,1.5,1)
4 * ConjectureRelativeSymbolWeight(ConstPrio,0.1, 100, 100, 100, 100, 1.5, 1.5, 1.5)
10 * ConjectureRelativeSymbolWeight(PreferNonGoals,0.5, 100, 100, 100, 100, 1.5, 1.5, 1)
2 * ConjectureRelativeSymbolWeight(SimulateSOS,0.5, 100, 100, 100, 100, 1.5, 1.5, 1)
10 * ConjectureSymbolWeight(ConstPrio,10,10,5,5,5,1.5,1.5,1.5)
1 * Clauseweight(ByCreationDate,2,1,0.8)
1 * Clauseweight(ConstPrio,3,1,1)
6 * Clauseweight(ConstPrio,1,1,1)
2 * Clauseweight(PreferProcessed,1,1,1)
6 * FIFOWeight(ByNegLitDist)
1 * FIFOWeight(ConstPrio)
2 * FIFOWeight(SimulateSOS)
8 * OrientLMaxWeight(ConstPrio,2,1,2,1,1)
2 * PNRefinedweight(PreferGoals,1,1,1,2,2,2,0.5)
10 * RelevanceLevelWeight(ConstPrio,2,2,0,2,100,100,100,100,1.5,1.5,1)
8 * RelevanceLevelWeight2(PreferNonGoals,0,2,1,2,100,100,100,400,1.5,1.5,1)
2 * RelevanceLevelWeight2(PreferGoals,1,2,1,2,100,100,100,400,1.5,1.5,1)
6 * RelevanceLevelWeight2(SimulateSOS,0,2,1,2,100,100,100,400,1.5,1.5,1)
8 * RelevanceLevelWeight2(SimulateSOS,1,2,0,2,100,100,100,400,1.5,1.5,1)
5 * rweight21_g
3 * Refinedweight(PreferNonGoals,1,1,2,1.5,1.5)
1 * Refinedweight(PreferNonGoals,2,1,2,2,2)
2 * Refinedweight(PreferNonGoals,2,1,2,3,0.8)
8 * Refinedweight(PreferGoals,1,2,2,1,0.8)
10 * Refinedweight(PreferGroundGoals,2,1,2,1.0,1)
20 * Refinedweight(SimulateSOS,1,1,2,1.5,2)
1 * Refinedweight(SimulateSOS,3,2,2,1.5,2)
```

BliStr on 1000 Mizar@Turing training problems



BliStr on 400 Mizar@Turing testing problems



Thanks

- Thanks for your attention!
- If interested, come to AITP: `http://aitp-conference.org`
- ATP/ITP/Math vs AI/Machine-Learning people, Computational linguists