

# *Trends in Reasoning (and AI?)*

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# Computers Checking Large Math Proofs



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## Scientists Deliver Formal Proof of Famous Kepler Conjecture

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An international team of mathematicians led by University of Pittsburgh Professor **Thomas Hales** has delivered a formal proof of the **Kepler conjecture**, a famous problem in discrete geometry. The team's paper is published in the journal *Forum of Mathematics, Pi*.



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# Computers Proving Open Conjectures Automatically

PROVER9 ASSISTED PROOF OF THE WEAK AIM CONJECTURE

## PROVER9 ASSISTED PROOF OF THE WEAK AIM CONJECTURE

Michael Kinyon



AITP 2021, 6 September 2021

▶ 0:02 / 2:16:01



# Computers Proving Math Theorems Automatically



## ENIGMA - The Rise of Computronium



Inbox x



**Josef Urban** <josef.urban@gmail.com>

Jul 26, 2021, 9:47 AM



to Undisclosed, bcc: Stephan, bcc: Michael, bcc: Lawrence, bcc: Michael, bcc: Jesse, bcc: T

I am happy to announce that the ENIGMA system ([1],[2],[3],[4],[5],[12],[18],[25]) of the E ([8],[9],[13]) lineage, helped by its Deepire ([22],[23],[24]) Vampiric ([26],[27]) cousin, has reached today the landmark of 75% automatically proved Mizar top-level problems.

This is measured in a setting when the premises for the proof can be selected from the library either by a human or by a machine.

# Leibniz's/Hilbert's/Russell's Dream: Let Us Calculate!

Solve all (math, physics, law, economics, society, ...) problems  
by  
reduction to logic/computation



[Adapted from: *Logicomix: An Epic Search for Truth* by A. Doxiadis]

## *How Do We Automate Math and Science?*

- **What is mathematical and scientific thinking?**
- Pattern-matching, analogy, induction from examples
- Deductive reasoning
- Complicated feedback loops between induction and deduction
- Using **a lot of previous knowledge** - both for induction and deduction
- We need to develop such methods on computers
- Turing: Logic is a *game* in which Math, Computing & Science is played
- **So let's develop strong AI for large logic, math, science!**

# Applications – Verified Operating Systems

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## Minimal Example – Proving Equivalence of Two Programs

```
(* simple list reversal - runs in quadratic time *)
primrec rev :: "'a list => 'a list" where
"rev [] = []" |
"rev (x # xs) = rev xs @ [x]"

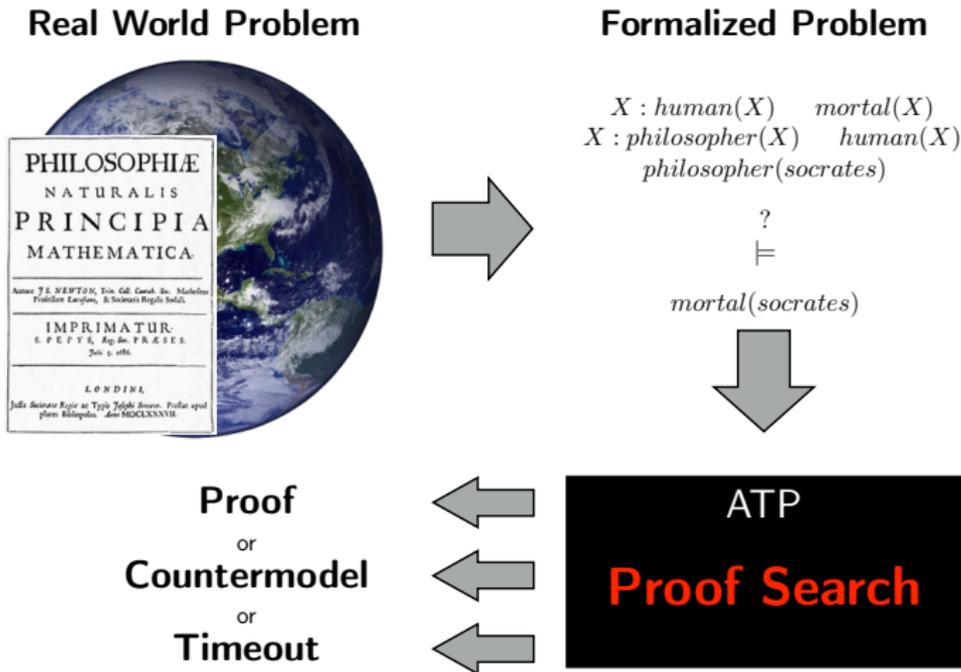
(* more advanced list reversal - runs in linear time *)
primrec itrev:: "'a list => 'a list => 'a list" where
  "itrev [] ys = ys" |
  "itrev (x#xs) ys = itrev xs (x#ys)"

strategy CDInd=Thens [Conjecture,Fastforce,Quickcheck,DInd]
strategy DInd_Or_CDInd = Ors [DInd, CDInd]

lemma "itrev xs [] = rev xs"
find_proof DInd_Or_CDInd
apply (subgoal_tac "\forall y. itrev xs y = Demo.rev xs @ y")
apply fastforce
apply (induct xs)
apply auto
done
```

# The Technology: Automated Theorem Provers

## Theorem Proving: Big Picture



3

## Feedback loops between reasoning and learning

- **70% improvement** of the best prover by ML/RL
- From 14933 proofs to 25397 proofs
- 75% of the Mizar corpus reached in July 2021 - higher times and many runs:

[https://github.com/ai4reason/ATP\\_Proofs](https://github.com/ai4reason/ATP_Proofs)

	$S$	$S \odot M_9^0$	$S \oplus M_9^0$	$S \odot M_9^1$	$S \oplus M_9^1$	$S \odot M_9^2$	$S \oplus M_9^2$	$S \odot M_9^3$
solved	<b>14933</b>	16574	20366	21564	22839	22413	23467	22910
$S\%$	+0%	+10.5%	+35.8%	+43.8%	+52.3%	+49.4%	+56.5%	+52.8%
$S+$	+0	+4364	+6215	+7774	+8414	+8407	+8964	+8822
$S-$	-0	-2723	-782	-1143	-508	-927	-430	-845

	$S \odot M_{12}^3$	$S \oplus M_{12}^3$	$S \odot M_{16}^3$	$S \oplus M_{16}^3$
solved	24159	24701	25100	<b>25397</b>
$S\%$	+61.1%	+64.8%	+68.0%	<b>+70.0%</b>
$S+$	+9761	+10063	+10476	+10647
$S-$	-535	-295	-309	-183

## *Future Potential - Science*

- Use strong AI/reasoning and formal verification for:
- **Science**
  - Routinely verify complex math, software, hardware?
  - Make all of math/science computer-understandable?
  - Strong AI assistants for math/science?
- **Examples**
  - Automatically understand/verify/explain all arXiv papers?
  - Can we train a superhuman system like AlphaGo/Zero for math/physics? What will it take?
  - Can we prove that the Amazon Cloud cannot be hacked?
  - The same for critical government/private IT systems?

## *Future Potential - Society*

- Use strong AI/reasoning and formal verification for:
- **Society**
  - Leibniz's dream: **Let us Calculate!** (solve any dispute)
  - J. McCarthy: **Mathem. Objectivity and the Power of Initiative**
  - AI/reasoning assistants for law/regulations
  - Verification of financial, transport/traffic systems, ...
  - **Explainable** and very securely **verified** systems
- **Examples**
  - Prove that two Paris metro trains will never crash?
  - Prove that a trading system doesn't violate regulations?
  - Prove that a new law is inconsistent with an old one?
  - Automatically debunk fallacies in political campaigns?

## *Misconceptions*

- Rule based vs statistical vs symbolic:
- "symbolic" includes practically all human-written programs!
- this is a huge AI target! by no means "old" or "outdated by sub-symbolic"
- Alphazero is a rule based system. Anything programmed by humans is rule based. But, unlike many other systems, it also creates rules by itself.
- Push back against the superimportance of data:
- alphazero learns from scratch by self play
- the same for our math problem solving - with the benefit of getting very unusual solutions
- unclear how much the current large language models generalize, understand logic, compute ...