AI4REASON:
Artificial Intelligence for Large-Scale Computer-Assisted Reasoning

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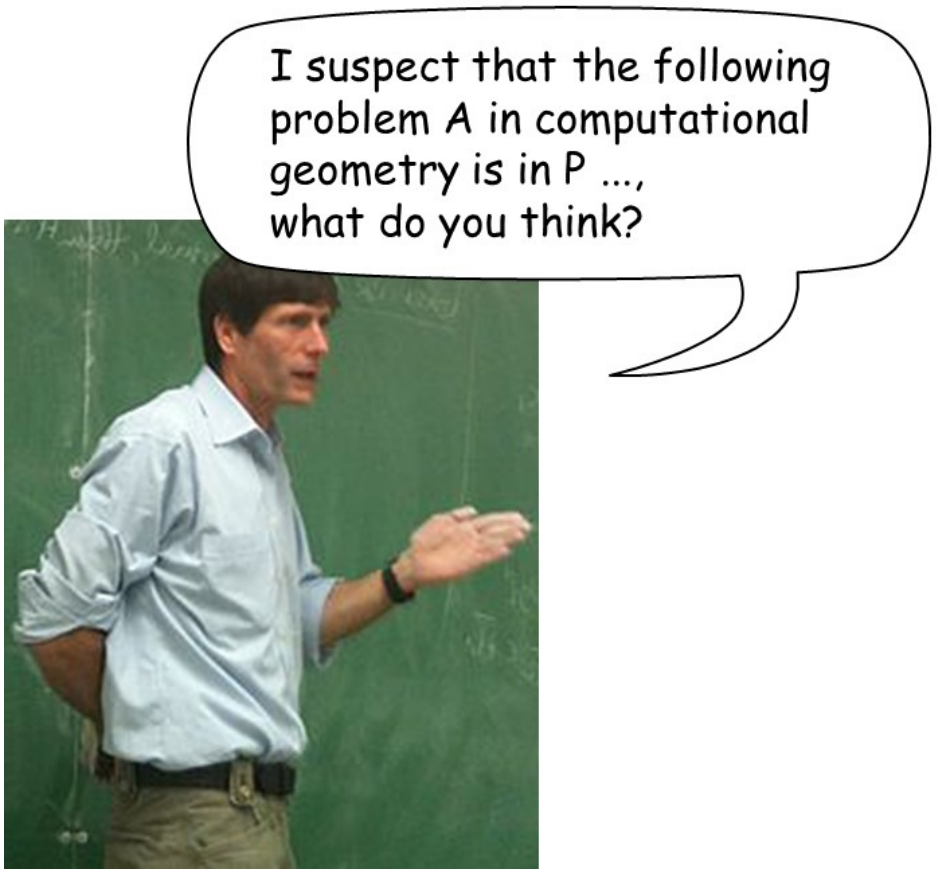
Not So Distant Future



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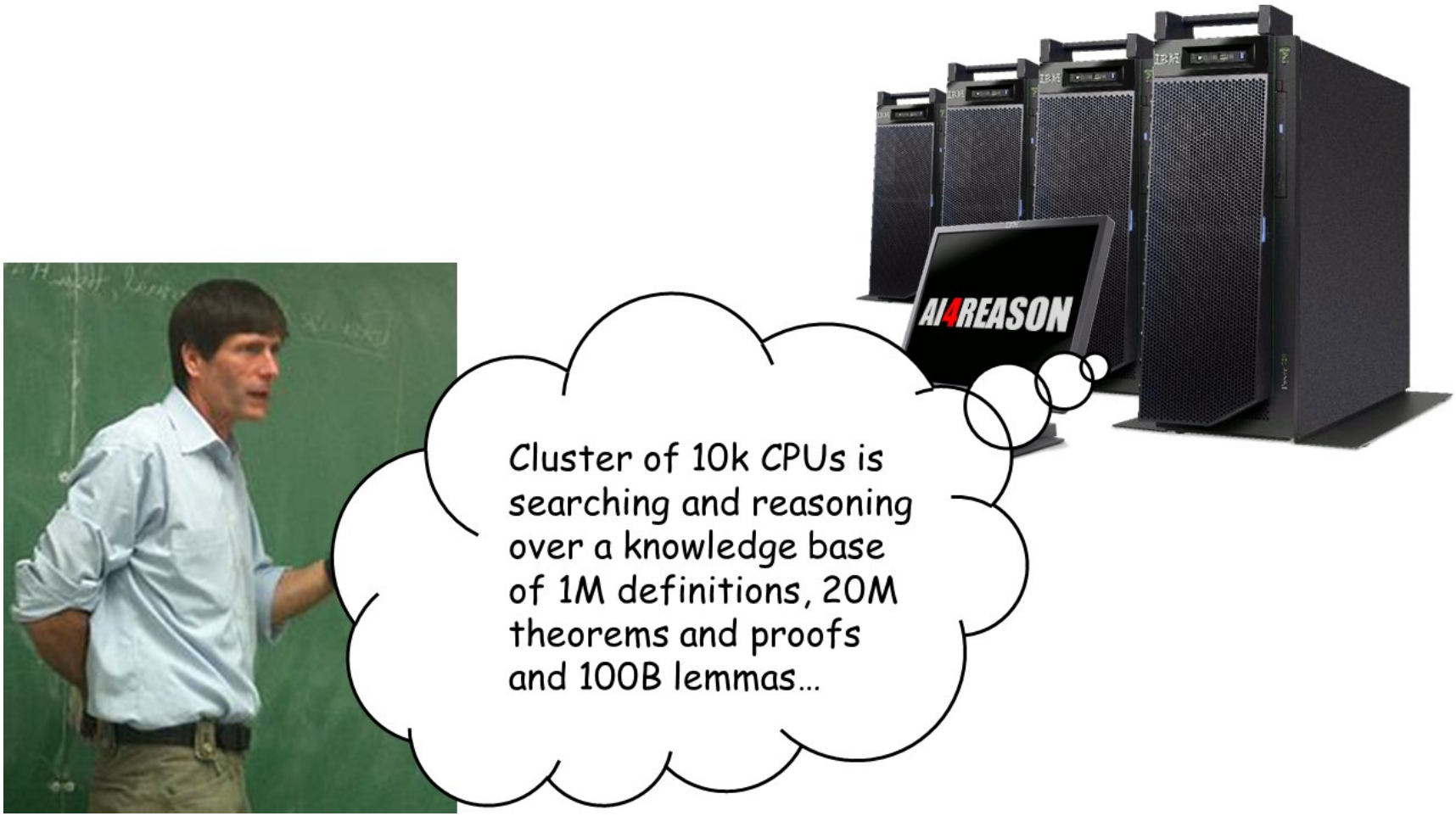

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Indeed, it is similar to a less known problem B number 13501 in my knowledge base. We can use a similar polynomial reduction to planar graphs as in B, and for the resulting constraint-solving problem we use a modified version Y of the $O(n^9)$ algorithm X published last year in Proc. of Indian Conf. on Graph Theory.

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Here is my verified formal proof with 100k basic inference steps. Here are two high-level versions of the proof, one for experts and one for textbooks.

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Btw., A, B and X, Y generalize to a far-reaching conjecture that could solve a long-standing open problem.

Let's write an ERC proposal about exploring them!

How Distant?

- 15 - 50 years, depending on our efforts
- Today's numbers about 100x smaller:
  - 10k-30k computer-understandable definitions
  - 200k-300k (small) theorems and proofs
  - 1B-10B primitive lemmas
- Covers roughly the Bc level in Math/CS, PhD level still far

- The main bottleneck:

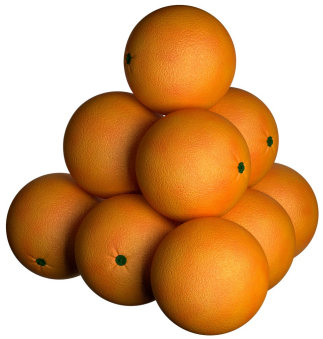
WEAK AUTOMATION OF REASONING
OVER LARGE COMPUTER-UNDERSTANDABLE CORPORA

- This is where a breakthrough is necessary

Example: The Flyspeck project

- Kepler conjecture (1611): The most compact way of stacking balls of the same size in space is a pyramid.

$$V = \frac{\pi}{\sqrt{18}} \approx 74\%$$



- Proved by Hales in 1998, 300-page proof + computations
- Big: Annals of Mathematics gave up reviewing after 4 years
- Formal proof finished in 2014, 20000 theorems & proofs
- All of it computer-understandable and verified in HOL Light:
- `polyhedron s /\ c face_of s ==> polyhedron c`
- However, this took 20 – 30 person-years!

Why Is Current Large-Theory Automation So Weak?

- Until recently, no large complex corpora
- Focus on *human engineering of search* for small problems
- But the search space in large corpora is huge
- Detailed human engineering of search does not scale!

- Our work:
  - Introduced AI/ML/ATP over large formal corpora as a research topic [Urban04,Urban06]
  - 40% of Flyspeck and Mizar problems solved recently by combining ML/AI/ATP methods [KaliszykU14,KaliszykU15]:
  - T. Hales: *"one of the most promising technologies on the horizon"*

What Is Needed: The AI4REASON Plan of Attack

WP1 AI for finding relevant knowledge in large formal corpora:

- How to capture similarity and analogy of ideas?
- How to learn from proofs, counter-examples and theories?

WP2 AI-based guiding methods for reasoning tools:

- How to efficiently apply the learned guidance in ATPs/ITPs?
- How to automatically learn the best reasoning strategies?

WP3 AI for suggesting plausible conjectures and concepts:

- What makes a good conjecture for a given problem?
- What concepts are good for a given problem?

WP4 Self-improving AI interleaving learning and deduction:

- How to explore easier problems to learn for harder ones?
- How to develop theories and gain most useful knowledge?

WP5 Auto-formalization, deployment, cross-corpora reuse:

- Learn formalization from aligned informal/formal corpora
- Deploy the methods as strong online services
- Develop AI methods for aligning different formal corpora

WP 1: High-Level Premise Selection

- Raise performance: 40% now vs 56% if perfect
- Task 1: Feature Characterizations
  - abstract and semantic characterizations of math objects (unification, term patterns, validity in (pseudo-)models)
  - modern ML methods: LSA, LDA, Word2vec/Glove, neural embeddings, their semantic/math adaptations
- Task 2: Sophisticated Machine Learning Techniques
  - scaling up kernel methods, random forests, deep learning
  - ensemble methods, boosting, stacking, etc.
- Task3: Dynamic Premise Selection Methods
  - extract additional features from short ATP/ITP searches
  - use for ML guidance, repeat/loop the focusing
  - abstract (structural) matching combined with lemma/conjecture introduction (cf. WP3)

WP 2: Internal Proof Guidance

- Guide ATPs and ITPs internally by learned methods
- Task 4: Export of Proof Search Data for Learning
  - modifications of leanCoP, ITPs, E prover for providing training data
- Task 5: Accepting External Guidance in Theorem Provers
  - plug in effecient learning methods directly into the infernce cores of ATP and ITP systems
- Task 6: Inventing ATP Strategies and Learning Their Use
  - Automatically invent good parameters and their combinations, DSL programs guiding sophisticated ATPs like E, Vampire, Prover 9

WP 3: Lemmatization, Conjecturing, Concept Introduction

- Task 7: Lemma Mining and Reuse
- Task 8: Conjecture Generating Methods
- Task 9: Model and Concept Generation

WP 4: Self-Improving AI Systems Interleaving Deduction and Learning

- Task 10: Integrating new High-Level Guiding Methods
- Task 11: Integrating Internal Proof-Guiding Methods
- Task 12: Integrating Lemmatization and Conjecturing

WP 5: Auto-formalization, deployment, cross-corpora reuse

- Task 13: Deployment for Formal Proof Communities
- Task 14: Reuse Across Formal Corpora
- Task 15: Aligning Informal and Formal Corpora

AI4REASON Mantra: Relevance/Automation At Each Level

- Always do only what is *relevant* in current context
- Avoid exhaustive search as much as possible
- Learn methods and their contextual relevance from corpora
- Always evaluate your ideas on large corpora
- Learn from many experiments, this is experimental science
- Automate your exploration using your relevance heuristics
- Always think how to automatically learn your own methods

... and follow Alan Turing's thinking (1950, AI):

"We may hope that [learning] machines will eventually compete with men in all purely intellectual fields."

Current Team, Institution and Collaborations

- me, C. Brown, J. Jakubuv, J. Vyskocil, 2 PhD students
- K. Chvalovsky + 2 PhD students paid from Czech funding
- Basis of the Prague Automated Reasoning Group (2003):
- Czech Institute for Informatics, Robotics and Cybernetics:
- New EU-funded (50M) national research institute at CTU

- External scientific advisors:
  - Prof. Stephan Schulz (DHBW Stuttgart)
  - Prof. Robert Veroff (U. of New Mexico)
  - Prof. Tom Heskes (Radboud U. Nijmegen)
- Tight collaboration with C. Kaliszyk's group in Innsbruck
- International collaboration with 6 other groups in place
- 20+ incoming/outgoing research visits in the first 2 years